Direct Performance-Based Seismic Design: Avant-garde and code-compatible approaches

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ABSTRACT: Current force-based seismic design codes use design spectra and system-specific behavior factors to satisfy two pre-defined structural limit-states: Serviceability and Life Safety. Instead, performance-based seismic design aims to design a structure to fulfill any number of target performance objectives, defined as user-prescribed levels of structural response, loss or casualties to be exceeded at a maximum rate less than a given mean annual frequency (MAF). Even at its simplest structural response basis, the inverse probabilistic nature of the requirements has not yet allowed a satisfying solution without cumbersome cycles of re-design and re-analysis. An alternative approach is proposed, relying on a new format for visualizing seismic performance, termed Yield Frequency Spectra (YFS). YFS offer a unique view of the entire solution space for structural performance, as indexed by the MAF of exceeding arbitrary ductility (or displacement) thresholds, versus the base shear strength of a structural system with given yield displacement and backbone capacity curve. YFS can be instantly computed for any system that is satisfactorily approximated by a single-degree-of-freedom oscillator, as in any nonlinear static procedure. Thus, stated performance objectives are directly related to the strength and stiffness of the structure while fully incorporating aleatory and epistemic sources of uncertainty, as needed to achieve any required level of confidence. The combination of ductility (or displacement) demand and its exceedance MAF is readily determined, allowing a satisfactory initial design to be realized in a single step. Using a simple safety factor approach given the period and the hazard curve slope, the benefits can also apply to contemporary seismic codes, essentially introducing true performance-based capabilities in a traditional format.

KEY WORDS: Earthquake engineering; Performance-based design; Probabilistic methods; Safety; Uncertainty.

1 INTRODUCTION

Earthquake engineering is a fascinating scientific field where multiple disciplines come together to mitigate the threat of one of the deadliest natural hazards on Earth. Of immediate concern is the entirety of the built environment that houses the majority of human societal and economic activities. Earthquakes thus threaten immeasurable wealth, representing past investments in existing infrastructure and ever-increasing future ones as new structures are added or renovated yearly. Recent seismic events, e.g., Northridge (1994), Kobe (1995), China (2008), Tohoku (2011), have shown that despite considerable advances in research, we still have a long way to go: While modern buildings reduce the rate of fatalities, the staggering monetary losses and functionality disruption from seismic events can cripple entire cities, or even countries.

At the core of earthquake engineering stand the dual problems of assessment and design. Assessment is the direct process of estimating structural behavior given the structure and the hazard. Design is the inverse problem, whereby a structure, its members and properties are sought to assure a desired behavior under the given seismic hazard. As typically befitting such dualities, the direct path of assessment is by far the simpler of the two. Thus, while the earthquake engineering field has benefited enormously from recent advances in computer science and technology, these have been realized in an asymmetric way. Important solutions have mostly been achieved for the assessment problem (e.g., Fajfar and Dolsek [1]), introducing the concept of performance quantification within a probabilistic context as a viable approach. Therein, a structure's properties may be taken into account together with the seismic hazard provided by probabilistic seismic hazard analysis (PSHA) to provide relatively accurate estimates of the distributions of structural response, repair cost, time-to-repair or even human casualties. While most of these capabilities are still only available at the academic level, some of the benefits are slowly finding their way into the new generation of seismic codes dealing with the assessment of existing structures (EC8 [2], ASCE/SEI 41-06 [3]).

On the design front, though, no such revolution has been realized. Current seismic design codes follow a classical force-basis paradigm that has served the civil engineering community well for many decades: Design lateral loads are prescribed through a design (pseudo)acceleration spectrum together with a system-specific "behavior" R or q factor to account for inelasticity. The elastic design spectrum is chosen to represent a certain seismic hazard for a given site that is uniform over all periods and typically corresponds to a probability of exceedance of 10% in 50 years. This is considered to be equivalent to a Life Safety limit-state, whereby a structure may be severely damaged yet allows the occupants to evacuate without serious injury. To take into account the effects of plasticity that allow us to trade strength for ductility, elastic design spectral accelerations are divided by a q-factor larger than 1.0. This is meant to be calibrated to maintain the desired level of safety for each lateral load-resisting system type, incorporating both ductility and overstrength in view of local construction practice. Life Safety is essentially satisfied by checking member resistance (force/moment) against the effects of the prescribed lateral loads. Serviceability is similarly satisfied by checking displacements (interstory drifts) for a suitably reduced design spectrum, equivalent to a more frequent level of seismic loads. Thus, elastic static or dynamic analysis is employed together with the design spectrum and the q-factor to enable a simplified design to be reached with a process that is intuitive to engineers.

While conceptually simple and undoubtedly useful, the classical approach suffers from many apparent drawbacks that need to be addressed. For example, the only allowance for nonlinear effects is through the obscure reduction/behavior factor while uncertainties are incorporated via safety factors at the input level of material and load values instead of the output response. When considering severe nonlinear response under earthquake loading, all such approximations become wishful thinking rather than scientific fact. Still, theoretical and practical difficulties have not allowed achieving the much sought-after leap to performance-based seismic design, whereby a structure would be *directly designed* to satisfy a range of performance objectives with specific allowable exceedance rates. Presently, the only approach for achieving the desired performance is through a cumbersome trial-and-error process using cycles of nonlinear analysis and linearized redesign that will slowly converge to a satisfactory, albeit largely non-optimal, solution.

Nevertheless, as assessment essentially forms the basis of design, the foundation is now there and the necessary tools are nearly complete. It is our belief that the much desired breakthrough is at hand: A direct performance-based design method that will revolutionize the state-of-the-art in earthquake engineering research. An attempt to fulfill this vision forms the basis of the paper.

