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A CODE-COMPATIBLE APPLICATION OF YIELD FREQUENCY SPECTRA FOR DIRECT PERFORMANCE-BASED DESIGN

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ABSTRACT

An analytical formulation is offered for performance-based seismic design following a direct procedure based on code-like requirements. The approach builds upon the use of the yield displacement as a robust system characteristic. A new format for displaying seismic demands known as Yield Frequency Spectra is introduced to quantitatively link performance objectives with the base shear seismic coefficient, for a fixed value of yield displacement. The closed-form solutions developed in the SAC/FEMA project are inverted to provide analytical solutions for estimating the design base shear strength implied by typical code requirements, involving both strength and displacement limitation checks. It is shown that the consideration of aleatory and epistemic sources of uncertainty result in an increase of 30% or more in the strength required to achieve the stated performance objectives, depending on the site and structure characteristics. While the blanket safety factors embodied in the seismic code may cover this difference, they do so inconsistently. Instead, the simple expressions derived can offer uniform safety at no additional complexity.

INTRODUCTION

As a result of economic damage in the 1994 Northridge and 1995 Hyogo-Ken Nambu (Kobe) earthquakes, significant attention has been directed at augmenting the life safety performance objective, characteristic of traditional codes, with additional criteria to limit economic losses in more frequent earthquakes. Basic notions of performance-based design, elaborated in the Vision 2000 report (SEAOC 1995), are widely accepted and are now incorporated in mainstream documents such as ASCE/SEI 41/06 (ASCE 2013). Several approaches have been suggested, mainly conforming to the displacement-based design paradigm, as presented by Moehle (1992), Priestley (2000) and Aschheim (2002). They invariably incorporate some form of a single-degree-of-freedom (or first-mode) representation for use in preliminary design. More importantly, they use as starting point an estimate of the yield displacement, rather the fundamental period of the structure, the former being a more stable parameter for a given structural configuration (Aschheim 2002). On the other hand, though, they are deterministic in focusing design on a specific intensity of shaking represented by a design response spectrum associated with a specified hazard level. The hazard level is typically set at a 10% probability of exceedance in 50 years, equivalent to a 475-year mean return period, which is equivalent to a $-\ln(1-0.10)/50 = 0.0021$ mean annual frequency (MAF) of exceedance.

When facing the significant uncertainty associated with ground motions, modeling and structural response, deterministic methods are inherently limited. Cornell et al. (2002) showed that in the presence of variability due to either aleatory or epistemic sources, the determination of performance at a single level of “design” intensity is unconservative. This is because the more

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frequent appearance of significant damage at lower levels of intensity is not taken into account, thus biasing the analysis. Thus, the use of the "design" intensity results in damage levels that have a higher mean annual frequency of occurrence (greater than 0.0021 for the typical hazard level). Rather, to achieve uniform levels of safety in the presence of uncertainties, additional hazard and structural response data are needed, in order to consider site and structure characteristics. At present, modern seismic codes use blanket safety factors, typically embodied into the definition of the strength reduction factor R (or behavior factor q) and other design requirements that provide inconsistent levels of safety, even for different buildings within the same class and site. Still, despite the apparent advantages, a fully probabilistic performance-based seismic design approach is difficult to achieve in practice. Design is an inverse problem that, in the case of earthquakes, is based on the non-invertible nonlinear relationships between seismic intensity and structural demands. Thus, iterations are needed, in which each cycle involves re-design of the structure and its full performance-based assessment via nonlinear static or dynamic procedures (e.g., Krawinkler et al. 2006).

As a partial solution, Vamvatsikos et al. (2013) proposed "Yield Frequency Spectra" (YFS) as a rapid means to establish the strength required for a preliminary design to provide a desired level of confidence in satisfying one or more performance objectives related to system drift and ductility demands. YFS provide a visual representation of a system's performance that quantitatively links the MAF of exceeding any displacement value (or ductility μ) with the system yield strength (or seismic coefficient at yield, C_y). As with other methods, an "equivalent" single-degree-of-freedom model is utilized to establish the preliminary design, which may be based on current code criteria. Fig.1 presents an example of YFS developed for an elastic-perfectly-plastic oscillator. In this case, three performance objectives are specified (the red "x" symbols) while curves representing the site hazard convolved with the system fragility are plotted for fixed values of C_y. Thus, the minimum acceptable C_y that fulfils the set of performance objectives for the site hazard can be determined. Still, YFS need a full set of hazard curves for every period of interest for the specific site. For cases where such information is not available, we derive, instead, a set of *closed-form expressions* that provide YFS-like results for code-compatible applications using only smoothed design spectra and an estimate of the slope of the hazard curve in the region of interest.

