# SEISMIC FRAGILITY AND VULNERABILITY ASSESSMENT USING SIMPLIFIED METHODS FOR THE GLOBAL EARTHQUAKE MODEL

Dimitrios Vamvatsikos, and Athanasia K. Kazantzi

National Technical University of Athens 9 Heroon Polytechneiou, 157 80 Athens, Greece {divamva,kazantzi}@mail.ntua.gr

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**Abstract.** The Global Earthquake Model (GEM) has commissioned the preparation of analytical vulnerability guidelines for general use. Within this framework, a distinct modeling and analysis method hierarchy has been proposed, whereby both detailed and reduced-order models can be analyzed using nonlinear static or dynamic methods. Each subsequent reduction in complexity increases the speed of application, yet generates additional error that needs to be considered in the form of epistemic uncertainty. The available choices represent different levels of compromise between the accuracy achieved and the associated effort needed, meant to suit users having different levels of expertise and resource availability. Our particular focus will be on the middle path that is expected to become the most popular choice, combining (a) a simplified stick model of the structure with (b) a static pushover analysis with accurate record-to-record dispersion information. The entire procedure is cast within an appropriate probabilistic framework that can effortlessly incorporate all the epistemic and aleatory uncertainty sources to become a viable path for evaluating structural fragility for a building class.

#### **1** INTRODUCTION

The Global Earthquake Model (GEM) is a grand effort to offer a comprehensive opensource tool for loss assessment on a large scale. A vital component of it is the definition of physical vulnerability for different classes of buildings (Figure 1), a task that has been undertaken by an international consortium of researchers. Separate thrusts of the project are geared towards defining building vulnerability functions based on (a) empirical data (b) expert opinion and (c) structural analysis. Our focus will be on the latter part with emphasis on low- and mid-rise structures and specifically on the development of guideline documents for estimating analytical fragility functions for assessing structural damage.

The analytical vulnerability guidelines are being developed as a hierarchy of approaches that can accommodate different levels of expertise. Our proposal thus accommodates several paths towards reasonable approximations that strike different compromises between the time committed and the accuracy achieved. The present work deals with the case where the analyst has the skills and time to perform nonlinear static analysis. The minimum target is for a structural engineer with Master's level training and the ability to create simplified nonlinear structural models, to be able to determine the vulnerability functions pertaining to structural response, damage or loss with reasonable effort. This has been defined as 20-40 man-hours for any single structure, and 80-160 man-hours for a class of buildings.

Estimating vulnerability for a class of buildings involves a number of steps that need to be undertaken in series. First and foremost, a set of "index" buildings needs to be selected to represent the class. These are typically 3 to 7 distinct structures having different macrocharacteristics, such as number of stories, level of construction quality and degree of vertical or plan asymmetry. Subsequently, appropriate structural models need to be generated and structural analysis undertaken to determine their response to seismic loads. Fragility curves, i.e., probability-valued functions of the seismic intensity for exceeding specified damage states for each story or an entire building, are then estimated. The final step is the estimation of losses. At each step all sources of uncertainty need to be quantified and propagated to the final result, taking into account the variability of ground motions, single building properties, population diversity and our own methodological errors.

Our focus will be on the modeling and analysis stages, with particular emphasis on providing a solid basis for evaluating fragility curves for a single building without loss of any probabilistic information. Among a wealth of offered paths to achieve this, we are only going to discuss the option of applying nonlinear static analysis on simplified stick models of the structures, representing an excellent compromise between accuracy and simplicity.



Figure 1: Conceptual framework for seismic risk assessment adopted by GEM (source: http://en.wikipedia.org/wiki/Global\_Earthquake\_Model).