2 CURRENT STATE OF THE ART

During the last decade, the use of probabilistic tools for assessing the reliability of structural systems under seismic excitations has attracted considerable research worldwide, prompted by the damages detected in many structures following severe seismic events. It is worth noting that, despite the recent advances in the earthquake engineering research field, seismological and structural response-related uncertainties are among the largest faced by engineers [4], while their proper quantification remains a daunting task. In view of this, significant effort has been put towards the development of probabilistic methodologies to properly quantify structural performance under the influence of uncertainties. At present, the scientific community seems to effectively tackle the seismic performance-based assessment challenge [1]. However, when it comes to design, considerable further evolution is needed to address uncertainties, aging, directness and practicality.

2.1 Aleatory and epistemic uncertainty

Both assessment and design become more challenging when coupled with a probabilistic framework, to account for the various uncertainty sources [4] that are intrinsic in all engineering applications. These uncertainties can be due to the inherently random nature (aleatory), e.g., of seismic loads, or can be attributed to our incomplete knowledge (epistemic), for example regarding the modeling or the actual properties of an existing structure. The fundamental difference is that epistemic uncertainty can be reduced, for example by conducting additional measurements, tests or experiments. However, aleatory randomness is naturally occurring in a process, e.g. the sequence and magnitude of earthquakes, and cannot be alleviated. In general, most existing performance-based assessment formats treat these two flavors in a unified way (despite some objections, e.g., [8]) prompting us to refer to them collectively as simply “uncertainty”.

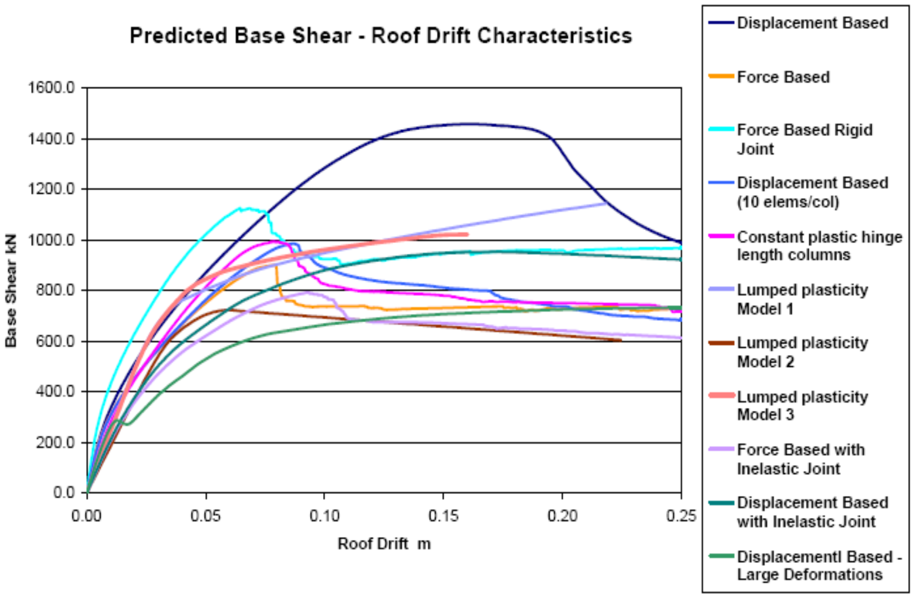

Figure 1. The effect of alternative modeling choices on the capacity curve of a 5-story reinforced concrete building, as estimated via first-mode static pushover (from Zeris et al. [5]).

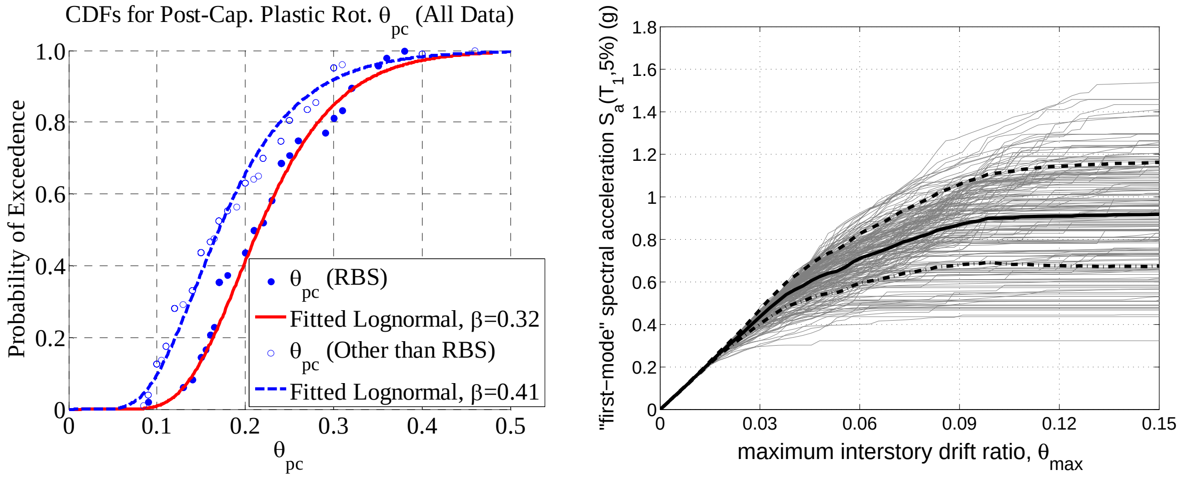

Figure 2. The effect of beam-plastic hinge uncertainties on the performance of a 9-story steel frame: (a) cumulative distribution functions of the hinge ultimate rotation (Lignos and Krawinkler [6]) and (b) the estimated dispersion of the median response in spectral acceleration versus maximum interstory drift terms (Vamvatsikos and Fragiadakis [7]).

