

Direct performance-based seismic design using yield frequency spectra

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Abstract. Yield Frequency Spectra (YFS) are employed to enable the direct design of a structure subject to a set of performance objectives. YFS offer a unique view of the entire solution space for structural performance. This is measured in terms of the mean annual frequency (MAF) of exceeding arbitrary ductility (or displacement) thresholds, versus the base shear strength of a structural system with given yield displacement and backbone capacity curve. Using publicly available software tools or closed-form solutions, YFS can be nearly instantaneously computed for any system that can be satisfactorily approximated by a single-degree-of-freedom oscillator, as in any nonlinear static procedure application. Thus, stated performance objectives can be directly related to the strength and stiffness of the structure. The combination of ductility (or displacement) demand and its mean annual frequency of exceedance that governs the design is readily determined, allowing a satisfactory design to be realized in a single step.

Keywords: performance; seismic design; analytical solution; yield frequency spectra; nonlinear analysis

1 INTRODUCTION

Performance-based seismic design (PBSD) has been entering the earthquake lingo more and more in recent years. It hinges upon the concept of designing a structure to fulfill target performance objectives, typically defined as not exceeding given structural response levels with a mean annual frequency higher than the prescribed one. At its most advanced form, one would require specific non-exceedance rates of economic losses or even casualties, echoing the definition of decision variables that are embedded in the Cornell-Krawinkler framework (Cornell and Krawinkler 2005) adopted by the Pacific Earthquake Engineering Research (PEER) Center.

Despite the apparent significance of this goal, few steps have been taken towards developing such a design process. This comes as no surprise since resolving an inverse problem of design, where the functional relationship between the design variables and the performance objectives is not invertible, essentially needs iteration. Each iteration for a nonlinear structure means a cycle of redesign and reanalysis, where the latter is a full-blown performance-based assessment involving nonlinear static or dynamic runs. It is no wonder then that most attempts to represent PBSD have mostly come back to discuss assessment instead (see for example fib 2012, FEMA 2006). Any method built on this paradigm essentially needs to become an iterated assessment procedure. Conceptual support for such a design paradigm is provided by Krawinkler et al. (2006). Many researchers have also chosen to improve upon the efficiency of the re-design to achieve a fast convergence, often leading to the use of numerical optimization. For example, Mackie and Stojadinovic (2007) have suggested this approach for bridges while Fragiadakis and Papadrakakis (2008), Franchin and Pinto (2012) and Lazar and Dolsek (2012) have all used optimization techniques for the performance-based seismic design of reinforced-concrete structures. A more comprehensive review of such methods can be found in Fragiadakis and Lagaros (2011).

Despite the undoubtable usefulness of currently suggested approaches, their implementation is not trivial by far. The link between a performance objective and the resulting design is fundamentally obscure, coming out of numerous steps of numerical analysis. Instead, the Yield Frequency Spectra (YFS) are proposed as a design aid, being a direct visual representation of a system's performance that factually links the mean annual frequency (MAF) of exceeding any displacement value (or ductility μ) with the system yield strength (or seismic coefficient C_y). Figure 1 presents such an example for an elastic-perfectly-plastic oscillator, showing the simplicity of prescribing three performance objectives and calculating the required yield strength (seismic coefficient) for the given limiting ductilities. Being an "exact" (within some tolerance) solution for a given single-degree-of-freedom system, they are the ideal starting point for any practical PBSD application, potentially solving the performance-based design problem in a single step.

