Tomb Raiders of the Lost Accelerogram: A Fresh Look on a Stale Problem

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Abstract. Throughout recorded history, accelerograms have displayed an unfortunate tendency to become unrecorded and lost. Statistically speaking, even after the advent of low-cost accelerometers, the ground motion retains an almost 100% chance of staying unobserved at any given point. One may only place some limits on the peak amplitude of ground motion by observing its effects, or lack thereof. To do so, seismologists run to the mountains, looking for fragile geological features, such as precariously balanced rocks. Structural engineers take a slightly more cinematic and sinister approach. They put on their fedora hats (or tank top and shorts, for video game enthusiasts) and go tomb raiding, searching for rocking rigid bodies that may have survived or toppled in graveyards, tombs, mausoleums, churches, and temples. Yet how is one to best make sense of such low-entropy (and sometimes contradictory) uncertain information? Let's have some fun by blowing an old problem to smithereens, perhaps needlessly bringing to bear all the tools of contemporary earthquake engineering, ranging from ground motion prediction models and correlation structures to rocking body fragilities and Bayesian analysis.

Keywords: seismic intensity, tombstone, overturning.

1 Introduction

Perhaps starting from the discovery of pottery and that very first red face looking at a toppled vase somewhere in a prehistoric settlement, there has been considerable interest on understanding how rigid bodies rock and overturn. The fact that pots, rocks, boulders and tombstones overturn due to earthquakes, has lent even greater importance to their use as evidence for the intensity of ground motion. Field reports of earthquake damage have long recorded the toppling of tombstones and other graveyard objects, dating at least from 1926 [1], using such observations to approximate seismic intensities, most prominently in Japan [2-5].

Seismology routinely employs fragile geologic structures, and especially precariously balanced rocks, to place limits on the seismic intensity that has or has not been exceeded at a given site [6-9]. This has been of particular importance for planned or existing nuclear waste repositories and powerplant sites, where the capability of dating such rock formations is probably the only way to validate the results of seismic hazard analysis with return periods in the thousands of years. Since the earliest studies circa 1992 recognizing the usefulness of such rock formations [6], seismologists have refined their approaches considerably [10-12], nowadays employing state-of-art probabilistic concepts to fine-tune their seismic hazard predictions [13,14].

On the other hand, despite being early in the game of rigid body rocking [15-17] and having recognized the importance of incorporating uncertainty, structural engineers have lagged a bit behind when it comes to applying refined probabilistic concepts to the same problem. To date, most investigations of historically observed earthquake-induced overturning remain focused on the modeling side [18-22], which should not surprise us given how important it is for achieving some measure of accuracy. Still, whenever some sensitivity analysis is performed, this invariably results in reaching the limits of deterministic approaches (applied with or without a thin veneer of probability). In turn, this often leads to rather bleak assessments that trying to derive any meaning from overturning observations is probably useless [20]. Still, being structural engineers at heart, the authors could not resist poking a bit more at this age-old problem. It may very well be that the situation is hopeless. Still, why not bring all the modern guns of PBEE to bear, introducing standardized probabilistic models for rocking response [23], modern analytical tools [24], and novel fitted expressions for overturning fragilities [25] in the hope of understanding how far we can go, even with an idealized model? At the very least, even if nothing better comes to pass, we will learn the true limits of our predictive capabilities. With this rather sobering thought, let us aim at the problem and let fly.

2 The single tombstone problem

Let us first consider a single 2D rigid block, using the idealized model of Housner [15] to represent a tombstone. In this we assume no bouncing, or sliding, but only pure rocking behavior. While the onset of rocking (uplift initiation) is a relatively easier and more secure issue to check, as it is theoretically fully predicted by the peak ground acceleration, at the same time it may not be easy to detect, unless one observes some movement or fresh damage of the toe of the block in question (e.g., due to post-uplift impact at the base). Instead, let us go directly for the obvious case of overturning. As the IM we shall choose the peak ground velocity (PGV), which is a fairly efficient and sufficient choice [26]. Let O and S represent the overturned versus standing (i.e., non-overturned) conditions of a tombstone.

