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# A TARGETED NONLINEAR DYNAMIC PROCEDURE TO EVALUATE THE SEISMIC PERFORMANCE OF STRUCTURES

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**Abstract.** *The Targeted Nonlinear Dynamic Procedure is introduced to offer a practical evaluation of the seismic performance of structures. Building upon the SAC/FEMA closed-form probabilistic framework it can incorporate all important sources of variability and can be calibrated for conservatism. The hazard curve is combined with the nonlinear dynamic analysis performed for each limit-state using one or two levels of the intensity measure. This is either the elastic first-mode spectral acceleration or the more sufficient inelastic spectral displacement. From the suite of ground motion records only "targeted" subsets are used that are optimally selected to estimate the median and dispersion of the structural response. The simple factored demand and capacity checking format employed allows for a seamless integration with current engineering practice, while rational safety factors add the required degree of conservatism to account for epistemic uncertainties both for ductile and brittle modes of failure. Using a four-story reinforced concrete frame as an example, the proposed approach is shown to provide a relatively simple means to account for important sources of variability in nonlinear response history analysis. It offers powerful analysis options to a knowledgeable user in a format that can be upgraded incrementally and can provide an excellent introduction to sophisticated analysis techniques with more precisely controlled levels of conservatism.*

# 1 INTRODUCTION

The assessment of the seismic performance of structures is a fundamental problem in performance-based earthquake engineering (PBEE) that necessitates a compromise between cost and accuracy. Each method that has been proposed achieves a different balance between these two requirements, offering different advantages for the consideration of the engineer-user.

At the lower end of the hierarchy of methods, nonlinear static procedures (NSPs) offer considerable computational simplicity but at a certain cost in fidelity. They have limitations when predicting various engineering demand parameters (EDPs) for most structures. Also, as currently formulated, the nonlinear static procedures cannot treat uncertainty explicitly in estimating EDPs. These limitations are evident from comparisons with results from nonlinear response history analyses (NRHA).

NRHA methods lie at the opposite end of the scale, offering improved accuracy at the cost of requiring results from many ground motions to properly account for record-to-record variability. One example is incremental dynamic analysis (IDA [1]) which offers unparalleled accuracy but necessitates multiple nonlinear dynamic analyses under a suite of ground motion records scaled to several levels of the intensity measure (IM). Simpler NRHA implementations in the guidelines do exist, but they are mostly concerned with finding a single “central” value of response (e.g. a mean or median) for a certain seismic hazard level while completely ignoring the effect of record-to-record variability and epistemic uncertainty. This has led to the familiar prescriptions such as taking the maximum of three records or the mean of seven to determine a representative “central” response level. More recent developments, such as the Scaled NDP [2] also offer dispersion estimates but still do not provide a modern PBEE basis. Thus, practical implementation of complex NRHA methods, such as IDA, is impeded by the computational cost, while the simpler versions often seem to lack proper rigor in the wake of recent PBEE developments. We aim to offer a better alternative that strikes a favorable compromise between these two ends.

From the results of the work on the ATC-76-6 project [3] and its direct predecessors (FEMA 440/440A [4,5]), a Targeted Nonlinear Dynamic Procedure (Targeted NDP) is formulated conceptually as outlined herein. Although experience with this procedure is limited, it is expected to be more broadly applicable than predecessor nonlinear static procedures with regard to structural irregularities, component hysteretic behavior, and site-dependent ground motion characteristics (e.g. type of source mechanism and proximity to fault). At the same time, it is simpler and considerably less costly than IDA while obviously paying for it with lower accuracy. The mitigating factor though is that it is formulated to take into account epistemic uncertainties, including its inherent errors, and wherever possible shift them in such a way as to incur higher conservatism throughout its range of application. Furthermore, this implied conservatism is, to a certain degree, user-defined and can be tuned to properly cover a variety of safety requirements.

# 2 PROPOSED METHODOLOGY AND APPLICATION

In brief, the Targeted NDP the method is composed of 6 basic steps. At its basis lie the formation of a multi-degree-of-freedom model and its equivalent single-degree-of-freedom model, the use of subsets of records and the determination of response via NRHA:

1. Inelastic MDOF and equivalent SDOF models
2. Seismic shaking hazard and intensity measure selection
3. Stability assessment
4. Record set and subset selection
5. Nonlinear response history analyses and statistics of EDPs.

6. Safety checking

The procedure utilizes a multiple-degree-of-freedom model of the structure capable of capturing inelastic behavior and degradation at a component and system level. First mode pushover analysis of the model produces a capacity boundary for an equivalent-single-degree-freedom representation of the structure that is used to assess the potential for dynamic instability. Based on the characteristics of the structural mode, the seismic shaking hazard at the site of the structure is evaluated. This offers the fundamental consideration of uncertainty and naturally conveys the expected seismic threat level.

In the procedure described herein, two options for an intensity measure are available for scaling the ground motion records for use in nonlinear response history analyses of the MDOF model. The traditional approach, best suited when site specific ground motion records are available, is to scale records to match the spectral acceleration associated with a specific hazard level at the fundamental period of vibration of the model. This approach is sometimes referred to as $S_a(T_1)$ scaling. The alternative option is to use the peak displacement of the ESDOF model at the hazard level as the basis for scaling the ground motion records. This option allows use of a larger range of ground motion records and is sometimes referred to as $S_{di}$ scaling.

At present we will focus on the application of the more familiar basis of $S_a(T_1)$ scaling: Two subsets of scaled ground motions are selected and median EDP’s and related dispersions are estimated based on the results of nonlinear response history analyses of the MDOF model. Finally, this data is used to develop statistical representations of EDP demand and related capacities and perform safety checking in the Demand and Capacity Factored Design format introduced by FEMA-350/351 [6,7].

In the following pages the methodology will be presented in detail using the assessment of a 4-story reinforced concrete moment-resisting frame designed for California as an illustrative example.

## 2.1 Inelastic MDOF and equivalent SDOF models

First, a model representing the mechanical behavior of the structure to be assessed is created. The model must be capable of representing important potential inelastic mechanisms and degradation of strength and stiffness as well as P-Delta effects. The building under consideration is a 4-story reinforced concrete moment-resisting frame (RCMRF) building designed according to the 2003 IBC [8] that was developed as an archetype in FEMA-P695 [9] (see Haselton [10]).

The structural model was formed using the OpenSees platform and it is shown in Figure 1. It has accurate representation of the nonlinear behavior of beams and columns while it also incorporates nonlinear modeling of the beam-column joints. A leaning column has been included to supply any additional P-Delta forces. This is a first-mode dominated structure with a fundamental period of $T_1$ = 0.86 sec.