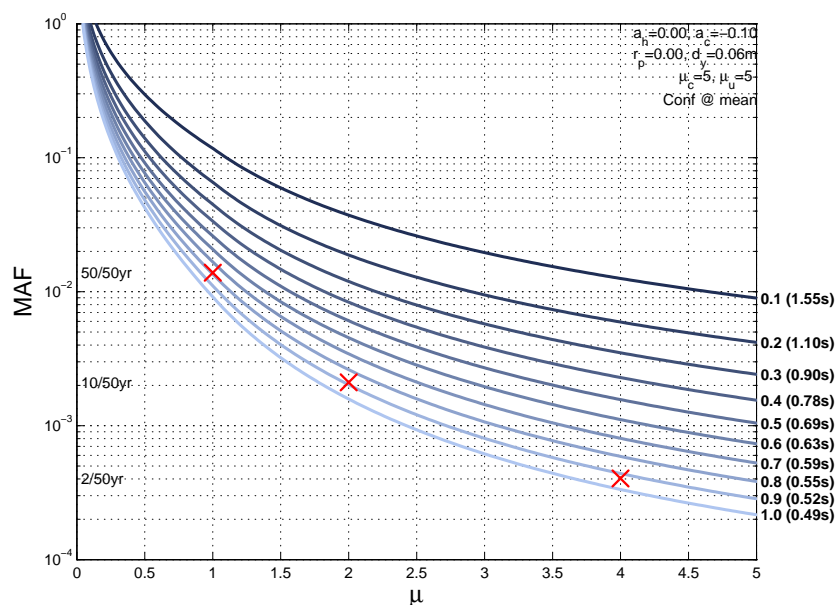


Figure 1. YFS contours at $C_y = 0.1, 0.2, \dots, 1.0$ for an elastoplastic system ($\delta_y = 0.06\text{m}$) at Van Nuys, CA, overlaid by the design points of three performance objectives for $\mu = 1, 2, 4$ at 50%, 10% and 2% in 50yrs exceedance rates, respectively. The third objective governs with $C_y \approx 0.93$ and a period of $T \approx 0.51\text{s}$.

2 BASIS OF DESIGN

Far from finding fault with current proposals, it should be recognized that the design of a multi-degree-of-freedom structure will always involve some level of iteration. Thus, a truly direct performance-based design will probably never be realized. The complexity of the system (and the problem) will usually see to that. The real question then becomes: How could one cut through the design/analysis cycles and start from an initial design that is close enough to the final one to minimize said iterations? The obvious shortcut, which actually forms the basis of all current seismic codes, is to go through the SDOF system approximation. This will also be our approach.

The essential ingredients of our approach to PBSD are (a) the site hazard and (b) some assumption about the system's behavior (e.g. elastic, elastoplastic etc). Comprehensive site hazard representation that is compatible with current design norms can be achieved by the seismic hazard surface, a 3D plot of the MAF of exceeding any level of spectral acceleration for the full practical range of periods (Figure 2). This is the true representation of the seismic loads for any given site. More familiar pictures can be produced from the hazard surface by taking cross-section (or contours). Cutting

horizontally at given values of MAF will provide the corresponding uniform hazard spectra (UHS). For example, at $P_o = -\ln(1-0.1)/50 = 0.0021$, or a 10% in 50yrs probability of exceedance (Figure 3a), one gets the spectrum typically associated with design at the ultimate limit-state (or Life Safety). Taking a cross-section at a given period T produces the corresponding $S_a(T)$ hazard curve (Figure 3b). Now compounding this information with the capacity curve (i.e. force-deformation relationship envelop) of the system is where things start getting interesting.

To illustrate the problem in more detail, let us first attempt a “perfect” elastic design. Suppose that an elastic oscillator of given mass M needs to be designed to not exceed a displacement δ_{lim} more often than a given MAF of P_o , for example $P_o = 0.0021$ for a code-compatible safety requirement. We are essentially asking for the stiffness, or equivalently the period of this oscillator. Note here that a strength requirement would be quite straightforward to resolve, as one would simply take a horizontal line at $S_a = F/m$ in Figure 3a and seek the period (or periods) at the intersection(s) with the corresponding uniform hazard spectrum. A displacement threshold though is slightly trickier as it requires some iteration:

1. Select an initial period T .
2. Extract $S_a(T)$ from the UHS at P_o .
3. Calculate new period as $T = 2\pi\sqrt{\delta_{lim}/S_a}$.
4. Go to step 2 until the period converges (i.e. does not change significantly).

