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SIMPLIFIED MODELS FOR THE NONLINEAR ANALYSIS OF ARSW STRUCTURES UNDER SEISMIC LOADING

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Abstract

Automated Rack Supported Warehouses (ARSW) are the state of the art in storage technology, as they provide substantial savings in terms of cost, space and energy with respect to traditional solutions. Despite their lightness, ARSWs carry very high live loads, by far higher than their self-weight, in contrast to what happens in typical civil engineering structures. Thus, standard design approaches are not applicable, especially when one considers lateral loading, i.e. seismic and wind loading.

In the frame of the STEELWAR project, the behavior factor (q) as well as the seismic fragility shall be assessed for a number of archetype warehouses. FEM modelling for such structures is a tedious task; they consist of hundreds or thousands steel members and nodes connected to each other through simple and semirigid joints. Modern computers accompanied with efficient computational algorithms can handle linear systems with ease and thus, linear analysis can be performed by including all structural components in the analysis model. Problems arise when one considers nonlinear phenomena i.e. material and geometric nonlinearity. Simulations that take into account all ARSW members and their nonlinear response may lead to prohibitive computational costs, while introducing convergence and numerical stability problems. As a direct remedy, a reduced-order physical model is proposed that enables accurate assessment of nonlinear behavior without compromising convergence performance.

Keywords: Simplified models, steel racks, pallet racking systems, automated rack supported warehouses, nonlinear analysis.

1 INTRODUCTION

Pallet rack is a material handling storage aid system designed to store materials on pallets (or “skids”). Although there are many varieties of pallet racking, all types allow for the storage of palletized materials in horizontal rows with multiple levels. Forklift trucks are usually an integral part of any pallet rack system as they are usually required to place the loaded pallets onto the racks for storage.

1.1 Structural Components

The structural design of a warehouse (geometry, materials, cross-sections etc.) varies depending on the material handling system, the designer’s preferences and the owner’s requirements. However, the following structural components are commonly considered:

a) Upright frame

Also known as built-up column, this component is in-plane with the cross-aisle direction of the warehouse (Figure 1). It consists of two or three vertical elements known as uprights, which are usually made of cold-formed open cross-section. The uprights are connected with diagonal and horizontal bracings typically made of “C” cross-sections, which transfer shear forces by uniaxial compression-tension mechanism. Their assembly strategy defines different upright frame types (D, Z, K, X etc. [1]).

b) Beam

Similarly to steel frames, beams carry the pallet loads and transfer them to the upright frames (Figure 1). Usually, the have connecting claws that ensure a decent connection to the frames without the use of bolts or screws. They are made of hollowed cross-section of high bending resistance and thus, their weakest point is the beam-to-upright connection.

c) Down-aisle vertical bracing

In many cases the loose connection between uprights and beams is not capable to resist the lateral loads and a bracing system is assembled in down-aisle direction. Purpose of this system is to prevent soft-story collapse mechanism [2] and limit the displacements induced by earthquake excitations.

1.2 Automated Rack Supported Warehouses

At present, Automated Rack Supported Warehouses (ARSW) or clad rack warehouses are usually built by manufacturers specialized in structural systems for logistics with the same or similar cold formed profiles used for warehouse storage pallet racks although in the case of ARSW the rack forms the load bearing structure of the whole building by itself.

The research made up to now is mainly limited to steel storage racks which are a much smaller scale of automated warehouses ([3], [4]). Automated storage systems, which will probably be the future of the warehouse sector, have not been investigated to such an extent so far. Moreover, in Europe (and in the world) there is no official reference document specific for the design of Automated high-rise warehouses. Designers are obliged to work with a total lack of specific references and of commonly accepted design rules and procedures. As a result, these structures are vulnerable in extreme load scenarios, such as high wind speeds and seismic actions (Figure 2).

The aforementioned lack of knowledge and bibliography raises the demand for further research. Today, a variety of methods (Pushover, IDA etc.) exist to determine the nonlinear characteristics (overstrength, ductility, energy dissipation) of a structure, which require computationally expensive numerical analyses. By examining the FEA model of an ARSW, it is obvious that a nonlinear simulation of the whole structure is nearly impossible. Objective of the present paper is to develop simplified models for the representation of complete warehouses in order to check their nonlinear response to seismic motion.

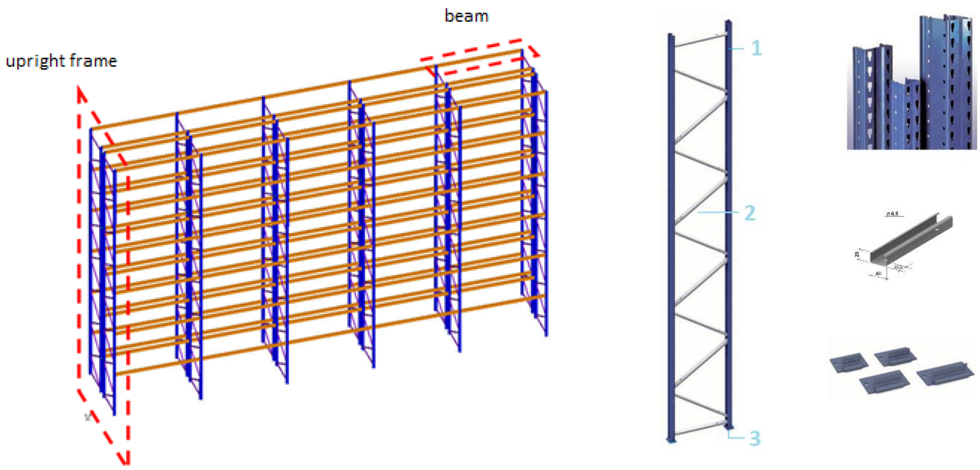

Figure 1: Configuration of a racking system: portal frame (left), upright frame's components (right)

Figure 2: Configuration of a racking system: portal frame (left), upright frame's components (right)

2 MODEL SIMPLIFICATION PROCEDURE

As mentioned before, the full simulation of an Automated Rack Supported Warehouse consists of hundreds of thousands of elements and nodes yielding to an extremely computationally cumbersome model, which is not only hard to be designed in a CAD program, but also nearly impossible to be solved nonlinearly. These problems motivate the search for an equivalent model which will be computationally efficient but also respect the behavior of the real structure. The solution suggested in the present papers is to substitute the built-up columns and the roof for simple beam elements whose degrees of freedom will be way lesser.