Conventional static pushover analysis was performed using a first-mode load pattern to develop the capacity curve of the structure shown in Figure 2. The structure was pushed well into the nonlinear range, beyond its maximum base shear capacity, to allow observation of the negative stiffness region. The resulting pushover curve is in turn fitted via a trilinear elastic-hardening-softening backbone that will serve as the capacity boundary of the equivalent SDOF (ESDOF) model that will be used for stability assessment.

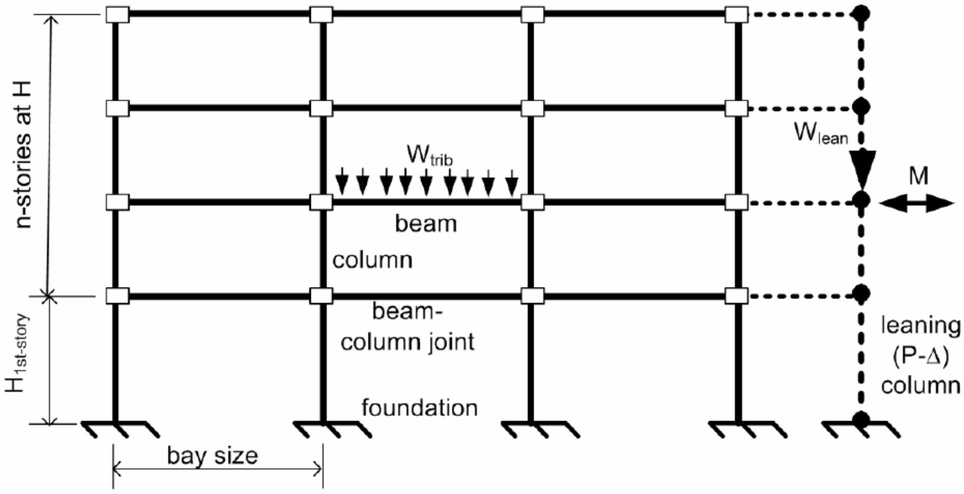


Figure 1: The four-story RC moment resisting frame building (reproduced from [8]). The bay width is 9.1m (30ft) and the story heights are 4.6m (15ft) for the first story and 4.0m (13ft) for the higher ones.

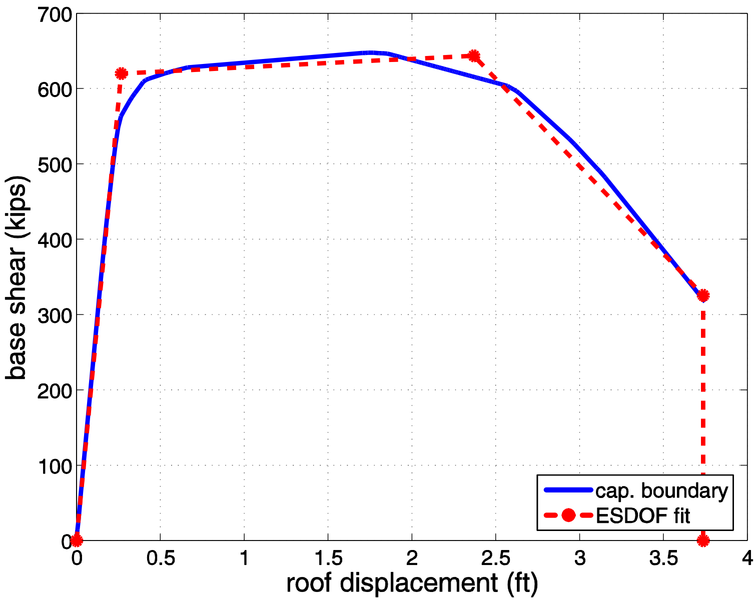


Figure 2: Capacity boundary (pushover curve) of the equivalent single-degree-of-freedom system fitted to the results of a first-mode pushover of the 4-story reinforced concrete moment frame building model.

## 2.2 Seismic hazard and intensity measure selection

As the next step, the seismic shaking hazard at the site of the structure is defined, allowing for the analysis results to be expressed probabilistically. Hazard may be determined from a site-specific probabilistic seismic hazard analysis. For sites within the United States, $S_a(T_1, 5\%)$ hazard data are available on the USGS website (www.usgs.gov).

Figure 3 presents an example of such data for the 4-story reinforced concrete frame structure at $T_1$ = 0.86 sec, which corresponds to the first-mode period. Mean hazard information is preferred over median data because the higher mean hazard curve also incorporates the important effect of epistemic uncertainty in the seismic hazard, as described in Cornell et al. [11]. Use of the mean hazard curve effectively allows the considerable uncertainty in seismic hazard to be addressed at this stage in a transparent way that simplifies the assessment process and avoids the need to explicitly account for seismic hazard uncertainty in subsequent steps.

Determining the seismic hazard goes hand-in-hand with selecting the intensity measure that will be used to scale the ground motion records for NRHA. Two possible choices have been considered for application with the Targeted NDP, namely the elastic spectral acceleration $S_a$ and the inelastic spectral displacement $S_{di}$.

Scaling ground motions to $S_a(T_1, 5\%)$ is most appropriate where site-specific hazard data, including ground motion records, are available. Since seismic hazard data are readily available in terms of $S_a$, it naturally offers a very simple assessment path. Still, it is plagued by issues of sufficiency, in so far that it cannot describe adequately important seismological parameters. This may introduce a bias in the estimation of performance that is generally conservative. On the other hand, as stated by Tothong and Cornell [12], $S_{di}$ offers advantages over $S_a(T_1, 5\%)$ in that $S_{di}$ removes the so-called peak-valley effects associated with period elongation during nonlinear response. It can reduce the potential bias in scaling the amplitude of ground motions, thus simplifying record selection by avoiding strong emphasis on other ground-motion record properties such as epsilon [13], $M_w$, and distance. It is thus less restrictive than $S_a(T_1, 5\%)$ scaling as far as the selection of acceptable ground motion records. The disadvantage of the use of $S_{di}$ as an intensity measure is the need to compute a custom-made $S_{di}$ hazard curve that depends upon the characteristics of the ESDOF system. This may be achieved, e.g., by a probabilistic seismic hazard analysis performed using attenuation relationships for $S_{di}$ (e.g., Tothong and Cornell [14]), or by using the ESDOF itself to determine the $S_{di}$ from the $S_a(T_1, 5\%)$ hazard [3].

We will presently not expand further into this process which can cut down on the conservative bias induced by $S_a(T_1)$ to provide a closer estimation that is more suitable for accurate assessment. Thus we will proceed with $S_a$ as the intensity measure, which is also the simpler version of the Targeted NDP.

