

COLLAPSE MARGIN AND BEHAVIOR FACTOR EVALUATION FOR EUROCODE-DESIGNED CONCENTRIC BRACED FRAMES

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Abstract. Modern structures are designed to trade strength for damage through the stable accumulation of plastic deformations in predefined “energy-dissipating” sacrificial elements. Logistically, this is taken into account by the use of the ubiquitous behavior (or strength reduction) factors that incorporate the effects of both ductility and overstrength to allow a simple and essentially elastic process of design. Despite the importance of such factors, their estimation is not subject to any rigorous rules, leaving large margins of uncertainty, especially for newly introduced lateral-loading systems where experience is lacking.

In the US, the estimation of behavior factors has been largely standardized by the introduction of the FEMA P695 guidelines. On the other hand, Eurocode 8 has not been paired with a similar compatible document to allow the seamless introduction of new systems. As an attempt to investigate the potential for introducing such a set of guidelines, an investigation of concentrically-braced frames is undertaken. The aim is to evaluate the currently offered margin of safety against collapse and discuss the possible basis one could use to found a reliable estimation for behavior factors, while accounting for issues of spectral shape, record selection and intensity measure sufficiency.

1 INTRODUCTION

The process followed in order to evaluate the behavior factor is based on the methodology that is described in the report of FEMAP695^[1] and it is proposed to be used in combination with seismic codes in order to provide the minimum acceptable criteria of design. This methodology is consistent with the demand of a low probability of collapse, while the structure is subjected to Maximum Considered Earthquake ground motions. In a hazard analysis, the maximum considered earthquake is an earthquake which is expected to occur once in 2500 years and has a probability of exceedance 2% in 50 years.

In order to evaluate the behavior factor, the execution of nonlinear static and dynamic analysis is necessary. Nonlinear static analysis (pushover analysis) is performed in order to determine the maximum base shear, V_{max} , the overstrength factor, Ω , and sometimes the ductility corresponding to loss of strength, μ_T . In Figure 1 an example of a pushover curve and the aforementioned quantities is illustrated.

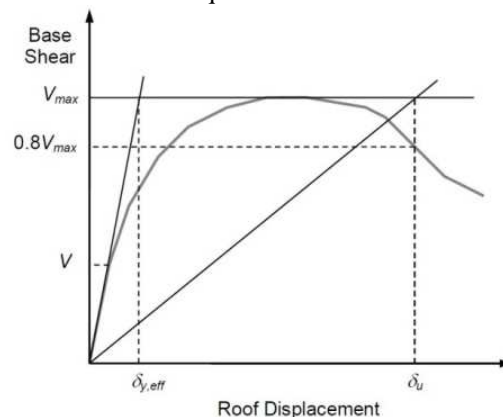


Figure 1 Example of a pushover curve (FEMAP695)

The overstrength factor, Ω is defined as the ratio of maximum force, V_{max} , developed in pushover over the design base force V as:

$$\Omega = \frac{V_{max}}{V} \quad (1)$$

The ductility, μ_T , is defined as the ratio of maximum displacement before strength loss to the yield displacement:

$$\mu_T = \frac{\delta_u}{\delta_{y,eff}} \quad (2)$$

Nonlinear dynamic analysis is executed in order to find the median collapse acceleration, \hat{S}_{CT} , and the collapse margin ratio, CMR for a maximum considered spectral acceleration S_{MT} . The average spectral acceleration S_{CT} is obtained from the results of Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell, 2002)^[2] in which independent seismic records are properly scaled to an incremental intensity until the structure reaches collapse. An example of results of a group of records is illustrated in Figure 2. The vertical axis is the spectral acceleration of the record and the horizontal axis is the maximum story drift. The collapse margin ratio can be obtained from the equation:

$$CMR = \frac{\hat{S}_{CT}}{S_{MT}} \quad (3)$$

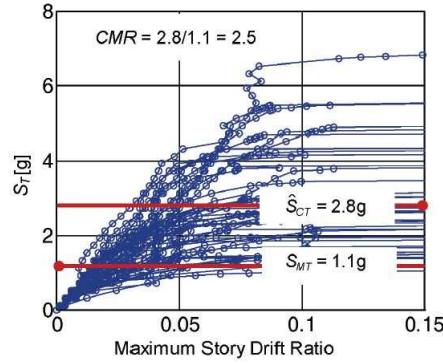


Figure 2 Example of a set of IDA Curves (FEMAP695)