The formula employed at step 3 is simply the result of solving for T the well-known relationship between the (pseudo) spectral acceleration and the spectral displacement. In an actual structural design setting this would probably be replaced by an eigenvalue analysis of the intermediate design resulting from loads consistent with the $S_a(T)$ of the preceding step 2.

A simpler solution exists that achieves the same results without any iteration. It involves the pre-calculation of a set of values of displacement consistent with the UHS spectrum at P_o for any period T that can then be interpolated to estimate the required period for any desired δ_{lim} . An intuitive graphical representation of this is actually the displacement spectrum, $S_d(T)$, which allows a direct non-iterative solution of the elastic design problem for any limit-state of interest. Unsurprisingly, it is the starting point of most (if not all) displacement-design procedures (Priestley et al. 2007). Note that the seismic design codes typically do not enter this line of reasoning, despite being based on the acceleration rather than the displacement spectrum. This is achieved by virtue of prescribing an initial period that is considered to be close enough to the expected value for a given type of structure, thus foregoing the need for iterations (and eigenvalue analysis) for most rudimentary design cases.

The aforementioned process is much compounded for application to a nonlinear system. Then, for a given capacity curve shape (or system type) we are asked to estimate the yield strength and the period T for not exceeding a limiting displacement δ at a rate higher than P_o . Even for an SDOF system, the introduction of yielding, ductility and the resulting record-to-record response variability fundamentally change the nature of the problem. This is best represented in the familiar coordinates of intensity measure (IM), here being the first mode spectral acceleration $S_a(T)$, and engineering demand parameter (EDP), i.e., the displacement δ . The structural response then appears in the form of incremental dynamic analysis (IDA, Vamvatsikos and Cornell 2002) curves as shown in Figure 4 for a $T = 1s$ system with a capacity curve having positive and then a negative post-yield stiffness. Cornell et al (2002) have shown that response variability means that additional hazard levels beyond P_o need to be considered in evaluating the system’s performance. The reason is that values lower than the average response for the seismic intensity corresponding to P_o appear more frequently (i.e. correspond to a higher hazard rate in Figure 3b). Hence, they tend to contribute significantly more to the system’s rate of exceeding $\delta = \delta_{lim}$. Formally, this relationship may be represented by the following integral (Jalayer 2003, Vamvatsikos and Cornell 2004):

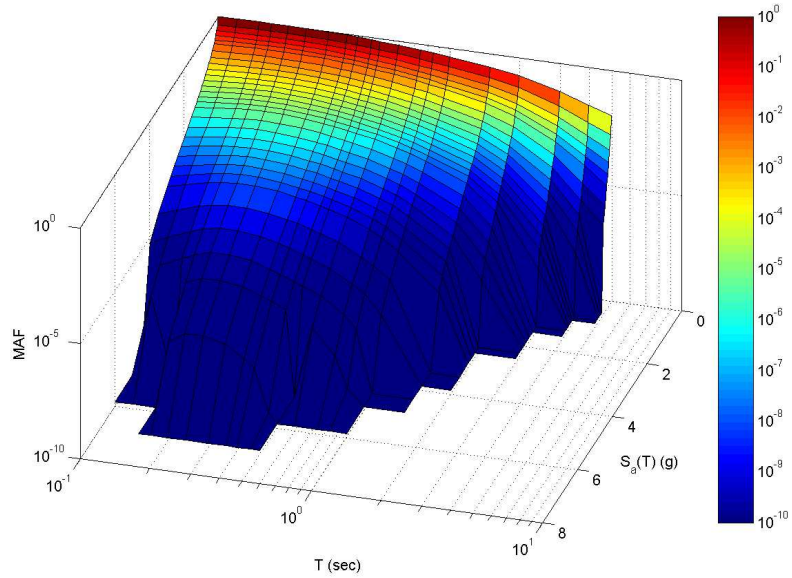


Figure 2. Spectral acceleration hazard surface for Van Nuys, CA.