2.1 Elastic Properties of equivalent beam element in Cross-Aisle direction

The stiffness matrix of a prismatic homogeneous two-dimensional beam element with doubly symmetric cross-section depends on the material properties (i.e. E and G), length, cross-section area, moment of inertia and shear area. It is worth mentioning that shear deformation is usually neglected because in beams with typical lengths and cross sections this phenomenon is insignificant. However, in the case of upright frames, the "cross section" does not remain perpendicular to the neutral axis [5] and so shear must be considered (Figure 3). If one neglects the effects of shear deformation, the equivalent element may be 10 to 30% more stiff, depending on the characteristics of the structure and the distribution of loads. Moreover, compatibility of deformations leads to fixed restraints on the equivalent beam.

Obviously, the equivalent element is made of the same material and has the same length, thus:

$$\begin{aligned} E_{eq} &= E \\ G_{eq} &= G \\ L_{eq} &= L \end{aligned} \tag{1}$$

Cross-section area of the simplified column is equal to the sum of upright's cross-section areas:

$$A_{eq} = \sum_{i=1}^{N} A_i \tag{2}$$

, where N is the total number of uprights and A_i is the cross-section area of i-upright.

In general, the equivalent moment of inertia of a built-up column consisted of N uprights is given by:

$$I_{eq} = \sum_{i=1}^{N} A_i \cdot h_i^2 \tag{3}$$

, where h_i is i-upright's distance from the center of gravity. For example, the upright frame shown in Figure 4 has moment of inertia equal to:

$$I_{eq} = A_c \left(-\frac{h_0}{2}\right)^2 + A_c \cdot 0 + A_c \left(\frac{h_0}{2}\right)^2 = A_c \frac{h_0^2}{2}$$

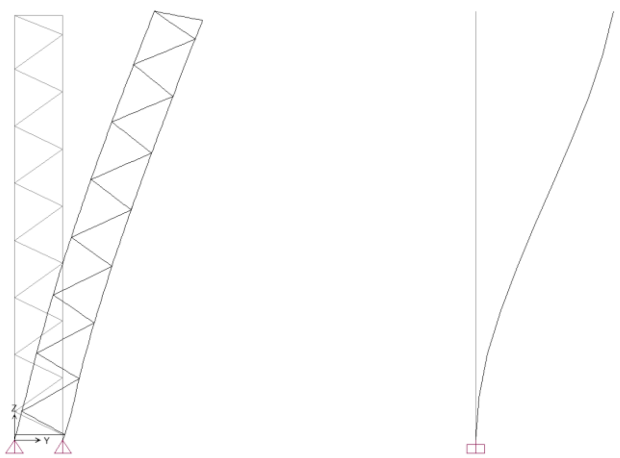

Figure 3: In built-up columns (left) "cross sections" do not remain perpendicular to the neutral axis. This effect must be considered when assigning equivalent element's properties (right)

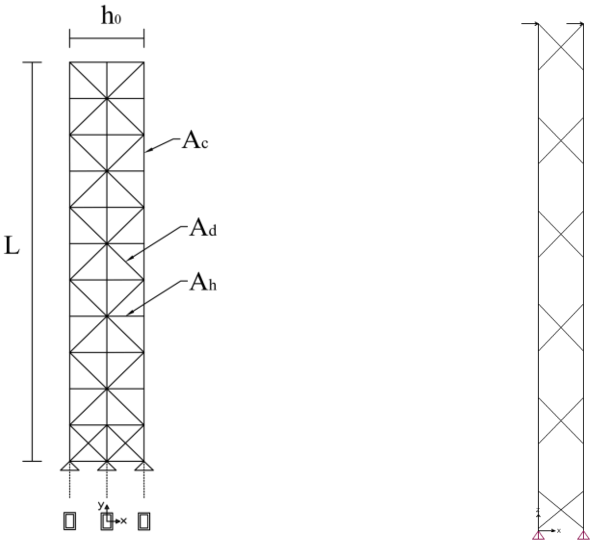

Figure 4: Example of a X-type column with 3 uprights (left) and an upright frame with arbitrary bracing (right)

Finally, shear area depends on the geometry and the type of the upright frame (X, D, Z and K systems. Closed form solutions can be easily derived for common systems, by considering a segment of the upright frame and enforce static equilibrium [6]. However, in the case of arbitrary bracing configuration like the one shown in Figure 4, no formula exists for the calculation of shear area. In order to overcome this difficulty, the following approximate procedure is introduced:

1. Isolate the column under consideration and pin the nodes at one end.
2. Apply a point load at the free end. If P is the applied load, then the corresponding displacement of the free end δ_{tot} is given by:

$$P=\frac{12}{4+\Phi}\frac{EI_{eq}}{L^3}\delta_{tot} \tag{4}$$

, where $\Phi=\dfrac{12EI_{eq}}{GA_{eff}L^2}$

3. Solve Eq. (4) for A_{eff} :

$$A_{eff}=\frac{P\left(3EI\right)L}{\left(3EI\right)\delta\text{-}PL^3}/G \tag{5}$$

2.2 Elastic Properties of equivalent beam element in Down-Aisle direction

In down-aisle direction the uprights of an upright frame are independent and thus they behave as springs in parallel. This assumption is valid as the bracings are commonly pinned to the uprights. As a result, we aggregate the moment of inertias of all uprights:

$$I_{eq,DA}=\sum_{i=1}^{N}I_{i,DA} \tag{6}$$

, where $I_{i,DA}$ is i-th's upright moment of inertia in down-aisle direction. Moreover, if the uprights are assumed partially fixed to the base plate by employing rotational springs, the sum of their stiffnesses has to be applied on equivalent beam's restrained end.

