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DIRECT PERFORMANCE-BASED SEISMIC DESIGN OF STRUCTURES USING YIELD FREQUENCY SPECTRA

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ABSTRACT

Yield Frequency Spectra (YFS) are employed to enable the direct design of a structure subject to a set of performance objectives. YFS offer a unique view of the entire solution space for structural performance. This is measured in terms of the mean annual frequency (MAF) of exceeding arbitrary ductility (or displacement) thresholds, versus the base shear strength of a structural system with given yield displacement and backbone capacity curve. Using publicly available software tools or closed-form solutions, YFS can be nearly instantaneously computed for any system whose performance is characterized by response quantities that can be satisfactorily approximated by an equivalent nonlinear single-degree-of-freedom oscillator. Thus, stated performance objectives can be directly related to the strength and stiffness of the structure. The combination of ductility (or displacement) demand and its mean annual frequency of exceedance that governs the design is readily determined, allowing a satisfactory design to be realized in a single step.

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ABSTRACT

Yield Frequency Spectra (YFS) are employed to enable the direct design of a structure subject to a set of performance objectives. YFS offer a unique view of the entire solution space for structural performance. This is measured in terms of the mean annual frequency (MAF) of exceeding arbitrary ductility (or displacement) thresholds, versus the base shear strength of a structural system with given yield displacement and backbone capacity curve. Using publicly available software tools or closed-form solutions, YFS can be nearly instantaneously computed for any system whose performance is characterized by response quantities that can be satisfactorily approximated by an equivalent nonlinear single-degree-of-freedom oscillator. Thus, stated performance objectives can be directly related to the strength and stiffness of the structure. The combination of ductility (or displacement) demand and its mean annual frequency of exceedance that governs the design is readily determined, allowing a satisfactory design to be realized in a single step.

Introduction

Performance-based seismic design (PBSD) has received significant attention following large economic losses in the 1994 Northridge and 1995 Hyogo-Ken Nambu Earthquakes. Rather than focusing only on a life-safety performance level, PBSD targets multiple performance objectives, each typically defined as not exceeding a prescribed structural response level with a mean annual frequency higher than specified. At its most advanced form, one would seek specific non-exceedance rates of economic losses or even casualties, echoing the definition of decision variables that are embedded in the Cornell-Krawinkler framework (Cornell and Krawinkler [1]) adopted by the Pacific Earthquake Engineering Research (PEER) Center.

Despite the apparent significance of this goal, few steps have been taken towards developing such a design process. This comes as no surprise since design is an inverse problem. Because the functional relationship between the design variables and the performance objectives is not invertible, iteration is required. This is costly as each iteration for a nonlinear structure means a cycle of re-design and re-analysis, where the latter is a full-blown performance-based assessment involving nonlinear static or dynamic runs. It is no wonder then that most attempts to represent PBSD have mostly come back to discuss assessment instead (see for example fib [2],

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FEMA-445 [3]). Any method built on this paradigm essentially becomes an iterated assessment procedure. Conceptual support for such a design paradigm is provided by Krawinkler et al. [4]. Many researchers have also chosen to improve upon the efficiency of the re-design to achieve a fast convergence, often leading to the use of numerical optimization. For example, Mackie and Stojadinovic [5] have suggested this approach for bridges while Fragiadakis and Papadrakakis [6], Franchin and Pinto [7] and Lazar and Dolsek [8] have all used optimization techniques for the performance-based seismic design of reinforced-concrete structures (see also Fragiadakis and Lagaros [9] for a comprehensive review).