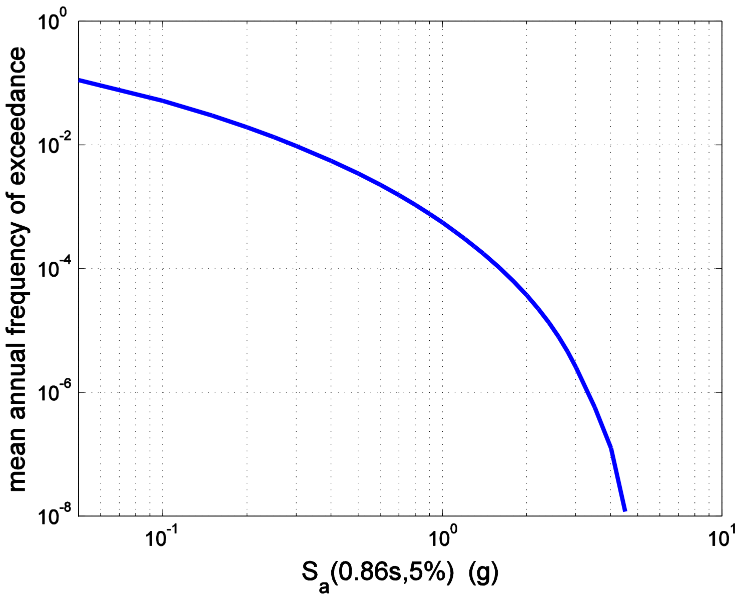


Figure 3: Seismic hazard curve for the intensity measure $S_a(T_1, 5\%)$ for $T_1$=0.86s.

### 2.3 Stability assessment

Lateral instability occurs when lateral resistance degrades due to damage caused by seismic shaking. NRHA of the MDOF system is considered the most accurate approach for assessing the likelihood of lateral instability. However, several simpler approaches based on pushover analysis may be used for this purpose—it seems that evaluation of lateral instability by pushover analysis may be more robust than evaluation of nonlinear response via NSP more generally, possibly due to the response at collapse being dominated moreso by a single mode than at lower levels of nonlinear response. Each of these pushover-based approaches makes use of the capacity boundary determined from nonlinear static pushover analysis.

One approach uses the percentile IDA curves obtained from either explicit NRHA of the ESDOF oscillator or SPO2IDA [15] relationships (the rightmost panels of Figures 4 and 5) to assess the median and distribution of collapse capacity—collapse due to lateral instability is considered to occur when the 16/50/84% IDA curves have zero slope, which indicates the corresponding percentile value of pseudo-spectral acceleration at which lateral displacements increase without limit. In case of the NRHA analyses, the record suite that will be selected for the formation of subsets for NRHA in the following section may be used for this purpose. In case of SPO2IDA though, no records are needed, as the software tool [16] is capable of instant delivery of the needed collapse capacity values in the form of the 16/50/84% percentile values of the allowable strength reduction factor.

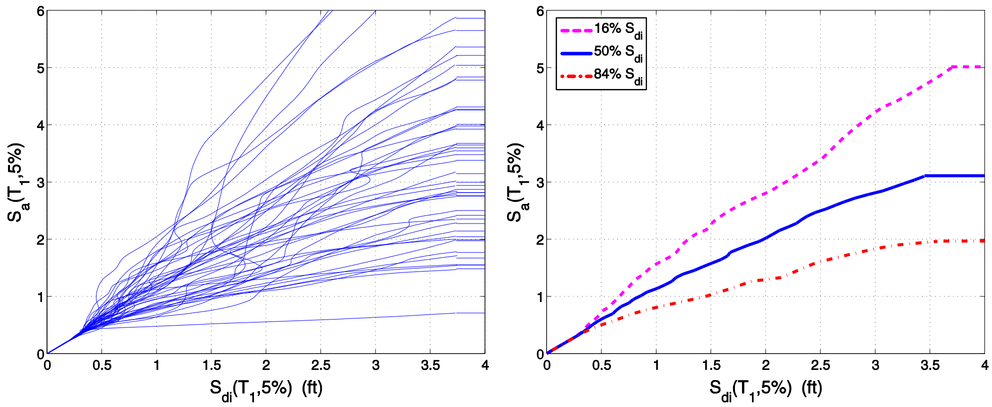


Figure 4: Complete set of 44 IDA curves and the summary 16th, 50th, 84th percentile IDA curves in $S_a$-$S_{di}$ coordinates as produced by nonlinear response history analysis for the equivalent single-degree-of-freedom fitted to a 4-story reinforced concrete moment frame building model.

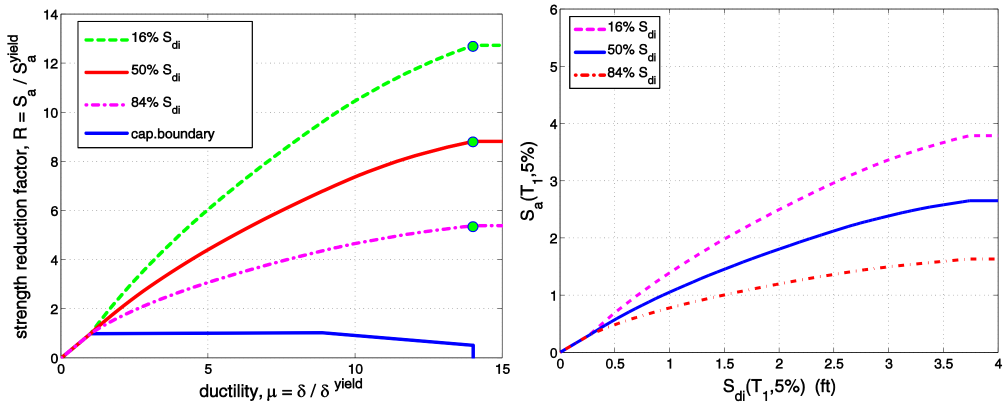


Figure 5: Percentile IDA curves in $R$-$\mu$ and $S_a$-$S_{di}$ coordinates as produced by SPO2IDA [14] for the equivalent single-degree-of-freedom fitted to a 4-story reinforced concrete moment frame building model.

