Influence of parameter uncertainties on the seismic performance of oscillators via SPO2IDA

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ABSTRACT: The effects of the parameter epistemic uncertainties on the predicted seismic performance are investigated for single-degree-of-freedom systems. The oscillators examined are 5%-damped systems having a complex quadrilinear backbone with pinching hysteresis, including an elastic, a hardening, a negative stiffness and a residual plateau branch, terminating with a final drop to zero strength. The six uncertain properties considered are the post-yield hardening ratio, the end-of-hardening ductility, the slope of the descending branch, the residual strength, the ultimate ductility and the period. Using Monte-Carlo simulation and the SPO2IDA tool for rapid performance estimation, we calculate the increased variability in the predicted limit-state capacities due to the epistemic uncertainty in each parameter. Thus, we are able to evaluate the influence of modeling assumptions to the predicted versus the actual seismic performance of the oscillator and understand the effect of the variability of each parameter to the total system variability at each limit-state.

1 INTRODUCTION

Structures in seismic areas are being analyzed, built and designed with only a summary treatment of the uncertainty that is present in the model, the material properties and the loads by using generic safety factors. In general, uncertainty has been a thorny issue that is only implicitly included by most seismic codes, perhaps the first explicit treatment appearing in the FEMA-350 (SAC/FEMA 2000) guidelines. In the wake of the SAC/FEMA project there has been a widespread adoption of the notion of uncertainty in the earthquake engineering community and novel methodologies have already appeared on how to include the uncertainty in performance estimation. Prominent examples are the work by Cornell et al. (2002) for the SAC/FEMA format or the work of Baker & Cornell (2003) which is builds upon the Pacific Earthquake Engineering Research (PEER) Center framework.

Still, while methods to deal with uncertainty do exist, there is only so much we can do about estimating it. When dealing with complex nonlinear structures it is practically impossible to escape from Monte-Carlo methods, with every single simulation involving time-consuming static or dynamic nonlinear analysis (e.g. Kwon & Elnashai 2006). Actually, one of the most comprehensive methods for seismic performance evaluation is incremental dynamic analysis (IDA, Vamvatsikos & Cornell 2002), which would essentially require multiple nonlinear dynamic analyses under a suite of ground motion records. In essence, a Monte Carlo simulation performing a full IDA for each point in the parameter space would be prohibitively resource intensive, even for the simplest of structures. The necessary simulations would necessitate the execution of numerous IDAs, each for twenty or thirty records, resulting to millions of nonlinear dynamic analyses, thus rendering such efforts extremely cumbersome.

Circumventing this significant computational obstacle is usually done by assuming ad hoc values for the dispersions caused by uncertainties, e.g. in the model properties, and either implicitly taking them into account or explicitly including them in the guidelines, as in FEMA-350. Actually, even in the latter case these are meant to serve as reasonable placeholders that, unfortunately, in the absence of a more rational and proven values, tend to become the de facto standard.

One solution to this problem has emerged by the recent appearance of a simple and accurate approximation to IDA. Using the flexible SPO2IDA tool (Vamvatsikos 2002) it has now become possible to rapidly estimate the performance of single-degree-of-freedom (SDOF) oscillators and first-mode-dominated structures, from elasticity all the way to collapse. Having such a tool at our disposal we can easily perform all the necessary simulations and actually provide an accurate estimate of the effect of uncertainty on the demand and capacity of structures. Using Monte Carlo on top of SPO2IDA we propose to study the effect of the uncertainty in the force-deformation envelope and period of an oscillator to the resulting normalized limit-state capacities as a precursor to the application to actual buildings.

2 INCREMENTAL DYNAMIC ANALYSIS

Incremental Dynamic Analysis (IDA) is a powerful analysis method that offers thorough seismic demand and capacity prediction capability (Vamvatsikos & Cornell 2002). It involves performing a series of nonlinear dynamic analyses under a multiply scaled suite of ground motion records. By selecting proper Engineering Demand Parameters (EDPs) to characterize the structural response and choosing an Intensity Measure (IM), e.g. the 5% damped first-mode spectral acceleration $S_a(T_1, 5\%)$, to represent the seismic intensity, we can generate the IDA curves of EDP versus IM for each record and the 16%, 50% and 84% summarized curves. On such curves the desired limit-states can be defined by setting appropriate limits on the EDPs. Thus the corresponding capacities and their probabilistic distribution are estimated. Such results combined with probabilistic seismic hazard analysis (Vamvatsikos & Cornell 2002) allow the estimation of mean annual frequencies (MAFs) of exceeding the limit-states thus offering a direct characterization of seismic performance.

Nevertheless, IDA comes at a considerable cost, even for simple structures, necessitating the use of multiple nonlinear dynamic analyses that are usually beyond the abilities and the computational resources of the average practicing engineer. Therefore, a simpler and faster alternative is always desirable.

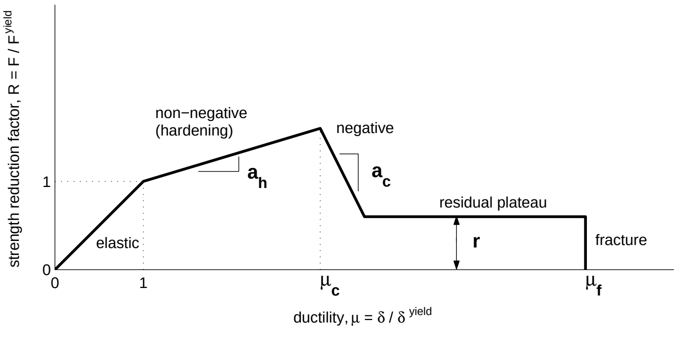

Figure 1. The normalized SPO2IDA backbone and its five controlling parameters.