Despite the usefulness of currently suggested approaches, their implementation is not trivial. The link between a performance objective and the resulting design is obscure, coming out of numerous steps of numerical analysis. As an alternative, so-called “Yield Frequency Spectra” (YFS) are proposed as a design aid, being a direct visual representation of a system’s performance that quantitatively links the mean annual frequency (MAF) of exceeding any displacement value (or ductility μ) with the system yield strength (or seismic coefficient C_y). YFS are plotted for a specified yield displacement; thus, periods of vibration represented in YFS vary with C_y . Fig. 1 presents an example for an elastic-perfectly-plastic oscillator. In this case, three performance objectives are specified (the red “x” symbols) while curves representing the site hazard convolved with the system fragility are plotted for fixed values of C_y . Of course, increases in C_y always reduce the MAF of exceeding a given ductility value. Thus, the minimum acceptable C_y (within some tolerance) that fulfils the set of performance objectives for the site hazard can be determined for a given single-degree-of-freedom system. This strength is used as a starting point for the PBSD of more complex structures. The performance-based design problem potentially can be solved in a single step with a good estimate of the yield displacement.

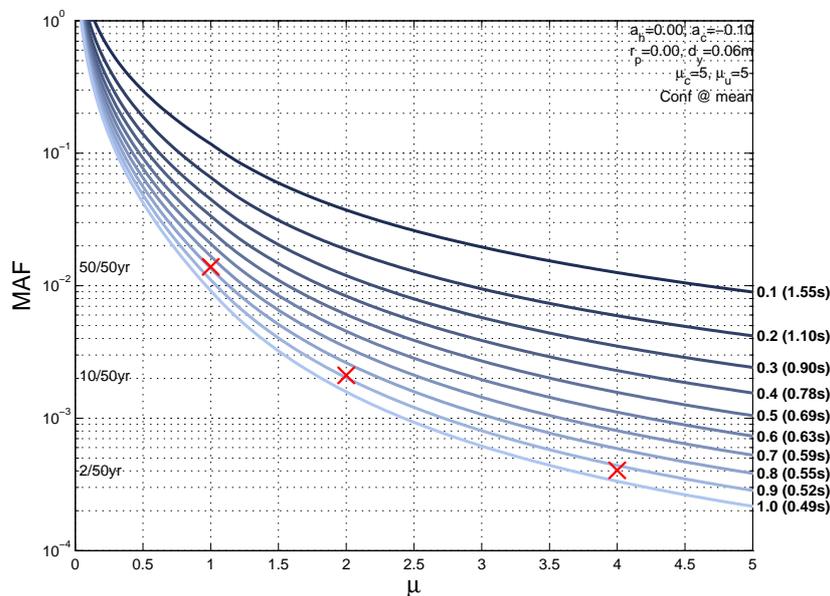


Figure 1. YFS contours at $C_y = 0.1, \dots, 1.0$ determined for an elastoplastic system ($\delta_y = 0.06\text{m}$) at Van Nuys, CA, along with red “x” symbols that represent three performance objectives ($\mu = 1, 2, 4$ at 50%, 10% and 2% in 50yrs exceedance rates, respectively). The third objective governs with $C_y \approx 0.93$. The corresponding period is $T \approx 0.51\text{s}$.

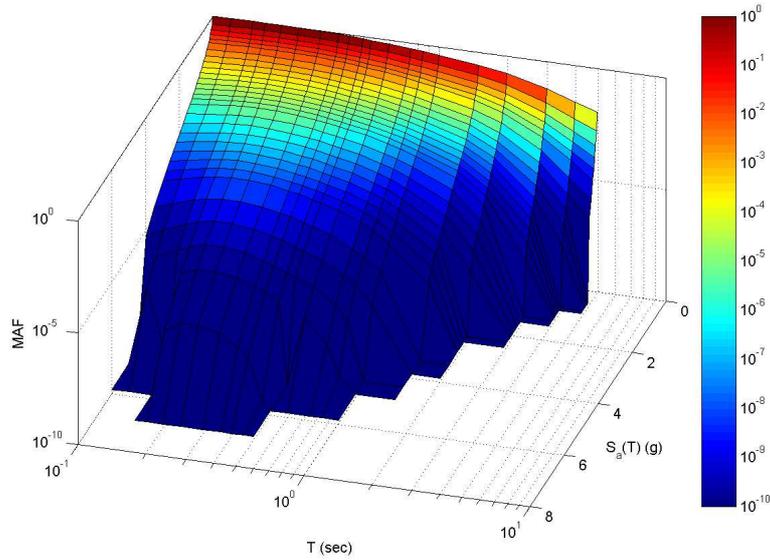


Figure 2. Spectral acceleration hazard surface for Van Nuys, CA.