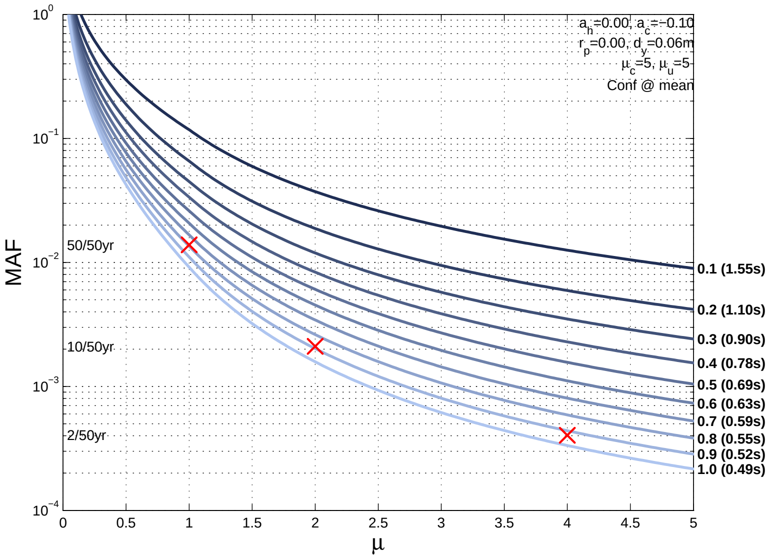


Figure 1. YFS contours at C_y = 0.1,0.2,…,1.0 for an elastoplastic system (δ_y = 0.06m) for a site in Los Angeles, CA, overlaid by the design points of three performance objectives for μ = 1, 2, 4 at 50%, 10% and 2% in 50yrs rates, respectively. The oscillator must have sufficient strength to satisfy each performance objective; the third objective governs with base shear coefficient of $C_y \approx 0.93$ and a period of $T \approx 0.51$s.

SAC/FEMA FRAMEWORK

Variability arising from epistemic and aleatory uncertainty is explicitly addressed in a design-friendly formulation developed in the SAC/FEMA project (Cornell et al. 2002). This formulation allows the engineer to establish that a certain performance level or limit-state is satisfied (demand < capacity) at an arbitrary confidence level. Estimating the probability of violating a certain performance objective or limit-state (i.e. demand > capacity) requires knowledge of the seismic hazard at the site, which is represented by the hazard curve, $H(s)$. $H(s)$ is obtained from a probabilistic seismic hazard analysis (Cornell, 1968), and is parameterized by the seismic intensity measure IM = s, typically selected to be the spectral acceleration at the first mode period, $S_a(T_1)$.

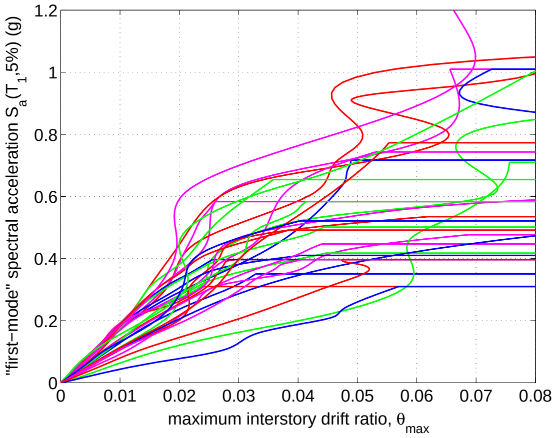


Figure 2. Example of IM vs EDP relationship in the form of IDA curves (Vamvatsikos and Cornell 2002). The EDP is the maximum drift over time and over all stories and the IM is $S_a(T_1)$.

Scalar representations of demand, D, and capacity, C, of the structure, may be expressed in terms of an engineering demand parameter (EDP). In the absence of uncertainty, violation of the limit state (LS) is a deterministic yes or no and it is simply checked as $C < D$. As an illustration, one could evaluate whether the maximum interstory drift demand of the structure exceeds a limiting value, often referred to as the EDP basis (in contrast to the alternative, but equivalent, IM basis). In reality, though, the relationship of EDP and IM is subject to considerable uncertainty (Fig.2). Then, the conditional failure probability, also known as the fragility, $P(C < D|s)$ is used instead. By convolving the fragility with the seismic hazard, $H(s)$, the mean annual frequency (MAF) of limit-state exceedance λ_{LS} can be estimated as (Jalayer 2003):

$$\lambda_{\mathrm{LS}} = \int_0^{+\infty} P(C < D\,|\,s)\,\left|\mathrm{d}H(s)\right| \qquad (1)$$

To simplify the above integration, an approximate closed-form solution was derived (Cornell et al., 2002), introducing the following assumptions and approximations:

1. Within a range of s of interest, the (mean) seismic hazard curve is approximated using a local power law fit:

$$H(s) \approx k_0\, s^{-k_1} = k_0 \exp(-k_1 \ln s) \qquad (2)$$

   where k_0 and k_1, the intercept and slope of the log-hazard curve, are positive real numbers.
2. The limit state capacity EDP-value (e.g. the maximum interstory drift that when exceeded signals violation of the limit-state), follows a lognormal distribution with median $\hat{\theta}_c$ and dispersion $\beta_{\theta c}$.
3. The EDP demand at a specific value of the IM = s is assumed to have a lognormal distribution (generally accurate for displacement and ductility quantities away from global collapse, NIST 2010) with a constant dispersion of $\beta_{\theta d}$ and a conditional median given by:

$$\hat{\theta}(s) \approx a\, s^b \qquad (3)$$

where a and b are positive real numbers.
Under these conditions, a simple closed form approximation to the mean annual frequency (MAF) of exceeding the limit state ($D > C$) is possible:

$$\lambda_{\mathrm{LS}} = H\left[\left(\frac{\hat{\theta}_c}{a}\right)^{\frac{1}{b}}\right]\exp\left[\frac{k_1^2}{2b^2}\left(\beta_{\theta c}^2 + \beta_{\theta d}^2\right)\right] \tag{4}$$