So far, several researchers have concluded that, among all uncertainties, the earthquake signature has the most profound influence on the structural reliability, especially in the case of performance levels associated with high structural and non-structural damages [9-11]. This outcome mainly stems from limited findings concluding that the uncertainty in several model input variables (mainly the material properties of the structure) has a relatively small effect on the overall response uncertainty. However, little progress has been made towards holistically quantifying the effect of uncertainty on the structural performance, and especially its components related to the choice of *model type* (Figure 1), *analysis type* and *material aging*. Details of soil, foundation and structural modeling may heavily influence the outcome of any analysis method. With models under cyclic loading still under development [6], the evaluation of the structural response within an analytical context remains a difficult task that can lead to incorrect response estimates and consequently to structural designs with unsatisfactory or non-homogeneous seismic reliability. This is particularly true in cases where simplified modeling assumptions and analysis options have been employed and the structure approaches its collapse capacity, where large deformations and complicated degradation come into play (Villaverde [12]), as prominently shown in Figure 2.

2.2 Performance-based seismic assessment

Performance-based earthquake engineering has recently emerged to quantify in probabilistic terms the performance of structures using metrics that are of immediate use to both engineers and stakeholders [13]. Using a variety of nonlinear analysis methods, such as the static pushover or the nonlinear timehistory analysis, and adopting a proper probabilistic framework for propagating both aleatory and epistemic uncertainties to the final results, this concept is best exemplified by the Cornell-Krawinkler framing equation adopted by the Pacific Earthquake Engineering Research (PEER) Center [14]:

$$\lambda(D_V) = \iiint G(D_V \mid D_M)\left|\mathrm{d}G(D_M \mid E_{DP})\right|\left|\mathrm{d}G(E_{DP} \mid I_M)\right|\left|\mathrm{d}\lambda(I_M)\right| \quad (1)$$

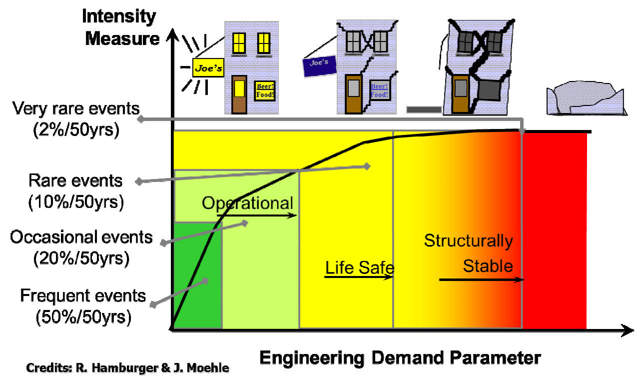

Figure 3. The structural response (E_{DP}) versus seismic intensity (I_M) relationship conceptualized via incremental dynamic analysis in a performance-based earthquake engineering framework.

D_V is a single or a vector of decision variables, such as cost, time-to-repair or human casualties that are meant to enable decision-making by the stakeholders. D_M represents the damage measures, typically discretized in a number of damage states (e.g. red/yellow/green, see Figure 3) of structural and non-structural elements and even building contents. E_{DP} contains the engineering demand parameters such as interstory drift or peak floor acceleration that the engineers are accustomed to using when determining the structural behavior. I_M is the seismic intensity, for example represented by the 5%-damped first-mode spectral acceleration $S_a(T_1,5\%)$. The relationship of E_{DP} and I_M is the result of structural analysis and it is best established through incremental dynamic analysis (IDA [15]), using multiple ground motion records scaled to different levels (and exceedance frequencies) of intensity as conceptually shown in Figure 3. The function $\lambda(y)$ provides the mean annual frequency (MAF) of exceedance of values of its operand y, thus making $\lambda(I_M)$ the seismic hazard, while $G(x)$ is the complementary cumulative distribution function (CCDF) of its variable x. Considerable research efforts have targeted the proper selection of the I_M, showing that scalars or vectors incorporating elastic spectral shape information can offer improved accuracy[16-18].

Defining performance without involving any decision variable D_V or the closely related damage measure D_M makes sense for practicing engineers, leaving engineering-level quantities, such as EDP, to express performance. This may be best achieved by moving to the familiar territory of limit-states and appropriately modifying the PEER framework, as shown by Vamvatsikos and Cornell [18]. Defining D_V and D_M to be simple indicator variables that become 1.0 when a given limit-state (LS) is exceeded, transforms Equation 1 to estimate λ_{LS}, the MAF of violating LS:

$$\lambda_{\mathrm{LS}} = \iint G(E_{DP} \mid I_M)\ \mathrm{d}\lambda(I_M) \tag{2}$$

Using either of the two equations presents a different basis for estimating performance as the MAF of exceeding a stated objective. For example, the latter could be the well-known 10% probability of exceedance in 50 years for Life Safety which, roughly corresponds to a MAF of exceedance of 0.10/50 = 0.2%. The difference in employing Equation 1 versus Equation 2 appears in the metrics used to define Life Safety itself. In the first case, this is expressed in terms of the decision variables, e.g., by requesting no casualties and property damage less than 5% of the total investment. In the second case, this could be a more familiar limitation, e.g. 2%, for the maximum or residual interstory drift ratio observed during the earthquake. Both approaches find excellent uses but they are presently limited to assessment, i.e. the forward derivation of a given structure's performance. A proper performance-based design would mean at least inverting such equations to allow deriving the desired properties of the structure that would satisfy a given value of λ_{LS}, for example the 0.2% per annum to successfully fulfill the Life Safety requirements.

2.3 Force-based versus performance-based seismic design

In principle, most current design codes may be considered to be performance-based in the limited sense that they relate certain design criteria to specific limit states [20]. According to most current codes, e.g. Eurocode 8 [2] or IBC2012 [21], for a seismic design to be considered adequate, the analyzed structure should satisfy appropriate objectives for life safety and post-earthquake occupancy. These performance objectives, usually require the structure to remain elastic in the event of a minor earthquake, to resist moderate earthquake events while sustaining repairable damages and finally, to withstand major seismic events without local or global collapse. This is typically achieved via the traditional "force-basis" by assigning design loads (i.e. lateral forces) tied to specific hazard levels. A structure built to withstand these forces is assumed to satisfy the performance requirements.