For the specification of IDA curves, FEMA P695 uses the 5% damped first mode spectral acceleration as an intensity measure. Although $S_a(T_1, 5\%)$ seems to be effective for low and medium period buildings and low to moderate levels of ductility, it presents large dispersion for long period buildings and large ductilities. This is mainly due to the fact that this intensity measure is unable to represent the spectral acceleration of the important higher modes as well as the first mode's elongation due to the damage appearing in the severely inelastic range. Consequently, when IDA curves are presented in $S_a(T_1, 5\%)$ terms, there is a demand for a great number of records in order to get reliable results, while there is significant potential for conservative bias when close to collapse.

Taking into consideration all the above, as well as the fact that the response of multi-degree-of-freedom systems are sensitive to multiple periods T_i , the use of an intensity measure which takes into account the elastic spectral accelerations through a specific range of periods is essential. The geometric mean of spectral accelerations, AvgSa, has thus been proven to be a sufficient and efficient intensity measure^{[3],[4]}:

$$AvgS_a(T_1, \dots, T_n) = \left(\prod_{i=1}^n S_a(T_i) \right)^{1/n} \quad (4)$$

$$\ln AvgS_a(T_1, \dots, T_n) = \frac{1}{n} \sum_{i=1}^n \ln S_a(T_i) \quad (5)$$

T_1, \dots, T_n are the periods of interest, typically chosen to cover the significant higher modes and the elongation of the first mode period.

Alternatively, one can keep using $S_a(T_1)$ by appropriately correcting for bias via empirical factors. The FEMAP695 methodology recognizes that the ability of structure against collapse as well as the evaluation of

margin ratio CMR are largely affected by the spectral shape of the records. In order to consider the influence of spectral shape, the collapse margin ratio, CMR, is properly modified through the use of the spectral shape factor SSF using the equation:

$$ACMR_i = SSF_i \times CMR_i \quad (6)$$

Spectral shape factor is a function of the fundamental period, T, the ductility, μ_T , and the Seismic Design Category. The values of spectral shape factor for the Seismic Design Category D_{max} are given in Table 1.

Table 1 Spectral Shape Factor for the Seismic Design Category D_{max}

T(sec)	Ductility, μ_T							
	1	1.1	1.5	2	3	4	6	≥ 8
≤ 0.5	1.00	1.05	1.10	1.13	1.18	1.22	1.28	1.33
0.6	1.00	1.05	1.11	1.14	1.20	1.24	1.30	1.36
0.7	1.00	1.06	1.11	1.15	1.21	1.25	1.32	1.38
0.8	1.00	1.06	1.12	1.16	1.22	1.27	1.35	1.41
0.9	1.00	1.06	1.13	1.17	1.24	1.29	1.37	1.44
1.0	1.00	1.07	1.13	1.18	1.25	1.31	1.39	1.46
1.1	1.00	1.07	1.14	1.19	1.27	1.32	1.41	1.49
1.2	1.00	1.07	1.15	1.20	1.28	1.34	1.44	1.52
1.3	1.00	1.08	1.16	1.21	1.29	1.36	1.46	1.55
1.4	1.00	1.08	1.16	1.22	1.31	1.38	1.49	1.58
≥ 1.5	1.00	1.08	1.17	1.23	1.32	1.40	1.51	1.61

Acceptable values of collapse margin ratio, CMR, are based in the total uncertainty and in the values of acceptable probability of collapse. They presume that the distribution of intensity measures is lognormal, with an average value, S_{CT} and a dispersion equal to the total uncertainty of the system, β_{TOT} . Collapse uncertainty is a function of the quality ratings associated with the design requirements, test data, and nonlinear models, as well as record-to record uncertainty. FEMA P695 offers acceptable values of collapse margin ratio ACMR10% and ACMR20% based on the total uncertainty and the values of acceptable probability of collapse taken as 10% and 20% accordingly. ACMR10% is, generally speaking, the maximum acceptable value that may be considered if the specimen is part of a group of buildings used in the assessment, while ACMR20% is the maximum value that may be considered for an individual building.