3 IDA AND SPO2IDA

A fast and accurate approximation has been recently proposed for IDA, both for single and multi-degree-of-freedom systems utilizing information from the force-deformation envelope (or backbone) to generate the summarized 16%, 50% and 84% IDA curves by using elaborate fitted equations (Vamvatsikos & Cornell 2006).

The approximation is based on the study of numerous SDOF systems having varied periods, moderately pinching hysteresis and 5% viscous damping, while they feature backbones ranging from simple bilinear to complex quadrilinear with an elastic, a hardening and a negative-stiffness segment plus a final residual plateau that terminated with a drop to zero strength as shown in Figure 1 (Ibarra 2003; Ibarra et al. 2005). The oscillators were analyzed through IDA and the resulting curves (Fig. 2) were summarized into their 16, 50, and 84% fractile IDA curves (Fig.3) which were in turn fitted by flexible parametric equations (Vamvatsikos & Cornell 2006). Having compiled the results into the SPO2IDA tool, available online (Vamvatsikos 2002), we can get an accurate estimate of the performance of virtually any oscillator without having to perform the costly analyses, almost instantaneously recreating the fractile IDAs in normalized coordinates of $R = S_a(T, 5\%)/S_{ay}(T, 5\%)$ (where $S_{ay}(T, 5\%)$ is the $S_a(T, 5\%)$ value to cause first yield) versus ductility μ.

A typical example of applying SPO2IDA appears in Figures 3 and 4 where the 16, 50, and 84% fractile IDA curves of a 5% damped oscillator with moderately pinching hysteresis are estimated using both IDA and SPO2IDA. The oscillator's period is T = 0.9s and its backbone has a hardening segment with stiffness a_h = 30% of the elastic up to ductility μ_c = 2, followed by a_c = −200% negative stiffness segment plus a residual plateau that has a strength r = 50% of the yield strength and ends at μ_f = 5. The accuracy achieved by SPO2IDA is remarkable everywhere on the IDA curves, even close to collapse.

SPO2IDA is in fact a powerful R–μ–T relationship that will provide not only central values (mean and median) but also the dispersion due to record-to-record aleatory randomness of the strength reduction R factor given μ. Such dispersions are of primary importance for the performance evaluation of structures and they are usually represented by their β-value (SAC/FEMA 2000), i.e. by the standard deviation the natural logarithm of R given μ, which can be calculated from the fractile IDAs as

$$\beta_R = \ln R_{50\%} - \ln R_{16\%} \tag{1}$$

where $\hat{R} = R_{50\%}$ and $R_{16\%}$ are the 50% (median) and 16% R-values of capacity. Obviously, since we are mostly interested in the lower values of the capacity, it makes sense to estimate any β-value using the median and the lower fractile (16%) rather than the higher one (84%).

This tool has been extended to first-mode dominated MDOF structures (Vamvatsikos & Cornell 2004), enabling the accurate estimation of the fractile IDA curves even close to collapse without needing nonlinear dynamic analyses. In addition it has been shown to only slightly increase the error in our estimation, resulting to an accuracy that is equivalent to a full IDA using ten ordinary ground motion records. Thus it can render performing the multiple IDAs quite effortless, offering an efficient and very simple method for estimating the uncertainty associated with the normalized limit-state capacities of an oscillator, given the variability in its backbone parameters and its period.

Still, SPO2IDA has several limitations which will consequently limit the scope of this study. First of all, the only EDP supported is ductility. Therefore, our results are meaningful only for limit-states that can be defined on terms of ductility or other directly related EDPs, such as the maximum deformation. Furthermore, when creating SPO2IDA we assumed only a moderately pinching hysteresis, not allowing us to considered at all the uncertainty in the shape of the hysteretic loop. Such issues remain to be resolved.

4 METHODOLOGY

Having SPO2IDA available we can easily perform a classic Monte Carlo by randomly varying the parameters of the oscillator according to their distribution. The uncertain parameters considered are the five backbone variables a_h, μ_c, a_c, r and μ_f plus the period T of the oscillator. For a given oscillator, i.e., for given default or mean values of the parameters, all we need to do is create a large enough sample of possible realizations by drawing randomly from the distributions of the six parameters and use SPO2IDA on each alternate model. Thus we are able to directly estimate the fractile IDA curves without actually performing a single dynamic analysis. The final results are the distributions of the estimates of fractile demands and capacities, allowing the assessment of confidence intervals or dispersion β-values for the oscillator R-capacities given μ or the μ-demands given the normalized intensity R.