Epistemic uncertainty in the seismic hazard is treated by using the mean hazard curve as $H(s)$, rather than the median hazard curve. Uncertainties in demand and capacity are introduced by adopting a first-order assumption, in which epistemic uncertainty is considered to influence only the dispersions of demand and capacity, and not their medians. The potential for systematic biases in the medians is thus neglected. Examples where this might be significant are the contributions from non-structural components that are not represented in the structural model or the modeling of damping as a force developed due to the relative velocity of adjacent connected nodes in the structural model. Thus, epistemic uncertainties in capacity and demand, $\beta_{U\theta c}$ and $\beta_{U\theta d}$, may be incorporated, along with aleatory randomness, $\beta_{\theta c}$ and $\beta_{\theta d}$, (where record-to-record variability in demands generally dominates $\beta_{\theta d}$). These uncertainty terms can be combined with a square-root-sum-of-squares rule,

$$\beta_{T\theta}^2 = \beta_{\theta d}^2 + \beta_{\theta c}^2 + \beta_{U\theta d}^2 + \beta_{U\theta c}^2\,, \tag{5}$$

as long as demand and capacity are not correlated. Otherwise appropriate interaction terms need to be introduced (see Cornell et al. 2002). Introducing $\beta_{T\theta}^2$ in Eq.(4) in place of $\beta_{\theta c}^2 + \beta_{\theta d}^2$ will produce a mean estimate of the (lognormally distributed) MAF in consideration of all modeled uncertainties.

INVERTING THE SAC/FEMA FORMAT

Consider a structure that is designed to conform to a specific MAF, or performance objective P_o, for violating a given limit state. A typical example is a probability of exceedance of 10% in 50 years of violating the Life Safety limit state, which corresponds to a MAF of $P_o = -\ln(1 - 0.10)/50 = 0.00211$. Assume that the limit state violation is checked using a displacement EDP, for which the corresponding capacity δ_{lim} is considered to be log-normally distributed (rather than being a point value). Thus, using Eq.(4) with $\lambda_{\mathrm{LS}} = P_o$ allows the median EDP capacity (required to satisfy the performance objective) to be determined as:

$$\delta_{lim} = a \cdot \exp\left[\frac{k_1}{2b}\beta_{T\theta}^2 - \frac{b}{k_1}\ln\left(\frac{P_o}{k_0}\right)\right]. \tag{6}$$

In the context of the limited information available in most current seismic codes, Eq.(6) may be quite useful. For example, where a design spectrum is available along with a recommendation for the (local) hazard slope k_1, Eq.(6) can be used by fitting Eq.(2) at the first-mode spectral acceleration value S_{ao}, corresponding to the performance objective:

$$P_o = k_0\, S_{ao}^{\ -k_1} \Leftrightarrow \frac{P_o}{k_0} = S_{ao}^{\ -k_1}\,. \tag{7}$$

Introducing Eq.(7) into Eq.(6) provides an expression similar to the Demand-Capacity Factor Design format in SAC/FEMA (Cornell et al. 2002) if equality (rather than inequality) is adopted in the latter:

$$\delta_{lim} = aS_{ao}^{b} \cdot \exp\left[\frac{k_1}{2b}\beta_{T\theta}^2\right], \tag{8}$$

Note that the SAC/FEMA expressions are based on a local power law fit to the hazard curve in the range of interest for *s* [Eq.(2)], which can introduce non-negligible conservatism where the seismic hazard curve departs significantly from the fitted curve. One option for using Eq.(8) is to perform localized biased fitting in the region of interest (Vamvatsikos 2014). Alternatively, similar expressions have been developed using a more accurate second-order fit (Vamvatsikos 2013).

YIELD DISPLACEMENT BASIS

The response of a single-degree-of-freedom system having yield strength, F_y, and reactive weight, W, can be described in normalized terms, where the base shear coefficient at yield, C_y, is defined by

$$C_y = \frac{F_y}{W}. \tag{9}$$

While (pseudo) spectral acceleration is defined in relation to spectral displacement, in common usage the spectral acceleration associated with a yielding system, $S_{ay}(T)$, is equivalent to $C_y g$. For a system with a given yield displacement, δ_y, changes in C_y represent changes in both strength and stiffness, and hence result in a change in period. For yielding SDOF systems,

$$T = 2\pi\sqrt{\frac{\delta_y}{C_y g}}, \quad \text{or} \quad C_y = \frac{\delta_y}{g}\left(\frac{2\pi}{\mathrm{T}}\right)^2. \tag{10}$$

The normalized response of an oscillator having peak displacement δ is given by the ductility, μ,

$$\mu = \frac{\delta}{\delta_y} \tag{11}$$

with the ductility level μ_{lim} corresponding to a limit-state ductility value: $\mu_{lim} = \delta_{lim}/\delta_y$.