A second approach to assess the median collapse capacity does so by means of a maximum limit on strength reduction factor $R_{di}$, developed through a large number of SDOF simulations in FEMA P440A [5]. In this approach, $R_{di} = S_a / S_{ay}$, where $S_a$ = the median spectral acceleration causing instability and $S_{ay}$ = the strength of the oscillator, expressed as a spectral acceleration. Terms used in Equation 1 are defined in Figure 6.

$$R_{di} = \left(\frac{\Delta_c}{\Delta_y}\right)^a + b\frac{T_e}{3|\gamma|} + \frac{F_y}{F_r}(\Delta_u - \Delta_r)\sqrt[3]{T_e}$$
$$a = 1 - \exp(-dT_e) \tag{1}$$
$$b = 1 - \left(\frac{F_y}{F_c}\right)^2$$

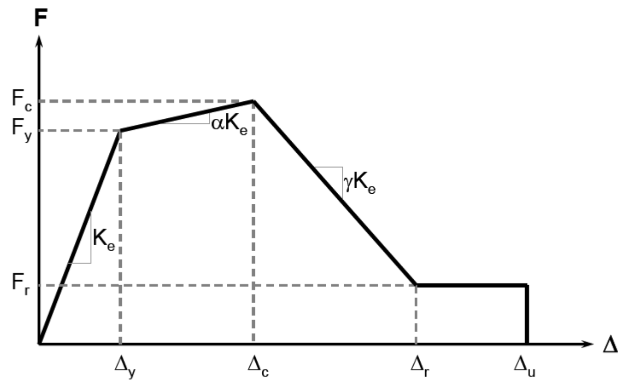


Figure 6: The backbone characteristics used in the FEMA 440A equation for maximum strength factor, $R_{di}$, to avoid dynamic instability.

## 2.4 Record Set and Subset selection

Nonlinear response history analyses of the MDOF model of the structure can be implemented at a given hazard level either by using a moderately large pool (greater than 30) of ground motion records or, more economically, by selecting subset(s) of ground motion records to estimate index values of the demand parameter distributions. The use of record subsets here follows the initial ideas presented by Azarbakht and Dolsek [17,18]. The ground motion records used in the nonlinear response history analyses are individually scaled to the target $S_a$ or target $S_{di}$ level, according to our choice of intensity measure.The target pseudo-spectral acceleration level $S_a(T_1)$, symbolized as $S_a^{\text{des}}$, is determined from the $S_a$ hazard curve at the desired mean annual frequency of exceedance.

When using $S_a$ as the intensity measure of choice, sufficiency (Luco and Cornell [19]) becomes an important issue. Thus, ground motion records for this approach ideally should be epsilon-consistent (Baker and Cornell, [13]) to obtain unbiased response quantity estimates. Nevertheless, ignoring this issue generally will lead to conservative estimates. A simple alternative to better control the level of conservatism is to select ground motions for which the scale factor applied to each natural (corrected) record is between 0.4 and 2.5. In our case, the far-field set of 44 records selected for FEMA-P695 [9] will be used. These motions are generally consistent with the smoothed elastic response spectrum for this site, needing scale factors within the suggested limits for reaching the design value of spectral acceleration, $S_a^{\text{des}}$, given by $S_a(T_1, 5\%)$ at the designated hazard level of 10% in 50yrs.

Median values of demand parameters are estimated using a subset of records whose spectra individually and collectively best approximate the median (50$^{\text{th}}$ percentile) spectrum; these records comprise record subset A. Where estimates of dispersion are sought, records are selected whose spectra individually and collectively best approximate a desired higher level of response spectrum (taken as the 84$^{\text{th}}$ percentile in this discussion); these records comprise subset B.

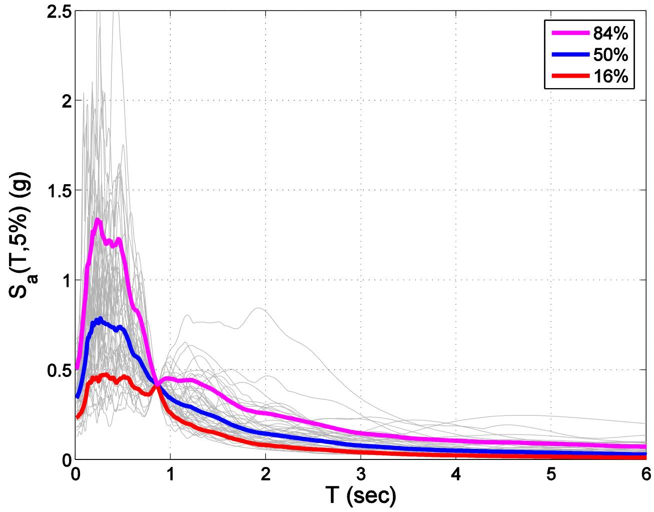


Figure 7: The 5%-damped 16,50,84% $S_a$ spectra of the 44 records scaled to match the set median at $T_1$=0.86sec.

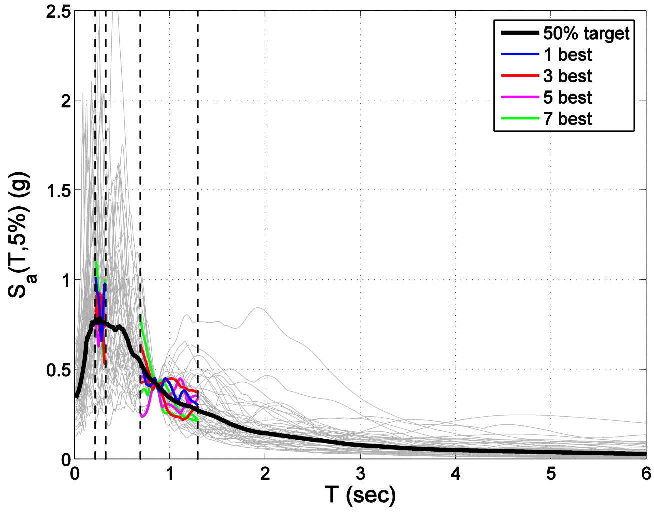


Figure 8: The 1,3,5 and 7 best record subsets A as selected to optimally match the 50% scaled-spectrum of Figure 7 in the regions of interest.

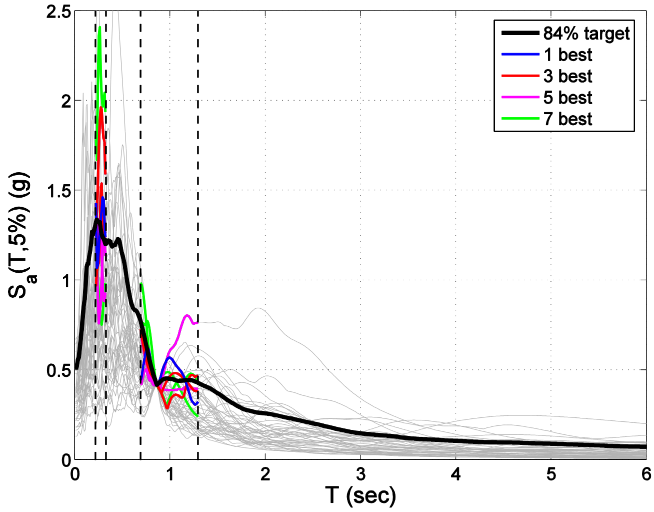


Figure 9: The 1,3,5 and 7 best record subsets B as selected to optimally match the 84% scaled-spectrum of Figure 7 in the regions of interest.