We are interested in estimating the probability $\mathrm{P}(PGV = x \mid \mathrm{O})$, i.e., the probability that PGV lies in a small interval "around" x given that overturning has occurred. Note that we shall henceforth loosely use $\mathrm{P}(PGV = x \mid \mathrm{O})$ to imply the mathematically more precise term of $\mathrm{P}(PGV \in [x, x + \mathrm{d}x] \mid \mathrm{O})$. We do this for reasons of convenience, as the resulting expressions are anyways meant to be evaluated by discretizing x in bins, as well as for reasons of promoting understanding of the subsequent derivations, which are easier to picture in terms of discrete probabilities rather than probability densities.

In all cases, it is easy to return to continuous functions and proper mathematical formalism by considering, for example, that

$$\mathrm{P}(PGV = x \mid \mathrm{O}) \simeq f_{PGV}(x \mid \mathrm{O}) \cdot \mathrm{d}x \tag{1}$$

where $f_{PGV}(x)$ is the probability density function of the *PGV* experienced at the site.

At a first glance it would be easy to say that $\mathrm{P}(PGV = x \mid \mathrm{O})$ is essentially the distribution of the PGV that characterizes overturning of the block, or its overturning fragility function, readily available by Kazantzi et al. [25]. That is until one realizes that the fragility is actually $\mathrm{P}(\mathrm{O} \mid PGV = x)$, or the probability of overturning given the PGV of *x*. In other words, it is the inverted conditioned quantity. Equating the two is a common fallacy that should be avoided. Perhaps it is better to explain this via a counter-example. Say for example, that we have a very slender block whose overturning fragility has a median "collapse" PGV value of PGV_{C50} = 0.01m/s, with an associated dispersion (standard deviation of the logs) of $\beta = 0.5$. If we see this block overturn and then come out to say that the median PGV of the ground motion that occurred is 0.01 m/s, there is something seriously wrong. This is even clearer if we try to set an upper bound at the 95% percentile, which comes to $PGV_{\mathrm{C50}} \times \exp(1.645\beta) = 0.023$m/s. Obviously, the higher percentiles of the block overturning capacity say nothing about the earthquake that occurred. The only thing that can reliably be said is that the PGV was larger than 0.01 m/s, or if we want to be on the safe side of things, larger than 0.023m/s, without any capability of assigning an actual confidence or probability. To be able to say something more meaningful than this, requires more information. This becomes clearer if we apply the Bayes rule:

$$\mathrm{P}(PGV = x \mid \mathrm{O}) = \frac{\mathrm{P}(\mathrm{O} \mid PGV = x)\,\mathrm{P}(PGV = x)}{\mathrm{P}(\mathrm{O})} \tag{2}$$

Of the above expression, the only term that is presently quantifiable is $\mathrm{P}(\mathrm{O} \mid PGV = x)$, this being the overturning fragility, or if you will, the cumulative distribution function (CDF) of the overturning capacity PGV_{C} [27]. $\mathrm{P}(PGV = x)$ and $\mathrm{P}(\mathrm{O})$ are meaningless without some information about the hazard. As it often happens whenever we hit an insurmountable wall trying to solve a problem, the reason is that we are lacking the proper frame of reference. In our case we clearly need to introduce information about the causal event and the graveyard site. Such information becomes readily available after every earthquake and there is no loss of utility in including it. At a minimum, this should include information about the event magnitude, *M*, source-to-site distance, *R*, site class, fault type etc. In simple terms, one should include as much information as needed to employ a suitable ground motion prediction equation (GMPE) from the literature, or in general a probabilistic model of the form $\mathrm{P}(PGV = x \mid M, R, site, fault)$, which nearly universally conforms to a lognormal distribution of PGV given the event, source and site characteristics. For brevity, we will henceforth only refer to it as $\mathrm{P}(PGV = x \mid MR)$, implying all other conditioning required by the GMPE at hand. Readers well-versed in probabilistic seismic hazard assessment (PSHA, [28]) will immediately recognize the need for having alternative GMPEs at the very least. This is easily treatable by a logic tree, having one branch per GMPE, each with its own weight. In the end, the resulting probability $\mathrm{P}(PGV = x \mid \mathrm{O})$ is estimated as a weighted average of the individual branch probabilities, each one coming from its own GMPE. We contend this extension to be trivial, therefore we shall not expand further on it.