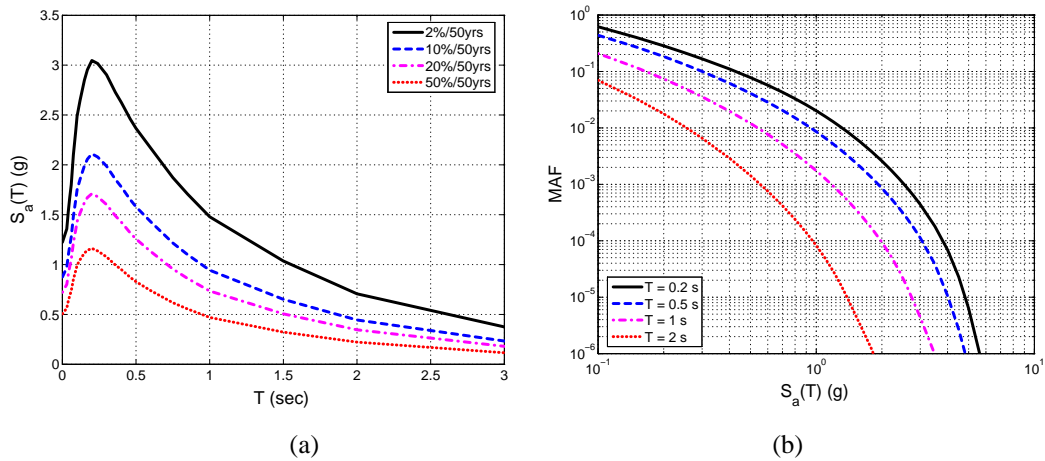


Figure 3. (a) Uniform hazard spectra and (b) spectral acceleration hazard curves for Van Nuys, CA.

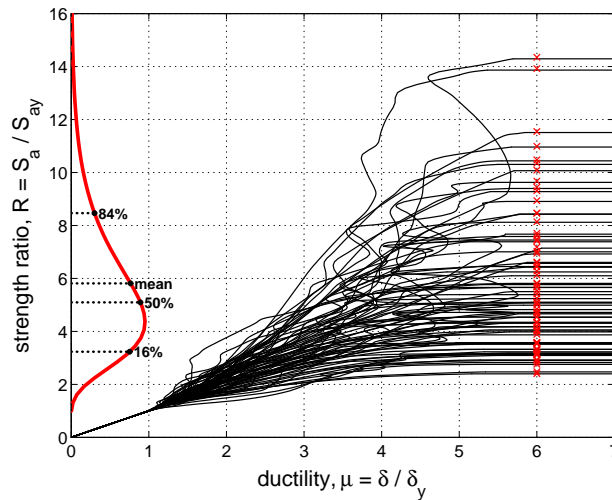


Figure 4. IDA curves for a $T = 1$ s oscillator with a degrading (in-cycle) capacity curve, showing the distribution of the spectral acceleration capacity, S_{ac} (normalized by the yield spectral acceleration, S_{ay}) and corresponding to the collapse ductility of $\mu = 6$.

$$\lambda(\delta) = \int_0^{+\infty} F(S_{ac}(\delta) | s) |dH(s)| \quad (1)$$

where $\lambda(\cdot)$ is the MAF of exceeding δ . $S_{ac}(\delta)$ is the random limit-state capacity, representing the minimum intensity level for a ground motion record to cause a displacement of δ to be exceeded (e.g., Figure 4). $F(\cdot)$ is the cumulative distribution function (CDF) of S_{ac} evaluated at a spectral acceleration value of s , and $H(s)$ is the associated hazard rate. The absolute value is needed for the differential of $H(s)$ because the hazard is monotonically decreasing, thus always having a negative slope.