Apart from the methodology followed in accordance with FEMAP695, the behavior of buildings is evaluated in terms of Mean Annual Frequency (MAF) of exceedance of a Collapse Prevention limit-state, which corresponds to a probability of exceedance 1% or 2% in 50 years. For the definition of MAF the establishment of a hazard curve of a specific area is necessary. For the case at hand, the seismic hazard curve was appropriately chosen to correspond to the elastic spectrum of Eurocode 8. To gain some insight into the actual collapse performance of the structure, it is examined whether the condition for $MAF_{collapse} < MAF_{lim}$ is met, where $MAF_{collapse}$ is the calculated MAF of collapse while MAF_{lim} is the Mean Annual Frequency with a probability of collapse 1% or 2% in 50 years.

2 EVALUATION OF THE BEHAVIOR FACTOR

An initial value for the behavior factor q can be estimated making use of pushover analysis as:

$$q = \frac{V_E}{V} = \Omega \cdot \mu_T \quad (7)$$

where Ω is the overstrength factor and μ_T is the ductility. As an attempt to approach a potential optimal value for the behavior factor, from the results of incremental dynamic analysis the following equation can be used instead:

$$q = \frac{ACMR}{Acc. ACMR} \cdot q_{design} \quad (8)$$

where q_{design} is the behavior factor actually used for design, ACMR is the value achieved in the building studied and Acc.ACMR the target (acceptable) value.

3 DESIGN OF BUILDINGS

Three steel buildings with three, six and twelve stories are designed according to the provisions of Eurocode 8^[5]. The plan dimensions are 54x36m for all the buildings and the story height is 3.5 m. Steel S355 was used throughout. The gravity loads used in the design of buildings are presented in Table 3. For the seismic loads we assumed ordinary importance (class II), Soil B, peak ground acceleration of $a_g = 0.24g$ (Zone II for Greece) and a behavior factor of $q = 4$.

Table 2 Design loads

Roof Loading	
Metal deck and concrete fill	3.11 kN/m ²
Superdead load	0.90 kN/m ²
Live loads	2.00 kN/m ²
Floor Loading	
Metal deck and concrete fill	3.11 kN/m ²
Superdead load	1.80 kN/m ²
Live loads	2.00 kN/m ²

In Figures 3-5 the floor plan of the designed buildings is illustrated. For the 3 and 6-story buildings, one braced bay on each side of the building is used. For 12-story building, two braced bays were used on each side of the building. In order to diminish the magnitude of P-Delta effects, a second version of the twelve story building is also studied, strengthened by placing two more braced bays in the centre of the building in each direction.