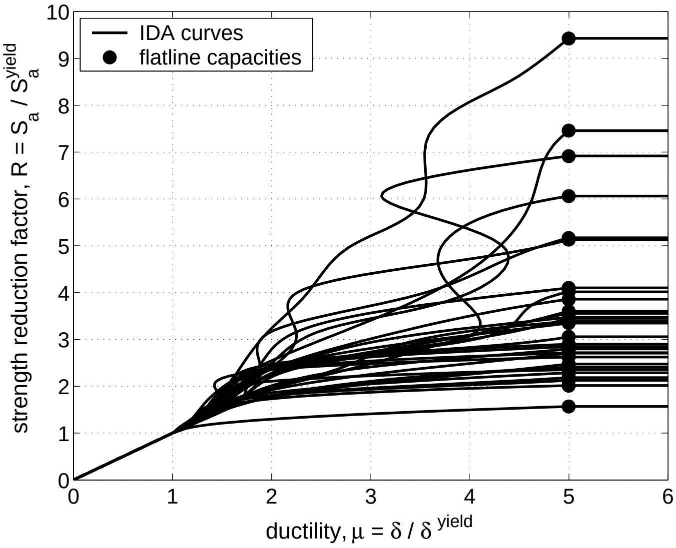

Figure 2. Thirty IDA curves and their flatline capacities for a T=0.9s system with the base backbone (a_h = 0.3, μ_c = 2, a_c = -2, r = 0.5, μ_f = 5).

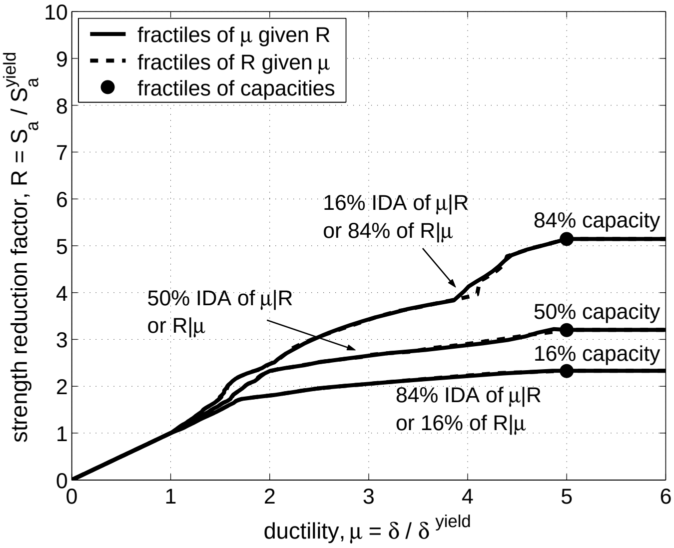

Figure 3. Summarization of the thirty IDA curves into fractile IDAs given μ or R.

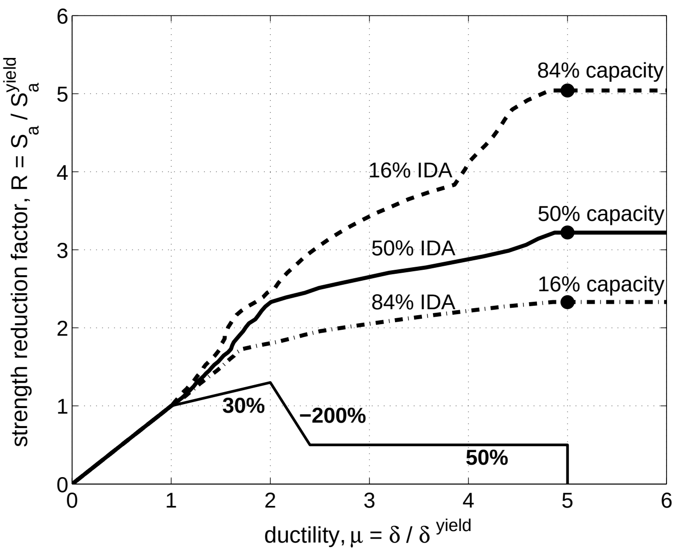

Figure 4. The fractile IDA curves from Figure 3 versus the static pushover curve.

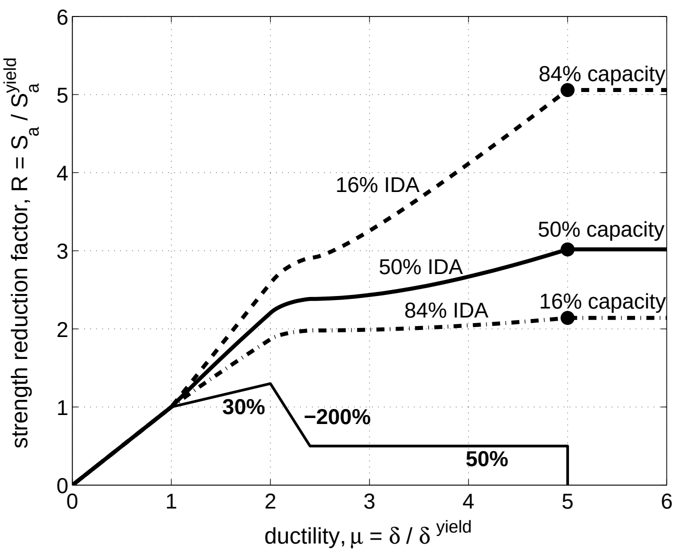

Figure 5. The fractile IDAs for the base backbone and T=0.9sec, as estimated by SPO2IDA.