The seismic code foregoes such considerations through implicit incorporation of two assumptions: (a) Using the strength reduction R or behavior factor q to account for the effect of yielding and ductility in the mean/median response, (b) ignoring the effect of dispersion, and assuming that the seismic loads consistent with P_o are enough to guarantee a similar (or lower) rate of non-exceedance of δ_{lim} . The error due to the above is “covered” by employing various implicit conservative approximations to account for the effect of the previous non-conservative assumptions, typically through the selection of R (or q) (see for example FEMA P695, FEMA 2009). Thus the inelastic design process becomes “identical”, at least in terms of the required steps, with the elastic design process described earlier.

Unfortunately, the magnitude of the assumptions is such that one can never be entirely sure of actually achieving the stated objective(s) with any confidence. The margin of safety depends on the site and the system characteristics. Even when safe, the design is typically far from optimal: Economy and safety are two competing objectives and, given the size of the uncertainty involved in code-based inelastic design, common sense necessitates erring on the side of caution, i.e. injecting conservativeness, for example through R . Consequently, the designer has no real clue on where exactly his/her design is sitting on this wide blurry margin between meeting and failing the presumed performance criteria. Even worse, as any calibration for safety has been performed on the basis of the standard code assumptions of what an acceptable performance is, it is not possible to accurately inject one’s own (stricter) criteria for a better performing structure. Any importance factors used to amplify the design spectrum are only a poor substitute. This has actually become common knowledge in the past few years, and it is the premise of performance-based design. In other words, this is where the search starts for ways to fully incorporate the actual performance of a given structural system and allow its design for any desired performance objective. Unfortunately, neither the problem nor the (so far) proposed solutions are simple.

As a complete replacement of this hazy picture, we aim to offer instead a practical and theoretically consistent procedure that can fully resolve the inelastic SDOF design problem, in the same way that the aforementioned iterative process and the associated displacement spectrum do for elastic design. This will be built upon (a) Eq. (1) for estimating structural performance, (b) the SPO2IDA R - μ - T relationship for estimating the probabilistic distribution of structural response given intensity and (c) a yield displacement basis for design, by virtue of being a far more stable system parameter compared to the period (Aschheim, 2002). In a graphical format, this solution will be represented by the yield frequency spectra.

3 ORIGIN, DEFINITION AND USE OF YFS

For a yielding system, the direct equivalent of elastic spectral acceleration or spectral displacement hazard curves are the inelastic displacement (or drift) hazard curves. These may be estimated by using Eq. (1) to estimate the MAF of exceeding any limiting value of displacement. They have appeared at least in the work of Inoue and Cornell (1990) and subsequently further discussed by Bazzurro and

Cornell (1998) and Jalayer (2003). While useful for assessment, they lack the necessary parameterization to become helpful for design. An appropriate normalization may be achieved for oscillators with yield strength and displacement of F_y and δ_y , respectively, by employing ductility μ , rather than displacement δ

$$\mu = \frac{\delta}{\delta_y}, \quad (2)$$

and the seismic coefficient C_y instead of the strength

$$C_y = \frac{F_y}{W}. \quad (3)$$

For SDOF systems C_y is numerically equivalent to $S_{ay}(T, \zeta) / g$, i.e. the spectral acceleration value to cause yield in units of g , at the period T and viscous damping ratio ζ of the system.