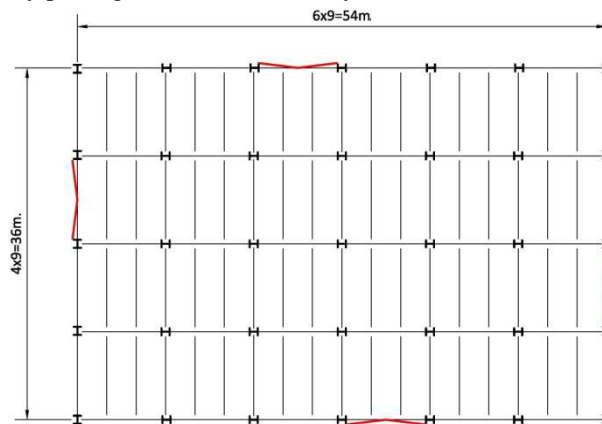


Figure 3 Floor plan of the 3 and 6-story building

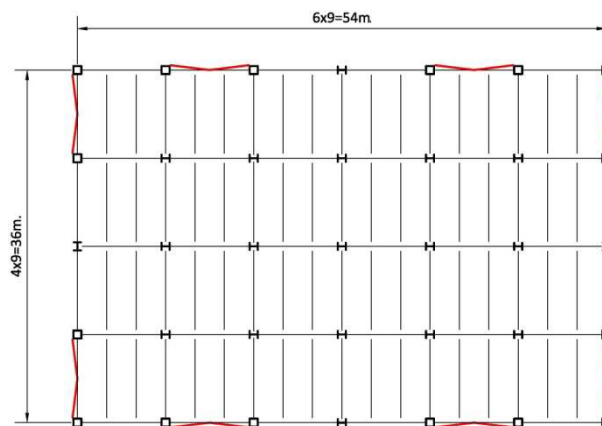


Figure 4 Floor plan of the 12-story building

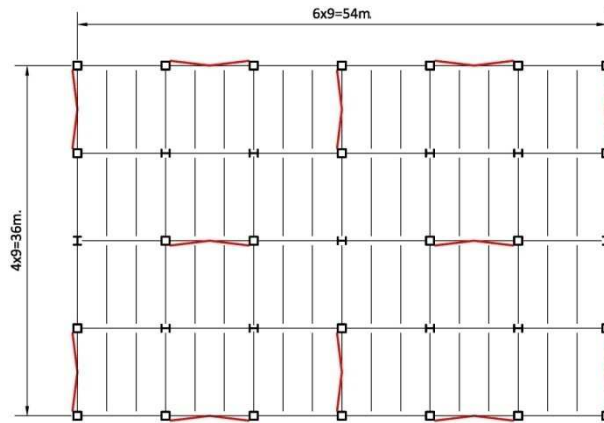


Figure 5 Floor plan of the strengthened 12-story building

Table 3 Member sizes for the 3-story building

Story	Braces	Columns	Beams
3	SHS140x10	HEB360	HEM600
2	SHS140x14.2	HEB360	HEA450
1	SHS140x16	HEB360	HEA360

Table 4 Members sizes for the 6-story building

Story	Braces	Columns	Beams
6	SHS140x7.1	HEA450	HEA360
5	SHS140x8	HEA450	HEA360
4	SHS140x14.2	HEB500	HEA450
3	SHS140x14.2	HEB500	HEA360
2	SHS150x16	HEM550	HEB450
1	SHS150x16	HEM550	HEA360

Table 5 Member sizes for the 12-story building

Story	Braces	Columns	Beams
12	SHS120X5	SHS600X20	HEA360
11	SHS120X5	SHS600X20	HEA360
10	SHS140X10	SHS600X20	HEA400
9	SHS140X10	SHS600X20	HEA360
8	SHS150X11	SHS600X20	HEA450
7	SHS150X11	SHS600X20	HEA360
6	SHS150X14.2	SHS600X25	HEA450
5	SHS150X14.2	SHS600X25	HEA360
4	SHS150X14.2	SHS600X30	HEB450
3	SHS150X14.2	SHS600X30	HEA360
2	SHS150X16	SHS600X35	HEB450
1	SHS150X16	SHS600X35	HEA360

Table 6 Members sizes for the strengthened 12-story building

Story	Braces	Columns	Beams
12	SHS100X5.4	SHS500X20	HEA360
11	SHS100X4	SHS500X20	HEA360
10	SHS120X7.1	SHS500X20	HEA400
9	SHS120X7.1	SHS500X20	HEA360
8	SHS140X8	SHS500X20	HEA450
7	SHS140X8	SHS500X20	HEA360
6	SHS140X10	SHS500X25	HEA450
5	SHS140X10	SHS500X25	HEA360
4	SHS140X10	SHS500X30	HEB450
3	SHS140X10	SHS500X30	HEA360
2	SHS140X12.5	SHS500X35	HEB450
1	SHS140X12.5	SHS500X35	HEA360

4 NON LINEAR ANALYSIS

In order for the nonlinear analysis to be executed, a two-dimensional model of a single building frame was implemented in Opensees. All beam-column connections were taken as pinned. Columns were assumed to be fixed at the base. A leaning column is also added to introduce the gravity loads not carried by the braced frame. All braces and their connection to the frame were simulated in accordance to the model proposed by Uriz and Mahin^[6]. An initial imperfection is introduced in the middle of each brace to enable buckling in compression. Finally, to represent the physical size and stiffening effect of the connections, rigid offsets were assumed at the beam-column connections and brace-to-framing connections. A non-linear rotational spring was used to simulate the behavior of gusset plates, the characteristics of which are calculated according to the relations proposed by Hsiao et al.^[7].

