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Assessment of structures subject to time-dependent degradation via equivalent constant rates

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Abstract. Analytical, closed-form solutions are presented for the computation of equivalent constant rates of limit-state exceedance for structures under seismic loads whose capacity is degrading with time. Seismic guidelines currently designate constant, time-independent probabilities or mean annual frequencies of exceedance that are assumed to remain invariable for the entire design life. These are at odds with the time-dependent, ever-increasing exceedance rates of ageing structures. Based on the concept of social equity and discounting of societal investments, the equivalent constant rate provides a basis for judging the safety of structures with time-dependent capacity by allowing comparisons with the code-mandated rates of limit-state exceedance. Starting from the simple SAC/FEMA solution and assuming a power-law degradation of capacity with time and a linear change in the combined epistemic and aleatory variability of capacity, we provide general solutions for the equivalent constant rate and for the limiting case of the average rate over the design life of the structure. The solutions are formulated both in the demand-based and in the intensity-based format, the latter being suitable for all limit-states, even close to global collapse. By using a 7-story reinforced concrete building as an example, we demonstrate the accuracy and the practicality of these approximations for the assessment of an existing structure.

Keywords: probabilistic; earthquake; ageing; capacity degradation; analytical solution

1 INTRODUCTION

The ageing of existing infrastructure has become a major issue in the field of design and analysis. Structures are still being designed without consideration of the ageing and the weathering that they are subject to. Design codes typically contain only some prescriptive requirements, e.g., mandating a certain thickness for concrete cover or specific corrosion protection measures for steel members. It is simply implied that such measures will let the structure retain the required capacity for the duration of its design life. In a seismic environment, it is precisely this unaccounted for, actual level of capacity that will determine whether a failure will occur or not. Thus, while both the seismic capacity and the weathering of structures have received a lot of research interest, it is the combined effect of the two that matters most in many situations.

On the seismic side, performance-based earthquake engineering (PBEE) has recently emerged to explicitly enable the assessment of structural performance under seismic loads. This is exemplified by the framework adopted by the Pacific Earthquake Engineering Research (PEER) Center to estimate the mean annual frequency (MAF) of exceeding designated limit-states in terms of costs, casualties, downtime or simply structural response (Cornell and Krawinkler 2000). Such approaches have been greatly simplified by Cornell et al. (2002) that have offered closed-form solutions to estimate the MAF of exceeding structural limit-states. While such significant advances have taken place in this field, they all incorporate the assumption that the structure will remain practically indefinitely in the same initial condition. In other words, the MAFs calculated for the initial state are assumed to be time-independent and to extend to the entire design life of the structure, without any consideration for weathering effects. This is a concept that is integrated in our design methodologies as we typically think of, e.g., the "*x*% in 50yrs" exceedance probability as a typical design objective for buildings. While the current PBEE approach may provide a comprehensive solution to seismic assessment, it also introduces a large question mark for structures that are subject to ageing and their capacity degrades with time.

Such effects may indeed not be critical for structures that are not exposed to severe environments, but they may be detrimental for many others. Recent literature has come to acknowledge this problem, especially in the case of bridges (for example Tao et al. 1994, Enright and Frangopol 1998, Stewart and Rosowsky 1998). Several ideas have been put forward to deal with such degrading-capacity problems in a rational way, often focusing on the life-cycle cost (e.g., Frangopol et al. 1997). Unfortunately, applying these methods is not trivial: Estimating life-cycle cost can be an elaborate operation, especially since one needs to address the cost and the frequency of maintenance schemes. Furthermore, inspection and maintenance actions are not the norm for most, if not all, buildings and many bridges. On the contrary, any weathering consideration is typically assumed to be provided via a proper initial design meant to maintain the desired level of safety throughout the design life.

Thus, it becomes attractive to maintain a code-compatible format based on the MAF by reconsidering the typical PBEE approach. When capacity degradation is present, the use of a constant-rate homogeneous Poisson process of limit-state exceedance events is not feasible anymore, since the actual rate increases with the age of the structure. Still, it is always useful to derive a constant rate that will, in some reasonable sense, be representative of the ever-changing risk faced by the building over its design life. In other words, we need some way to transform time-dependent rates to the usual time-invariant (constant) rates that are so ingrained in our design philosophy.