As we are in a post-earthquake environment, where a single event has occurred, all subsequent derivations need to be conditioned on it, so $\mathrm{P}(PGV = x \mid \mathrm{O})$ is now properly written as $\mathrm{P}(PGV = x \mid \mathrm{O}, MR)$, where the comma is used to imply the logical AND. Thus, Equation (2) now is transformed to

$$\mathrm{P}(PGV = x \mid \mathrm{O}, MR) = \frac{\mathrm{P}(\mathrm{O} \mid PGV = x, MR)\ \mathrm{P}(PGV = x \mid MR)}{\mathrm{P}(\mathrm{O} \mid MR)} \tag{3}$$

Assuming that PGV is a sufficient intensity measure [29] to describe overturning, implies that conditioning on PGV and other ground motion characteristics is the same as conditioning on PGV only. Lachanas et al. [26] have provided enough evidence in support, allowing us to state that

$$\mathrm{P}(\mathrm{O} \mid PGV = x, MR) \approx \mathrm{P}(\mathrm{O} \mid PGV = x) \tag{4}$$

As mentioned earlier, this is the block overturning fragility, evaluated at $PGV = x$. Of the remaining terms, $\mathrm{P}(PGV = x \mid MR)$ is clearly coming from the GMPE while $\mathrm{P}(\mathrm{O} \mid MR)$ is akin to a more typical risk integral: The probability of overturning given a certain event has occurred. This is easily treated by the total probability theorem, using PGV in its proper role as an interface variable between the structural and the seismological parts:

$$\mathrm{P}(\mathrm{O} \mid MR) = \int_{x'=0}^{x'=\infty} \mathrm{P}(\mathrm{O} \mid PGV = x', MR) \cdot \mathrm{P}(PGV = x' \mid MR)\, \mathrm{d}x' \tag{5}$$

The first and second terms inside the integral are identical to the first and second terms in the numerator of Equation (3), i.e., they are evaluations of the overturning fragility and the GMPE. Thus, we may rewrite Equation (3) as:

$$\mathrm{P}(PGV = x \mid \mathrm{O}, MR) = \frac{\mathrm{P}(\mathrm{O} \mid PGV = x) \cdot \mathrm{P}(PGV = x \mid MR)}{\int_{x'=0}^{x'=\infty} \mathrm{P}(\mathrm{O} \mid PGV = x') \cdot \mathrm{P}(PGV = x' \mid MR)\, \mathrm{d}x'} \tag{6}$$

For those familiar with Bayesian analysis, this is a fairly typical application whereby the prior distribution provided by the GMPE, $\mathrm{P}(PGV = x \mid MR)$, is updated to include the information provided by the block overturning (O), resulting in the posterior distribution of $\mathrm{P}(PGV = x \mid \mathrm{O}, MR)$. If we are interested in deriving the PDF of the PGV that occurred at the site, namely $f_{PGV}(x \mid \mathrm{O}, MR)$, we may simply apply the transformation of Equation (1) to all probabilities, other than those conditioned on $PGV = x$ (as they are CDFs) to receive the respective PDFs.