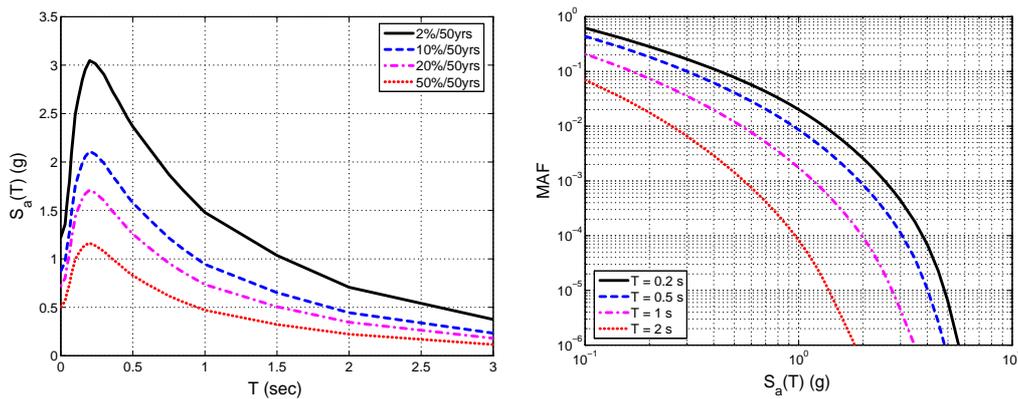


Figure 3. (a) Uniform hazard spectra and (b) S_a hazard curves for Van Nuys, CA.

Basis of design

Far from finding fault with current proposals, it should be recognized that the design of a multi-degree-of-freedom structure will always involve some level of iteration. Thus, a truly direct performance-based design will probably never be realized. To reduce the number of design/analysis cycles, we should ask what parameters are stable as one moves from the initial design to the final one? One obvious shortcut, which actually forms the basis of all current seismic codes, is to rely on an SDOF system approximation. We will use this approximation for representing system level displacement (and ductility) responses. A second shortcut is to rely on the stability of the yield displacement—the notion that the yield drift ratio of a bilinear approximation to the first mode pushover curve is stable with changes in strength. The changes in strength affect stiffness and drift (or ductility) demands.

The essential ingredients of our approach to PBSB are (a) the site hazard and (b) some assumption about the system's behavior (e.g. elastic, elastoplastic etc). Comprehensive site hazard representation that is compatible with current design norms can be achieved by the seismic hazard surface, a 3D plot of the MAF of exceeding any level of spectral acceleration for the full practical range of periods (Fig. 2). This is the true representation of the seismic loads for any given site. More familiar pictures can be produced from the hazard surface by taking cross-section (or contours). Cutting horizontally at given values of MAF will provide the corresponding uniform hazard spectra (UHS). For example, at $P_o = -\ln(1-0.1)/50 = 0.0021$, or a 10% in 50yrs probability of exceedance (Fig. 3a), one gets the spectrum typically associated with design at the ultimate limit-state (or Life Safety). Taking a cross-section at a given period T produces the corresponding $S_a(T)$ hazard curve (Fig. 3b). Now compounding this information with the capacity curve (i.e. force-deformation relationship envelope) of the system is where things start getting interesting.