Subsets are selected using the elastic response spectra determined for the entire suite of ground motion records, scaled either to the target $S_a$ value or to the target $S_{di}$ value. The scaled elastic response spectra determined for the entire pool are used to generate 50th percentile (median) and 84th percentile spectra of $S_a$ as a function of $T$. Subsets of ground motion records are selected based on how well the corresponding elastic spectra match the median or 84th percentile spectral amplitudes over a relevant period range. For records scaled to the target $S_a$, investigations in ATC-76-6 [3] suggest that spectral matching should be done over a period range defined by the two intervals, designed to capture both the higher mode effects and the expected lengthening of the "first-mode" after yielding. In the case of $S_{di}$ scaling, the period range need not extend up to and past the first-mode period as the use of $S_{di}$ is expected to directly take into account the lengthening of the "first-mode period" of the structure that occurs as inelastic response develops. Thus, the period range of interest becomes:

$$R_T = \begin{cases} [0.8T_i, 1.2T_2] \cup [0.8T_1, 1.5T_1], & \text{if } S_a \text{ - scaling} \\ [0.8T_i, 1.2T_2], & \text{if } S_{di} \text{ - scaling} \end{cases} \tag{2}$$

The lower period is $T_i$ determined as a function of the number of stories $N_{st} > 1$, where $i = \text{ceil}\left(\sqrt{N_{st}}\right)$, "ceil" being the ceiling function that rounds up to the nearest higher integer.

To properly describe the subset selection method, the following symbols are introduced: The operator $[.]_{D,x\%}$ denotes the $x$ percentile of dataset $D$, while the designations "sub" and "all" are used to distinguish the subset and the full set. Then, the selection method for $S_a$-scaling may be formally defined as follows: Minimize the sum of the absolute *relative* differences from the median of the elastic spectrum of the ground motion suite within the period range $R_T$. Formally:

$$\text{minimize} \quad \int_{R_T} \left| \frac{[S_a(T)]_{sub,x\%} - [S_a(T)]_{all,x\%}}{[S_a(T)]_{all,x\%}} \right| dT \tag{3}$$

A proper optimization to select the true optimal subsets would be cumbersome, involving a difficult combinatorial optimization problem. In its place, a simplified selection procedure is introduced that can be easily implemented, e.g. in a spreadsheet:

1. For a given $x$% (50/84%) fractile to be estimated, calculate the "signed" and "unsigned" objective values $S^i_{x\%}$ and $U^i_{x\%}$ for each $i$-th record by adapting Equation (3) with and without the absolute value, respectively:

$$S^i_{x\%} = \int_{R_T} \frac{S^i_a(T) - [S_a(T)]_{all,x\%}}{[S_a(T)]_{all,x\%}} dT \tag{4}$$

$$U^i_{x\%} = \int_{R_T} \left| \frac{S^i_a(T) - [S_a(T)]_{all,x\%}}{[S_a(T)]_{all,x\%}} \right| dT \tag{5}$$

2. Separate the records into two lists: A "negative" and a "positive" list according to the sign of the "signed" objective value. Sort each in ascending "unsigned" value.
3. Merge the two lists by selecting records alternatively from each: Start from the "positive" list and pick the record with the lowest "unsigned" value, then similary proceed with the "negative" list to pick the second record. Remove them from the lists.
4. Continue with step 3 until the desired subset size is reached.

The above algorithm should be applied twice, first for $x$ = 50% to determine subset A to estimate the median response and then for $x$ = 84% to determine subset B and allow determination of the response dispersion. The use of 7 to 11 records in each subset appears to be adequate for many cases. There is no restriction to the maximum size of subset A or B, apart from a logical limitation of about one third the total number of records employed. The reason is that the subsets should always be small enough so that no single record appears in both A and B. Thus, for a minimum pool size of 30 records, the maximum suggested corresponding subset size is 10 for each subset. Due to the approximate nature of the results obtained with record subsets, wherever greater accuracy is required, larger subsets may be selected from an even larger pool size, or the entire pool of records may be used, disregarding subsets entirely.

The results of applying the selection algorithm for the 4-story RC building appear in Figures 8 and 9, showing the progressive selection of the 1/3/5/7 best records for subsets A and B, respectively. Obvioysly, the proposed method adopts a structure-dependent fitting range and a simple selection algorithm exploiting the relative robustness of the percentile estimators to run more efficiently. It also employs an integral over relative differences rather than plain differences. Since higher differences generally appear at the lower end of the spectrum, this distinction means that plain differences will tend to weigh lower periods more heavily than relative differences. Thus, the presented algorithm strives for more equal weighting across periods. Of course, it is conceivable that the degree of nonlinearity in the structure itself should influence the relative weighting of different areas in the spectrum, favoring, for example, the periods above $T_1$ when deep in the post-yield range of response versus periods around, e.g., $T_2$ when close to linear elastic behavior. This points to the expectation that, for the sake of simplicity, the proposed selection method may not be equally efficient at all intensity levels.

### 2.5 Nonlinear response history analyses and EDP statistics

Having selected the two subsets A and B, the corresponding two groups of nonlinear response history analyses are peformed. Estimates of median values of each engineering demand parameter (EDP) are given as the median of the demand parameter values obtained using record subset A as follows:

$$EDP_{50} \cong [EDP_i]_{A,50\%} \tag{6}$$

However, the estimation of the 84% needs more care, as the errors can be much higher than with estimates of the median. Consider that for a given scale factor (or intensity level) record subsets A and B have sizes $N_A$ and $N_B$, respectively. Then, a relatively conservative estimate of the 84% can be made by taking the maximum of three distinct values:

1. $[EDP_i]_{B,50\%}$, the median of EDP values obtained using subset B
2. $[EDP_i]_{\max(A,B),50\%}$, the median of the $N_B$ largest EDP values from both subsets A and B
3. $[EDP_i]_{A+B,84\%}$, the 84% of the EDP values from subsets A and B pooled together.