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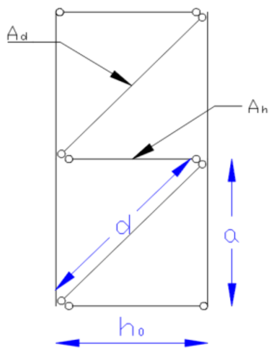
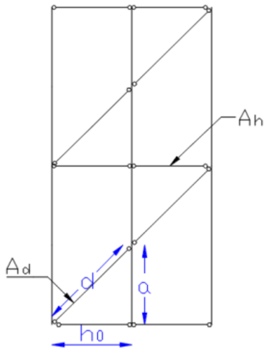
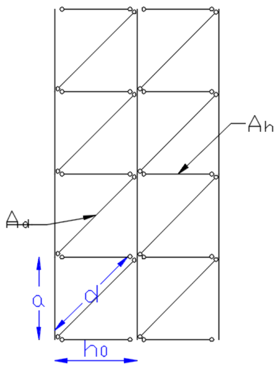
			
Nº uprights	2	3	3
A_{eq}	$2A_c$	$3A_c$	$3A_c$
I_{eq}	$A_c\frac{h_0^2}{2}$	$2A_ch_0^2$	$2A_ch_0^2$
A_{eff}	$\frac{\frac{EA_d}{G}\frac{h_0^2}{d^2}\frac{a}{d}}{1+\frac{h_0^3}{d^3}\frac{A_d}{A_h}}$	$\frac{\frac{EA_d}{G}\frac{h_0^2}{d^2}\frac{a}{d}}{1+\frac{h_0^3}{d^3}\frac{A_d}{A_h}}$	$2\cdot\frac{\frac{EA_d}{G}\frac{h_0^2}{d^2}\frac{a}{d}}{1+\frac{h_0^3}{d^3}\frac{A_d}{A_h}}$

Table 1: Equivalent properties of X-type columns

2.3 Nonlinear behavior of equivalent beam element

The nonlinear behavior of an upright frame can be distinguished in three main categories:

1. *Axial Failure*. This type of failure refers to flexural, local, distortional and lateral torsional buckling of the uprights [7]. It is common in rack-system technology to perform laboratory tests to evaluate uprights' compression resistance and thus, $N_{rd,upright}$ is usually a known value.
2. *Bending Failure*. Loads are not primarily carried by bending mechanisms, as bracings are considered to be pinned and uprights are usually simply supported to the foundation. However, when vertical bracing system is missing in down-aisle direction, horizontal loading may lead to development of bending moments in the uprights. However, their contribution is usually small with respect to the axial loads.
3. *Shear Failure*. Shear forces are transferred via axial tension-compression mechanism by bracings, which may fail due to buckling or tensile yielding.

The equivalent element must take into consideration all these failure mechanisms. For instance, open-source software OpenSees [8] provides the Two Node Link Elements (aka Link Elements, see Figure 5) with the capability to assign axial, rotational and shear springs.

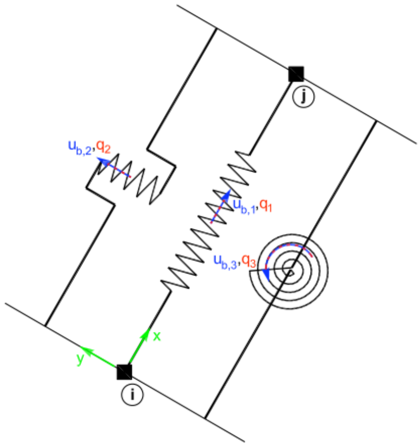


Figure 5: Concept of Two Node Link Element

One important characteristic of the equivalent element is the coupled behavior of the axial and rotational spring. Equivalent element's bending moment M_{eq} is linked to a set of axial forces N_b on the uprights, of opposite direction. As an example, for the Z-type upright frame shown in Figure 6 the following relation holds:

$$M_{eq} = N_b \cdot h_0 \tag{7}$$

On the other hand, equivalent axial force N_{eq} is related to a set of axial forces $N_{eq}/2$ on the uprights, of the same direction. Summing all together, if $N_{rd,upright}$ is upright's compression resistance and N, M the axial force and the bending moment acting on the equivalent element respectively, then the condition for axial failure is (same formulae can be derived for any number of uprights):

$$N_{rd,upright} = \frac{N_{eq}}{2} + \frac{M_{eq}}{h_0} \tag{8}$$

Eq. (8) indicates a linear interaction between moment and axial force of the equivalent element, which is conceptually illustrated in Figure 7.

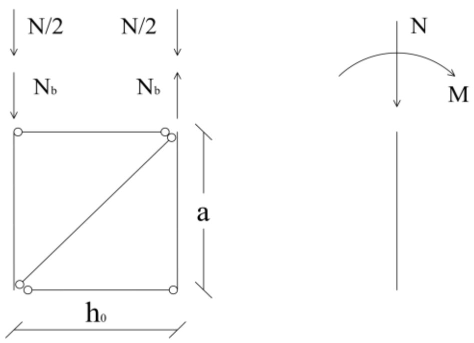


Figure 6: Relation between axial forces and bending moments of the two models

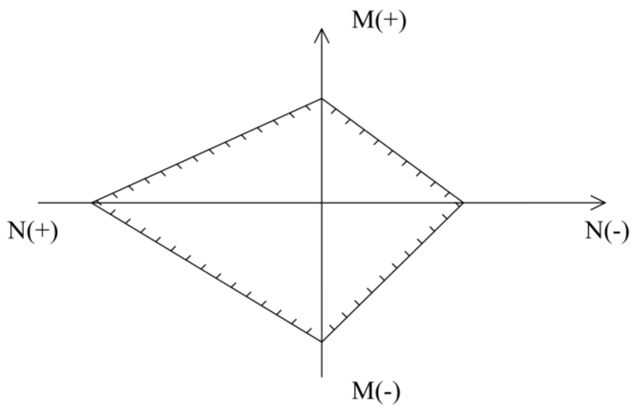


Figure 7: Interaction between the bending moment and axial force of the equivalent element

An important factor that dominates the nonlinear behavior of an upright frame in seismic loading is bracings' failure, as indicated in [9]. These structural components are responsible for the transfer of seismic shear forces to structure's foundation. If V_{eq} is equivalent element's shear force and $N_{rd,bracing}$ the axial resistance of the diagonal bracing shown in Figure 6, then failure occurs when:

$$N_{rd,upright} = \frac{V_{eq}}{\cos\varphi} \tag{9}$$

, where $\cos\varphi$ is the angle between upright and bracing. Eq. (9) holds for Z-type upright frames with two uprights, but it can be extended for any system.