Recently, it has been recognized that such design methodologies may often fail to the desired structural reliability as they ignore the variability in structural response and seismic hazard. Instead, they attempt to inject the necessary conservatism via safety factors applied at the material level, and via specific choices of formula constants that are propagated up to the final design. Nowhere in this process is there any indication of the actual level of reliability achieved, nor is there any differentiation according to the model or analysis method used. Furthermore, such methods have often been criticized for evaluating the design base shear on the premise of the elusive fundamental period of the structure. As a result, the design load is effectively tied to the strength (and stiffness) of the structure that frequently changes from one design cycle to the next, exacerbating the need to iterate. As an alternative, Priestley [22] and Aschheim [23] proposed evaluating the design base shear on the basis of the yield displacement. The latter is proven to be a stable parameter that is also more closely correlated to the set performance targets. On such premises, a displacement-basis for design is possible (e.g. Moehle [24], Priestley et al. [25]), whereby member sizing is based on the maximum nonlinear displacement of an equivalent single-degree-of-freedom system. Nevertheless, while such methods may offer an improvement over traditional force-based design, they still fail to connect seismic risk to design decisions.

The inconsistent probabilistic basis of all such approaches becomes an important liability. There is truly no "right" value for the safety factors that can *safely* and *economically* cover the entire building stock. Whenever a new design concept veers away from the beaten path, it may finds itself in the red zone *without violating the code*. Recent advances in geotechnical engineering have unveiled such a twist to traditional design. Because of difficulties with identification and repair of foundation elements, the general design philosophy in the realm of design codes has been to ensure that foundations remain elastic during seismic shaking. By contrast, inelastic deformations in the form of ductile plastic hinges are acceptable (desirable) in the superstructure. However, in some cases it appears economically beneficial to allow inelastic action below grade, as, for example, in the case of single column bents [26]. With reference to pile foundations, kinematic response imposed by seismic waves may lead to development of large bending moments at deep interfaces; thereby nonlinear action in the foundation may be unavoidable. The currently prevailing view is that heavily stressed regions in a pile must be designed to possess adequate ductility, because complete loss of flexural strength may result in loss of vertical load-carrying capacity. However, uncertainties exist with regard to soil-structure interaction and the actual foundation behavior during severe earthquakes. It would appear to be essential to detail piles and footings so as to be capable of a reasonable degree of ductile behavior. Developing an understanding and obtaining practical solutions to these issues will require extensive research in the years to come.

Attempting to offer a path for answering such difficult issues under a coherent framework, the main idea of performance-based design is to form a methodology for structural and geotechnical design, capable to achieve simultaneously multiple performance objectives (SEAOC Vision 2000 [27]). However, what mainly differentiates a holistic performance-based design from the currently used force/displacement-based paradigm is that, the design criteria should be explicitly expressed as performance objectives paired with specific allowable exceedance rates (or probabilities). To achieve this, any proposed method should account for nonlinear structural behavior and uncertainty.

3 CONCEPTUAL PERFORMANCE-BASED DESIGN

All design approaches are essentially methods to solve inverse problems, where the functional relationship between the design variables and the target objectives is not invertible, or even precisely known. Thus, iteration is required. This is especially costly with performance-based design: Each iteration for a nonlinear structure means a cycle of re-design and re-analysis, where the latter is a full-blown performance-based assessment involving nonlinear static or dynamic runs. It is no wonder then that most attempts to represent PBSD have mostly come back to discuss assessment instead (see for example fib [28], FEMA-445 [29]). Any method built on this paradigm essentially becomes an iterated assessment procedure. Conceptual support for such a design process is provided by Krawinkler et al. [30]. Many researchers have also chosen to improve upon the efficiency of the re-design to achieve a fast convergence, often leading to the use of numerical optimization. For example, Mackie and Stojadinovic [31] have suggested this approach for bridges while Fragiadakis and Papadrakakis [32], Franchin and Pinto [33] and Lazar and Dolsek [34] have all used optimization techniques for the performance-based seismic design of reinforced-concrete structures (see also Fragiadakis and Lagaros [35] for a comprehensive review).

Despite the usefulness of currently suggested approaches, their implementation is not trivial. The link between a performance objective and the resulting design is obscure, coming out of numerous steps of numerical analysis. As an alternative, so-called "Yield Frequency Spectra" (YFS) are proposed as a design aid, being a direct visual representation of a system's performance that quantitatively links the mean annual frequency (MAF) of exceeding any displacement value (or ductility μ) with the system yield strength (or seismic coefficient C_y). YFS are plotted for a specified yield displacement; thus, periods of vibration represented in YFS vary with C_y. Figure 4 presents an example for an elastic-perfectly-plastic oscillator. In this case, three performance objectives are specified (the red "x" symbols) while curves representing the site hazard convolved with the system fragility are plotted for fixed values of C_y. Of course, increases in C_y always reduce the MAF of exceeding a given ductility value. Thus, the minimum acceptable C_y (within some tolerance) that fulfils the set of performance objectives for the site hazard can be determined for a given single-degree-of-freedom system. This strength is used as a starting point for the PBSD of more complex structures. The performance-based design problem potentially can be solved in a single step with a good estimate of the yield displacement.