#### 2 SIMPLIFIED STICK MODELS

The development of structural models for each index building is an important issue when deriving analytical vulnerability functions. The complexity of detailed modeling offers undeniable accuracy, yet it often absorbs most of the effort due to the multiple representative buildings employed. Typically, a detailed 2D or 3D multi-degree-of-freedom model would be required for each index building. Appropriate representation of the nonlinear behavior of all identified lateral-load resisting components in the building (columns, beams, walls, braces etc.) is essential. Significant global or local geometric nonlinearities (e.g.,  $P-\Delta$  effects, brace buckling) also need to be included. In short, this is a modeling level that is roughly equivalent to the detail needed for assessing an individual building according to current seismic guidelines. Despite the inherent accuracy and reliability of such detailed models, their use in loss estimation for an entire class may not always be practical. The broad variability within the class means that individual details that have been painstakingly modeled will eventually disappear and only some macro-characteristics may dominate. Then, the simplicity of a 2D stick representation of a building becomes a cost-effective alternative.



Figure 2: A three-story stick model, showing column elements, floor masses  $M_1 - M_3$  and rotational springs to represent beam and foundation stiffness.

Our proposal for a simplified macro-model is based on the concept of "fishbone" models pioneered by Luco et al. [1] to represent moment-resisting frame buildings. This would reduce a frame to a single column-line, each story rotationally restricted by two half-beams that are roller-supported at their opposing ends. This idea has been further simplified and generalized to represent both flexural and shear buildings as shown in Figure 2. It retains the column-line, comprising *N* columns and *N* nodes (plus the foundation node) for *N* stories, each with 3 degrees of freedom (horizontal, vertical, rotational) in 2D space. The nodes are further restrained by *N* rotational springs representing the strength and stiffness of beams at each floor. All elements are nonlinear, at the very minimum having a capped elastoplastic behavior, i.e., an elastic perfectly-plastic relationship of force-deformation, moment-rotation or stress-strain that contains a hard-coded ultimate ductility to simulate component failure (Figure 3). Element characteristics can be easily derived using the aggregate stiffness of the columns, piers, walls or beams in each story together with the corresponding yield and ultimate displacements or rotations. Only translational story masses need be assigned to each node, while global P- $\Delta$ effects are explicitly taken into account.

By thus condensing the characteristics of each story into one column and one rotational spring, the stick model achieves remarkable economy. While it can capture many of the sali-

ent features of modern buildings, especially height, vertical irregularities and flexural versus shear behavior, it cannot take into account any effects related to the two neglected horizontal dimensions. For example, the effect of column compression/tension due to the overturning moment, or any shear lag effects within a single beam-line are not captured. In addition, as with any 2D structure, 3D interaction effects are not modeled. This is of little importance for plan-symmetric structures with distinct lateral load-resisting systems in the two horizontal directions. It becomes an issue for plan-asymmetric structures or wherever the appearance of mass/shear center eccentricity causes torsion. It may also introduce bias in the results if the system strength in the two horizontal directions is strongly interacting, thus making biaxial shaking significantly more detrimental than uniaxial.



Figure 3: The capped elastic-plastic backbone is the simplest recommended force-deformation or momentrotation backbone relationship for elements.

Such 3D effects can still be taken into account approximately by using theoretical or regression expressions to relate, e.g., the index of plan asymmetry to a reduction in the column and beam ductility capacities incorporated in the model. This is considered a far superior approach than using a direct "damage modifier", where the modification is applied on the EDP displacement (or acceleration) response of the model rather than its properties, leading to considerable difficulties in properly defining collapse. In other words, it is not easy to make such modifications influence the seismic intensity level causing collapse when applying them only in post-processing. By including them in the model properties, though, their integration becomes more natural. For example, for square-plan multi-bay space-frames with ductile members, it can be shown that a normalized plan eccentricity of e leads to an increase of elastic base shear in any of the two directions by a factor of 1+1.5e. Reducing the yield and ultimate ductility of both the beam spring and the column element in the corresponding eccentric stories yields a simple method to roughly account for this effect.

# **3** EVALUATION OF FRAGILITY CURVES USING NSP

While nonlinear dynamic analysis is steadily gaining ground as the standard method of analysis, at present its use largely remains within the academic community. The mainstay of current practical guidelines for seismic assessment is currently the nonlinear static procedure (NSP). There are several methodologies for estimating fragility curves for a structure that are based on NSP, using a pushover analysis to evaluate system performance. They are generally simple and relatively easy-to-use methods that are intuitive for engineers that have worked with nonlinear static analysis. Apart from any inaccuracies incurred by the well-known approximating nature of the pushover, they do share one major drawback: Despite trying to capture an inherently probabilistic quantity such as the fragility curve, they are in essence deterministic approaches, simply because of their root in classic pushover analysis.