The early seeds of such an approach can be found in the definition of the average rate of exceedance proposed, e.g., by Cornell and Bandyopadhyay (1996), Amin et al. (1999) and also indirectly by Torres and Ruiz 2007. The latter actually provide estimates for the expected number of exceedance events over a time interval, which, when divided by the interval length, produces the average rate. Nevertheless, it is not obvious that such indiscriminate averaging is the best solution. For example, from the owner's perspective, exceedance events closer to the end of a building's lifetime might not matter as much as events that happen while it is still new.

Seeking a more useful measure, we have turned to the equivalent constant rate (ECR), based on the concept of social equity, as introduced by Yeo and Cornell (2009a) and applied in Yeo and Cornell (2009b) for decision-making in an aftershock environment. This idea has been recast by Vamvatsikos and Dolsek (2011) considering the problem of ageing structures under seismic loads. They have derived simple closed-form expressions that are going to be presented in the following pages.

2 DEFINITION OF ECR FOR AGEING STRUCTURES

The derivation of Vamvatsikos and Dolsek (2011) is based on the concept of the societal cost of seismic damage, due to exceeding a limit-state LS in some time in the future, as introduced by Paté (1985). In order to properly account for the time-value of money between $\tau = 0$ and the time of LS exceedance, α is defined as the discount rate (adjusted for inflation) that is appropriate for societal investments in seismic-safety technologies, having typical values in the range of 3–5% (see Paté-Cornell 1984). A structure is considered to have a time-dependent rate of exceeding a certain limit-state due to ageing. Let $\lambda_{\mathrm{LS}}(\tau)$ be the instantaneous rate of LS violations at τ. Then, for this structure and limit-state LS and for a given design life, or period of interest, T_d, we define the ECR as the time-independent exceedance rate within $[0, T_d]$ that is "equivalent" to the time-dependent rate $\lambda_{\mathrm{LS}}(\tau)$.

Paralleling the definition in Yeo and Cornell (2009a), “equivalence” means that the expected investment for preventing violation of LS in the future is the same for both cases. As shown by Vamvatsikos and Dolsek (2011), the ECR is then estimated as:

$$\lambda_{\mathrm{LS}}^{\mathrm{ECR}} = \frac{a}{1-e^{-aT_d}} \int_0^{T_d} \lambda_{LS}(\tau) e^{-a\tau}\, \mathrm{d}\tau \tag{1}$$

In the sections to follow, we shall discuss the derivation of closed-form approximations to the above expression using two different but equally popular formulations, the IM-based and the EDP-based.

3 CLOSED FORM SOLUTIONS

3.1 IM-based formulation

In order to estimate the ECR via Eq. (1), we first need a closed form solution for evaluating $\lambda_{\mathrm{LS}}(\tau)$. This has been presented by Cornell et al. (2002) and explained in detail by Jalayer (2003), whose results we will present here. First, we assume that the IM-capacity is lognormally distributed and adopt the first-order assumption, i.e., we assume that the epistemic uncertainty does not influence the median value, only the dispersion around the median. This assumption is not entirely accurate, as shown, e.g., by Liel et al. (2009), Dolsek (2009) and Vamvatsikos and Fragiadakis (2010), but still reasonable. Furthermore, we assume that the hazard curve $H(s)$, i.e., the function of mean annual frequency of exceeding values of the IM, typically taken as the 5%-damped first-mode spectral acceleration $S_a(T_1,5\%)$, can be approximated in the area of the median IM capacity $\hat{s}_c$ by a power-law form, or, equivalently, a straight line in log-log coordinates:

$$H(s) \approx k_0\, s^{-k_1} = k_0 \exp(-k_1 \ln s) \tag{2}$$

Then, as shown in Jalayer (2003), the mean annual frequency of exceeding limit-state LS can be approximated as

$$\lambda_{\mathrm{LS}} = H(\hat{s}_c) \exp\left[\frac{1}{2} k_1^2 \left(\beta_{Sc}^2 + \beta_{USc}^2\right)\right] \tag{3}$$

where β_{Sc} is the record-to-record dispersion (the standard deviation of the log-data) around the median capacity $\hat{s}_c$ and β_{USc} represents the corresponding epistemic uncertainty. The reader is referred, for example, to Dolsek (2009) and Vamvatsikos and Fragiadakis (2010) for a comprehensive discussion of how these quantities, especially β_{USc}, can be estimated in the case of model parameter uncertainty.