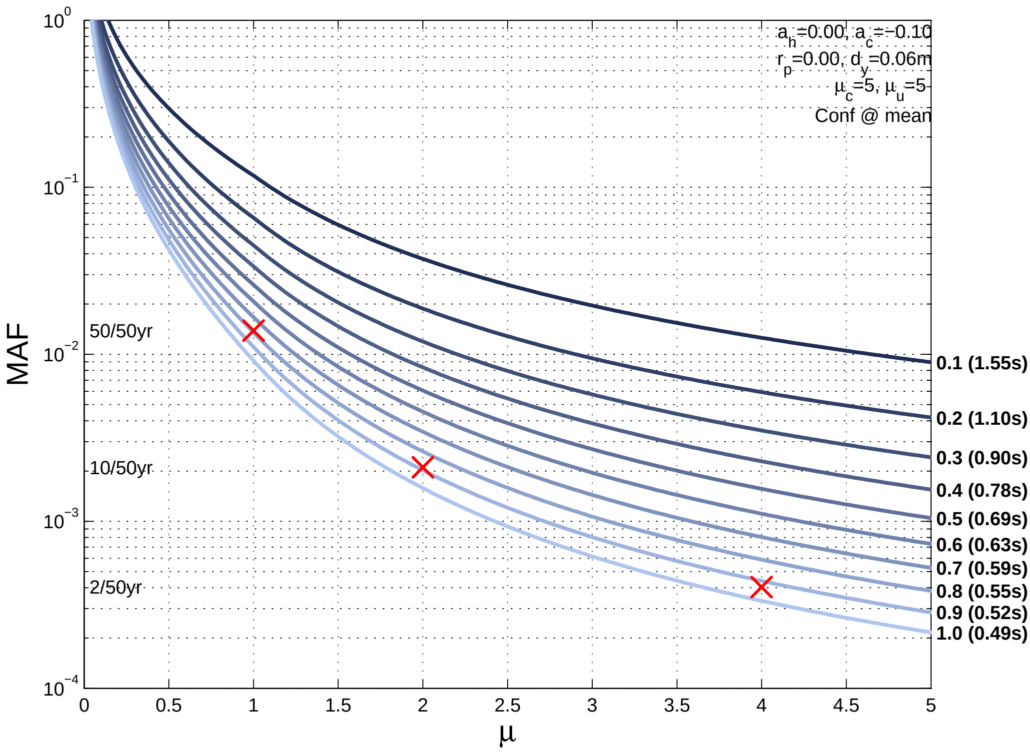

Figure 4. YFS contours at C_y = 0.1,…,1.0 determined for an elastoplastic system (δ_y = 0.06m) at Van Nuys, CA, along with red "x" symbols that represent three performance objectives (μ = 1, 2, 4 at 50%, 10% and 2% in 50yrs exceedance rates, respectively). The third objective governs with $C_y \approx 0.93$. The corresponding period is $T \approx 0.51$s.

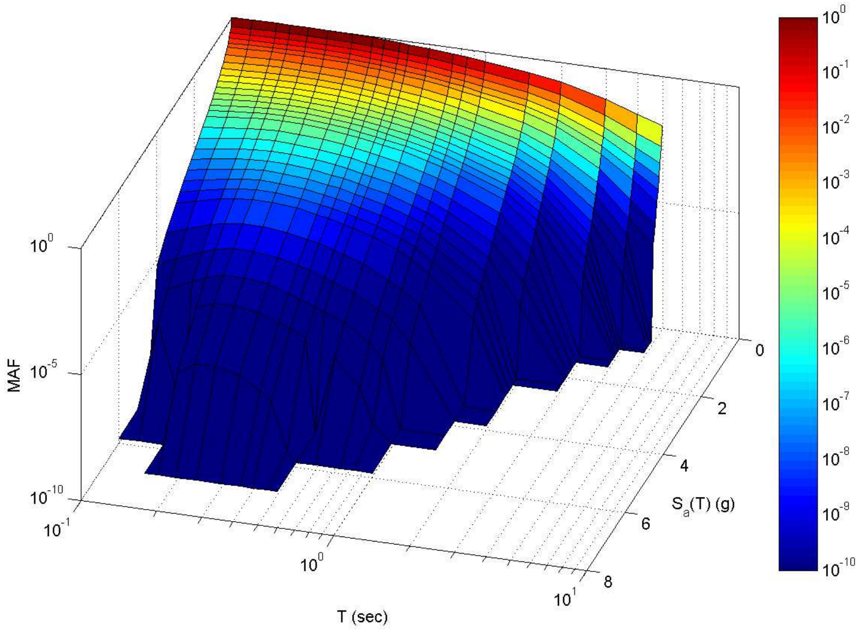

Figure 5. Spectral acceleration hazard surface for Van Nuys, CA.

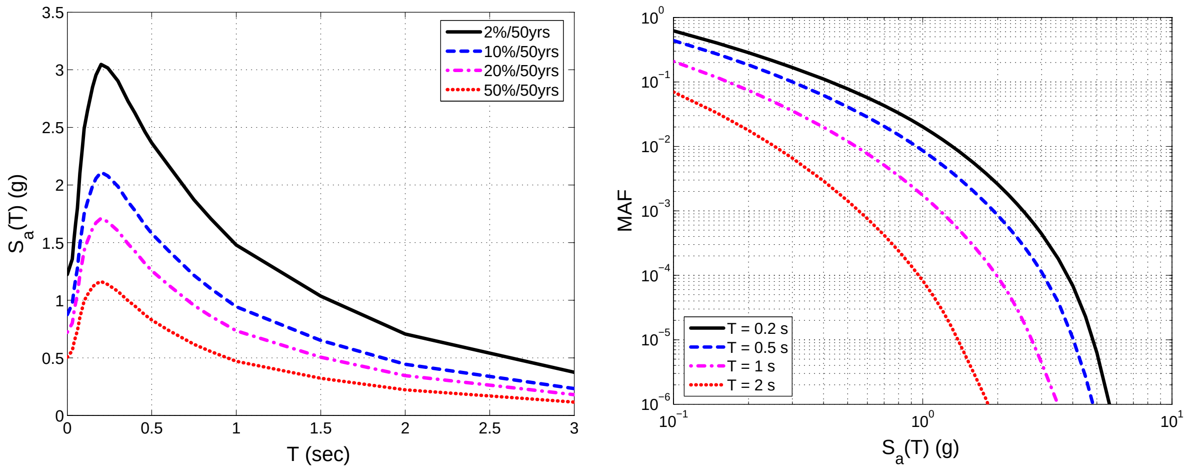

Figure 6. (a) Uniform hazard spectra and (b) S_a hazard curves for Van Nuys, CA.