In an identical way, we may easily derive the probability distribution of PGV when the block has remained standing, as S and O are complementary events:

$$\mathrm{P}(PGV = x \mid \mathrm{S}, MR) = \frac{[1 - \mathrm{P}(\mathrm{O} \mid PGV = x)] \cdot \mathrm{P}(PGV = x \mid MR)}{\int_{x'=0}^{x'=\infty} [1 - \mathrm{P}(\mathrm{O} \mid PGV = x')] \cdot \mathrm{P}(PGV = x' \mid MR)\, \mathrm{d}x'} \tag{7}$$

Just note that $\mathrm{P}(PGV = x \mid \mathrm{S}, MR) \neq 1 - \mathrm{P}(PGV = x \mid \mathrm{O}, MR)$. Simply put, a block remaining upright gives us completely different and unrelated information than when it overturns.

In the case where the causal M, R are not known, an uninformative prior can be employed, say assigning a uniform distribution to $\mathrm{P}(PGV = x \mid MR)$. Its support of $[v_{low}, v_{upp}]$ can be appropriately set to indicate the range of physically plausible values of PGV. For instance, a typical choice could be $v_{low} = 0$, and $v_{upp} = 2$m/s. This serves the same purpose as a truncation limit to $f_{GMPE}(x \mid MR)$, removing unreasonably large velocity values. Then, within the entire field of definition of the prior, we can eliminate $\mathrm{P}(PGV = x \mid MR)$ from both the nominator and the denominator of Eqs (6) and (7) to receive:

$$\mathrm{P}(PGV = x \mid \mathrm{O}, MR) = \begin{cases} \dfrac{\mathrm{P}(\mathrm{O} \mid PGV = x)}{\int_{x'=v_{low}}^{x'=v_{upp}} \mathrm{P}(\mathrm{O} \mid PGV = x')\, \mathrm{d}x'} & \text{if } x \in [v_{low}, v_{upp}] \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

$$\mathrm{P}(PGV = x \mid \mathrm{S}, MR) = \begin{cases} \dfrac{1 - \mathrm{P}(\mathrm{O} \mid PGV = x)}{\int_{x'=v_{low}}^{x'=v_{upp}} [1 - \mathrm{P}(\mathrm{O} \mid PGV = x')]\, \mathrm{d}x'} & \text{if } x \in [v_{low}, v_{upp}] \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

3 Truncation: Separating fantasy from reality

The lognormal distribution is present in all the equations derived in Section 2. It is the standard model for GMPEs, while it is also a very effective model for the PGV fragilities [23]. Unfortunately, it remains an imperfect model for both, as it comes with a wide support of $[0, +\infty)$. For the GMPE, this implies that any event, regardless of how strong or weak, has a non-zero probability of producing both a very high and a very low PGV. For the fragility, this means that any block has a non-zero probability of overturning at very low intensities, and of remaining standing at very high ones. The nature of the problem is such that all values of PGV within $[0, +\infty)$ will thus receive a non-zero probability of occurrence.

In most situations this is not a problem, as these are extremely unlikely combinations whose probability is negligible. Still, this will become an issue when “extreme” observations appear, such a nearby, large magnitude event that could not topple a low-stability (e.g., small and very slender) block, or conversely a far, small magnitude event that overturned a high-stability block. In such highly improbable cases, one should expect that other unmodeled factors may come into play. For example, in the case of the overturning of a theoretically stable system, there may be factors that have not been accounted for that can increase the severity of the ground motion beyond what is predicted by the GMPE employed, such as directivity or soft-soil effects. There can also be issues with the block itself, such as crushed/weak toes, or an uneven distribution of mass, which if properly modeled would render it less stable. If we let the lognormal distribution offer non-zero probabilities everywhere in $[0, +\infty)$, the equations of Section 2 will still try to offer plausible explanations based on the idealized models of the GMPE and the 2D block, typically falling off the mark. Instead, applying simple truncation limits of two to three log-standard-deviations away from the log-mean (signified as $\pm 2\sigma_{\ln}$ or $\pm 3\sigma_{\ln}$, respectively) to both the GMPE and the capacity distribution, safeguards the results against such improbable combinations and points the modeler in the proper direction of GMPE or model (rather than probability) updating. Actually, a $\pm 3\sigma_{\ln}$ truncation is fairly common in PSHA [30]; we are only extending it here to the PGV capacity.