According to Eq.(3), which can be assumed to hold in the elastic range as well, at the yield point we have:

$$a = \frac{\delta_y}{S_{ay}^b} = \frac{\delta_y}{\left(C_y \cdot g\right)^b}. \tag{12}$$

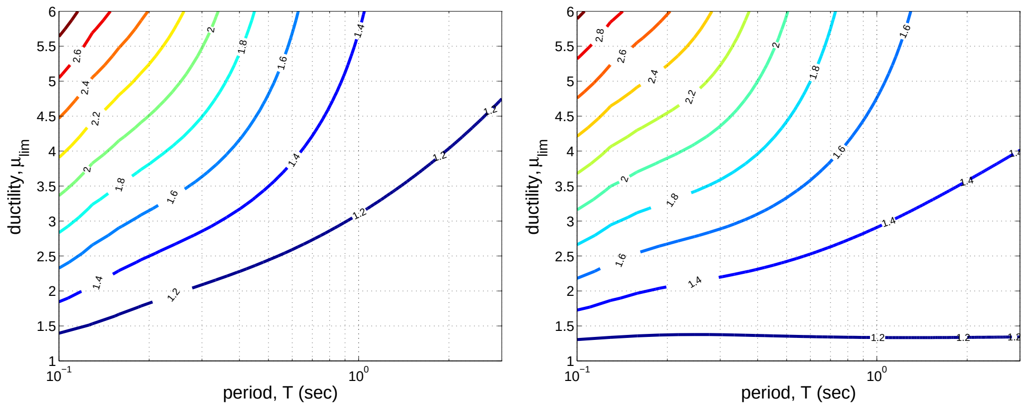
By introducing Eq.(12) into Eq.(8) and solving for C_y, the following fundamental result is obtained:

$$C_y = \frac{S_{ao}}{g\mu_{lim}^{1/b}} \cdot \exp\left[\frac{k_1}{2b^2}\beta_{T\theta}^2\right]. \tag{13}$$

Eq.(13) establishes the correspondence of the required base shear coefficient consistent with a design spectral acceleration S_{ao}. It cannot be used for direct design yet, as its implementation is iterative: C_y, S_{ao}, b and the dispersions depend on the structure's first-mode period. Still, it offers useful intuition on adopting a true performance basis (applied on the output, the structural performance) rather than the input (the seismic load). Rearranging Eq.(13) offers for some limit state (LS):

$$A_{LS} = \frac{C_y}{S_{ao}/(g\,\mu_{lim})} = \mu_{lim}^{1-1/b} \cdot \exp\left[\frac{k_1}{2b^2}\beta_{T\theta}^2\right]. \tag{14}$$

Where the equal displacement "rule" applies, μ_{lim} is numerically equivalent to the strength reduction factor R (or behavior factor q), at least for an equivalent single-degree-of-freedom system where no overstrength (relative to the design base shear) is present. Then, the left side of Eq.(14) becomes the ratio of the PBSD seismic coefficient at yield over the traditional code base shear requirement, at the LS design level. The incorporation of modeled uncertainty into Eq.(14) results in $A_{LS} > 1$. This is the required safety margin to account for the effects of aleatory and epistemic uncertainty. Seismic codes do not take this into account but instead rely on overstrength and added conservativeness, for example in the definition of R (or q) or the use of nominal rather than expected material properties.



(a) $\beta_{\theta c} = \beta_{U\theta c} = \beta_{U\theta d} = 0.0$ (b) $\beta_{\theta c} = \beta_{U\theta c} = \beta_{U\theta d} = 0.2$

Figure 3. The ratio A_{LS} of performance-basis over code-basis estimates for the seismic coefficient C_y considering (a) only the EDP|IM aleatory variability or (b) all pertinent demand and capacity dispersions. The seismic hazard slope is taken as $k_1 = 3$.

As Fig.3 shows, the effect of variability depends on period and ductility. For example, moderate- and long-period structures ($T > 0.5$ sec) deforming in the equal displacement range have $b = 1$. Then, according to Eq.(14) for $k_1 = 3$ (EN1998 recommendation) and a total dispersion of $\beta_{T\theta} = 0.5$, $A_{LS} \approx 1.45$, i.e., a 45% increase due to variability effects, almost regardless of μ_{lim}. For shorter periods, $b > 1$ leads to higher A_{LS} values, often exceeding 2.0 for moderate-to-high ductilities (Fig.3). In part, this is due to the nature of the R-μ-T relationship, for which the ratio of the peak displacement of the yielding system and that of its elastic companion increases without bounds as T approaches zero. Thus, at the extreme low periods such values should be curtailed. This is a well-known phenomenon that has been addressed, for example, in ASCE/SEI 41-06 by restricting the period used for calculating the inelastic displacement ratio C_1 to not less than 0.2 sec (an explicit consideration of kinematic aspects of soil-structure interaction is also suggested). Still, Fig.3 should help explain how catering for the effect of variability in a systematic rather than a judgmental fashion is beneficial for structural safety.