To illustrate the problem in more detail, let us first attempt a "perfect" elastic design. Suppose that an elastic oscillator of given mass M needs to be designed to not exceed a displacement δ_{lim} more often than a given MAF of P_o , for example $P_o = 0.0021$ for a code-compatible safety requirement. We are essentially asking for the stiffness, or equivalently, the period of this oscillator. Note here that a strength requirement would be quite straightforward to resolve, as one would simply take a horizontal line at $S_a = F/M$ in Fig. 3a and seek the period (or periods) at the intersection(s) with the corresponding uniform hazard spectrum. A displacement threshold though is slightly trickier as it requires some iteration:

1. Select an initial period T .
2. Extract $S_a(T)$ from the UHS at P_o .
3. Calculate new period as $T = 2\pi \sqrt{(\delta_{lim} / S_a)}$.
4. Go to step 2 until the period converges.

The formula employed at step 3 is simply the result of solving for T the well-known relationship between the (pseudo) spectral acceleration and the spectral displacement. In an actual structural design setting this would probably be replaced by an eigenvalue analysis of the intermediate design resulting from loads consistent with the $S_a(T)$ of the preceding step 2.

A simpler solution exists that achieves the same results without any iteration. It involves the pre-calculation of a set of values of displacement consistent with the UHS spectrum at P_o for any period T that can then be interpolated to estimate the required period for any desired δ_{lim} . An intuitive graphical representation of this is actually the displacement spectrum, $S_d(T)$, which allows a direct non-iterative solution of the elastic design problem for any limit-state of interest. Not surprisingly, it is the starting point of most (if not all) displacement-design procedures (e.g. Priestley et al. [10]). Note that the seismic design codes typically do not enter this line of reasoning, despite being based on the acceleration rather than the displacement spectrum. This is achieved by virtue of prescribing an initial period that is considered to be close enough to the expected value for a given type of structure, thus foregoing the need for iterations (and eigenvalue analysis) for most rudimentary design cases.

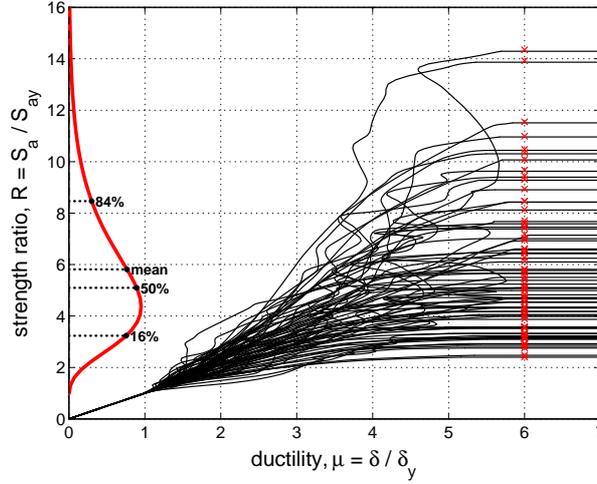


Figure 4. IDA curves for a $T = 1$ s oscillator with a degrading (in-cycle) capacity curve, showing the distribution of the spectral acceleration capacity, S_{ac} (normalized by the yield spectral acceleration, S_{ay}) and corresponding to the collapse ductility of $\mu = 6$.

The aforementioned process is much compounded for application to a nonlinear system. Then, for a given capacity curve shape (or system type) we are asked to estimate the yield strength and the period T for not exceeding a limiting displacement δ_{lim} at a rate higher than P_o . Even for an SDOF system, the introduction of yielding, ductility and the resulting record-to-record response variability fundamentally change the nature of the problem. This is best represented in the familiar coordinates of intensity measure (IM), here being the first mode spectral acceleration $S_a(T)$, and engineering demand parameter (EDP), i.e., the displacement response δ . The structural response then appears in the form of incremental dynamic analysis (IDA, Vamvatsikos and Cornell [11]) curves as shown in Fig. 4 for a $T = 1$ s system with a capacity curve having positive and then a negative post-yield stiffness. Cornell et al [12] have shown that response variability means that additional hazard levels beyond P_o need to be considered in evaluating the system's performance. The reason is that values lower than the average response for the seismic intensity corresponding to P_o appear more frequently (i.e. correspond to a higher hazard rate in Fig. 3b). Hence, they tend to contribute significantly more to the system's rate of exceeding $\delta = \delta_{lim}$. Formally, this relationship may be represented by the following integral (Jalayer [13], Vamvatsikos and Cornell [14]):