4 Application example

Let us consider a single slender block with of base width $b = 0.71$m, height $h = 3.61$m, resulting to a stability angle (or slenderness) of $\alpha = b/h = 0.2$ and size parameter $p = 2\text{s}^{-1}$. Based on the work of Kazantzi et al. [25], this has an overturning capacity in terms of the geometric mean PGV that is lognormally distributed with a median of $PGV_{\text{gmC50}} \approx 63$ cm/s, and dispersion (standard deviation of the logarithm of the data) of $\beta \approx 0.3$. Let us also consider two scenario events, having moment magnitudes of $M = 6$ and $M = 8$, both at a closest distance to the surface projection of the fault of $R = 10$km. The first event is a moderate one, while the second can be called extreme for most sites. The GMPE of Boore and Atkinson [31] was employed, assuming a site with shear wave velocity in the top 30m of $v_{s30} = 400$m/s and a reverse fault.

Fig. 1(top) compares the PDFs of the overturning capacities in terms of PGV_{gm} vis-à-vis the GMPE predictions and the results of Eqs (5) or (6), assuming the block overturned (O) or remained standing (S), respectively, for the $M = 6$ event. As expected, if no overturning is observed (S), a near identical PDF is derived, matching the initial estimated GMPE distribution. Essentially, this is a “no-information-added” event, as a relatively low intensity did not overturn a fairly large block, telling us nothing in the process. If instead the block overturned (O), the update points to considerably larger PGV values expected at the site, somewhere in the middle between the fragility estimate and the GMPE prediction. Fig. 1 (bottom) shows the same distributions, only for the much larger M = 8 event. Now, both the O and S observations offer valid updates, the first restricting the low end of PGVs that may have occurred, and the latter restricting the upper end (right tail). In both cases, though, the dispersion remains quite high. Fig. 2 shows an alternative to Fig. 1(top) when truncation is applied. While no change appears when the block remains standing, the overturning is a fairly unusual event, given the moderate magnitude of M = 6. Thus, the truncation of the upper tail of the GMPE has the most pronounced effect, setting an upper limit of PGV_{gm} ~ 65m/s, and severely cutting off the upper tail. Finally, Fig. 3 shows a case where no event knowledge is available. Then, an S observation only serves to derive a plausible upper bound, while the O observation conversely offers a lower bound.