As an illustrative example we are going to use the base backbone that appears in Figures 4-5 with default parameters $a_h = 0.3$, $\mu_c = 2$, $a_c = -2$, $r = 0.5$, $\mu_f = 5$ and a period of $T = 0.5$, 1.0, 1.5 or 2.0s. Each parameter is independently normally distributed with a mean equal to its default value and a coefficient of variation (c.o.v) equal to 0.3. Care needs to be exercised as the normal distribution assigns non-zero probabilities even for physically impossible values of the parameters, e.g., $T < 0$, or $a_h > 1$. Additionally, the equations comprising SPO2IDA were fitted for a specific range in the parameters: a_h in [0,0.9], μ_c in [1,9], a_c in [–4,-0.02], r in [0,0.9] and T in [0.1s,4s]. Therefore it makes sense to truncate the distribution of each parameter within some reasonable minimum and maximum that satisfies both the physical limits and the fitted range of SPO2IDA. We chose to do by setting hard limits at roughly 2.0 standard deviations away from the central value, thus cutting off only the most extreme cases.

Given the parameter distributions, we performed classic Monte Carlo simulations for N realizations, using SPO2IDA to obtain N different sets of IDA fractiles. Of primary importance here is the estimation of the variability caused by the parameter uncertainties in the median capacity for each limit-state, i.e. in the R-values produced by SPO2IDA. As proposed by Cornell et al. (2002), such dispersion caused by the uncertainty in the median capacity will be characterized by its β-value, β_U, which can be calculated directly as the standard deviation of the natural logarithm of the estimates of the median capacities

$$\beta_U = \sqrt{\frac{\sum\left(\ln R^i_{50\%} - \overline{\ln R_{50\%}}\right)^2}{N-1}} \tag{2}$$

where $R^i_{50\%}$ (i = 1,2,…,N) are the estimates of the median R-value of capacity for a given limit-state, one from each oscillator realization, and $\overline{\ln R_{50\%}}$ is the mean of the natural logarithm of the median R-values of capacity.

Of significant importance though is not just the value of β_U but also the value of the total dispersion β_{RU} caused by both the record-to-record randomness and the model uncertainty, which is used, e.g., in the SAC/FEMA framework to assess performance in the presence of uncertainty (Cornell et al. 2002). This is typically estimated as the square-root-sum-of-squares of the β-values associated with uncertainty and randomness:

$$\beta_{RU} = \sqrt{{\beta_R}^2 + {\beta_U}^2} \tag{3}$$

Such a value for every limit-state, or value of ductility, serves as a useful comparison of the relative contribution of randomness and uncertainty to the total dispersion. In general, we expect the high β_R to overshadow the lower β_U, especially since the latter is produced by a c.o.v of only 0.3 in the parameter values. The only exceptions to this rule will appear in the exact areas where the sensitivity of performance to the parameters is most important.

5 ILLUSTRATIVE RESULTS

In our final implementation we performed a sensitivity analysis of the resulting β_U values versus N, showing reliable results for practically any $N > 500$. Therefore we chose to use a conservative sample size of $N = 1000$, the full simulation needing only a minute on a Pentium IV processor.

5.1 *Single parameter uncertainty*

In order to better understand the effect of each parameter, additional simulations were performed by letting only one of the six parameters vary at a time around the default backbone values and having a mean oscillator period of $T = 1$s. The resulting β_U and β_{RU} values for each limit-state and each parameter appear in Figures 6-11.

In Figure 6, the end-of-hardening ductility μ_c seems to generate significant uncertainty especially for low ductilities, close to its central value of $\mu_c = 2$. On the other hand, in Figure 7, the hardening slope a_h has little or no influence on any limit-state. This is also echoed in recent sensitivity studies (e.g. Ibarra 2003; Vamvatsikos & Cornell 2006) that have shown the small influence of a_h on the oscillator performance. Surprisingly, Figure 8 shows that there is also little uncertainty in the capacities associated with the negative slope a_c. This is actually a consequence of having a high mean $|a_c|$ value of 200% that forces a sharp drop in the force-displacement envelope, something that is narrowing down the available options: having a slightly milder or sharper slope is not really a big issue, as the descent remains quite rapid, thus leaving little room for variability on the IDA curve. Only for relatively low negative slopes (e.g. $|a_c|$ of 30% or less) would the uncertainty rise.

Actually, where the strength drop starts (μ_c) and where it ends (r) seems more important: For any ductility μ beyond the start of the residual plateau in the base case we get a small increase in the uncertainty due to the variability in the plateau height r (Fig. 9). Still, the randomness β_R attains its highest values in that region, thus obscuring any contribution from r. In contrast, the variability in the end of the residual plateau μ_f contributes significant uncertainty to the R-values of capacities, practically for any limit-state at ductilities higher than the mean value of $\mu_f = 5$ (Fig. 10). Finally the period T in Figure 11 is adding a small amount to the total uncertainty that is steadily increasing for higher limit-states.

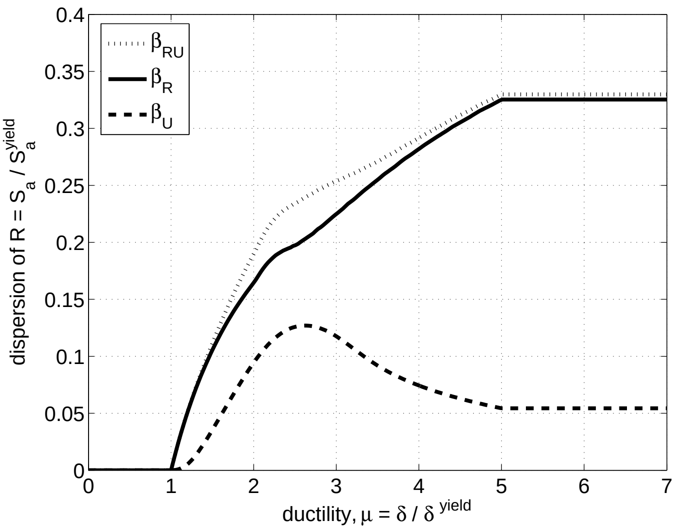

Figure 6. Dispersion β_U due to uncertainty in the end-of-hardening ductility μ_c versus β_R and the total dispersion β_{RU}.