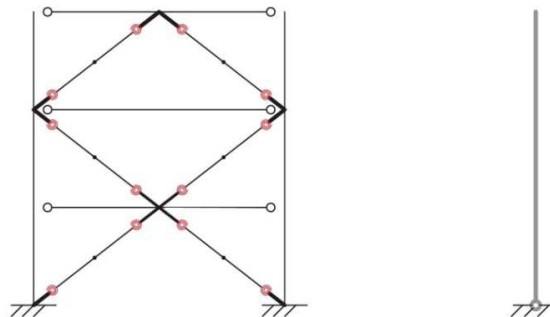


Figure 6 Two dimensional model in Opensees for the 3-story building. The leaning column (right) is constrained to follow the deformation of each story and carries the mass and loads of gravity frames

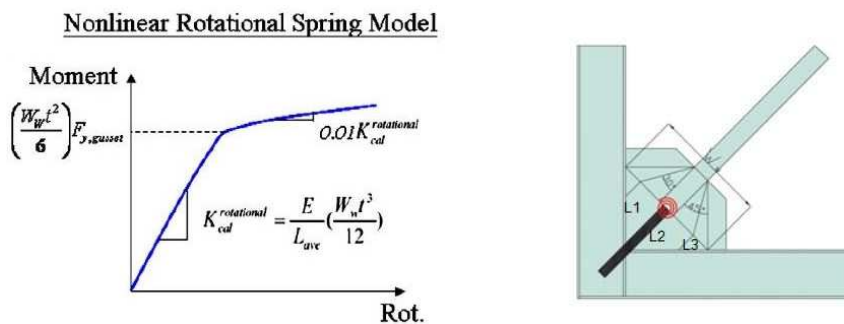


Figure 7 Conceptual modeling of connections in Opensees^{[6],[7]}

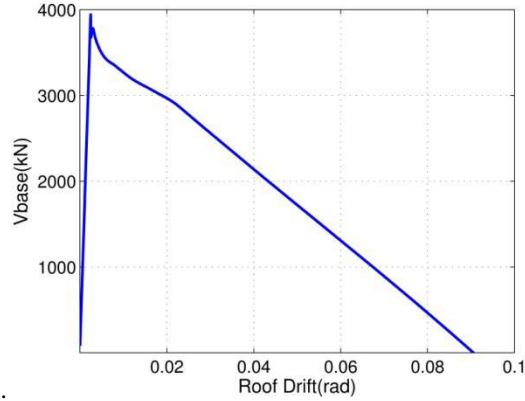


Figure 8 Pushover Curve of the 3story building

In Figure 8, the pushover curve of the 3-story building is illustrated. The system yielded at a roof drift ratio of 0.25%. The maximum strength at the base was 3943 kN and maximum drift which corresponds to the 80% of maximum strength was 1.33%. The design base force for the fundamental period $T_1=0.54$ sec is $V_d=2818.81$ kN. The overstrength factor was computed as $\Omega = 3943\text{kN} / 2818.81\text{kN} = 1.41$ and the ductility as $\mu_T = 0.0133 / 0.0025 = 5.41$

Nonlinear Incremental Dynamic Analysis is a powerful method of analysis which includes the execution of multiple nonlinear analysis under a set of ground motions, each scaled to an appropriate intensity measure. IDA results are presented via IDA curves, one for each record, plotted in terms of the intensity measure (IM) against the response engineering demand parameter (EDP). As regards the intensity measure, both the first mode spectral acceleration, $S_a(T_1)$ and the geometric mean of spectral accelerations, $AvgS_a$ are used. The results of the analysis are summarized in fractile curves which correspond to the 16, 50 and 84% of accelerations which lead to the collapse and aim to represent the distribution of data. The percentage 50% of the accelerations is the one which corresponds to the collapse acceleration \hat{S}_{CT} which is used in evaluation methods. In Figure 9 the results of IDA of the 3story building are illustrated. The average value of spectral accelerations which lead to the collapse is $S_{CT} = 2.53$ g. Following FEMA P695, the maximum considered elastic design acceleration is $S_{MT} = 1.0$ and the collapse margin ratio is computed as $CMR = S_{CT} / S_{MT} = 2.53$.