Up to this point, what has been proposed is not fundamentally different from the results presented by Ruiz-Garcia and Miranda (2007) on the derivation of maximum inelastic displacement hazard curves. What truly makes the difference is defining δ_y as a constant for a given structural system, following the observations of Aschheim (2002) on its stability as a design parameter. Then, C_y essentially becomes a direct replacement of the period T :

$$T = 2\pi \sqrt{\frac{\delta_y}{C_y g}}, \quad \text{or} \quad C_y = \frac{\delta_y}{g} \left(\frac{2\pi}{T} \right)^2 \quad (4)$$

For a given site hazard, system damping, δ_y , value of C_y (or period), and capacity curve shape (e.g. as normalized in terms of $R = F/F_y$ and μ), a unique representation of the system's probabilistic response may be gained through the displacement (or ductility) hazard curves produced via Eq. (1). By plotting such curves of $\lambda(\mu)$, for a range of μ_{lim} limiting values and a range of C_y , we can get contours of the inelastic displacement hazard surface for constant values of C_y . By considering the damping, δ_y and the capacity curve *shape* as stable system characteristics, such curves allow the direct evaluation of system strength and period, i.e. the C_y , of such a system for any combination of performance objectives defined as $P_o = \lambda(\mu_{lim})$, i.e., limiting values of ductility and their maximum tolerated exceedance MAF P_o as shown in Figure 1.

4 CALCULATION

In the following pages we will discuss how to practically evaluate Eq. (1) either analytically or numerically. Two options shall be offered, namely a numerical approach and a simple analytical approximation, both capable of achieving accurate point estimates.

4.1 Numerical approach

To get the YFS and the corresponding performance points, one needs to estimate $\lambda(\mu)$ for a range of μ_{lim} and C_y values. By plotting them on a graph and interpolating, any performance objective within the plotted range can be satisfied (see Figure 1). Alternatively, for each performance objective, one can estimate only $\lambda(\mu_{lim})$ for a trial value of C_y (or T), estimate the updated value of C_y and then iterate until convergence, in essence similarly to the elastic design algorithm presented previously.

Either way, to estimate the MAF of limit-state exceedance Eq. (1) can be numerically evaluated as discussed by Baker and Cornell (2005) using the following expression:

$$\lambda(\mu) \cong \sum_{\text{all } s_i} F(S_{ac}(\mu) | s_i) \Delta H(s_i), \quad (5)$$

where s_i are a number of IM values covering the entire hazard curve from the lowest to the highest non-zero MAF values available (at least 50 for reasonable accuracy) and $\Delta H(s_i) = H(s_i) - H(s_{i+1}) > 0$, due to the monotonically decreasing hazard.

There are only two points that deserve further clarification in the numerical estimation of YFS. First is the issue of damping. In order to directly connect $C_y = S_a / g$, this means that the S_a should be expressed in the same damping ratio as the system. Thus, if the system has different viscous damping ratio ζ than the damping ratio used to characterize the seismic hazard curve, typically 5%, some appropriate modification factor will need to be utilized. Second is the incorporation of uncertainty. If one desires to obtain a value of C_y consistent with the mean estimate of the displacement hazard vis-à-vis epistemic uncertainty, then (a) the mean hazard curve needs to be utilized (Cornell et al 2002) and (b) the dispersion of capacities from the R - μ - T (reduction factor, ductility, period) relationship has to be modified. Adopting the typical first-order assumption (Cornell et al 2002) it is assumed that epistemic uncertainty causes the S_{ac} values of capacity to become lognormally distributed with an unchanged median of \hat{S}_{ac} but increased overall dispersion (standard deviation of the log data) of

$$\beta_{TS_c} = \sqrt{\beta_{US_c}^2 + \beta_{Sc}^2}, \quad (6)$$

where β_{Sc} is the aleatory dispersion, incorporating the effect of the natural variability of S_{ac} (its record-to-record component provided directly by the R - μ - T), and β_{US_c} is the corresponding dispersion due to uncertainty in displacement demand and capacity. This may be approximated as the square-root-sum-of-squares of the corresponding uncertainty dispersions in μ_{lim} and in the system EDP demand itself, namely $\beta_{U\theta_d}$, $\beta_{U\theta_c}$, an assumption that (strictly speaking) loses some accuracy for short periods and close to the dynamic instability region. Any aleatory variability in the collapse capacity with dispersion β_{θ_c} , can also be incorporated in the same way into the R - μ - T relationship's demand aleatory dispersion. Using the above assumptions, Eq. (1) will provide an estimate consistent with confidence somewhat higher than 50%, the exact value depending on the overall dispersion.