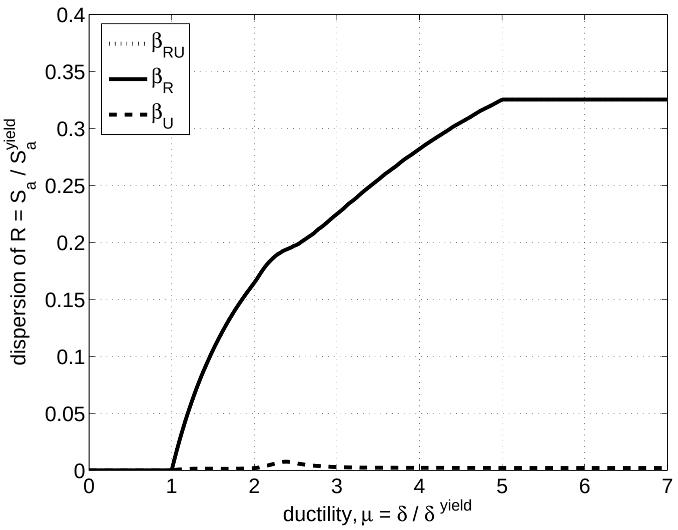

Figure 7. Dispersion β_U due to uncertainty in the hardening slope a_h versus β_R and the combined dispersion β_{RU}.

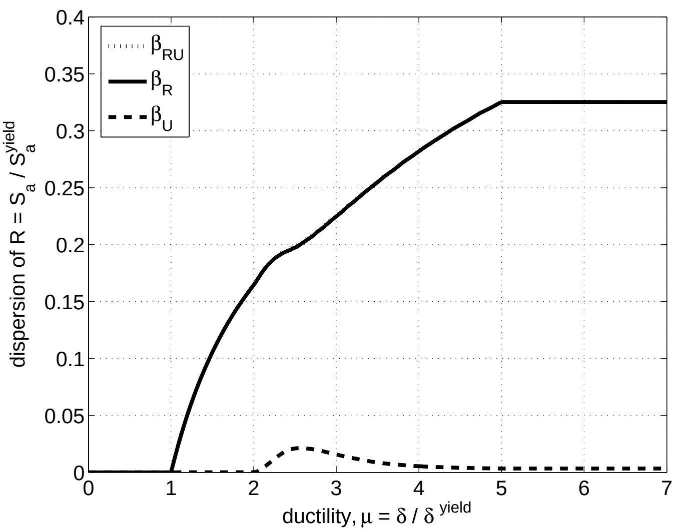

Figure 8. Dispersion β_U due to uncertainty in the negative slope a_c versus β_R and the combined dispersion β_{RU}.

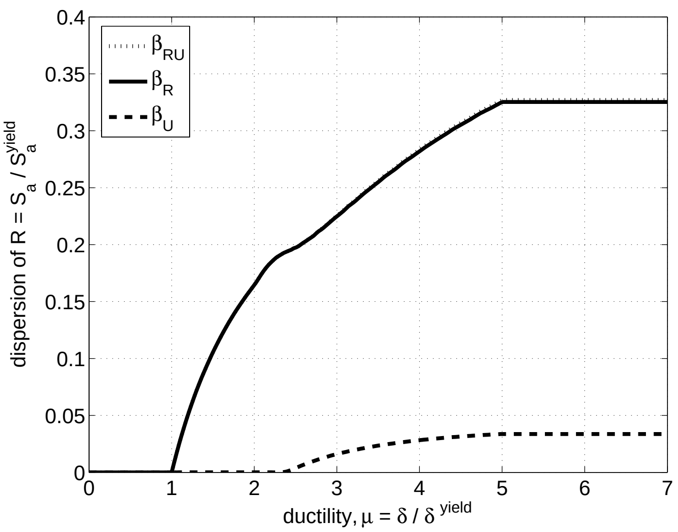

Figure 9. Dispersion β_U due to uncertainty in the residual plateau height r versus β_R and the combined dispersion β_{RU}.

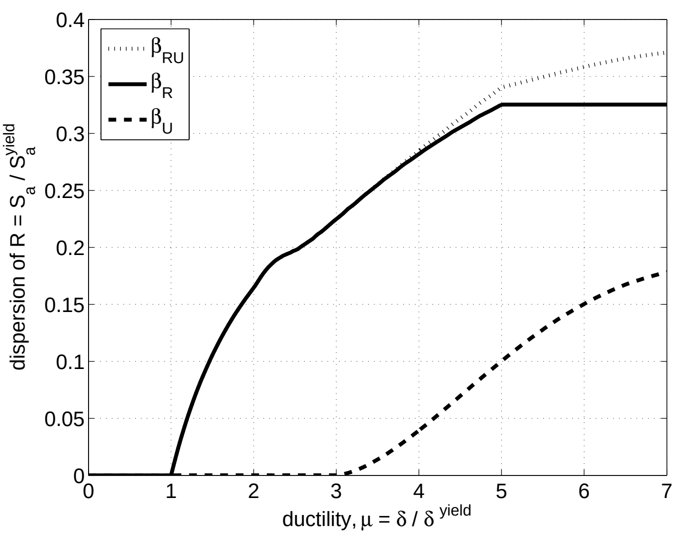

Figure 10. Dispersion β_U due to uncertainty in the ultimate ductility μ_f versus β_R and the combined dispersion β_{RU}.