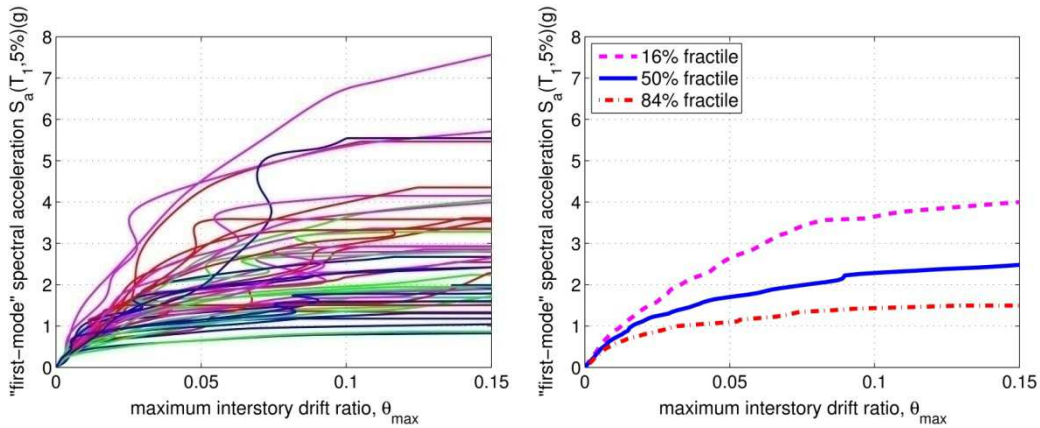


Figure 9 Individual record and fractile IDA curves for the 3story building ($IM=S_a(T_1)$)

5 COLLAPSE MARGIN AND BEHAVIOR FACTOR EVALUATION

In an attempt to evaluate the general behavior of buildings, the mean annual frequency of exceedance of the spectral acceleration which corresponds to collapse is calculated and compared to the maximum permitted level of mean annual frequency MAF_{lim} for probability of collapse 1% or 2% in 50 years. For the evaluation of mean annual frequency of exceedance, hazard surfaces of mean annual frequency of the area Van Nuys of California and Istanbul are used, the spectrum of which is adapted via scaling to the Eurocode 8 spectrum. Then the level of confidence is estimated on whether the mean annual frequency of exceedance of the collapse level, $MAF_{collapse}$ is lower than the MAF which corresponds to 1% or 2% in 50 years when considering the inherent uncertainty.

Table 7 Evaluation of behavior for 2% in 50yrs (MAFlim=0.0004) and Intensity Measure $S_a(T_1)$ using the California hazard data.

Building	sf	b_{TSa}	$S_a(T_1)$		$S_a(T_1) \cdot SSF$	
			MAF _{collapse}	CONF	MAF _{collapse}	CONF
3story	0.102	0.536	0.00015	97.95%	0.00007	99.93%
6story	0.081	0.487	0.00030	79.63%	0.00013	98.98%
12story	0.087	0.447	0.00036	68.12%	0.00017	96.91%
12story(strengthened)	0.084	0.483	0.00027	84.54%	0.00013	99.12%

Table 8 Evaluation of behavior for 2% in 50yrs (MAFlim=0.0004) and Intensity Measure $S_a(T_1)$ using the Istanbul hazard data

Building	sf	b_{TSa}	$S_a(T_1)$		$S_a(T_1) \cdot SSF$	
			MAF _{collapse}	CONF	MAF _{collapse}	CONF
3story	1.625	0.536	0.00010	99.00%	0.00004	99.98%
6story	1.267	0.487	0.00020	91.61%	0.00007	99.80%
12story	1.109	0.447	0.00027	82.71%	0.0001	99.07%
12story(strengthened)	1.302	0.483	0.00019	93.17%	0.00007	99.78%

Table 9 Evaluation of behavior for 2% in 50yrs (MAFlim=0.0004) and Intensity Measure AvgSa using the Istanbul hazard data