Typical NSP approaches determine a single demand value for any structural response variable (or engineering demand parameter, EDP) that corresponds to a given level of the seismic intensity measure (IM), as measured in terms of the spectral acceleration  $S_a(T)$  at period T. This is summarized in the so-called performance point, situated on the capacity curve at an estimated target displacement (typically of the roof). This single demand value can be estimated via two possible methods, namely displacement modification/coefficient method (Veletsos and Newmark [2]) or the equivalent linearization approach (Jacobsen [3]). In the first case, an  $R-\mu-T$  relationship is employed to provide an approximation to the mean or median value of ductility,  $\mu$ , of a nonlinear single-degree-of-freedom oscillator with period T that is subjected to a given level of intensity, defined by the strength ratio R. The latter is the ratio of the seismic force over the oscillator yield strength, or simply the seismic intensity in  $S_a$  terms over its value that causes yield. This method is the basis of most current US and EU guidelines, namely ASCE/SEI 41-06 [4] and EN1998-Part III [5]. The equivalent linearization method, instead, utilizes a lengthened period and an increased damping value to define a linear oscillator that can provide the needed (mean/median) displacement response. It has been popularized by the ATC-40 document and it was later shown to be able to deliver mean or median results of similar accuracy to the displacement modification approach (FEMA 440 [6]), as long as a direct physical interpretation is not a constraint when deriving the equivalent period and damping. Still, it remains an indirect approach that has not seen much use beyond ATC-40 [7]. Therefore it will not be the focus of our proposed approach.

Summing up, the constraints placed upon NSPs by the target displacement approximation method essentially limit its ability to provide a full distribution of response for a given level of intensity and hence capture the seismic input randomness. As a result, the static pushover itself is only used as a method to determine the central value (median or mean) of intensity measure (IM) capacity that anchors the fragility curve, while the dispersion around it is typically an assumed constant value, regardless of period or deformation. This has the undesirable effect of providing only a rough approximation of the considerable record-to-record variability, while not offering any insight into the additional dispersion due to aleatory and epistemic sources inherent in modeling, analysis and threshold values of EDP capacity.

Recent advances in R- $\mu$ -T relationships have offered at least two viable options for introducing record-to-record variability back into NSP estimates. The first is the work of Vamvatsikos and Cornell [8] on SPO2IDA, a spreadsheet-level tool that allows estimating the median and dispersion of ductility for complex quadrilinear capacity curves that may incorporate a negative stiffness segment, e.g. due to P- $\Delta$  or material in-cycle degradation and a residual strength branch, similar to the post-peak response of a braced or infilled frame. The second approach comes from Ruiz-Garcia and Miranda [9] that have offered R- $\mu$ -T (or, more precisely  $C_R$ - $\mu$ -T, where  $C_R = \mu(R)/R$ ) expressions with dispersion information for elastoplastic oscillators. Such tools offer substantially improved information that can be used to inject probability back into traditional NSPs.

#### 3.1 Probabilistic basis of fragility

Fragility curves have been around for a long time, dating back to the early work in the nuclear industry, e.g., Kennedy and Ravindra [10]. In our case, the probabilistic formulation that will be adopted to represent the fragility curve goes back to at least the concepts forming the backbone of the SAC/FEMA framework (Cornell et al [11]) that have also appeared in the earlier or later work of many researchers, for example Shinozuka et al [12], Choi et al [13] and Kazantzi et al. [14]. Following in their footsteps, the fragility function is defined as the probability function of the limit-state capacity C being exceeded by the demand D for a given intensity level (i.e. IM-value), s. It may be defined for an entire building or for any of its sto-

ries. In both cases, if demand and capacity are expressed in terms of intensity levels, then we get the simplest representation of fragility:

$$P_{LS}(s) = P(C < D \mid s) = P(s_c < s \mid s) = F(s_c \mid s)$$

$$\tag{1}$$

where  $s_c$  is the (random) IM-value of capacity that when exceeded signals violation of the limit-state and  $F[\cdot]$  is the cumulative distribution function (CDF) of its arguments. Essentially, the fragility curve then becomes the CDF of  $s_c$  evaluated at the intensity level s. It is usually assumed that  $s_c$  is lognormal, leading to the simple expression of:

$$P_{LS}(s) = \Phi\left(\frac{\ln s - \ln \hat{s}_c}{\beta_{Sc}}\right)$$
(2)

where  $\hat{s}_c$  is the median IM-value of capacity and  $\beta_{Sc}$  the corresponding dispersion (standard deviation of the log-data). While conceptually simple, these two parameters may become difficult to evaluate as the results of structural analysis are in terms of EDP given the level of IM, rather than vice-versa.

Therefore, a more intuitive format is based on the expression of both demand and capacity in terms of the engineering demand parameter that is used to test for limit-state violation. If  $\theta_c$  is the corresponding EDP capacity and  $\theta$  the structural demand (both random variables), then

$$P_{LS}(s) = P(C < D \mid s) = P(\theta_c < \theta \mid s)$$
(3)

Again, it is assumed that both demand and capacity are lognormal with medians  $\hat{\theta}(s)$ ,  $\hat{\theta}_c$  and dispersions  $\beta_{\theta d}$ ,  $\beta_{\theta c}$ , respectively.

Then, considering that the sum (or difference) of two normal variables is also normal, and assuming that demand and capacity are independent (see also the relevant discussion in Cornell et al. [11]) the following well-known result comes up (e.g., Kennedy and Ravindra [10]):

$$P_{LS}(s) = \Phi\left(\frac{\ln\hat{\theta}(s) - \ln\hat{\theta}_c}{\sqrt{\beta_{\theta d}^2 + \beta_{\theta c}^2}}\right)$$
(4)

Following the work of Cornell et al. [11], a power law approximation is assumed for the median EDP demand given IM, which is valid as long as the structure has not approached the global instability region:

$$\theta(s) \approx a \cdot s^b \tag{5}$$

Then, by introducing the above into Eq. (4) a simpler approximation may be derived for fragility that resembles the earlier IM formulation:

$$P_{LS}(s) = \Phi\left(\frac{\ln a + b\ln s - \ln \hat{\theta}_c}{\sqrt{\beta_{\theta d}^2 + \beta_{\theta c}^2}}\right) = \Phi\left[\frac{\ln s - \left(\ln \hat{\theta}_c - \ln a\right)/b}{\sqrt{\beta_{\theta d}^2 + \beta_{\theta c}^2}/b}\right]$$
(6)

Now, it becomes obvious by comparing Eq. (2) and (6) that the median and dispersion of the IM value of capacity,  $s_c$ , may be estimated as:

$$\hat{s}_c = \left(\frac{\hat{\theta}_c}{a}\right)^{1/b} \tag{7}$$

$$\beta_{Sc} = \frac{1}{b} \sqrt{\beta_{\theta l}^2 + \beta_{\theta c}^2} \tag{8}$$

In order to introduce the effect of epistemic uncertainty, it is assumed that demand and capacity maintain their medians but acquire additional dispersion of  $\beta_{U\theta d}$  and  $\beta_{U\theta c}$ , respectively. This is typically referred to as the "first-order assumption" and it causes the overall dispersion of Eq. (8) to become instead:

$$\beta_{TSc} = \frac{1}{b} \sqrt{\beta_{\theta d}^2 + \beta_{\theta c}^2 + \beta_{U \theta d}^2 + \beta_{U \theta c}^2}$$
(9)

The dispersions  $\beta_{\theta c}$ ,  $\beta_{U\theta d}$  and  $\beta_{U\theta c}$  are essentially parameters of the problem that need to be provided and cannot be easily determined by a simple computational analysis. For example, Kazantzi et al. [15], Liel et al [16], Dolsek [17], Vamvatsikos and Fragiadakis [18] offer a number of computational methods for estimating  $\beta_{U\theta d}$  when dealing with models having uncertain parameters. Similarly, the median EDP capacity is best determined by experimental data, post-earthquake surveys or expert judgment. On the other hand, *a*, *b* and  $\beta_{\theta d}$  can be reasonably approximated, ideally by multiple dynamic analyses (e.g. incremental dynamic analysis, IDA, Vamvatsikos and Cornell [19]) or, in many cases, with a simple static pushover. The latter will be the focus of the proposed method.