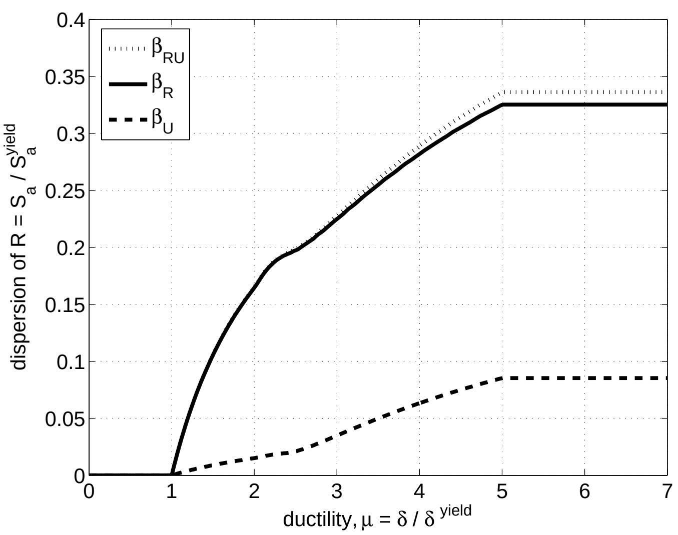

Figure 11. Dispersion β_U due to uncertainty in the oscillator period T versus β_R and the combined dispersion β_{RU}.

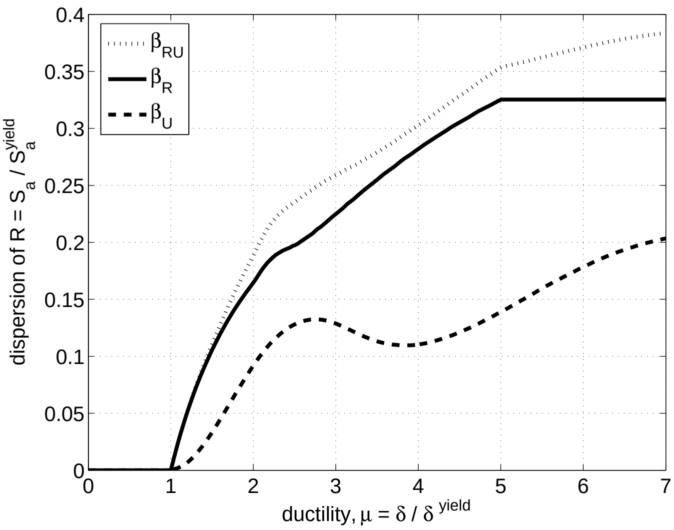

Figure 12. Dispersion β_U due to uncertainty in all six parameters versus β_R and the combined dispersion β_{RU} for oscillator period $T = 1$s.

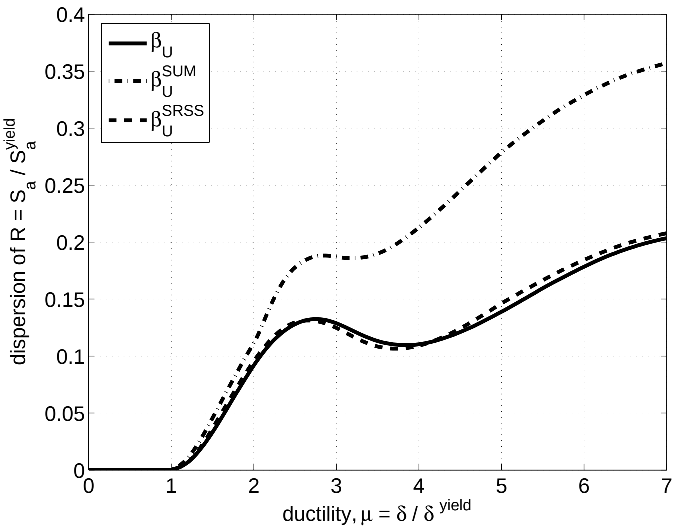

Figure 13. Total dispersion β_{RU} due to uncertainty in all six parameters versus the sum and the SRSS of individual β_U's computed for each parameter.

Regarding the uncertainty in the period, it should be noted though that the issue is more complex that what Figure 11 seems to imply. Our methodology is focusing specifically on the normalized capacities $R = S_a(T,5\%)/S_{ay}(T,5\%)$. Still, in any comprehensive performance estimation, when changing the period T we are also changing the seismic demand as we are moving to adjacent points in the $S_a(T,5\%)$ spectrum. Therefore, accurately including period uncertainty in performance estimation requires an integrated approach where at least some information about the spectral shape is included in the Monte Carlo procedure. For example, for a given ductility, increasing T will also increase R (Vamvatsikos & Cornell 2006), while it will generally decrease $S_a(T,5\%)$-demand, at least in the moderate and long periods. Therefore in that spectral region there is a negative correlation that we are not taking into account here by just focusing on the normalized R-values. Nevertheless, even without including the spectral information this approach can provide a useful approximation to the actual problem.