NON-ITERATIVE DIRECT DESIGN FORMULATION

The iterative formulation of Eq.(13) can be simplified for use with design codes if the hazard is described by a design spectrum and the local slope of the hazard curve. This is exactly the case of EN1998 (CEN 2004) for Europe, where a $k_1 = 3$ is generally suggested, while US codes can take advantage of hazard data provided by USGS to achieve the same goal. To remove the need to iterate due to the dependence of b and $\beta_{\theta d}$ on period, its influence is introduced explicitly into Eq.(13). In this formulation, the relevant design spectral acceleration, S_{ao}, for a given performance objective depends on the portion of the spectrum where the solution resides. Several distinct regions, typically referred to as constant acceleration, constant velocity, and constant displacement, are present in the range of period (approximately 0.2 to 2.5 sec) that is useful for many practical engineering applications. Each region can be represented using the general form

$$S_{ao}(T) = S_{amax}\left(\frac{T_c}{T}\right)^r, \quad T \in [T_a, T_b] \tag{15}$$

where T_c is a constant with units of sec and r = 0, 1, 2 for the constant acceleration, velocity and displacement parts of the spectrum, respectively. T_a and T_b (also having units of sec) define the extent of each segment of the spectrum along the period axis. Note that for conservativeness in the long period range, some codes mandate a minimum S_a regardless of period, thus introducing a final constant acceleration region at periods above a certain long-period threshold, typically higher than 4 sec. This also implies $r = 0$ above, albeit with a much decreased "S_{amax}".

Starting from Eq.(13), we assume a relatively constant (period independent) value for b, $\beta_{\theta d}$ and k_1 in each spectral segment. By introducing Eq.(15), replacing T by its equivalent from Eq.(10), and solving for C_y, we obtain:

$$C_y = \left\{ \frac{S_{amax}}{g\mu_{lim}^{1/b}} \cdot \left(\frac{T_c}{2\pi}\sqrt{\frac{g}{\delta_y}} \right)^r \exp\left[\frac{k_1}{2b^2}\beta_{T\theta}^2 \right] \right\}^{\frac{1}{1-0.5r}} \tag{16}$$

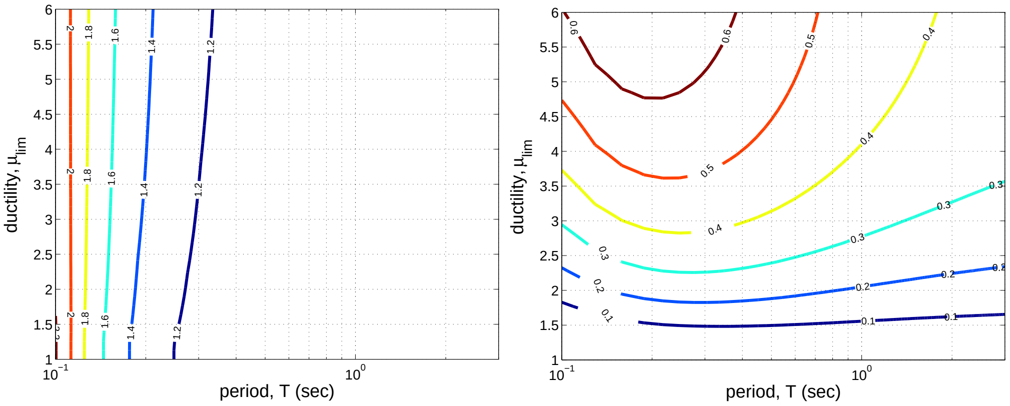
For convenience with units, Eq.(16) contains two instances of g. In the first instance, S_{amax} is normalized by g—where S_{amax} is already expressed in units of g, then this instance of g should be set to 1. In the second instance, g is required in the square root to reconcile the units of δ_y and T. If the former is in meters, this g is 9.81 m/s^2. The general forms of Eq. (16) can now be specialized for the three spectral regions. Setting $r = 0$ for the constant acceleration region we get

$$C_y = \frac{S_{amax}}{g\mu_{lim}^{1/b}} \cdot \exp\left[\frac{k_1}{2b^2}\beta_{T\theta}^2 \right] \tag{17}$$

A rough estimate of b and $\beta_{T\theta}$ in the constant acceleration region is given by $b \approx 1.2$ and $\beta_{T\theta} \approx 0.5$, assuming that $0.2 < T < 0.5$ sec and $\mu_{lim} > 2$. For $\mu_{lim} \leq 1$ the oscillator behaves elastically, therefore b equals 1.0 and a lower $\beta_{T\theta}$ should be adopted instead. For the constant velocity region, $r = 1$, thus

$$C_y = \left(S_{amax} \cdot \frac{T_c}{2\pi} \right)^2 \frac{1}{g\delta_y \mu_{lim}^2} \exp\left[k_1 \beta_{T\theta}^2 \right] \tag{18}$$

where b $\approx$ 1 for moderate to long periods and $\beta_{T\theta} \approx 0.45$.



(a) EDP – IM slope, b (b) EDP given IM dispersion, $\beta_{\theta d}$

Figure 4. Characterization of the logarithmic slope and dispersion of EDP demand given the seismic intensity for elastoplastic SDOF systems by adapting the expressions of Ruiz-Garcia and Miranda (2007).