#### **3.2** Estimation in a pushover setting without global collapse

The "central value" of roof (or generally control node) displacement response corresponding to any level of spectral acceleration intensity,  $S_a = s$  can be estimated as follows (e.g. EN1998 [5], ASCE/SEI 41-06 [4]):

$$\hat{\delta}_{roof} = \Gamma \hat{C}_R \frac{T^2}{4\pi^2} \cdot s \tag{10}$$

where  $\Gamma$  is the first-mode participation factor (estimated for the first-mode shape normalized by the roof displacement), *T* is the equivalent SDOF system period and  $\hat{C}_R$  is the median inelastic displacement ratio for the given strength ratio *R*. Considering that the pushover results offer practically a one-to-one mapping between any local EDP and the roof displacement, with the possible exception of isolated spots in the negative stiffness region, we can represent the median roof drift as a function of the corresponding median EDP, i.e.  $\hat{\delta}_{roof} = \delta_{roof}(\hat{\theta})$ . Then we can use the pushover results to easily estimate the  $\delta_{roof}$  corresponding to the median capacity value,  $\theta_c$ , and solve for the corresponding median seismic intensity:

$$\hat{s}_{c} = \frac{4\pi^{2}}{\Gamma \hat{C}_{R} T^{2}} \cdot \delta_{roof} \left( \hat{\theta}_{c} \right)$$
(11)

For the median  $C_R$ , one can use SPO2IDA for practically any shape of capacity curve, or resort to the simpler relationships provided by Ruiz-Garcia and Miranda [9], valid for elastoplastic systems:

$$\hat{C}_{R} = \frac{\hat{\mu}(R)}{R} = 1 + \frac{R - 1}{79.12 \, T^{1.98}} \tag{12}$$

An appropriate value for R for use with Eq. (12) should generally correspond to a value of seismic intensity close to the region of interest, or, in other words, close to the median EDP-

value of capacity. In terms of ductility, this maps to the value of  $\mu_{lim} = \delta_{roof}(\hat{\theta}_c)/\delta_y$ . A potential solution would be to prescribe  $R_{lim} = \mu_{lim}$  in the sense of the equal displacement rule, but this would grossly overestimate both *b* and  $\beta_{\theta d}$  for shorter periods. A much better solution is to set  $\mu = \mu_{lim}$  in Eq. (12) and solve the resulting quadratic expression for *R*. Since *b* and the dispersion  $\beta_{\theta d}$  tend to increase with *R* (rather than remain constant), it is best to take a point estimate (a form of biased fitting) at a reduced value, say at 85% of the resulting *R*-value

$$R_{lim} = \max\left\{0.425\left(1 - c + \sqrt{c^2 + 2c(2\mu_{lim} - 1) + 1}\right), 1.0\right\}, \text{ where } c = 79.12 \, T^{1.98}$$
(13)

A lower limit of 1.0 has been imposed for performance points close to yielding as the 85% reduction taken above may make  $R_{lim}$  become less than 1.0.

Thanks to the proportionality between  $C_R$  and  $d_{roof}/s_c$ , a useful local *b* can be estimated through Eq. (12) by interpolating in log-space between the yield point at  $(\mu,R) = (1,1)$  and the value of the median  $\mu$  at  $R_{lim}$ . This is equivalent to the ratio of the logs of the latter two values, which can be easily estimated from Eq. (12) above, by taking the log of both sides and then dividing by  $\ln R_{lim}$ :

$$b \cong \frac{\ln \hat{\mu}(R_{lim})}{\ln R_{lim}} = 1 + \frac{\ln\left(1 + \frac{R_{lim} - 1}{79.12 T^{1.98}}\right)}{\ln R_{lim}}$$
(14)

In the limiting case of  $R_{lim} = 1$ , signifying elastic response, the slope *b* is always set to 1.0. Strictly speaking, this is the *b*-slope corresponding to the roof displacement. In the case of inelastic response, proportionality should be locally valid between  $\delta_{roof}$  and  $\theta$  for the above *b* to be usable for the latter. Otherwise more careful interpolation will need to be performed close to the median value of  $\theta_c$ .