$$\lambda(\delta) = \int_0^{+\infty} F(S_{ac}(\delta) | s) |dH(s)| \quad (1)$$

where $\lambda(\cdot)$ is the MAF of exceeding δ . $S_{ac}(\delta)$ is the random limit-state capacity, representing the minimum intensity level for a ground motion record to cause a displacement of δ to be exceeded (e.g., Fig. 4). $F(\cdot)$ is the cumulative distribution function (CDF) of S_{ac} evaluated at a spectral acceleration value of s , and $H(s)$ is the associated hazard rate. The absolute value is needed for the differential of $H(s)$ because the hazard is monotonically decreasing, thus always having a negative slope.

The seismic code foregoes such considerations through implicit incorporation of two

assumptions: (a) Using the strength reduction R or behavior factor q to account for the effect of yielding and ductility in the mean/median response, (b) ignoring the effect of dispersion, and assuming that the seismic loads consistent with P_o are enough to guarantee a similar (or lower) rate of non-exceedance of δ_{lim} . The error due to the above is “covered” by employing various implicit conservative approximations to account for the effect of the previous non-conservative assumptions, typically through the selection of R (or q) (see for example FEMA P695 [15]). Thus, in the code environment, the inelastic design process becomes “identical”, at least in terms of the required steps, with the elastic design process described earlier.

Unfortunately, the magnitude of the assumptions is such that one can never be entirely sure of actually achieving the stated objective(s) with any confidence. The margin of safety depends on the site and the system characteristics. Even when safe, the design is typically far from optimal: Economy and safety are two competing objectives and, given the size of the uncertainty involved in code-based inelastic design, common sense necessitates erring on the side of caution, i.e. injecting conservativeness (for example, through R). Consequently, the designer lacks specific information on where exactly his/her design is sitting on this wide blurry margin between meeting and failing the presumed performance criteria. Even worse, as any calibration for safety has been performed on the basis of the standard code assumptions of what an acceptable performance is, it is not possible to accurately inject one’s own (stricter) criteria for a better performing structure. Any importance factors used to amplify the design spectrum are only a poor substitute. This has actually become common knowledge in the past few years, and it is the premise of performance-based design. In other words, this is where the search starts for ways to fully incorporate the actual performance of a given structural system and allow its design for any desired performance objective. Unfortunately, neither the problem nor the (so far) proposed solutions are simple.

As a complete replacement of this hazy picture, we aim to offer instead a practical and theoretically consistent procedure that can fully resolve the inelastic SDOF design problem, in the same way that the aforementioned iterative process and the associated displacement spectrum do for elastic design. This will be built upon (a) Eq. 1 for estimating structural performance, (b) the R - μ - T relationships for estimating the probabilistic distribution of structural response given intensity and (c) a yield displacement basis for design, by virtue of being a far more stable system parameter compared to the period (Priestley [16], and Aschheim [17]). In a graphical format, this solution is represented using yield frequency spectra.

Origin, Definition, and Use of YFS

For a yielding system, the direct equivalent of elastic spectral acceleration or spectral displacement hazard curves are inelastic displacement (or drift) hazard curves. These may be determined by using Eq. 1 to estimate the MAF of exceeding any limiting value of displacement. They have appeared at least in the work of Inoue and Cornell [18] and subsequently further discussed by Bazzurro and Cornell [19] and Jalayer [13]. While useful for assessment, they lack the necessary parameterization to become helpful for design. An appropriate normalization may be achieved for oscillators with yield strength and displacement of F_y and δ_y , respectively, by employing ductility μ , rather than displacement δ

$$\mu = \frac{\delta}{\delta_y}, \quad (2)$$

and the seismic coefficient C_y instead of the strength

$$C_y = \frac{F_y}{W}, \quad (3)$$

where W is the weight. For SDOF systems C_y is numerically equivalent to $S_{ay}(T, \xi) / g$, i.e. the spectral acceleration value to cause yield in units of g , at the period T and viscous damping ratio ξ of the system.