Care should be exercised though: the 3$^{rd}$ estimate is intended to be useful only for large samples, as it can become too conservative for small subset sizes, smaller than 20% of the total size $N_T$ (e.g. $N_B$ less than about 9 relative to a full set size of 44). On the other hand, it has to be used for larger subsets as the use of medians in the first two estimates will tend to underestimate the 84$^{th}$ percentile. Therefore, the combined, multi-part estimate of the 84% EDP is:

$$EDP_{84} \cong \begin{cases} \max\left([EDP_i]_{B,50\%}, [EDP_i]_{\max(A,B),50\%}\right), & \text{if } N_B < 0.2N_T \\ \max\left([EDP_i]_{B,50\%}, [EDP_i]_{\max(A,B),50\%}, [EDP_i]_{A+B,84\%}\right), & \text{if } N_B \geq 0.2N_T \end{cases} \tag{7}$$

Median and 84$^{th}$ percentile values are "counted" values, typically obtained by linear interpolation between the two closest values of the sample. Any collapses, assuming they are less than 10% of $N_T$, are still considered in the calculations as having an infinite EDP. The dispersion (aleatory randomness) in the demand parameter is thus estimated as

$$\beta_{DR} = \beta_{EDP|IM} \cong \ln(EDP_{84}) - \ln(EDP_{50}) \tag{8}$$

If the entire set of ground motion records is used to determine the MDOF response, then, allowing for modeling limitations, the evaluation procedure would be expected to introduce negligible additional uncertainty (error) in the EDP statistics. On the other hand, if the more efficient subsets are used, there is additional (non-negligible) error to consider that depends on the number of records employed in each subset and the EDP type itself. To represent this error, it is suggested that additional epistemic uncertainty dispersion is introduced in the safety checking given by

$$\beta_{SubU} = \begin{cases} 0.60 / \sqrt{\min(N_A, N_B)}, & \text{for global EDPs} \\ 0.75 / \sqrt{\min(N_A, N_B)}, & \text{for story-level EDPs} \\ 1.00 / \sqrt{\min(N_A, N_B)}, & \text{for component-level EDPs} \end{cases} \tag{9}$$

Examples of global EDPs are maxima over the entire building such as the maximum story drift or the peak roof drift over all stories; examples of story-level EDPs are story drifts and floor accelerations, and examples of component-level EDPs are member forces and section rotations.

In addition, to the two groups of NRHA corresponding to subsets A and B, in order to approximate the general shape of the IM to EDP relationship, one more group of analyses is needed. The determination of the mean annual frequency of exceedance of a demand parameter, or the required strength to ensure the capacity meets or exceeds the demand within limits implied by the mean annual frequency of exceedance at a given confidence level, necessitates the estimation of response for at least one additional seismic intensity level. When multiple intensity levels are being evaluated simulatenously (e.g., at the 50%, 10% and 2% in 50 year hazard levels) then the results at the next highest (if available) intensity level (lower probability, $P_o$) can be used for this purpose.

Alternatively, if the performance assessment is being made at only a single performance level (i.e., only one value of probability $P_o$), median response at a higher intensity level must also be determined, using a third subset of records comprising subset A scaled by a factor of 1.10 or larger. If the median capacity EDP-value for which we are testing is $EDP_C$ then if $IM^{des}$ (e.g. $S_a^{des}$ for $S_a$-scaling) is the level of IM where we previously run subsets A and B, we can now rerun subset A at $IM' = IM^{des} \cdot EDP_C / EDP_{50}$. By virtue of Equation (6), the new set of results yields its median, $EDP'_{50}$, to be used in the following section.

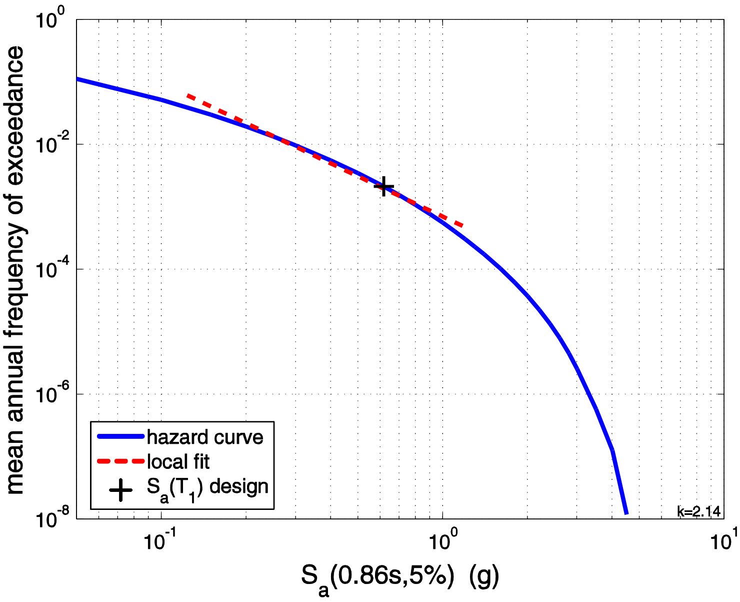


Figure 10: The mean $S_a$-hazard curve and the power-law approximation in the region of the 475 year intensity level.

## 2.6 Safety checking

Design decisions ultimately require ascertaining that a design satisfies performance expectations, and this often requires comparison of seismically induced demands with the capacities associated with the materials and components that form the structure. Although structural engineers are accustomed to treating uncertainty in demands and capacities in ultimate strength design for non-seismic loads, seismic design practice generally has neglected uncertainty, particularly on the demand side.

The Demand and Capacity Factored Design (DCFD) format developed by Cornell et al [11] is meant to be used as a check of whether a certain performance level has been violated. It cannot provide an estimate of the mean annual frequency of exceeding a given performance level. Instead, it was designed to be a checking format that conforms to the familiar Load and Resistance Factor Design (LRFD) format used in all modern design codes to check, e.g., member or section compliance. It can be represented by the following inequality:

$$FC_R \geq FD_{RPo} \cdot \exp\left(K_x \beta_{TU}\right) \Leftrightarrow \phi_R \cdot EDP_{C,50} \geq \gamma_R \cdot EDP_{50} \cdot \exp\left(K_x \beta_{TU}\right), \tag{10}$$

where $FC_R$ is the factored capacity and $FD_{RPo}$ is the factored demand evaluated at the probability $P_o$ associated to the selected performance objective. The subscript $R$ denotes that they only include aleatory uncertainties. Correspondingly, $EDP_C$ is the median EDP capacity defining the performance level (for example, 2% maximum interstory drift for a Life Safety limit-state) and $EDP_{50}$ is the median demand evaluated via subset A at the IM intensity level that has a probability of exceedance equal to $P_o$. For example, $P_o = 0.0021 \approx 1/475$ for a typical 10% in 50 years Life Safety performance level. The capacity and demand factors $\varphi_R$ and $\gamma_R$ are similar to the safety factors of LRFD formats and they are defined as:

$$\phi_R = \exp\left[-\frac{1}{2}\frac{k}{b_{EDP}}\beta_{CR}^2\right] \tag{11}$$

$$\gamma_R = \exp\left[\frac{1}{2}\frac{k}{b_{EDP}}\beta_{DR}^2\right], \tag{12}$$