4 PROPOSED BASIS FOR DESIGN

Far from finding fault with current proposals, it should be recognized that the design of a multi-degree-of-freedom structure will always involve some level of iteration. Thus, a truly direct performance-based design will probably never be realized. To reduce the number of design/analysis cycles, we should ask what parameters are stable as one moves from the initial design to the final one? One obvious shortcut, which actually forms the basis of all current seismic codes, is to rely on an SDOF system approximation. We will use this approximation for representing system level displacement (and ductility) responses. A second shortcut is to rely on the stability of the yield displacement—the notion that the yield drift ratio of a bilinear approximation to the first mode pushover curve is stable with changes in strength. The changes in strength affect stiffness and drift (or ductility) demands.

The essential ingredients of our approach to PBSD are (a) the site hazard and (b) some assumption about the system's behavior (e.g. elastic, elastoplastic etc). Comprehensive site hazard representation that is compatible with current design norms can be achieved by the seismic hazard surface, a 3D plot of the MAF of exceeding any level of spectral acceleration for the full practical range of periods (Figure 5). This is the true representation of the seismic loads for any given site. More familiar pictures can be produced from the hazard surface by taking cross-section (or contours). Cutting horizontally at given values of MAF will provide the corresponding uniform hazard spectra (UHS). For example, at $P_o = -\ln(1\text{-}0.1)/50 = 0.0021$, or a 10% in 50yrs probability of exceedance (Figure 6a), one gets the spectrum typically associated with design at the ultimate limit-state (or Life Safety). Taking a cross-section at a given period T produces the corresponding $S_a(T)$ hazard curve (Figure 6b). Now compounding this information with the capacity curve (i.e.

force-deformation relationship envelope) of the system is where things start getting interesting.

To illustrate the problem in more detail, let us first attempt a "perfect" elastic design. Suppose that an elastic oscillator of given mass M needs to be designed to not exceed a displacement δ_{lim} more often than a given MAF of P_o, for example P_o = 0.0021 for a code-compatible safety requirement. We are essentially asking for the stiffness, or equivalently, the period of this oscillator. Note here that a strength requirement would be quite straightforward to resolve, as one would simply take a horizontal line at $S_a = F/M$ in Figure 6a and seek the period (or periods) at the intersection(s) with the corresponding uniform hazard spectrum. A displacement threshold though is slightly trickier as it requires some iteration:

1. Select an initial period T.
2. Extract $S_a(T)$ from the UHS at P_o.
3. Calculate new period as $T = 2\pi \sqrt{(\delta_{lim} / S_a)}$.
4. Go to step 2 until the period converges.

The formula employed at step 3 is simply the result of solving for T the well-known relationship between the (pseudo) spectral acceleration and the spectral displacement. In an actual structural design setting this would probably be replaced by an eigenvalue analysis of the intermediate design resulting from loads consistent with the $S_a(T)$ of the preceding step 2.

A simpler solution exists that achieves the same results without any iteration. It involves the pre-calculation of a set of values of displacement consistent with the UHS spectrum at P_o for any period T that can then be interpolated to estimate the required period for any desired δ_{lim}. An intuitive graphical representation of this is actually the displacement spectrum, $S_d(T)$, which allows a direct non-iterative solution of the elastic design problem for any limit-state of interest. Not surprisingly, it is the starting point of most (if not all) displacement-design procedures (e.g. Priestley et al. [25]). Note that the seismic design codes typically do not enter this line of reasoning, despite being based on the acceleration rather than the displacement spectrum. This is achieved by virtue of prescribing an initial period that is considered to be close enough to the expected value for a given type of structure, thus foregoing the need for iterations (and eigenvalue analysis) for most rudimentary design cases.

The aforementioned process is much compounded for application to a nonlinear system. Then, for a given capacity curve shape (or system type) we are asked to estimate the yield strength and the period T for not exceeding a limiting displacement δ_{lim} at a rate higher than P_o. Even for an SDOF system, the introduction of yielding, ductility and the resulting record-to-record response variability fundamentally change the nature of the problem. This is best represented in the familiar coordinates of intensity measure (IM), here being the first mode spectral acceleration $S_a(T)$, and engineering demand parameter (EDP), i.e., the displacement response δ. The structural response then appears in the form of incremental dynamic analysis (IDA, Vamvatsikos and Cornell [15]) curves as shown in Figure 7 for a T = 1s system with a capacity curve having positive and then a negative post-yield stiffness. Cornell et al. [36] have shown that response variability means that additional hazard levels beyond P_o need to be considered in evaluating the system's performance. The reason is that values lower than the average response for the seismic intensity corresponding to P_o appear more frequently (i.e. correspond to a higher hazard rate in Figure 6b). Hence, they tend to contribute significantly more to the system's rate of exceeding $\delta = \delta_{lim}$. Formally, this relationship may be represented by the following integral (Jalayer [37], Vamvatsikos and Cornell [19]), that is exactly equivalent to Equation 2:

$$\lambda(\delta) = \int_0^{+\infty} F\left(S_{ac}(\delta) \,|\, s\right) \left|\mathrm{d}H(s)\right| \tag{3}$$

where $\lambda(\cdot)$ is the MAF of exceeding δ. $S_{ac}(\delta)$ is the random limit-state capacity, representing the minimum intensity level for a ground motion record to cause a displacement of δ to be exceeded (e.g., Figure 7). $F(\cdot)$ is the cumulative distribution function (CDF) of S_{ac} evaluated at a spectral acceleration value of s, and $H(s)$ is the associated hazard rate. The absolute value is needed for the differential of $H(s)$ because the hazard is monotonically decreasing, thus always having a negative slope.