5.2 *Multiple parameter uncertainty*

The next task was to let all six parameters vary simultaneously in order to estimate the resulting uncertainties for the default backbone at an oscillator period of $T = 1$s. The results are shown in Figure 12, where it is obvious that the trends appearing in each parameter are also present here: The influence of the μ_c uncertainty is driving β_U up for ductilities around 2, while the influence of μ_f is very clear for ductilities close to collapse. Interestingly enough, for ductilities within $\mu = 3$-4, i.e. in the early part of the default residual plateau, we see a large reduction in the uncertainty, resulting in only a minor increase of the total β_{UR}. It should be noted in general, that having a c.o.v of 0.3 has only produced a maximum β_U of 0.2 that appears only in the collapse limit-state.

Actually, for the lower limit-states, before the onset of negative stiffness, we have a negligible effect of the model uncertainty on the R-values of capacity. Still, as discussed previously, because of T, the actual uncertainty in the $S_a(T,5\%)$-values of capacity may not be as benign.

Seeing these encouraging results, it is only natural to attempt a reconstruction of the total uncertainty by combining the uncertainty caused by each parameter. Two different rules were tried, (a) the sum of the individual uncertainties for each limit-state (SUM), namely β_U^{SUM} versus (b) the square-root-sum-of-square (SRSS) β_U^{SRSS}. As seen in Figure 13, the SUM rule is quite conservative, producing overly conservative estimates of β_U while the SRSS rule is much more accurate. Based on our earlier observations it appears that we could estimate the total uncertainty dispersion simply by considering the dispersion in the ductilities μ_f and μ_c and maybe the period T. Of course, it should be emphasized that no correlation has been designated between the various parameters, a fact that is probably enhancing the accuracy of the SRSS rule.

In order to investigate the effect of the mean period to the uncertainty, in addition to T = 1s the simulations were also performed for periods T = 0.5s, 1.5s, 2s, as shown in Figures 14, 15 and 16, respectively. Apparently, our general observations are still valid for short and long periods as well. The shapes of the β_U curves are quite similar, indicating the strong influence of μ_c and μ_f on the system's performance. What is most striking about the results though is the increase in the overall R-value uncertainty as we are moving to longer periods. Lower periods lead to a consistent sharp drop in R-values, thus reducing the overall dispersion. Naturally, these observations may be reversed for the actual $S_a(T,5\%)$-values. Another interesting feature is that the relative contribution of μ_c and μ_f to β_U is changing with the period. For low periods μ_c and μ_f cause a similar magnitude of uncertainty in the results. In contrast, the longer periods cause an increase in the uncertainty attributed to μ_f. Again, it appears that the rapid reduction of R-values for shorter periods obscures the effect of the residual plateau to the IDA curves, while at longer periods, shorter versus longer plateaus have a significant difference.

In summary, there is a lot of potential, but there are several issues that need to be resolved before the above observations are generalized. For example, it is imperative to check the resulting uncertainties for higher c.o.v. values, map their influence on the epistemic uncertainty and find out when β_U starts approach β_R. Furthermore, we need to verify our conclusions for different base (mean) backbones, especially ones containing milder negative slopes and much higher or lower residual plateaus. Finally, the accuracy of this tool needs to be checked versus actual IDA results, to make sure that there is nothing in the fitted equations that may introduce bias.

6 CONCLUSIONS

An innovative tool has been presented that can rapidly estimate the uncertainty in the limit-state capacities of SDOF systems having a complex quadrilinear backbone with uncertain properties and varying period. The resulting capacity dispersion due to uncertainty is generally increasing with ductility, thus becoming much larger for higher limit-states. The only exception are ductilities at the early part of the backbone residual plateau where, despite the uncertainty, the system response becomes stable enough to warrant a generous reduction in the resulting uncertainty. The most influential parameters are the end-of-hardening ductility and the ultimate ductility. In general, the results are only mildly dependent on the mean oscillator period, except perhaps in the shorter periods where, surprisingly, the effect of the most parameters is reduced. In all cases though, for relatively moderate coefficients of variation in the parameters, the dispersion in the capacities due to uncertainty remains lower than the respective variability due to the ground motion.

Finally, it is shown that despite the interaction between the oscillator parameters, at least when they are independently distributed the uncertainties in capacity due to each individual parameter can be combined via an SRSS rule to accurately estimate the total. Thus, by simply including the uncertainty in the end-of-hardening ductility, the ultimate ductility and perhaps the period we can get a good estimate of the final result for any limit-state. All in all, the proposed tool is an excellent resource for accurate estimation of the seismic performance of structures having uncertain properties, for the first time providing specific results for each limit-state that can be used in the place of the generic code-prescribed values.

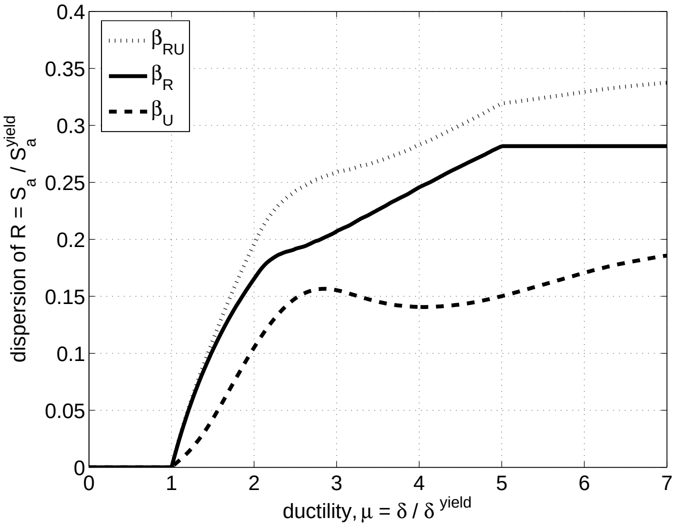

Figure 14. Dispersion β_U due to uncertainty in all six parameters versus β_R and the combined dispersion β_{RU} for oscillator period $T = 0.5$s.