Numerical results obtained with the above equations need to be validated by checking that the corresponding period from Eq.(10) is within the period limits of the corresponding spectral region. Due to the constant (and discontinuous) b and $\beta_{\theta d}$ assumptions involved, it is possible that for some values of δ_y and μ_{lim}, valid solutions for C_y may be obtained in both the constant acceleration and constant velocity regions. In such cases, it is advised to select the constant velocity solution for corner periods $T_c \geq 0.5$ sec and the constant acceleration solution otherwise, simply by virtue of where the corresponding b and $\beta_{\theta d}$ assumptions are most accurate. Alternatively, one may simply use more accurate estimates of b and $\beta_{T\theta}$ that may be derived, for example, from the work of Vamvatsikos and Cornell (2006) or Ruiz-Garcia and Miranda (2007), as shown in Fig.4 for the latter.

For the constant displacement region, Eq.(16) cannot be solved for a unique C_y. Rather, it becomes an identity, for which all feasible C_y's (or systems) in this region are acceptable. To wit: all elastic oscillators in this region have the same spectral displacement; due to the equal displacement "rule" (enforced by $b = 1$), oscillators of varied strengths having a given yield displacement have the same peak displacement, and thus the same ductility demand. Mathematically acceptable solutions can be obtained for each period, although realistic solutions may be constrained to certain ranges of period or yield displacement. But even the mathematical solutions represent ideal conditions—in reality the spectrum never truly conforms to the $r = 2$ shape, nor is the value of b equal to exactly 1 for any structure, regardless of period, in this spectral region. Thus, using the actual uniform hazard spectrum together with accurate estimates for b and $\beta_{T\theta}$ via Eq.(13), will tend to constrain the appearance of this issue of C_y indeterminacy to a small part of the long-period spectrum. Practically speaking, one can always choose to be conservative by selecting a C_y that would appear by extending the constant-velocity region into the constant displacement range, essentially assuming $r = 1$ rather than the value of 2.0, as illustrated in Fig.5 for an EN1998-compatible spectrum. Alternatively, one may consider using Eq. (16) with $r = 1.5$ or 1.8 as a better approximation. Either way, one would rarely expect to encounter such flexible systems in most realistic applications.

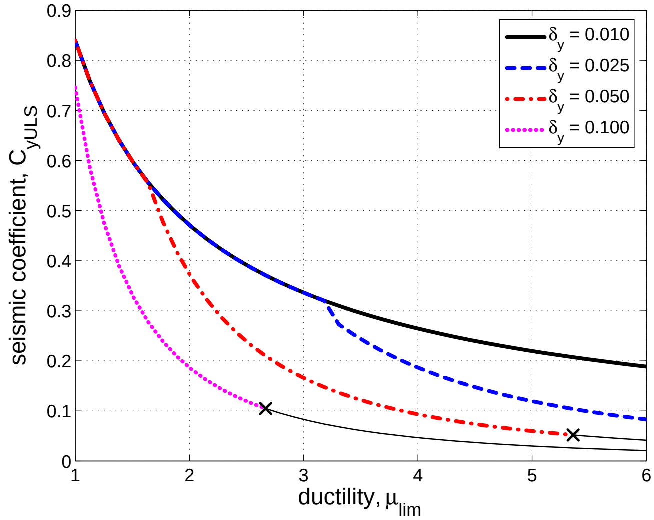


Figure 5. Approximate C_y seismic coefficient estimates using Eq.(17) and Eq.(18) for a EN1998 design spectrum: soil type C ($T_c = 0.6$sec), peak ground acceleration $a_g = 0.24$g and seismic hazard slope $k_1 = 3$. Distinct "elbows" indicate where the constant-velocity range is entered, while the X's show where the solution verges into the constant-displacement range, thus becoming a conservative approximation.

CODE-COMPATIBLE APPLICATION APPROACH

Seismic codes generally prescribe two types of safety checks, namely for strength and displacement. In other words, design is typically concerned with minimum strengths required to assure life safety, and the provision of sufficient stiffness to avoid (a) excessive interstory drifts with associated damage to non-structural components and (b) the risk of collapse from P-Delta effects. Towards this end, the design spectrum defines the seismic intensity level for the desired mean annual frequency of exceedance. Design forces are obtained by reducing this acceleration by a strength reduction factor, R, or behavior factor, q, to account for inherent ductility and overstrength of the seismic force-resisting system. This generic framework can be applied to fit into the performance-based approach that has been developed so far. The goal is to be able to derive a preliminary design C_y value that is consistent with code objectives.

When performing Strength Limitation (SL) checking, the portion of the reduction factor attributed to ductility (R_d) can be taken as the ductility limit μ_{lim} in a performance-based design context. Any part relating to overstrength (Ω) can be applied at a later stage to reduce the required base shear strength. Thus, if $R = \Omega \times R_d$, we may set $\mu_{limSL} = R_d$ and estimate the value of C_{ySL} using Eq.(17) and Eq.(18). Obviously, the value of S_{amax} should correspond to the elastic spectrum, i.e., not divided by R (or q).

Interstory drift limits provided in building codes ensure that a minimum lateral stiffness is provided. Displacement Limitation (DL) checks are performed either for explicitly reduced intensity levels corresponding to a higher MAF, as in the case of EN1998 (CEN 2004) or at the same intensity (and MAF) as SL checks, this being the case of ASCE/SEI 7/10 (2010). This distinction becomes important when deciding on the slope of the hazard curve to adopt. Typically, lower intensities are associated with a lower k_1 value, due to the monotonically decreasing concave (negative curvature) shape of realistic hazard curves in log-log space. Thus an optimal application of EN1998 should take that into account. Of course, unless site-specific hazard information is available, the point is mute.