Even with a closed form estimate for λ_{LS}, Eq. (1) is not amenable to analytical solution, unless we assume simplified expressions for the, potentially time-dependent, parameters in estimating λ_{LS} via Eq. (3). First of all, we will assume that $\hat{s}_c$ does not degrade dramatically. Then, we can use the same approximation for the hazard curve for all times τ. Thus, in the form of Eq. (2) we will assume that the constants k_0 and k_1 are time-independent. In order to apply this practically, we cannot fit the hazard curve via Eq. (2) just at the $\hat{s}_c$ point as proposed by Cornell et al. (2002). The problem is the potential overestimation of the hazard rate for low S_a values. Since the hazard rates there are exponentially higher, this is the area with the most significant contribution to the MAF integral. Actually, as discussed by Vamvatsikos and Cornell (2004), both a global fit and a tangential point-fit at a given $\hat{s}_c$ may sometimes be appropriate and other times in gross error. Extensive testing Dolsek and Fajfar (2008) has shown that a better, more robust approximation can be achieved through a local area-fit.

For example, for a given $\hat{s}_c$ it is better to fit within [0.25 $\hat{s}_c$, \$1.25 $\hat{s}_c$], rather than right at $\hat{s}_c$. This is a good strategy to achieve a robust estimation even just at the $\hat{s}_c$ point for $\tau = 0$, namely $\hat{s}_c^0$, with Eq. (3). As discussed later in the example's section, if we slightly extend this interval around $\hat{s}_c^0$ to cover even lower values then, depending on the degradation rate of the capacity, we can also achieve a good fit for multiple $\hat{s}_c$ values as τ grows. Therefore, such a local area-fit strategy helps both ways.

In approximating $\lambda_{\mathrm{LS}}^{\mathrm{ECR}}$, the form of the degradation for the $\hat{s}_c$ capacity of the structure over time τ is also important. Initially, we will assume that capacity degradation occurs (or continues) immediately from $\tau = 0$, without delay, and it may follow either a linear or a power-law form. Since, the latter choice will make no difference on the approximations we need to employ to make Eq (1) integrable analytically, we will go with the second and more powerful option. Thus, we assume that

$$\hat{s}_c(\tau) \approx \hat{s}_c^0 - \gamma\,\tau^{\delta} = \hat{s}_c^0 \cdot \left(1 - \frac{\gamma\,\tau^{\delta}}{\hat{s}_c^0}\right) \tag{4}$$

where, as stated earlier, $\hat{s}_c^0$ is the median S_a capacity at time $\tau = 0$. If $\delta = 1$ we have the linear degradation case (Torres and Ruiz 2007), while $\delta > 1$ denotes an accelerating degradation case and $\delta < 1$ a decelerating one. Each of these may be appropriate for different situations and case studies.

Regarding the dispersions $\beta_{Sc}(\tau)$ and $\beta_{USc}(\tau)$, we may either assume that they are constant or take them to vary linearly with τ. Again, both of these assumptions make no difference for the analytical treatment of Eq. (1), thus we will use the second and more powerful option:

$$\beta_{TSc}{}^2(\tau) = \beta_{TSc}^0{}^2 + c_\beta \cdot \tau \tag{5}$$

where

$$\beta_{TSc}^0 = \sqrt{\left(\beta_{Sc}^0\right)^2 + \left(\beta_{USc}^0\right)^2} \tag{6}$$

is the combined aleatory and epistemic uncertainty in the median S_a capacity at time $\tau = 0$. Note that the above formulation allows us to deal with various phenomena that are inherent in ageing. For example, the uncertainty associated with the degradation process may change over time, probably increasing. While at time zero there is no degradation-related uncertainty (assuming that we know the present state of the structure, or no degradation has occurred yet), once it starts its influence will increase with time. The above framework allows us to capture this fundamental effect with relative ease.