Concluding, the substitution of an upright frame for simple beam elements does not reduce model's capabilities to simulate any type of structural failure. OpenSees' Link Elements comprise of two rigid linear segments and three or six springs in the center for 2D and 3D analysis respectively. These springs have to produce the same stiffness matrix as the classic finite beam elements. Table 2 shows the stiffness of each spring for the 2D case.

Type of beam	**EULER-BERNOULLI**	**TIMOSHENKO**
Axial Spring	$\frac{EA}{L}$	$\frac{EA}{L}$
Shear Spring	$\frac{12EI}{L^3}$	$\frac{1}{1+\Phi} \cdot \frac{12EI}{L^3}$
Bending Spring	$\frac{EI}{L}$	$\frac{EI}{L}$
P-delta input value (OpenSees users)	-0.1	$\frac{-0.1}{(1+\Phi)^2}$

Table 2: Springs' stiffnesses for 2D Two Node Link Element

3 ANALYSIS OF AN ARSW FRAME WITH SHEAR FAILURE

The nonlinear behavior of a single ARSW frame is examined in static and dynamic analysis. The uprights are class 1 steel sections and thus not expected to participate in structure's failure mechanism. This test case focuses solely to shear failure of the simplified model or equivalently to bracings' failure.

3.1 Configuration of test case and structural characteristics

The ARSW frame under consideration (geometry illustrated in Figure 8) consists of 2 external single X-type upright frames and 4 internal double X-type upright frames connected to a "truss" roof. The section and the material of the uprights and diagonal bracings vary in height while the horizontal bracing has constant properties (see Table 3 for more details). The roof is comprised of double angle sections 45x45x4 with steel grade S355.

Regarding the connections, it was assumed that the base plates do not offer additional stiffness and thus the uprights were simulated as pinned to the foundation and the roof. Horizontal and diagonal bracings were also assumed pinned as they are commonly connected to the uprights by 1 or 2 bolts. A common practice in the design of pallet racking systems is to reduce diagonal bracings' cross-section area to take into account the looseness of their connection. The magnitude of this reduction can be quite significant (e.g. 80%) and is verified by experimental shear tests. However, for this particular test case the existence of class 1 uprights and diagonals may lead to the safe assumption that this reduction is negligible and thus it was not considered.

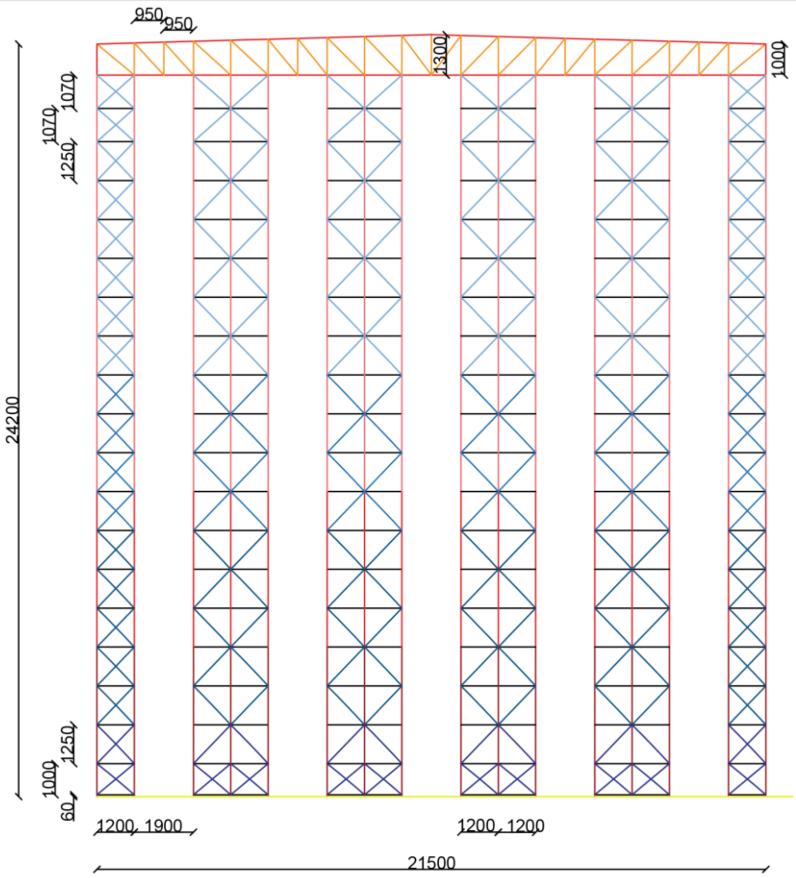


Figure 8: Configuration and geometric properties of ARSW frame test case (units in mm)

Height (m)	**Uprights (single)**	**Diagonal (single)**	**Diagonal (double)**	**Horizontal (both)**
0.00-2.31	RHS (S355) 120x80x10	L (S355) 40x40x5	L (S355) 40x40x5	DC (S355) 80x50x3
2.31-4.81	RHS (S355) 120x80x10	L (S275) 40x40x4	RHS (S355) 30x30x2.5	DC (S355) 80x50x3
4.81-9.75	RHS (S355) 120x80x6	L (S275) 40x40x4	RHS (S355) 30x30x2.5	DC (S355) 80x50x3
9.75-13.56	RHS (S355) 120x80x4	L (S275) 35x35x4	RHS (S275) 30x30x2.5	DC (S355) 80x50x3
13.56-23.2	RHS (S355) 120x80x4	L (S235) 30x30x4	RHS (S235) 30x30x2	DC (S355) 80x50x3