4.2 Analytical approach

As an alternative to numerical integration, Vamvatsikos (2013) has provided an accurate closed-form solution for the MAF of inelastic response that can be inverted analytically. The first step is to locally fit the hazard curve $H(s)$ by a second-order power-law relationship:

$$H(s) = k_0 \exp(-k_2 \ln^2 s - k_1 \ln s), \quad (7)$$

with $k_1, k_2 > 0$ and $k_2 \geq 0$. The latter indicates the (local) hazard curvature, its introduction being the major improvement over the original SAC/FEMA formulation by Cornell et al. (2002). This improved fitting, despite being "local" in nature, encompasses a large enough range of values. Thus, it allows the back-estimation of values of the IM for a required value of MAF, something that was not practical for the previous formulations. This enables the accurate inversion of any assessment formulation that is based on such a fit.

The EDP-capacity is assumed to be lognormal, with median $\hat{\theta}_c$ and dispersions (standard deviation of the log data) equal to β_{θ_c} and $\beta_{U\theta_c}$ due to aleatory and epistemic sources, respectively. The distribution of EDP-demand given the seismic intensity IM is also assumed to be lognormal with a constant

dispersion (standard deviation of the log data) equal to $\beta_{\theta d}$ and $\beta_{U\theta d}$, regardless of the level of intensity s , and a conditional median $\hat{\theta}(s)$ that depends on the IM via a power-law:

$$\hat{\theta}(s) \approx a \cdot s^b \quad (8)$$

This is typically obtained by a linear regression in log-log coordinates and, in the framework of IDA it can be thought of being an approximation of the median IDA curve. When fitting away from the global instability region, the above approximation is accurate enough to allow for a useful approximation of the required EDP-capacity to achieve a certain performance level (i.e., MAF) of P_o :

$$\hat{\theta}_c = a \cdot \exp \left[\frac{b}{2k_2} \left(-k_1 + \sqrt{\frac{k_1^2}{\phi'} - \frac{4k_2}{\phi'} \ln \frac{P_o}{k_0 \sqrt{\phi'}}}} \right) \right] \quad (9)$$

where

$$\phi' = \frac{1}{1 + 2k_2(\beta_{\alpha d}^2 + \beta_{\alpha}^2 + \beta_{U\alpha d}^2 + \beta_{U\alpha}^2) / b} \quad (10)$$

For use in obtaining point-estimates of C_y values, a few variable replacements are needed. First, let the median EDP capacity $\hat{\theta}_c$ be replaced by the desired displacement capacity, or limit, δ_{lim} and let $\mu_{lim} = \delta_{lim} / \delta_y$ be the corresponding ductility. Now, the one thing connecting Eq. (9) to elastic structural properties is the coefficient a of the median IDA curve. According to Eq. (8), which can be assumed to hold in the elastic range as well, the yield point can be expressed as:

$$a = \frac{\delta_y}{S_{ay}^b} = \frac{\delta_y}{(C_y \cdot g)^b}, \quad (11)$$

where g is the acceleration of gravity. Note that for the *second* form of the above equation to hold, S_{ay} must be expressed in the system's damping ratio. If this is different than the damping ratio used to characterize the seismic hazard curve, typically $\zeta = 5\%$, some appropriate modification factor will need to be utilized to express the hazard curve in terms of the proper damping ratio. By introducing the above Eq. (11) into Eq. (10) and performing some algebraic manipulations, the following expression appears:

$$C_y = \frac{1}{g \mu_{lim}^{1/b}} \cdot \exp \left[\frac{1}{2k_2} \left(-k_1 + \sqrt{\frac{k_1^2}{\phi'} - \frac{4k_2}{\phi'} \ln \frac{P_o}{k_0 \sqrt{\phi'}}}} \right) \right] \quad (12)$$

where g only serves to make sure that the units come out right. If the hazard curve has been fitted with S_a in units of g , then $g = 1$ should be used.