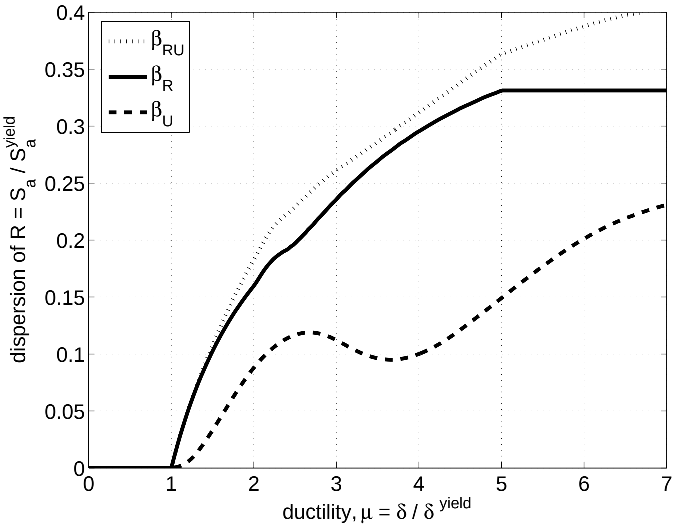

Figure 15. Dispersion β_U due to uncertainty in all six parameters versus β_R and the combined dispersion β_{RU} for oscillator period $T = 1.5$s.

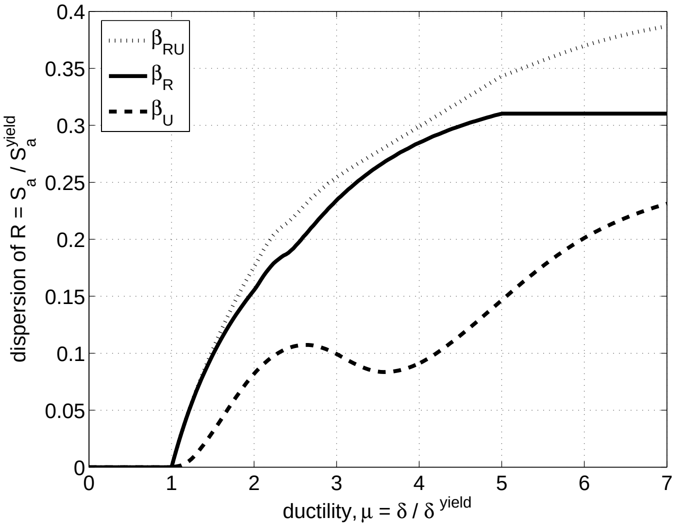

Figure 16. Dispersion β_U due to uncertainty in all six parameters versus β_R and the combined dispersion β_{RU} for oscillator period $T = 2$s.

REFERENCES

Baker, J.W. & Cornell, C.A. 2003. Uncertainty specification and propagation for loss estimation using FOSM methods. *PEER Report 2003/07,* Pacific Earthquake Engineering Research Center, University of California, Berkeley.

Cornell, C.A., Jalayer, F., Hamburger, R.O. & Foutch, D.A. 2002. The probabilistic basis for the 2000 SAC/FEMA steel moment frame guidelines. *ASCE Journal of Structural Engineering* 128(4): 526–533.

Ibarra, L.F. 2003. Global collapse of frame structures under seismic excitations. *PhD Dissertation*, Department of Civil and Environmental Engineering, Stanford University, Stanford, CA.

Ibarra, L.F., Medina R.A. & Krawinkler H. 2005. Hysteretic models that incorporate strength and stiffness deterioration. *Earthquake Engineering and Structural Dynamics,* 34(12): 1489–1511.

Kwon, O.-S. & Elnashai, A. 2006. The effect of material and ground motion uncertainty on the seismic vulnerability curves of RC structure. Engineering Structures, 28(2): 289–303.

SAC/FEMA 2000. Recommended seismic design criteria for new steel moment-frame buildings. *Report No. FEMA-350*, SAC Joint Venture, Federal Emergency Management Agency, Washington DC.

Vamvatsikos, D. 2002. SPO2IDA software for all periods. http://blume.stanford.edu/pdffiles/Tech%20Reports/TR151_spo2ida-allt.xls [Mar. 20, 2007].

Vamvatsikos, D. & Cornell C.A. 2002. Incremental Dynamic Analysis. *Earthquake Engineering and Structural Dynamics,* 31(3): 491–514.

Vamvatsikos, D. & Cornell C.A. 2005. Direct estimation of the seismic demand and capacity of MDOF systems through Incremental Dynamic Analysis of an SDOF Approximation. *ASCE Journal of Structural Engineering,* 131(4): 589–599.

Vamvatsikos, D. & Cornell C.A. 2006. Direct estimation of the seismic demand and capacity of oscillators with multi-linear static pushovers through Incremental Dynamic Analysis. *Earthquake Engineering and Structural Dynamics,* 35(9): 1097–1117.