$\beta_{DR}$ is the demand record-to-record (aleatory) variability, determined according to Equation 8 from the results of NRHAs of subsets A and B. $\beta_{CR}$ is the aleatory variability in the EDP-value of limit-state capacity that can be determined, e.g., from experimental tests. The parameter $k$ is the slope of the hazard curve when plotted in log-log coordinates and depends on local seismicity and site characteristics. The slope may be derived by taking a straight-line approximation to the hazard curve in log-log coordinates, as shown in Figure 10. The approximation should closely fit the hazard curve in the region [$0.25S_a^{\mathrm{des}}$, $1.5S_a^{\mathrm{des}}$], as suggested by Dolsek and Fajfar [20]. On the other hand, following Jalayer and Cornell [21], $b$ is estimated by taking advantage of the additional group of NRHAs performed with subset A at increased intensity $IM'$. Specifically, let $EDP'_{50}$ denote the median demand parameter at the higher $IM'$ level, then the log-slope, $b$, of the median demand parameter curve is estimated as follows:

$$b_{EDP} = \frac{\ln(EDP'_{50}) - \ln(EDP_{50})}{\ln(IM' / IM^{des})} \tag{13}$$

The epistemic uncertainty in demand and capacity is introduced by the total uncertainty dispersion

$$\beta_{TU} = \sqrt{\beta_{DU}^2 + \beta_{CU}^2}\ . \tag{14}$$

The two components of $\beta_{TU}$ are $\beta_{DU}$ and $\beta_{CU}$ to address epistemic uncertainty in the demand, e.g., due to modeling parameters, and also on the EDP capacity. Any value determined or provided for $\beta_{DU}$ should be inflated by $\beta_{SubU}$, which represents epistemic uncertainty associated with using record subsets to estimate demand, as given by Equation (9). The final $\beta_{DU}$ value should be estimated by taking the square root of the sum of the squares of the individual components. Figure 11 provides an illustrative breakdown of the sources of variability included in the assessment and how they influence the parameters of Equation (10).

Finally, to ensure the factored capacity, $FC_R$, exceeds the factored demand, $FD_{RPo}$, with the designated MAF at a confidence level of $\alpha$, we include $K_x$. This is the standard normal variate corresponding to the desired level $\alpha$. Values of $K_x$ are widely tabulated, and can also be easily calculated by the NORMINV function in Excel. For example NORMINV(0.9,0,1) yields $K_x$ = 1.28 for $\alpha$ = 90%, while NORMINV(0.5,0,1) produces $K_x$ = 0 for $\alpha$ = 50% confidence.

Equation (10) allows a user-defined level of confidence to be incorporated in the assessment. This is a quality that may prove to be very useful since different required levels of confidence can be associated with ductile versus brittle modes of failure or local versus global collapse mechanisms. The significant consequences of a brittle or a global failure often necessitate a higher level of safety and can be accomodated with an appropriate higher value of $K_x$. This is fundamental in the practical application of the FEMA-350/351 [6,7] guidelines where different suggested values of the confidence level are tabulated for a variety of checking situations.

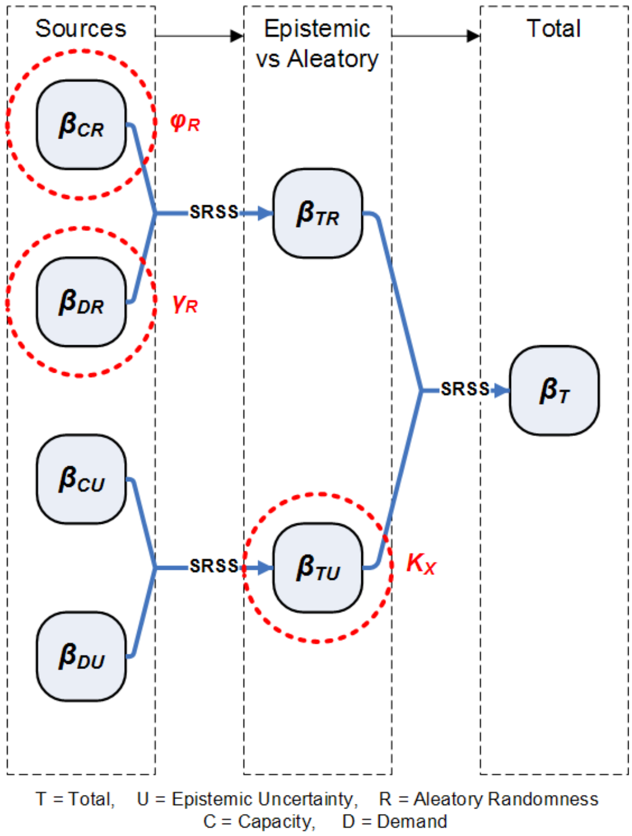


Figure 11: Breakdown of the total dispersion $\beta_T$ to its four contributing sources. The circled items are the components used for the targeted NDP format.

## 3 ILLUSTRATIVE ASSESSMENT

The interstory drift capacity limit of 2% is subject to both aleatory randomness and epistemic uncertainty. The aleatory randomness is associated with natural variability of earthquake occurrence while epistemic uncertainty is associated with our incomplete knowledge of the seismotectonic setting (i.e. the hazard), the building and its properties. Uncertainty in the hazard is addressed by using mean rather than median hazard information (Cornell et al. [10]). The latter two sources cause the true capacity limit to be lognormally distributed with dispersions (standard deviation of the natural log of the data) that are assumed here to be $\beta_{CU}$ = 30% and $\beta_{CR}$ = 20%. Furthermore, based on relevant results (e.g., Liel et al [22], Dolsek [23] and Vamvatsikos & Fragiadakis [24]), we set the base value of $\beta_{DU}$ at 20% as a possible estimate of dispersion due to epistemic uncertainty, associated mainly with the modeling parameters. Since we are using subsets A and B with 7 records each and assessment is peformed via a global EDP (interstory drift) this base value is inflated by $\beta_{SubU}$ = 0.6 / sqrt(7) = 23%. Other sources of uncertainty that have not been accounted for may increase such estimates considerably.