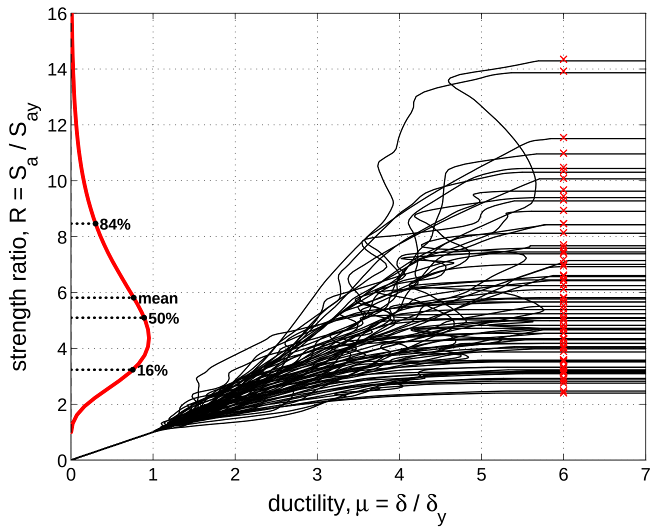

Figure 7. IDA curves for a T = 1s oscillator with a degrading (in-cycle) capacity curve, showing the distribution of the spectral acceleration capacity, S_{ac} (normalized by the yield spectral acceleration, S_{ay}) and corresponding to the collapse ductility of μ = 6.

The seismic code foregoes such considerations through implicit incorporation of two assumptions: (a) Using the strength reduction R or behavior factor q to account for the effect of yielding and ductility in the mean/median response, (b) ignoring the effect of dispersion, and assuming that the seismic loads consistent with P_o are enough to guarantee a similar (or lower) rate of non-exceedance of δ_{lim}. The error due to the above is "covered" by employing various implicit conservative approximations to account for the effect of the previous non-conservative assumptions, typically through the selection of R (or q) (see for example FEMA P695 [38]). Thus, in the code environment, the inelastic design process becomes “identical”, at least in terms of the required steps, with the elastic design process described earlier.

Unfortunately, the magnitude of the assumptions is such that one can never be entirely sure of actually achieving the stated objective(s) with any confidence. The margin of safety depends on the site and the system characteristics. Even when safe, the design is typically far from optimal: Economy and safety are two competing objectives and, given the size of the uncertainty involved in code-based inelastic design, common sense necessitates erring on the side of caution, i.e. injecting conservativeness (for example, through R). Consequently, the designer lacks specific information on where exactly his/her design is sitting on this wide blurry margin between meeting and failing the presumed performance criteria. Even worse, as any calibration for safety has been performed on the basis of the standard code assumptions of what an acceptable performance is, it is not possible to accurately inject one’s own (stricter) criteria for a better performing structure. Any importance factors used to amplify the design spectrum are only a poor substitute. This has actually become common knowledge in the past few years, and it is the premise of performance-based design. It other words, this is where the search starts for ways to fully incorporate the actual performance of a given structural system and allow its design for any desired performance objective. Unfortunately, neither the problem nor the (so far) proposed solutions are simple.

As a complete replacement of this hazy picture, we aim to offer instead a practical and theoretically consistent procedure that can fully resolve the inelastic SDOF design problem, in the same way that the aforementioned iterative process and the associated displacement spectrum do for elastic design. This will be built upon (a) Equation 3 for estimating structural performance, (b) the R-μ-T relationships for estimating the probabilistic distribution of structural response given intensity and (c) a yield displacement basis for design, by virtue of being a far more stable system parameter compared to the period (Priestley [22], and Aschheim [23]). In a graphical format, this solution is represented using yield frequency spectra.

5 ORIGIN, DEFINITION, AND USE OF YFS

For a yielding system, the direct equivalent of elastic spectral acceleration or spectral displacement hazard curves are inelastic displacement (or drift) hazard curves. These may be determined by using Equation 3 to estimate the MAF of exceeding any limiting value of displacement. They have appeared at least in the work of Inoue and Cornell [39] and subsequently further discussed by Bazzurro and Cornell [40] and Jalayer [37]. While useful for assessment, they lack the necessary parameterization to become helpful for design. An appropriate normalization may be achieved for oscillators with yield strength and displacement of F_y and δ_y, respectively, by employing ductility μ, rather than displacement δ

$$\mu = \frac{\delta}{\delta_y} \tag{4}$$

and the seismic coefficient C_y instead of the strength

$$C_y = \frac{F_y}{W} \tag{5}$$

where W is the weight. For SDOF systems C_y is numerically equivalent to $S_{ay}(T,\xi)$ / g, i.e. the spectral acceleration value to cause yield in units of g, at the period T and viscous damping ratio ξ of the system.

Up to this point, what has been proposed is not fundamentally different from the results presented by Ruiz-Garcia and Miranda [41] on the derivation of maximum inelastic displacement hazard curves. What truly makes the difference is defining δ_y as a constant for a given structural system, following the observations of Priestley [22] and Aschheim [23] on its stability as a design parameter. Then, C_y essentially becomes a direct replacement of the period T:

$$T = 2\pi\sqrt{\frac{\delta_y}{C_y g}} \tag{6}$$

or

$$C_y = \frac{\delta_y}{g}\left(\frac{2\pi}{\mathrm{T}}\right)^2 \tag{7}$$

For a given site hazard, system damping, δ_y, value of C_y (or period), and capacity curve *shape* (e.g. as normalized in terms of $R = F/F_y$ and μ), a unique representation of the system’s probabilistic response may be gained through the displacement (or ductility) hazard curves produced via Equation 3. Damping, δ_y and the capacity curve shape are considered as stable system characteristics. By plotting such curves of $\lambda(\mu)$, for a range of μ_{lim} limiting values and a range of C_y, we can get contours of the inelastic displacement hazard surface for constant values of C_y. These contours allow the direct evaluation of system strength and period—i.e., the C_y required to satisfy any combination of performance objectives defined as $P_o = \lambda(\mu_{lim})$, where each limiting value of ductility μ_{lim} is associated with a maximum MAF of exceedance P_o, as shown in Figure 4.