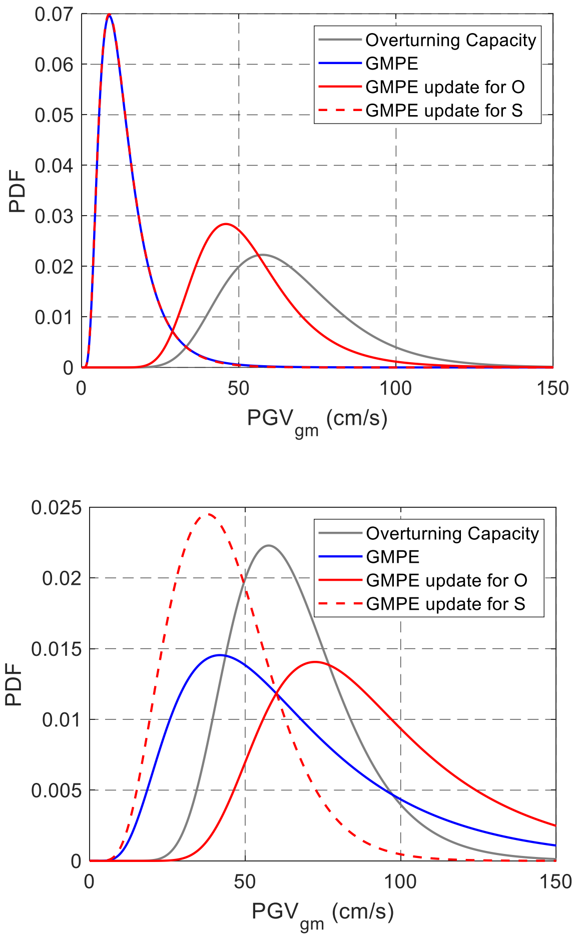

Fig. 1. PDF of the overturning capacities in terms of PGV_{gm} vis-à-vis the GMPE and its updates via Eqs (5) or (6) for the M = 6 (top), and the M = 8 (bottom) events.

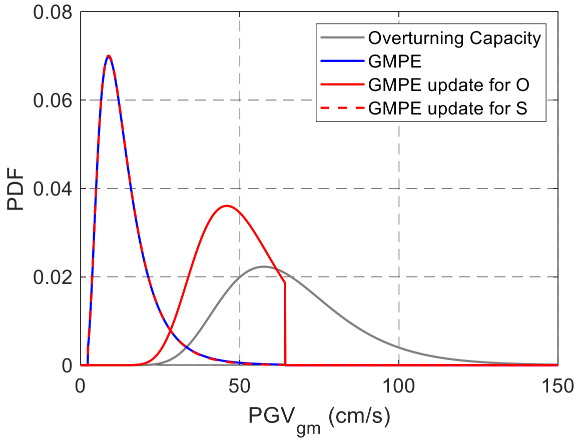

Fig. 2. PDF of the overturning capacities in terms of PGV_{gm} vis-à-vis the GMPE and its updates via Eqs (5) or (6) for the M = 6 event. Truncation at $\pm 3\sigma_{\ln}$ is applied.

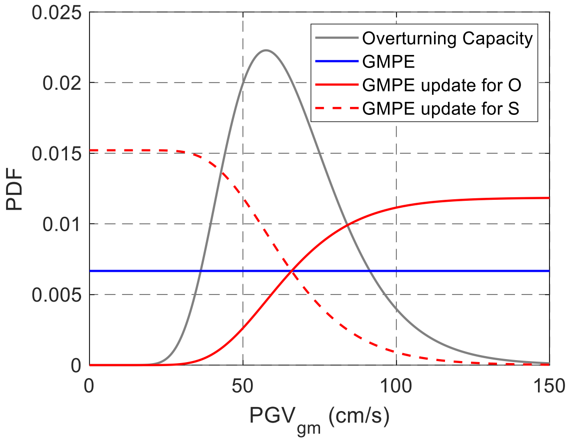

Fig. 3. PDF of the overturning capacities in terms of PGV_{gm} vis-à-vis the uninformative prior (no event knowledge) and its updates via Eqs (8) or (9).

5 A graveyard of identical tombstones

Let us consider multiple “identical” tombstones at a given site, meaning that they are all of the same type, as expressed in terms of size and slenderness in the idealized rigid block model of Housner [15]. All tombstones experience the same ground motion record, assuming the graveyard is relatively compact in size. In theory, if the idealized block model was indeed a perfect description of the actual tombstone rocking behavior, all tombstones should be at the same condition after any given event. In other words, they should all be found either standing up or lying overturned. In reality, there will be uncertainties due to imperfect modeling of each individual block, as we have disregarded the actual ground support conditions, coefficient of restitution, incidence angle of the ground motion relative to the block itself, 3D effects etc.