Up to this point, what has been proposed is not fundamentally different from the results presented by Ruiz-Garcia and Miranda [20] on the derivation of maximum inelastic displacement hazard curves. What truly makes the difference is defining δ_y as a constant for a given structural system, following the observations of Priestley [16] and Aschheim [17] on its stability as a design parameter. Then, C_y essentially becomes a direct replacement of the period T :

$$T = 2\pi \sqrt{\frac{\delta_y}{C_y g}}, \quad \text{or} \quad C_y = \frac{\delta_y}{g} \left(\frac{2\pi}{T} \right)^2 \quad (4)$$

For a given site hazard, system damping, δ_y , value of C_y (or period), and capacity curve *shape* (e.g. as normalized in terms of $R = F/F_y$ and μ), a unique representation of the system's probabilistic response may be gained through the displacement (or ductility) hazard curves produced via Eq. 1. Damping, δ_y and the capacity curve shape are considered as stable system characteristics. By plotting such curves of $\lambda(\mu)$, for a range of μ_{lim} limiting values and a range of C_y , we can get contours of the inelastic displacement hazard surface for constant values of C_y . These contours allow the direct evaluation of system strength and period—i.e., the C_y required to satisfy any combination of performance objectives defined as $P_o = \lambda(\mu_{lim})$, where each limiting value of ductility μ_{lim} is associated with a maximum MAF of exceedance P_o , as shown in Fig. 1.

At a certain level, YFS can be considered as a building- and user-specific extension of concepts behind the IBC 2012 [21] risk-targeted design spectra. Whereas the latter are meant to offer a uniform measure of safety, they only do so for one limit-state (global collapse), one specific target probability (1% in 50 years) and a given assumed fragility regardless of the type of lateral-load resisting system. On the contrary, YFS can target any number of concurrent limit-states, each for a user-defined level of performance (or safety), and employ building-specific fragility functions, as implied by the supplied capacity curve shape. The practical estimation of YFS is thus based on the case-by-case solution of the integral in Eq. 1. This involves a comprehensive evaluation for a number of SDOF oscillators with the same capacity curve shape and yield displacement but different periods and yield strengths. If a numerical approach is employed, then we can obtain the comprehensive view shown in Fig. 1 at the cost of a few minutes of computer time. Alternatively, if one seeks only the value of C_y corresponding to each performance objective, then an analytical approach can be used that offers accurate results with only a few iterations (Vamvatsikos et al [22]).