Table 3: Cross sections of upright frames. (RHS: Rectangular Hollowed Section, L: Angle, DC: Double Channel)

3.2 Reduced order models

Three models of decreasing accuracy will be examined:

1. *Fiber Model:* All structural members suspected to participate in structure's failure mechanism (i.e. uprights, diagonal and horizontal bracings) are simulated as force-based fiber elements [10] with 3 integration points. Especially for the diagonal bracings in compression, an imperfection L/200 is assumed, where L is the length of the element. For the rest elements classic Euler-Bernoulli beams where used.
2. *Truss Model:* In this case, uprights are assumed to behave linearly (class 1 sections), while the bracings are simulated by nonlinear truss elements. Their material law can be derived by EN1993 formulae or by isolating each bracing, simulate it by fiber elements, perform compression and tension arithmetic tests and use them to find an equivalent stress-strain diagram. For the rest elements classic Euler-Bernoulli beams where used.
3. *Link Model:* Here, an entire upright frame is substituted for a Two Node Link Element that includes shear failure. As mentioned before, Link Elements include axial, rotational and shear springs which may have nonlinear material laws. Here we will mainly focus on the characteristics of the shear spring.

To determine the elastic properties of the Link Elements first we have to substitute the upright frames for elastic Timoshenko Elements, using Eq (1) to Eq (6). Afterwards, spring's elastic stiffnesses can be readily evaluated using Table 2.

Next, the nonlinear behavior of the shear spring will be approximated. In initial configuration (Figure 9), shear forces are transferred through the diagonal bracings, while the horizontal bracing is unstressed due to symmetry. We define $N_{(+)}$, $N_{(-)}$ diagonal bracing's strength in tension and compression respectively and N_h buckling resistance of the horizontal bracing. The following relations hold:

- $N_{(+)} > N_{(-)}$, as in compression buckling phenomena are witnessed.

- $N_h > N_{(-)}$, as $h_0 < d$ and taking into account that horizontal bracing commonly is comprised of equal or even heavier section than the diagonal.

Thus, the first member exceeding its ultimate strength will be the diagonal bracing in compression, which corresponds to a *"yield shear strength"*:

$$V_y = 2N_{(-)}\frac{h_0}{d} \tag{10}$$

Figure 10 shows the failure mechanism of a X-type upright frame. One can claim that the system has transformed from X-type to Z-type and thus the shear area has changed. This will be referred as *"shear degradation"*. Link element's reduced shear stiffness after the degradation will be:

$$k_{y,shear} = \frac{12EI}{\left(1+\Phi_y\right)L^3} \tag{11}$$

, where Φ_y is calculated using the shear stiffness formula for the Z-type columns given in Table 1. Shear spring's *"yield deformation"* is derived by combining Eq (10), Eq (11) and:

$$\delta_y = \frac{V_y}{k_{y,shear}} \tag{12}$$

The upright frame has transformed from a X-type to a Z-type and thus, horizontal bracing is now compressed. The next structural member that will fail depends on their tensile and compressive strength. If $N_h < \cos\varphi N_{(+)} = \frac{h_0}{d} N_{(+)}$, the horizontal bracing in compression will fail first, otherwise the diagonal bracing in tension. Following a similar to Eq. (10), (11) and (12) procedure the complete force-displacement diagram can be calculated for each spring.

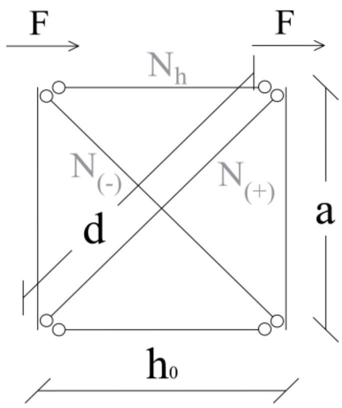


Figure 9: Shear transfer mechanism in initial configuration

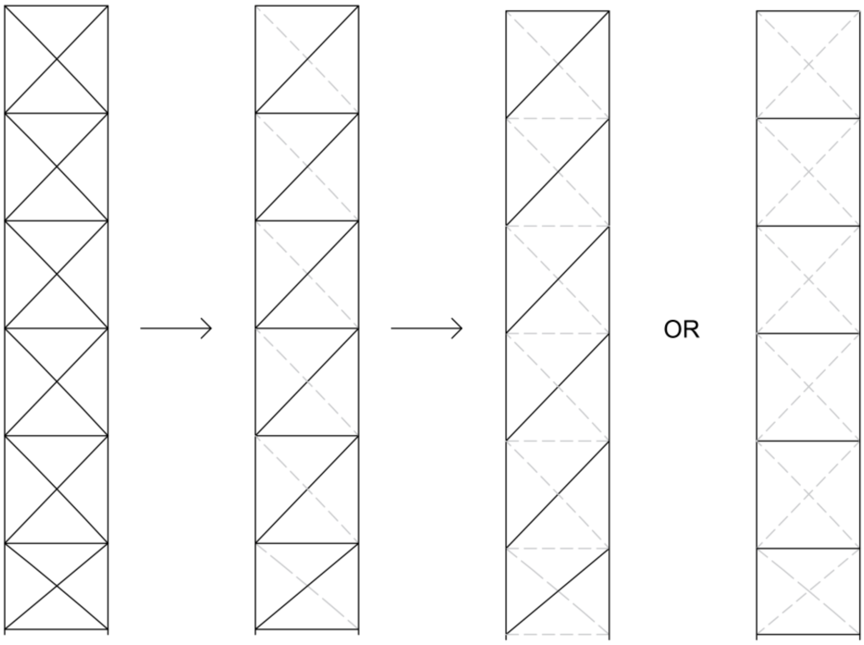


Figure 10: Failure sequence of X-type upright frame's bracing

3.3 Modal Analysis

First, Modal Analysis is performed to validate the Link Model in the elastic region. The masses are assumed to be lumped, of magnitude 10 kN at each level. The results for the five first eigenmodes are shown in Figure 11 and Figure 12. As it is observed, the Link Model predicts well even the higher modes of the system, and so, it is expected to give accurate results in linear dynamic analysis.