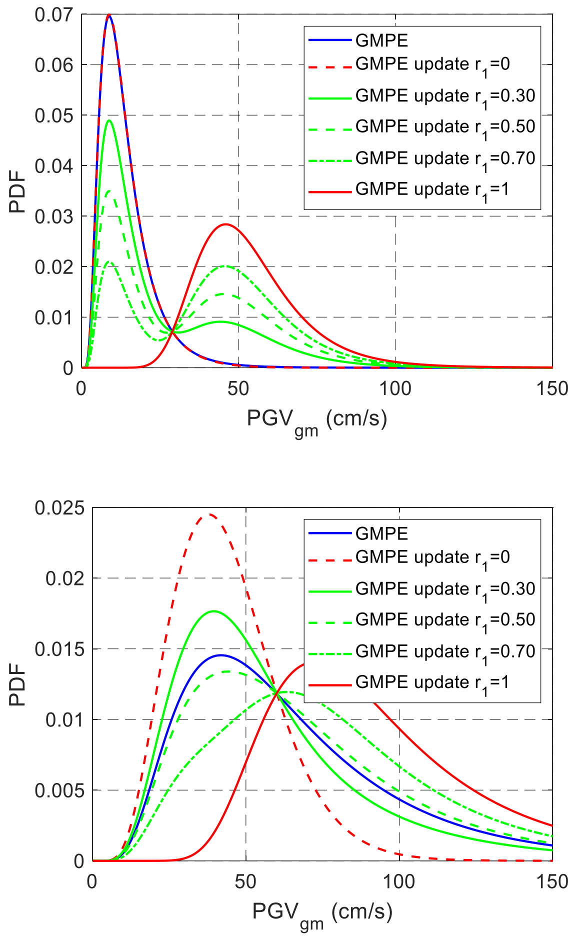

Fig. 4. PDF of the GMPE prediction and its updates via Eqs (10) for various percentages of "identical" tombstones found overturned, given the $M = 6$ (top) and the $M = 8$ (bottom) events.

Say, then, that we observe only a percentage r_1 of tombstones ($0 \le r_1 \le 1$) that have overturned. We should expect that r_1 will take the limiting values of 0 or 1 only in the case of an extremely weak or an extremely strong ground motion, respectively. Otherwise, it should lie somewhere between the two bounds. This is indicative of the additional unmodeled uncertainty that is not captured, e.g., by the Kazantzi et al. [25] probabilistic model. We propose to account for this via a logic tree, whereby an arbitrary block of the given type and subject to the same ground motion will overturn with probability r_1, and remain standing with probability $1 - r_1$. Then, the resulting estimate of the probability distribution of the occurred seismic intensity is:

$$\mathrm{P}(PGV = x \mid r_1, MR) = r_1 \cdot \mathrm{P}(PGV = x \mid \mathrm{O}, MR) + (1 - r_1) \cdot \mathrm{P}(PGV = x \mid \mathrm{S}, MR) \quad (10)$$

where both probability terms required for the evaluation are directly derived from Eqs (6) and (7), or (8) and (9), depending on whether event information is available or not.

Fig. 4 is the direct analogue of Fig. 1, considering the same two events but multiple tombstones and different r_1 ratios. Clearly, if $r_1 = 0$ or 1 one simply recovers the results of having a single block at S or O, respectively. As the ratio moves from 0 to 1, the PDF similarly fluctuates between the two extremes, with the case of least information (and highest dispersion) being the utterly ambiguous $r_1 = 0.5$.

6 Concluding discussion

A fresh look was promised on the stale problem of seismic intensity prediction via toppled/untoppled rigid rocking blocks, and hopefully it has been delivered. If there is any direct conclusion from the results observed so far, is that having some seismological characteristics of the event, sufficient to allow prediction of intensity via a ground motion prediction equation, will positively complement the overturning observations and help deliver a better result. Otherwise, especially where historical observations are concerned and no event data is available, only rough upper/lower bounds may be placed on the ground motion intensity. Inherent randomness limits the predictive capability of single or multiple "identical" blocks, while further epistemic uncertainty (i.e., non-suitability of the model employed) can limit their usefulness even further. Essentially, this is as far as blocks of a single type can go. Still, graveyards are filled with tombstones of different shapes and sizes. Whether these can help us improve our predictions will be the setting of our upcoming endeavors.

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