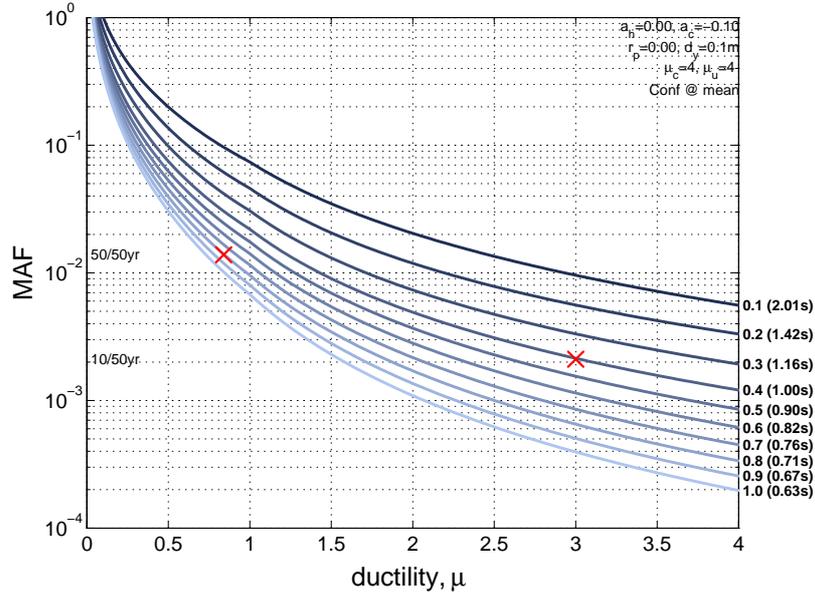


Figure 5. YFS contours at $C_y = 0.1, \dots, 1.0$ for designing a 4-story steel frame at Van Nuys, CA. The red “x” symbols represent two performance objectives ($\mu = 0.84, 3$ at 50% and 10% in 50yrs exceedance rates, respectively). The first objective governs with $C_y \approx 0.81$ corresponding to a period of $T \approx 0.71$ s.

Application Example

For showcasing the methodology, a 4-story steel moment resisting frame will be designed for a site in Van Nuys, CA (Fig. 2). It has uniform story height of 3.6m, total height of $H = 14.4$ m and $L = 9$ m beam spans. Let us adopt an interstory drift limit for serviceability (SLS) of $\theta_{lim} = 0.75\%$ and a limiting ductility of 3.0 for the ultimate limit-state (ULS). The allowable exceedance probabilities are 50% and 10% in 50yrs, respectively. Equal interstory drifts are assumed to occur throughout the height of the structure, at least in the elastic region. According to Aschheim [17], a simple way to calculate the yield roof drift (or any story yield drift) of a regular steel moment resisting frame is

$$\theta_y = \frac{\varepsilon_y}{6} \left(\frac{h}{d_{col} COF} + \frac{2L}{d_{bm}} \right) \quad (5)$$

where ε_y is the yield strain of steel, h the story height, L the beam span, COF the column overstrength factor and d_{col} , d_{bm} the column and beam depth, respectively. Let $\varepsilon_y = 0.18\%$ (for $f_y = 355$ MPa steel), $h = 3.6$ m, $L = 9$ m, $COF = 1.3$ (suggested values are 1.2 – 1.5), $d_{col} = 0.6$ m, $d_{bm} = 0.70$ m. Then, $\theta_y = 0.9\%$, and the limiting ductility for SLS becomes $\mu_{limSLS} = 0.84$. For a typical first-mode participation factor $\Gamma = 1.3$, the equivalent SDOF yield displacement is

$$\delta_y = \frac{\theta_y H}{\Gamma} = 0.10\text{m}$$

Let the dispersions due to epistemic uncertainty be 20% and 30% for SLS and ULS, respectively and assume that the system response is roughly elastoplastic. As expected for a moment-resisting steel frame the SLS governs. By employing the estimated YFS of Fig. 5 (for a confidence level consistent with the mean MAF estimate) the result is $C_y = 0.81$ corresponding to a period of $T = 0.71\text{sec}$. At this point, we can consider the beneficial effects of overstrength and further reduce C_y . For example, by employing a conservative value of, say, 1.50, the suggested seismic coefficient would become 0.54. This value can now be applied either within a force-basis or a displacement-basis for design. In the first case, we can use this as in typical code design to determine the lateral loads to be applied on the frame and then proceed as usual. The end result may not be perfect, but it is close to fully satisfying the stated objectives, something that is not as straightforward when using just a design spectrum as the point of entry.

Conclusions

Yield Frequency Spectra have been introduced as an intuitive and practical approach to performing approximate performance-based design. They are a simple enough concept to come with an accurate analytical solution, yet they also enable considering an arbitrary number of objectives that can be connected to the global displacement of an equivalent single-degree-of-freedom oscillator. For this relatively benign limitation, our approach can help deliver preliminary designs that are close to their performance targets, requiring only limited re-analysis and re-design cycles to reach the final stage.

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