The above equation is a powerful approximation as long as it is used away from the region of global collapse, where the basic assumption of Eq. (8) does not hold. While the application of Eq. (12) may seem straightforward, some iteration may be needed due to the dependence of the hazard curve (and the corresponding fit) to the period. It rarely takes more than 3 iterations for the algorithm to converge within 5% of the C_y value required for any performance objective. Still, overall errors up to 15% can be encountered vis-à-vis the more accurate numerical approach due to the approximations involved in deriving Eq. (12).

5 EXAMPLE OF APPLICATION

For showcasing our methodology, let us use it to design a 4-story steel moment resisting frame for a site in Van Nuys, CA (Fig. 2), with a story height of 3.6m, a total height of $H = 14.4\text{m}$ and $L=9\text{m}$ beam spans. Let us adopt an interstory drift limit for serviceability (SLS) of $\theta_{lim} = 0.75\%$ and a limiting ductility of 3.0 for the ultimate limit-state (ULS). The allowable exceedance probabilities are 50% and 10% in 50yrs, respectively. We shall assume equal drifts occur throughout the height of the structure, at least in the elastic region. According to Aschheim (Spectra, 2002), a simple way to calculate the yield roof drift (or any story yield drift) of a regular steel moment resisting frame is

$$\theta_y = \frac{\varepsilon_y}{6} \left(\frac{h}{d_{col} COF} + \frac{2L}{d_{bm}} \right), \quad (13)$$

where e_y is the yield strain of steel, h the story height, L the beam span, COF the column overstrength factor and d_{col} , d_{bm} the column and beam depth, respectively. Let $e_y = 0.18\%$ (for $f_y = 355\text{MPa}$ steel), $h = 3.6\text{m}$, $L = 9\text{m}$, $COF = 1.3$ (suggested values are 1.2 – 1.5), $d_{col} = 0.6\text{m}$, $d_{bm} = 0.70\text{m}$. Then, $\theta_y = 0.9\%$, and the limiting ductility for SLS becomes $\mu_{limSLS} = 0.84$. For a typical first-mode participation factor $\Gamma = 1.3$, the equivalent SDOF yield displacement is

$$\delta_y = \frac{\theta_y H}{\Gamma} = 0.10\text{m}.$$

Let the dispersions due to epistemic uncertainty be 20% and 30% for SLS and ULS, respectively and let's assume that the system response is roughly elastoplastic. As expected for a moment-resisting steel frame the SLS governs. Using either the analytical or the numerical approach we get a result of $C_y = 0.675$ and a period of $T = 0.77\text{sec}$. At this point, we can consider the beneficial effects of overstrength and further reduce C_y . For example, if we use a conservative value of, say, 1.50, the suggested seismic coefficient would become 0.45. This value can now be applied either within a force-basis or a displacement-basis for design. In the first case, we can use this as in typical code design to determine the lateral loads to be applied on the frame and then proceed as usual. The end result may not be perfect, but it is close to fully satisfying the stated objectives, something that is not doable when using just a design spectrum as the point of entry.

6 CONCLUSIONS

The Yield Frequency Spectra have been introduced as an intuitive and practical approach to performing approximate performance-based design. They are a simple enough concept to come with an accurate analytical solution, yet they also enable considering an arbitrary number of objectives that can be connected to the global displacement of an equivalent single-degree-of-freedom oscillator. For this relatively benign limitation, our approach can help deliver preliminary designs that are close to their performance targets, requiring only limited reanalysis and design cycles to reach the final stage.

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