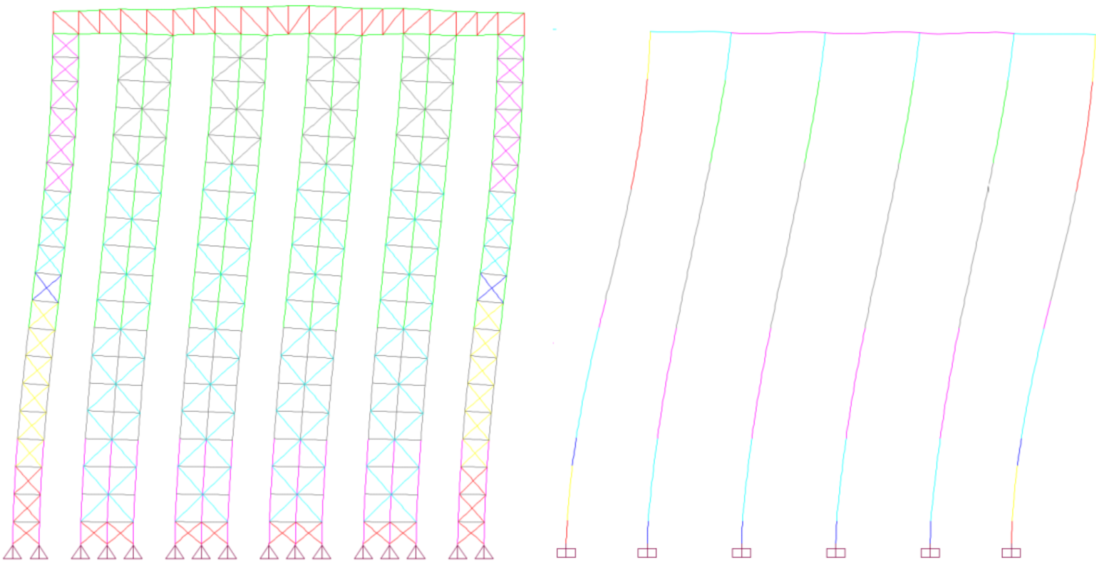


Figure 11: First Eigenmode of Full Model (T1,1=0.775 sec) and Link Model (T1,2=0.784 sec)

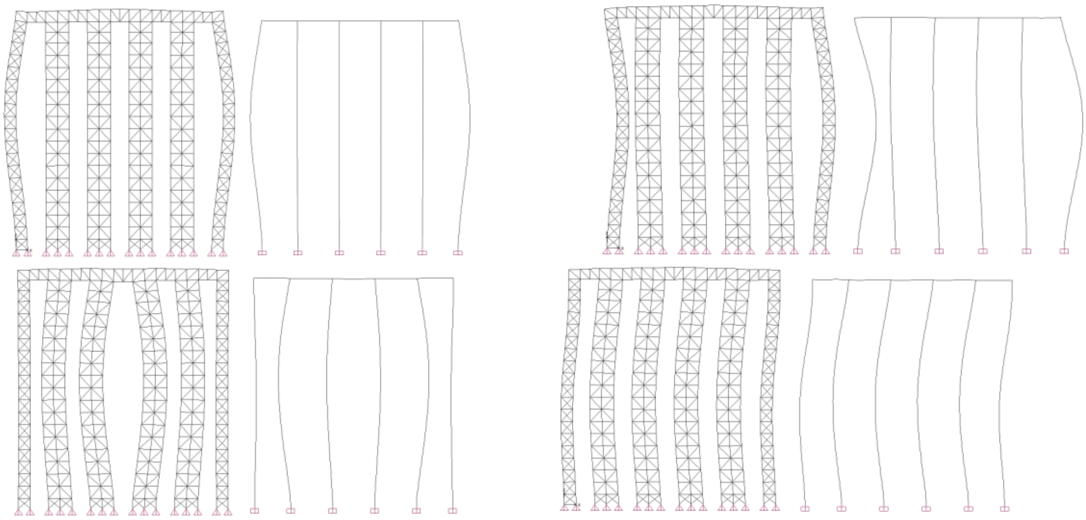


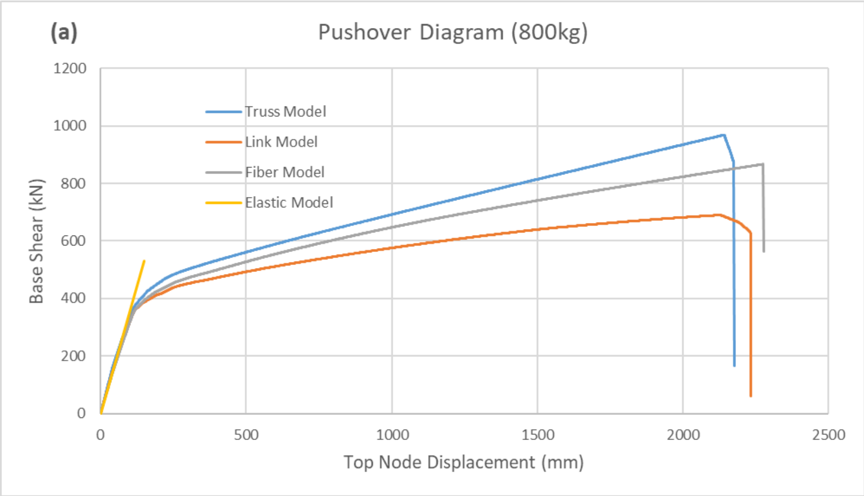
Figure 12: 2nd, 3rd, 4th and 5th mode shapes. 5% maximum relative error

3.4 Pushover Analysis

Next, Static Pushover Analyses were performed for the three models, assuming triangular distribution for the lateral loads. The analyses were executed until the system reached 10% roof drift or stability and non-convergence problems occurred. The Base-Shear vs Top Node Displacement diagrams are illustrated in Figure 13a and Figure 13b for 800 kg and 2000 kg pallet load respectively.