To achieve analytical integrability, a single-parameter exponential function is adopted to match the time-dependence of degradation under the influence of seismic hazard. The resulting parameter ϕ is defined as

$$\phi = -\frac{k_1}{\rho T_d} \ln\left[1 - \frac{\gamma\left(\rho T_d\right)^{\delta}}{\hat{s}_c^0}\right] \tag{7}$$

where ρ is a user-selected parameter, with recommended values in the interval [0.85, 1].

By introducing the above approximations in Eq. (1), the latter becomes:

$$\lambda_{\mathrm{LS}}^{\mathrm{ECR}} = \lambda_{\mathrm{LS}}^{0} \frac{a}{\alpha - \phi'} \cdot \frac{1 - e^{-(a-\phi')T_d}}{1 - e^{-aT_d}} \tag{8}$$

where

$$\phi' = \phi + \frac{k_1^2 c_\beta}{2} \tag{9}$$

The above equation is a simple, basic formula to estimate the ECR for a structure with a starting rate of limit-state exceedance $\lambda_{\mathrm{LS}}{}^0$ at time $\tau = 0$ for a rate of capacity degradation equal to ϕ' and a societal discount rate α. Actually, Eq. (8) is very robust numerically and its behavior has a simple interpretation, as it is directly controlled by the difference between the discount rate and the degradation rate:

- If $\alpha > \phi'$ then discounting is faster than degradation, and ECR increases sublinearly with time,
- if $\alpha = \phi'$ then discounting and degradation are equally fast, and ECR increases linearly with time,
- if $\alpha < \phi'$ then discounting is slower and ECR increases superlinearly with time.

In the special case where $\alpha \to \phi'$, Eq. (8) becomes

$$\lambda_{\mathrm{LS}}^{\mathrm{ECR}} = \lambda_{\mathrm{LS}}^{0} \cdot \frac{aT_d}{1 - e^{-aT_d}} \tag{10}$$

An extension to the above formats is needed for ageing processes that do not act immediately on the structure, i.e., starting (or continuing) from the time designated as $\tau = 0$ and progressing for the entire T_d life, but instead take a number of years T_i to initiate. This is the typical case of new reinforced concrete structures where there is a certain time needed for carbonation or for the chloride ions to pass through the concrete matrix before reaching the reinforcement and starting off its corrosion (Val and Stewart 2009). In such cases, we can easily adapt the proposed expressions to account for T_i to get:

$$\lambda_{\mathrm{LS}}^{\mathrm{ECR}} = \frac{\lambda_{\mathrm{LS}}^{0}}{1 - e^{-aT_d}} \left\{ 1 - e^{-aT_i} + \frac{ae^{-aT_i}}{\alpha - \phi'} \cdot \left[1 - e^{-(a-\phi')T_{di}} \right] \right\} \tag{11}$$

where

$$\phi' = -\frac{k_1}{\rho T_{di}} \ln\left[1 - \frac{\gamma \left(\rho T_{di}\right)^{\delta}}{\hat{s}_c^0} \right] + \frac{k_1^2 c_\beta}{2} \tag{12}$$

$$T_{di} = T_d - T_i \tag{13}$$

3.2 EDP-based formulation

As discussed in previous sections, λ_{LS} can also be defined on the basis of an EDP-formulation. Therefore, using a closed-form solution of λ_{LS} in EDP-terms, we can develop a $\lambda_{\mathrm{LS}}^{\mathrm{ECR}}$ approximation on the same basis. Actually, Cornell et al. (2002) have developed exactly such a solution, where the mean annual frequency of exceeding limit-state LS can be approximated via an analytical formula if we use some well-known approximations.

First of all we need the approximation of the hazard by a power-law, as expressed by Eq. (2). In addition, we need a power-law approximation of the relationship between the intensity S_a and the median response θ. This is typically obtained by a linear regression in log-log coordinates and, in the framework of IDA, it can be thought of as an approximation of the median IDA curve:

$$\hat{\theta}(s) \approx a\, s^b \tag{14}$$

b is approximately one for moderate and long periods while a can be estimated by a few nonlinear dynamic analyses. Alternatively, they can both be easily estimated through IDA (Vamvatsikos and Cornell 2002). Still, the approximation needs to be used with care when fitting close to the global-instability region, as discussed for example in Jalayer (2003). In general, the conditioning on no-collapse that is needed to make it work will preclude the derivation of a closed-form solution. Otherwise, as shown in Cornell et al. (2002), λ_{LS} is found to be:

$$\lambda_{\mathrm{LS}} = H\left[\left(\frac{\hat{\theta}_c}{a}\right)^{\frac{1}{b}}\right] \exp\left[\frac{k_1^2}{2b^2}\left(\beta_\theta^2 + \beta_{U\theta}^2\right)\right] \tag{15}$$

where $\hat{\theta}_c$ is the median EDP-capacity and β_θ, $\beta_{U\theta}$ represent the associated record-to-record randomness and epistemic uncertainty dispersions, respectively, for EDP demand and capacity combined. Using this fundamental result, we can derive the ECR using similar steps as in the IM-formulation for any limit-state that lies away from the near-collapse region.

Following our previous derivation, $\lambda_{\mathrm{LS}}^{\mathrm{ECR}}$ is defined according to Eq. (1). To make it amenable to analytical treatment, we need to make similar approximations like for the IM-based case. Again, most important is the degradation-law for the median capacity of the structure over time τ. We will assume one more time a power-law form

$$\hat{\theta}_c(\tau) \approx \hat{\theta}_c^0 - \gamma\,\tau^\delta = \hat{\theta}_c^0 \cdot \left(1 - \frac{\gamma\,\tau^\delta}{\hat{\theta}_c^0}\right) \tag{16}$$

where $\hat{\theta}_c^0$ is the median θ capacity at time $\tau = 0$. Regarding the dispersions $\beta_\theta(\tau)$ and $\beta_{U\theta}(\tau)$, we may either assume that they are constant (Torres and Ruiz (2007), or take them to linearly change with time τ. Again, both of these assumptions make no difference for the analytical integrability of Eq. (1), thus we will use the second option:

$$\beta_{T\theta}{}^2(\tau) = \beta_{T\theta}^{0\ 2} + c_\beta \cdot \tau \tag{17}$$

where

$$\beta_{T\theta}^0 = \sqrt{\left(\beta_\theta^0\right)^2 + \left(\beta_{U\theta}^0\right)^2} \tag{18}$$

is the combined aleatory and epistemic uncertainty dispersion in the EDP demand and capacity at time $\tau = 0$.

Following the same steps that we used before, we arrive at the exact same result of Eqs. (8) and (11) for zero or non-zero T_i respectively. Only the constants change to become

$$\lambda_{\mathrm{LS}}^{0} = H\left[\left(\frac{\hat{\theta}_c}{a}\right)^{\frac{1}{b}}\right]\exp\left(\frac{k_1^2}{2b^2}\beta_{T\theta}^{0\ 2}\right) \tag{19}$$

$$\phi = -\frac{k_1}{b\rho T_d}\ln\left[1-\frac{\gamma\left(\rho T_d\right)^{\delta}}{\hat{\theta}_c^0}\right] \tag{20}$$

$$\phi' = \phi + \frac{k_1^2 c_\beta}{2b^2} \tag{21}$$

where T_{di} should replace T_d in Eq. (20) for non-zero weathering initiation time T_i. Each one of our observations from the IM-based formula also applies here. As long as we avoid the near collapse region and respect the limits of our approximations, both the IM and the EDP formulations should yield equivalent $\lambda_{\mathrm{LS}}^{\mathrm{ECR}}$ results.

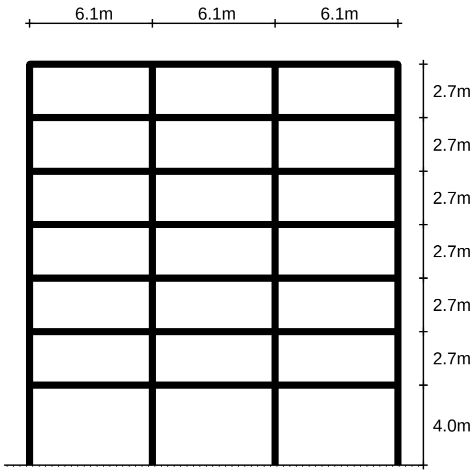

Figure 1. The 7-story Van Nuys reinforced concrete frame.