At a certain level, YFS can be considered as a building- and user-specific extension of concepts behind the IBC 2012 [21] risk-targeted design spectra. Whereas the latter are meant to offer a uniform measure of safety, they only do so for one limit-state (global collapse), one specific target probability (1% in 50 years) and a given assumed fragility regardless of the type of lateral-load resisting system. On the contrary, YFS can target any number of concurrent limit-states, each for a user-defined level of performance (or safety), and employ building-specific fragility functions, as implied by the supplied capacity curve shape. The practical estimation of YFS is thus based on the case-by-case solution of the integral in Equation 3. This involves a comprehensive evaluation for a number of SDOF oscillators with the same capacity curve shape and yield displacement but different periods and yield strengths. If a numerical approach is employed, then we can obtain the comprehensive view shown in Figure 4 at the cost of a few minutes of computer time. Alternatively, if one seeks only the value of C_y corresponding to each performance objective, then an analytical approach can be used that offers accurate results with only a few iterations (Vamvatsikos et al [42]).

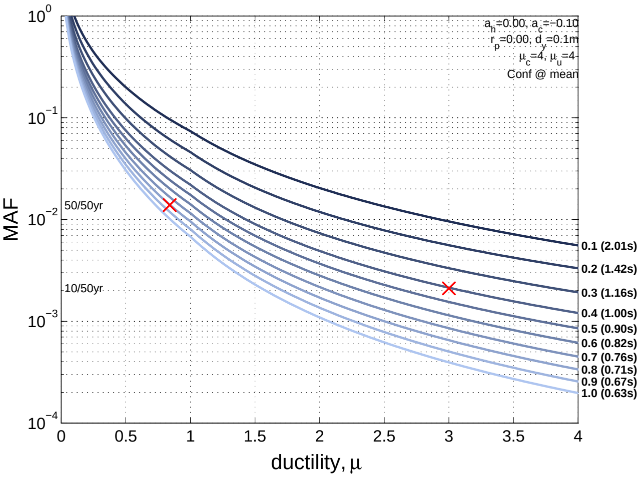

Figure 8. YFS contours at C_y = 0.1,…,1.0 for designing a 4-story steel frame at Van Nuys, CA. The red “x” symbols represent two performance objectives (μ = 0.84, 3 at 50% and 10% in 50yrs exceedance rates, respectively). The first objective governs with $C_y \approx 0.81$ corresponding to a period of $T \approx 0.71$s.

6 APPLICATION EXAMPLE

For showcasing the methodology, a 4-story steel moment resisting frame will be designed for a site in Van Nuys, CA (Figure 5). It has uniform story height of 3.6m, total height of H = 14.4m and L = 9m beam spans. Let us adopt an interstory drift limit for serviceability (SLS) of $\theta_{lim} = 0.75\%$ and a limiting ductility of 3.0 for the ultimate limit-state (ULS). The allowable exceedance probabilities are 50% and 10% in 50yrs, respectively. Equal interstory drifts are assumed to occur throughout the height of the structure, at least in the elastic region. According to Aschheim [22], a simple way to calculate the yield roof drift (or any story yield drift) of a regular steel moment resisting frame is

$$\theta_y = \frac{\varepsilon_y}{6}\left(\frac{h}{d_{col}COF} + \frac{2L}{d_{bm}}\right) \tag{8}$$

where ε_y is the yield strain of steel, h the story height, L the beam span, COF the column overstrength factor and d_{col}, d_{bm} the column and beam depth, respectively. Let ε_y = 0.18% (for f_y = 355MPa steel), h = 3.6m, L = 9m, COF = 1.3 (suggested values are 1.2 – 1.5), d_{col} = 0.6m, d_{bm} = 0.70m. Then, θ_y = 0.9%, and the limiting ductility for SLS becomes μ_{limSLS} = 0.84. For a typical first-mode participation factor Γ = 1.3, the equivalent SDOF yield displacement is

$$\delta_y = \frac{\theta_y H}{\Gamma} = 0.10\text{m} \tag{9}$$

Let the dispersions due to epistemic uncertainty be 20% and 30% for SLS and ULS, respectively and assume that the system response is roughly elastoplastic. As expected for a moment-resisting steel frame the SLS governs. By employing the estimated YFS of Figure 8 (for a confidence level consistent with the mean MAF estimate) the result is $C_y = 0.81$ corresponding to a period of T = 0.71sec. At this point, we can consider the beneficial effects of overstrength and further reduce C_y. For example, by employing a conservative value of, say, 1.50, the suggested seismic coefficient would become 0.54. This value can now be applied either within a force-basis or a displacement-basis for design. In the first case, we can use this as in typical code design to determine the lateral loads to be applied on the frame and then proceed as usual. The end result may not be perfect, but it is close to fully satisfying the stated objectives, something that is not as straightforward when using just a design spectrum as the point of entry.

7 CONCLUSIONS

Yield Frequency Spectra have been introduced as an intuitive and practical approach to performing approximate performance-based design. They are a simple enough concept to come with an accurate analytical solution, yet they also enable considering an arbitrary number of objectives that can be connected to the global displacement of an equivalent single-degree-of-freedom oscillator. For this relatively benign limitation, our approach can help deliver preliminary designs that are close to their performance targets, requiring only limited re-analysis and re-design cycles to reach the final stage.

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