Assessment based on $S_a(T_1)$ follows the basic steps discussed in the previous section. The first step is to determine the $S_a(T_1, 5\%)$ design point corresponding to the 10% in 50 years design level. This corresponds to a mean annual frequency (MAF) of $\lambda = -\ln(1 - 0.10)/50 = 0.00211$ or 1/475 years, resulting in $S_a^{des} = 0.62$g. The stability assessment performed via SPO2IDA (Figure 5b) clearly shows that there is insigificant chance of global collapse at this intensity level, as the 16% value of collapse capacity is approximately 1.7g >> 0.62g.

Now, a straight line is fitted to the hazard curve in log-log coordinates within the "region of interest", defined by $\lambda_{Sa} = k_0 \cdot S_a^{-k}$. This region, according to the suggestions in Dolsek &

Fajfar [20], should be defined over the interval [$0.25S_a^{des}$, $1.5S_a^{des}$]. The region is extended further into the lower intensities since these are the ones that have the higher probabilities of exceedance. The resulting fit appears as a red dashed line in Figure 10, corresponding to a slope of $k = 2.14$. From its relationship with the hazard curve we can tell directly that we will be overestimating the hazard more or less for any value of $S_a$. This observation implies that the $S_a$-based approach will result in a conservative evaluation.

Ground motion records are individually scaled to the $S_a^{des}$ level in the first set of nonlinear dynamic analyses. The maximum interstory drift response $\theta_{max}$, determined in each nonlinear dynamic analysis is the only engineering demand parameter (EDP) of interest in this example. Out of the total 14 nonlinear response history analyses, only one did not converge, signifying the occurrence of global dynamic instability. Such results are not to be excluded from the analyses. They may be considered to correspond to infinite EDP reponse and thus contribute to the estimation of the percentile values of the sample; use of percentiles is robust to infrequent infinite values. The median of the maximum interestory drift values obtained with subset A is $\theta_{max,50} = 0.0115$. Subset A together with B provide $\theta_{max,84} = 0.0153$ via Equation (7). Thus, dispersion can be estimated as

$$\beta_{DR} = \beta_{\theta max|Sa} = \ln\theta_{max,84} - \ln\theta_{max,50} = \ln(0.0153) - \ln(0.0115) = 29\%$$

If, on the other hand, collapse has been observed for more than 10% of the records, the probability of collapse should be considered explicitly with an alternative format (see Jalayer, [25]).

To estimate the slope of the median $\theta_{max}$ versus $S_a$ diagram, a second set of nonlinear dynamic analyses were done using ground motion records scaled to $1.10 \cdot S_a^{des}$ to determine the median value $\theta_{max,50(1.10)}$. Using the full set of 44 records resulted in a median value of $\theta'_{max,50} = 0.0124$. These median values allow the slope of the median EDP curve to be estimated as

$$b = \frac{\ln(\theta'_{max,\ 50}) - \ln(\theta_{max,\ 50})}{\ln(1.10)} = \frac{\ln(0.0124) - \ln(0.0115)}{\ln(1.10)} = 0.83 \,.$$

In this case, an assumption of $b = 1$, as originally suggested by FEMA-350/351 [6,7], would be slightly unconservative.

Finally, factored demand and factored capacity are estimated as:

$$FD_{RPo} = \theta_{max,50} \exp(0.5 \cdot k \cdot \beta_{\theta max|Sa}{}^2/b) = 0.0115 \exp(0.5 \cdot 2.14 \cdot 0.29^2 / 0.83) = 0.0128$$

$$FC_R = \theta_{max,C} \exp(-0.5 \cdot k \cdot \beta_{CR}{}^2/b) = 0.02 \exp(-0.5 \cdot 2.14 \cdot 0.2^2 / 0.83) = 0.0190$$

If exceedance of the 2% maximum interstory drift is assumed to involve a ductile mechanism for this building that may only produce some local problems, the probability of exceedance can be evaluated (for the purposes of this example) at a 50% confidence level. This corresponds to a lognormal standard variate of $K_x = 0$, effectively discounting in its entirety the detrimental effect of epistemic uncertainty. Thus, the evaluation inequality becomes:

$$FC_R > FD_{RPo} \cdot 1,$$

or equivalently, 0.0190 > 0.0128, which is satisfied. A result of $FC_R = FD_{RPo}$, would have indicated that the capacity is equal to the demand induced in the building, on average, at least once every 475 years, at a 50% level of confidence. The actual result indicates that the capacity exceeds the demand induced in the building at a longer mean recurrence interval.

In some circumstances a higher level of confidence for a given recurrence interval may be desired in evaluating factored capacities, especially if involving a brittle or a global collapse mechanism that will have severe consequences on the building occupants. For illustration purposes, let us assume that this is indeed the case in this situation. For a confidence level of 90%, the lognormal standard variate is $K_x$ = 1.28 and the evaluation inequality becomes:

$$FC_R > FD_{RPo} \exp\left(K_x \cdot \beta_{TU}\right) = 0.0128 \cdot \exp(1.28 \cdot \mathrm{sqrt}(0.3^2+0.2^2+0.23^2)) = 0.0221$$

Since the factored capacity of 0.0190 is lower than the factored demand of 0.0221 the result is not satisfactory at the 90% confidence level. This clearly indicates that while the structure may be adequate at the 10% in 50yrs level if it presents a ductile low-consequence failure, like plastic beam hinging, it should not be considered safe if the failure mode examined is, e.g., a shear failure with important consequences.

## 4 CONCLUSIONS

The Targeted Nonlinear Dynamic Procedure has been introduced as an alternative to the nonlinear static procedure, offering a higher level of accuracy in seismic performance assessments within a practical format. It uses both a realistic MDOF model and its equivalent SDOF to achieve inexpensive calculations. Lateral stability assessment is performed on the equivalent SDOF, while the expensive MDOF is selectively used for nonlinear response history analysis. This is achieved by employing small subsets of ground motion records appropriately selected to match the 50/84% elastic spectra to efficiently estimate the median and the dispersion of seismic demand. The final results are integrated with seismic hazard information using the practical SAC/FEMA format to check for limit-state violations.

It is a powerful alternative to typical pushover-based methods that properly incorporates both epistemic and aleatory sources of variability while it offers a user-controlled level of confidence in safety checking. Thus, it allows proper consideration of different modes of failure (e.g., brittle versus ductile) according to their expected consequences. While not as accurate as modern PBEE approaches, it is nevertheless designed to err on the conservative side and offer an inexpensive and user-friendly method that can serve as an introduction to more sophisticated analysis techniques.

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