All models respond linearly and elastic until a roof displacement approximately equal to 150 mm is achieved. After this "limit state" is exceeded, the structure is highly nonlinear, and Pushover's slope decreases exponentially. This behavior was attributed to the successive failure of the diagonal bracings in tension. At about 500mm the slope has dropped to 7.2% of the elastic branch for the Link Model, 5.5% for the Truss Model and 7.7% for the Fiber Model. It is mentioned that steel's strain hardening was chosen equal to 5% which is roughly Pushover's residual slope.

Pallet load scenarios 800 and 2000 kg have a vast difference in post-capping behavior. In latter, P-delta phenomena are of major importance and the structure is not able to achieve high ductility and overstrength.



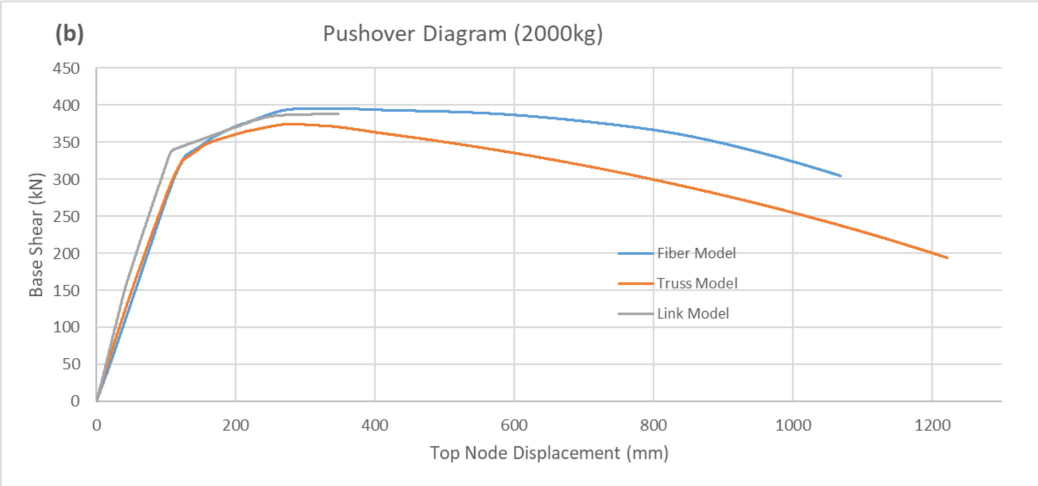
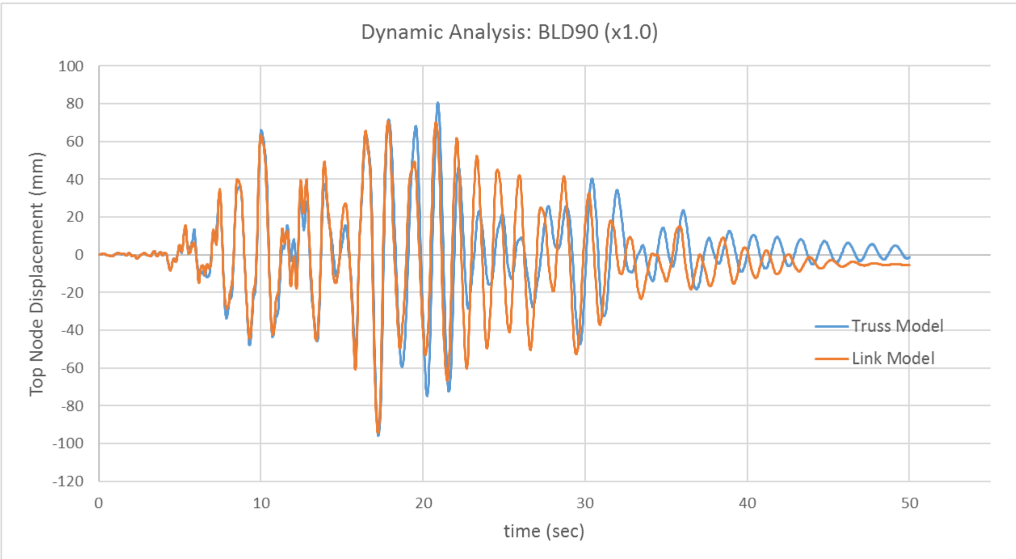


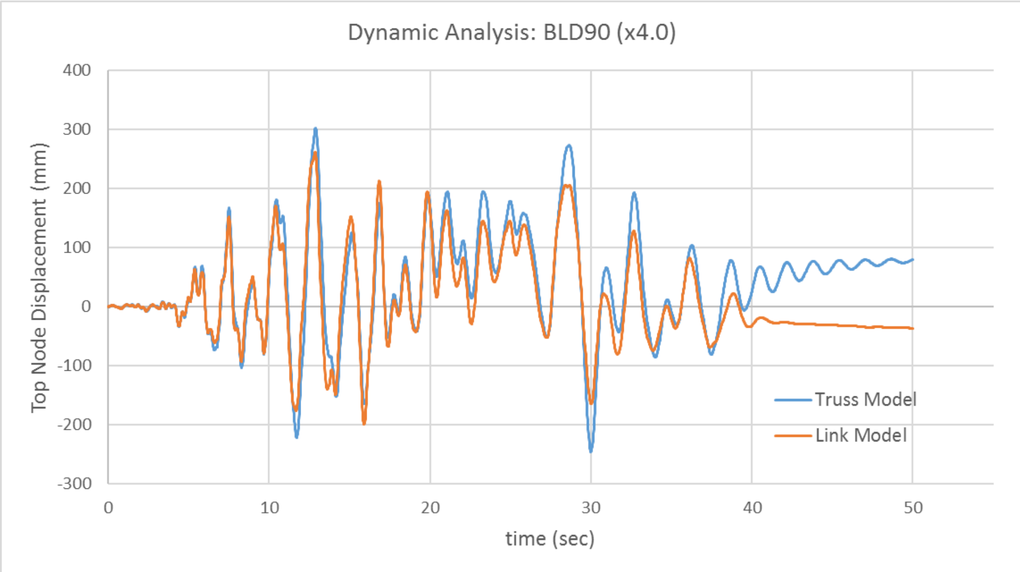
Figure 13: Base Shear vs Top Node Displacement of Pushover Analysis for the three models under examination; (a) 800 kg pallet load and (b) 2000 kg pallet load