When the interstory drift threshold is set at θ_{lim}, the corresponding limit on roof drift (or roof drift ratio) can be estimated if the drift profile over the height is known. If the drift profile is unknown, an estimate of the coefficient of distortion (COD) may be used. For a maximum interstory drift of θ_{max}, the COD is defined by Moehle (1992) as COD = $\theta_{max}/(\delta_y/H)$. The COD can derived relatively easily for structures whose drift profiles are dominated by the first mode by inspection of the first-mode shape, considering a “flexural” or “shear” mode of deformation, or a combination thereof (Miranda 1999). For systems in which higher mode effects contribute significantly to interstory drifts, a value of COD can be estimated on the basis of prior dynamic analyses. Having COD available, the equivalent SDOF ductility limit corresponding to DL is:

$$\mu_{limDL} = \frac{\theta_{lim}}{\theta_y} = \frac{\theta_{lim} H}{\Gamma \delta_y COD} \qquad (19)$$

where H is the height of the structure and Γ the appropriate first-mode participation factor. Typically, one would expect that μ_{limDL} will not be much different from 1.0. Actually, for the higher DL checking intensity of ASCE/SEI 7/10 we expect μ_{limDL} somewhat higher than 1.0, while for the more frequent low intensities mandated by EN1998 for DL, the opposite would typically be observed.

To estimate C_{yDL}, Eq.(17) and Eq.(18) can be used by employing the appropriate value for S_{amax}. For ASCE/SEI 7/10 this is the same as the SL value while for EN1998 it is multiplied by a factor of 0.4 – 0.5 depending on the importance class of the building. Additionally $b = 1$ by definition, while $\beta_{\theta d}$ may be set to zero, even though for tall structures it may attain higher values due to $S_a(T_1)$ not capturing the variability of higher modes. Aleatory variability for capacity and epistemic uncertainty contributions for both demand and capacity may be introduced via the $\beta_{\theta c}$, and $\beta_{U\theta d}$, $\beta_{U\theta c}$ terms.

When different intensity levels, and thus values of k_1, are involved for DL and SL checking, it is not clear at the outset which check will control the design. For a given yield displacement, strength in excess of that required to satisfy the limit state will result in greater stiffness and lower ductility and drift demands. Thus, having determined the required values of yield strength coefficient for the two checks, the larger of the two yield strengths ensures that the structure will satisfy both performance objectives, at this preliminary design stage: $C_{ymax} = \max(C_{yDL}, C_{ySL})$. In order to perform the actual design itself, one may now use C_{ymax} to define the level of lateral loads. If applying a code-like design process, the overstrength Ω should be used to reduce the estimated C_{ymax}; alternatively, a collapse mechanism analysis may be used for design The basic idea is that the strength obtained in a nonlinear static pushover analysis should equal $\alpha_1 C_{ymax} \cdot W$, where α_1 is the modal mass ratio participating in the first mode. Thus, a traditional force-based design process may be used where $\alpha_1 C_{ymax}/\Omega$ plays a similar role as the S_a/R or S_a/q in establishing the design base shear strength, V. Thus, the Equivalent Lateral

Force method (an equivalent linear static analysis) would use $V = \alpha_1 \cdot C_{ymax} W/\Omega$. For Modal Response Spectrum Analysis, the entire design spectrum would be scaled to $\alpha_1 \cdot C_{ymax} \cdot g/\Omega$ at the corresponding period T of Eq.(10). Thus, C_{ymax} can be used in the context of a design process that engineers are already familiar with.

If the estimate of yield displacement used in the preliminary design is inaccurate, the fundamental period of the resulting design, T_1, will differ from that determined using Eq.(10). Small differences would not be considered a sufficient reason to modify the loading. However, should a significant difference emerge, the process can be repeated with an improved estimate of the yield displacement, given by

$$\delta_y = \left(\frac{T_1}{2\pi}\right)^2 C_{y\max} g \,. \tag{20}$$

The design process described above makes use of a number of approximations that allow uncertainty to be explicitly considered in preliminary design. Consequently, the resulting design may not precisely satisfy the intended performance objectives, although it is likely to be much closer than the result of a traditional design process. If strict adherence to the specified drift or ductility performance limits is needed, or values of other EDPs are required, a complete performance-based assessment of the final design should be done. A practical methodology for assessment is described in GCR-10-917-9 (NIST 2010).

EXAMPLE

To illustrate the methodology, the basic seismic design parameters are determined for a 4-story steel special moment-resistant frame using the provisions of EN1998. The building is of moderate importance (Importance Class III) and is founded on Soil Type C (shear wave velocity in top 30m of v_{s30} = 180 – 360m/s), with a spectrum anchored at a peak ground acceleration of a_g = 0.30g. Each story has a height of 3.6 m, for a total height, H = 14.4 m. Columns are spaced at 9 m on center. The smoothed design response spectra is defined by periods of T_b = 0.2 s, T_c = 0.6 s and T_d = 2 s (recommended in EN1998 for a high-seismicity Type 1 spectra), where the constant acceleration plateau, spanning between T_b and T_c, is followed by a constant velocity region up to T_d, at which the constant displacement region begins. For Soil Type C, the maximum plateau acceleration S_{amax} is 0.86g for SL and 40% of that for DL (a 60% reduction for Importance Class 3).