4 EXAMPLE OF APPLICATION

To show an example of applying the proposed equations on an existing building, we used the transverse frame of the Holiday Inn Hotel in Van Nuys, CA (Jalayer 2003). It is a 7-story hotel located in California's San Fernando Valley, northwest of downtown Los Angeles. The hotel was designed in 1965 according to the 1964 Los Angeles City Building Code, and built in 1966. In plan, the building is rectangular, 19.2m by 45.7m, 3 bays by 8 bays, 7 stories tall (Figure 1). The structural system is a reinforced concrete moment-frame with flat-plate slabs, but the reinforcing steel lacks ductile detailing.

We have estimated its response and capacity statistics at time zero using IDA. We choose to focus on the Global Instability limit-state, a choice that necessitates the use of the IM-based formulation as discussed earlier. By postprocessing the IDA results we came up with an initial median S_a-capacity of $\hat{s}_c^0$ = 1.07g and an associated record-to-record dispersion β_{Sc} = 0.33. Regarding uncertainties, it was simply assumed that β_{USc} = 0.40 at time zero. We should normally repeat such calculations to determine $\hat{s}_c$, β_{Sc} and β_{USc} for the corroded state of this structure at several ages of this building within the design life period of T_d = 50 yrs, for example at times τ = 10,20,30,40 and 50 yrs. This entails determining the changes in the structure due to weathering, appropriately modifying the model to account for them and performing IDA to estimate the distribution of limit-state capacity. A comprehensive case-study that actually implements these steps can be found in Celarec et al. (2011).

In our case and in order to simplify our example, we decided to simulate the degraded state of the building by adopting some deterministic degradation rules for all needed quantities with time τ and then randomizing by adding some white noise. Thus, we assumed that due to reinforcement corrosion the $\hat{s}_c$ capacity degrades with time, β_{Sc} slightly decreases due to the lower record-to-record variability at lower S_a-values while β_{USc} increases due to extra epistemic uncertainty, e.g., due to the corrosion process itself. All such degradation processes start immediately from τ since at this time the structure is already several decades old, thus it is reasonable to assume that $T_i = 0$. The final results appear in Table 1.

Figures 2 through 4 show the fitting of the hazard curve, where a biased fit was preferred for improved precision, the power-law fitting of the mean IM capacity and the linear fit of its total dispersion over time. With these we have everything we need to apply Eq. (8). For α = 3% discounting and over the period of T_d = 50yrs, we found $\lambda_{\mathrm{LS}}^{\mathrm{ECR}}$ = 0.0100. The error is in the order of 15%. In general, though, our experience from a range of tests with the same structure shows that the error may reach up to 20% or 25%, especially when the hazard curve fit lies in its steeper, more rapidly-changing part and the changes in $\hat{s}_c$ are larger over T_d. This may be better understood by looking at Figure 5 where we show the estimates of λ_{LS} found via numerical integration versus the approximations calculated analytically via Eq. (3). The closed-form approximation remains accurate enough when we are allowed to fit the hazard curve locally. Still, as $\hat{s}_c$ moves away from its initial value with time, the constant k_1-slope assumption that we have made for all τ will start to hurt us and drive the λ_{LS} estimate away from its true value. Although we never directly estimate $\lambda_{\mathrm{LS}}(\tau)$ for any time other than $\tau = 0$, its estimation via a constant k_1 is actually incorporated into our analytical solutions. Nevertheless, in practice we do not expect dramatic changes with ageing, therefore, within such rational limits, the proposed formulas can be considered reasonably accurate for practical use.

Table 1. Capacity characteristics for the example building over a period of 50 yrs, as determined by IDA for $\tau = 0$ and random generation for $\tau > 0$.

τ [yrs]	$\hat{s}_c(\tau)$ [g]	$\beta_{Sc}(\tau)$	$\beta_{USc}(\tau)$
0	1.08	0.33	0.40
10	1.04	0.31	0.42
20	0.99	0.30	0.46
30	0.91	0.29	0.47
40	0.85	0.30	0.51
50	0.80	0.28	0.53

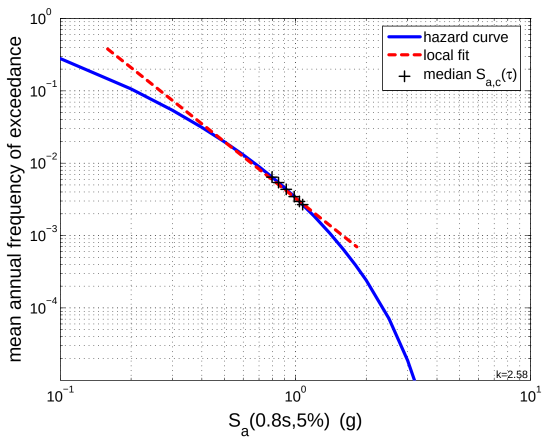

Figure 2. Biased linear fit of the hazard curve at T = 0.8s in log-log coordinates.