3.5 Pushover Analysis

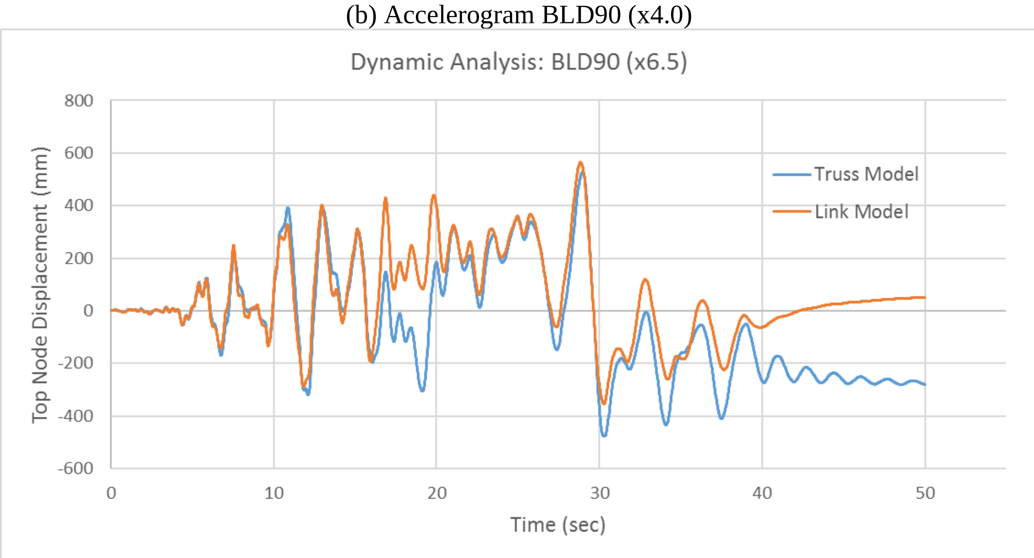
Nonlinear dynamic analyses were executed for the Truss and Link Model, and the time-history results are displayed in Figure 14a to 14d. Structural properties, geometry and distribution of masses were the same as in Pushover Analysis of 800 kg pallet load. In addition, both systems were assumed undamped, and thus residual oscillations are expected. It was observed that the Link Model was encouraging accurate, as it was able to predict adequately well the maximum displacement of the system.



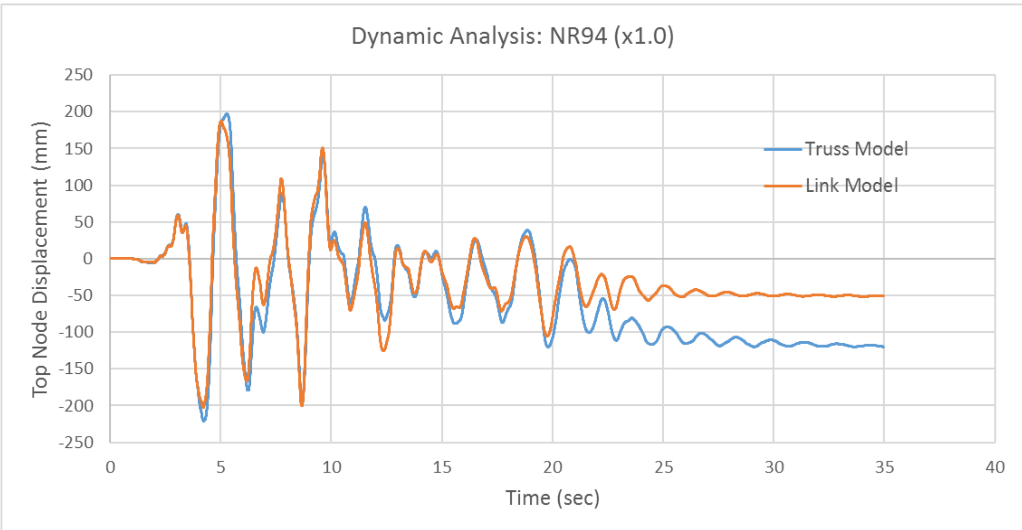
(a) Accelerogram BLD90 (x1.0)



(b) Accelerogram BLD90 (x4.0)



(c) Accelerogram, BLD90 (x6.5)



(d) Accelerogram NR94 (x1.0)

Figure 14: Results of time history analyses for the Truss and Link Model

4 CONCLUSIONS

The simplified method developed in present thesis was tested on a 2D Automated Rack Supported Warehouse frame for linear, nonlinear static and nonlinear dynamic analyses. In elastic region, the reduced-order model can predict extremely well even the higher eigenmodes of the real structure. A question remains about Linear Buckling Analysis, as upright's local buckling between bracings cannot be considered.

Concerning nonlinear analyses, the introduced model uses Two Node Link Elements to simulate the nonlinear response of upright frames and Timoshenko Beam Elements for the roof. In this specific test case, uprights were considerably stiff, so they did not participate in structure's plastic mechanism. Thus, we concentrated on bracings' compression and tension failure, which corresponds to shear degradation for the equivalent link element. As it was observed, X-column was "transformed" to Z-type after buckling of the diagonal bracing occurs and the system loses shear stiffness.

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