Finally, the needed conditional demand dispersion can also be estimated from either SPO2IDA [8], or the work of Ruiz-Garcia and Miranda [9]. Then, the dispersion of  $\delta_{roof}$  is the same as the dispersion of  $C_R$ , as they are proportional:

$$\sigma_{\ln \delta_{roof}} = \sigma_{\ln C_R} = 1.957 \left[ \frac{1}{5.876} + \frac{1}{11.749(T+0.1)} \right] \left\{ 1 - \exp\left[ -0.739(R_{lim} - 1) \right] \right\}$$
(15)

If proportionality holds in the vicinity of  $s_c$  between  $\delta_{roof}$  and  $\theta$ , then they both share the same dispersion. This is a reasonable assumption that generally makes sense for most situations. Otherwise, it is best to estimate  $\beta_{\theta d}$  as one half of the difference between the 16 and 84 percentiles of  $\theta$  at the given intensity level:

$$\beta_{\theta d} = \frac{\ln \theta_{84} - \ln \theta_{16}}{2} = \frac{\ln \theta \left[ \hat{\delta}_{roof} \exp(\sigma_{\ln \delta_{roof}}) \right] - \ln \theta \left[ \hat{\delta}_{roof} \exp(-\sigma_{\ln \delta_{roof}}) \right]}{2}$$
(16)

where  $\theta[\cdot]$  represents the inverse mapping offered by the pushover between roof drift and the EDP of choice.

Note that when multiple EDPs are used to define a limit-state, where exceeding any one of them signals violation, the above framework shows that they will generally offer the same  $\beta_{\theta d}$  dispersion to the fragility curve. As long as no significant differences are introduced by the other aleatory and epistemic contributions to the dispersion between the different EDPs, then we only need to define the appropriate median intensity capacity that corresponds to the EDP that governs, i.e., use the one whose  $\theta_c$  median capacity corresponds to the lowest roof displacement. This is for example the case of using the story drifts to determine the limit-state:

Separate checks for each story can be conveniently replaced by a single check of the maximum drift over all stories, as long as all stories have about the same drift capacity and the limit-state is considered to be violated if any story drift exceeds it (which is the usual choice). Even if the dispersions corresponding to each EDP differ significantly, the above statements are still true as long as the dispersion corresponding to the governing EDP is the highest. Otherwise, all EDPs may need to be taken into account, for example via the formulation discussed in Choi et al [13], to incorporate issues of correlation. It is noted however that such considerations are mostly an issue for tagging applications, where assigning a single damagestate to an entire building makes sense. Seismic loss assessment is best performed through a fine-grained application of fragility, at the story or even component level that allows a more accurate calculation of cost. Furthermore, structural, non-structural and content damage is best considered separately rather than through a common damage-state. Thus, it is not envisioned that multiple EDPs will become important when defining such localized damage-states.

In summary, a simple algorithm for estimating fragility curves for an elastoplastic system and for any limit-state defined by a single scalar EDP under the assumptions presented above can be cast as follows:

- 1. Run a static pushover analysis in order to obtain the capacity curve and the corresponding results for the EDP needed for limit-state definition.
- 2. Fit the pushover curve via an elastoplastic idealization
- 3. From the pushover results estimate  $\delta_{roof}(\hat{\theta}_c)$ , i.e., the median roof displacement corresponding to the median EDP capacity.
- 4. Estimate the median IM capacity,  $\hat{s}_c$  from Eq. (11) and (12)
- 5. Estimate b from Eq. (14)
- 6. Estimate  $\beta_{\theta d}$  from Eq. (15) or (16)
- 7. Evaluate the fragility dispersion according to Eq. (8) or (9).