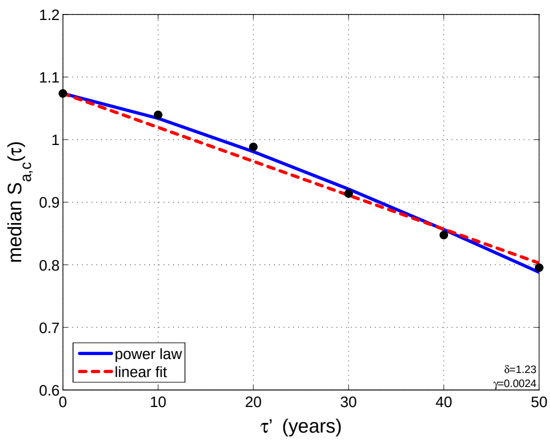

Figure 3. Linear and power-law fit to the median s_c capacity versus time.

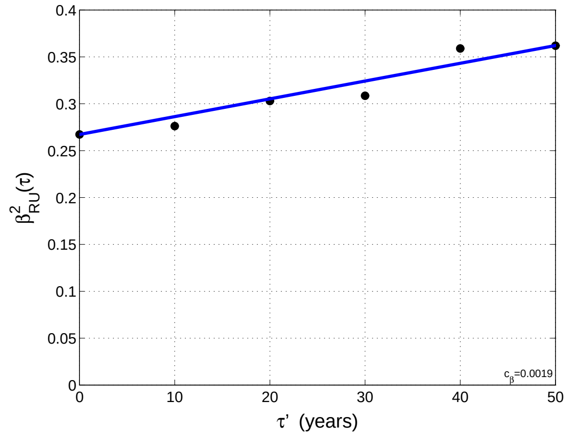

Figure 4. Linear fit of β_{TSc} with time.

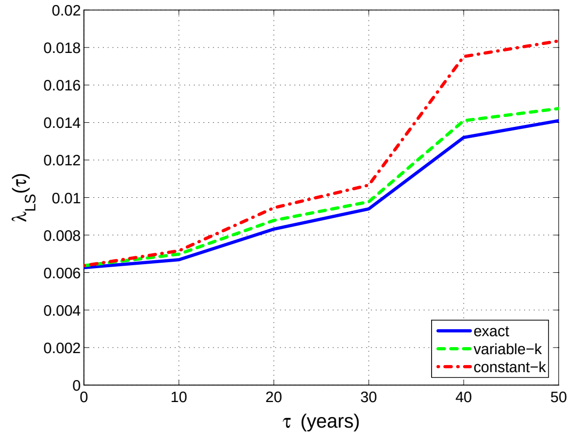

Figure 5. The exact $\lambda_{LS}(\tau)$ versus its approximations via Eq. (3) assuming a time-dependent or a constant hazard-curve slope.

5 CONCLUSIONS

We have presented closed-form expressions for the estimation of the equivalent constant rate of limit-state exceedance for ageing structures with degrading, time-dependent capacity. Based on the idea of societal investment discounting and social equity, the equivalent constant rate is a time-invariant measure that allows direct comparison with code-mandated values typical of seismic guidelines.

For each case, two equivalent formulations are possible. The first is based on the engineering demand parameter, i.e., the response, and while more intuitive it is only useful for limit-states away from global instability. The second is directly based on the intensity measure and is both simpler and more robust to use, being applicable to the full range of structural behavior. Using as a basis the SAC/FEMA approximation of the mean annual frequency of limit-state exceedance, simple analytical formulas can be derived. As long as the structural capacity does not degrade excessively and the rate of degradation does not change too rapidly with time, these solutions represent an accurate and practical alternative for characterizing the seismic performance of degrading structures.

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