The DL check is considered first. For ductile non-structural components, the interstory drift limit is θ_{lim} = 0.75%, which should be satisfied under a quasi-static equivalent lateral force distribution (neglecting higher mode contributions). For a medium ductility class, the maximum allowable behavior factor is 4.0 to also serve as the limiting ductility for SL. According to Aschheim (2002), a simple estimate of the yield roof drift of a regular steel moment resisting frame is

$$\theta_y = \frac{\varepsilon_y}{6}\left(\frac{h}{d_{col} COF} + \frac{2L}{d_{bm}}\right) \tag{21}$$

where ε_y is the yield strain of steel, h the story height, L the beam span, COF the column overstrength factor and d_{col}, d_{bm} the column and beam depths, respectively. For S355 steel, the characteristic yield strength is 355MPa and the average about 20% higher: f_y = 426MPa. Then, ε_y = 0.20%, h = 3.6 m, L = 9 m, COF = 1.3, d_{col} = 0.6 m, d_{bm} = 0.70 m, and the estimated yield roof drift is θ_y = 1.03%. Thus, the limiting ductility is μ_{limDL} = 0.75/1.03 = 0.73. For a COD of 1.5 (as suggested by Moehle 1992 for regular low-rise frames), assuming Γ = 1.3, the yield roof displacement of the MDOF structure is

$$\delta_y = \frac{\theta_y H}{\Gamma COD} = 0.76\text{m} \tag{22}$$

Table 1 summarizes all the parameter values to be used for design.

Design for the strength limitation is now considered. Since the code suggested period falls within the constant velocity region for Soil Type C, we begin in this region. The logarithmic slope of the IM-EDP relationship is b = 1 (equal displacement range) while the logarithmic dispersion is $\beta_{\theta d}$ = 0.37, using Fig.3 for a period in the middle of the constant velocity range. The total dispersion is

$$\beta_{T\theta} = \sqrt{\beta_{\theta d}^2 + \beta_{\theta c}^2 + \beta_{U\theta d}^2 + \beta_{U\theta c}^2} = 0.51$$

resulting in a yield strength coefficient and corresponding period of

$$C_{ySL} = \left(\frac{S_{amax}^{SL}}{g} \cdot \frac{T_c}{2\pi}\right)^2 \frac{g}{\delta_y \mu_{limSL}^2} \exp\left[k_1 \beta_{T\theta}^2\right] = 0.12$$

$$T_{SL} = 2\pi \sqrt{\frac{\delta_y}{C_{ySL} g}} = 1.60\text{s}$$

Since this period is within the limits of the constant velocity region, the solution is considered valid.

The process is even simpler for DL, as b = 1 and $\beta_{\theta d}$ = 0 in the elastic range. Correspondingly, $\beta_{T\theta}$ = 0.26 and thus:

$$C_{yDL} = \left(\frac{S_{amax}^{DL}}{g} \cdot \frac{T_c}{2\pi}\right)^2 \frac{g}{\delta_y \mu_{limDS}^2} \exp\left[k_1 \beta_{T\theta}^2\right] = 0.31$$

$$T_{DL} = 2\pi \sqrt{\frac{\delta_y}{C_{yDL} g}} = 0.99\text{s}$$

which is within the constant velocity region, where these equations are valid. As expected for flexible steel frames, the Displacement Limitation check governs and the required seismic coefficient is C_{ymax} = 0.31. Adjusting for overstrength Ω > 1 would proportionally reduce the design base shear.

Results determined above correspond to a mean estimate of the MAF of exceeding the limit-state. A confidence level higher than that associated with the mean may be desired. For example, considering the potential consequences of violating the performance limits, we may seek a 90% confidence level for the SL and a 75% confidence level for the DL. Although not presented herein, this is easily accommodated within the proposed approach.

Table 1. Design criteria for the example of EN1998 for Strength and Displacement Limitations checking (ultimate limit-state and damage-limitation state in EN1998 terminology). Note that P_o is not needed but it is listed for completeness.

Parameter	Notation	SL	DL
Probability of exceedance or MAF	P_o	10%/50yrs	10%/10yrs
Seismic hazard slope	k_1	3.00	2.50
Limiting ductility value	μ_{lim}	4.00	0.73
Demand dispersions	$\beta_{\theta d}$, $\beta_{U\theta d}$,	0.37, 0.20	0.10, 0.15
Capacity dispersions	$\beta_{\theta c}$, $\beta_{U\theta c}$	0.20	0.15
Constant spectral acceleration ordinate	S_{amax}	0.86g	0.34g

CONCLUSIONS

A method is presented to establish parameters for preliminary seismic design. Multiple seismic performance levels can be addressed, and modeled epistemic and aleatory uncertainties can be considered with only basic hazard information. The preliminary design can be established with very few or no iterations as the design method recognizes the stability of the yield displacement, while addressing limits on system ductility and drift consistent with current code requirements. Evaluation of the first mode period establishes whether the design base shear coefficient should be revised. Performance of the preliminary design can then be quantified using nonlinear dynamic analyses.

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