Use of SPO2IDA only changes steps 4-6, where instead of Eq. (12), (14) and (15) to estimate  $C_R$ , *b* and  $\beta_{\theta d}$ , respectively, direct numerical results are taken from the SPO2IDA tool.

In any case, the overall fragility curve is defined by the median IM capacity and the corresponding total dispersion, according to Eq. (2) in this very simple scheme that needs no iterations and no assumptions about record-to-record variability. Moreover, it is fully compatible with nonlinear dynamic analysis results, as long as the assumptions of the pushover analysis hold, the R- $\mu$ -T relationship is accurate enough for the capacity curve employed (see also De Luca et al. [20] on fitting) and the limit-state is not close to the region of global dynamic instability.

# 3.3 Introducing global collapse information

The phenomenon of global dynamic instability may only appear if adequate modeling of P- $\Delta$  and/or material in-cycle degradation has been employed, or, at the very least, some ultimate ductility capacity has been imposed a posteriori on the pushover results. Such modeling options generate a distinctive plateau on the IDA curves in IM versus EDP co-ordinates (for example the region beyond the ductility of 5 in Figure 4a). This corresponds to an explicit simulation of the results of global collapse. Then, estimation of the fragility for limit-states that occur close to this region cannot be reliably performed with the aforementioned proce-

dure. To be more precise, if the corresponding median intensity capacity  $\hat{s}_c$  is higher than the 16% spectral acceleration collapse capacity (i.e., an *R*-value of about 2.3 in Figure 4b), the power-law approximation of Eq. (5) is no longer accurate. Thus, additional steps need to be taken when defining the fragility curve to ensure that collapse (which by default violates all limit-states) is taken into account.

First, the values of  $C_R$ , *b* and  $\beta_{\theta d}$  should only be based on intensities that directly precede the 16% value of capacity. In other words, they should be based on non-collapse responses (ideally, higher intensities should also be included by taking into account only non-collapse data points but this definition is only usable when discrete dynamic analyses are available). Then, the probability of collapse needs to be directly incorporated by conditioning on collapse and non-collapse (Jalayer and Cornell [21]):

$$P_{LS}(s) = P(C < D \mid s, NC) \cdot (1 - P_C(s)) + P_C(s)$$
(17)

where  $P_C(s)$  is the probability of collapse, or simply, the fragility of the global collapse limitstate and P(C < D|s, NC) is the fragility curve determined with the procedure presented earlier.

Note that due to the nature of global instability, simplified assumptions fail to deliver the desired fidelity if an EDP basis is chosen for evaluating the collapse fragility  $P_C(s)$ . The appearance of multiple "infinite" values of EDP due to individual collapses (Figure 4a) means that the distribution of EDP at a given intensity cannot be characterized by a lognormal. On the contrary, an IM basis is perfectly adequate, as lognormality still holds. Thus, the collapse fragility can only be defined via Eq. (1), whose parameters are directly provided by SPO2IDA. Note here that this is different from other definitions of collapse where the model happily goes on forever (i.e., the IDA curves never flatline in Figure 4a) and collapse is retro-actively defined by some EDP capacity. Such cases may not offer the fidelity and accuracy of the above formulation, yet they have the advantage of being conveniently handled with the originally presented approach, without needing to separately account for global instability.



Figure 4: (a) Forty IDA curves and (b) their summarization into 16/50/84% fractiles for an SDOF system. The EDP is ductility and the IM is  $S_a(T_1)$  normalized by its yield-level value. The *x*% fractiles EDP|IM curves are practically identical to the (1-*x*)% IM|EDP curves (Vamvatsikos and Cornell [19]).

#### 4 CONCLUSIONS

A simplified method has been presented for extracting fragility curves via the static pushover method preferentially coupled with simplified stick models for optimal application to large sets of buildings. The methodology allows the accurate estimation of the record-torecord variability and the introduction of any additional source of uncertainty, without needing to resort to ad hoc assumptions. It is thus simple and reliable enough to be applicable to a large number of buildings and become useable for estimating fragility not just for a single but for an entire class of structures, as required for application with the Global Earthquake Model.

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