Building	sf	b_{TSa}	Avg S_a	
			MAF _{collapse}	CONF
3story	1.625	0.505	0.00005	99.89%
6story	1.267	0.475	0.00008	99.24%
12story	1.109	0.427	0.00008	99.15%
12story(strengthened)	1.302	0.464	0.00007	99.60%

Table 10 Evaluation of behavior for 1% in 50yrs (MAFlim=0.0002) and Intensity Measure $S_a(T_1)$ using the California hazard data

Building	sf	b_{TSa}	$S_a(T_1)$		$S_a(T_1) \cdot SSF$	
			MAF _{collapse}	CONF	MAF _{collapse}	CONF
3story	0.102	0.536	0.00015	78.55%	0.00007	97.45%
6story	0.081	0.487	0.00030	24.88%	0.00013	83.89%
12story	0.087	0.447	0.00036	12.99%	0.00017	72.05%
12story(strengthened)	0.084	0.483	0.00027	33.98%	0.00013	85.48%

Table 11 Evaluation of behavior for 1% in 50yrs (MAFlim=0.0002) and Intensity Measure $S_a(T_1)$ using the Istanbul hazard data

Building	sf	b_{TSa}	$S_a(T_1)$		$S_a(T_1) \cdot SSF$	
			MAF _{collapse}	CONF	MAF _{collapse}	CONF
3story	1.625	0.536	0.00010	90.35%	0.00004	99.31%
6story	1.267	0.487	0.00020	62.70%	0.00007	96.55%
12story	1.109	0.447	0.00027	40.71%	0.0001	89.87%
12story(strengthened)	1.302	0.483	0.00019	65.59%	0.00007	96.07%

Table 12 Evaluation of behavior for 1% in 50yrs (MAFlim=0.0002) and Intensity Measure AvgS_a, using the Istanbul hazard data

Building	sf	b _{TSa}	AvgS _a	
			MAF _{collapse}	CONF
3story	1.625	0.505	0.00005	98.43%
6story	1.267	0.475	0.00008	93.12%
12story	1.109	0.427	0.00008	92.51%
12story(strengthened)	1.302	0.464	0.00007	95.52%

The evaluation of the behavior factor is performed in accordance with the FEMAP695 methodology. The aim is to find the adjusted Collapse Margin Ratio ACMR which is obtained by nonlinear analysis and to compare it with an acceptable ACMR depending on total uncertainty. As previously mentioned, the ACMR20% is the minimum acceptable value when the specimen is an individual building, while ACMR10% is the minimum acceptable value when a group of buildings is examined. In this case, due to the absence of a sufficient number of buildings in order to categorize them into groups, the ACMR10% is used. The total uncertainty β_{TOT} is either estimated from IDA analysis or taken as 0.525 according to FEMA P695 for good quality data and analysis.

Table 13 Evaluation of the behavior factor for Sa(T₁) and $\beta_{TOT}=0.525$

Building	T ₁	μ_T	S _{MT}	S _{a50%}	CMR	SSF	ACMR	β_{TOT}	Acc.ACMR	Check	q
3story	0.54	5.41	1.00	2.53	2.53	1.26	3.19	0.525	1.96	Pass	6.51
6story	1.14	4.83	0.47	0.93	1.96	1.35	2.65			Pass	5.41
12story	1.80	2.77	0.30	0.52	1.73	1.30	2.25			Pass	4.60
12story (str)	1.70	2.94	0.32	0.64	2.01	1.31	2.64			Pass	5.39

Table 14 Evaluation of the behavior factor for Sa(T₁) and β_{TOT} estimated from IDA

Building	T ₁	μ_T	S _{MT}	S _{a50%}	CMR	SSF	ACMR	β_{TOT}	Acc.ACMR	Check	q
3story	0.54	5.41	1.00	2.53	2.53	1.26	3.19	0.536	1.97	Pass	6.47
6story	1.14	4.83	0.47	0.93	1.96	1.35	2.65	0.487	1.86	Pass	5.70
12story	1.80	2.77	0.30	0.52	1.73	1.30	2.25	0.447	1.77	Pass	5.09
12story (str)	1.70	2.94	0.32	0.64	2.01	1.31	2.64	0.483	1.86	Pass	5.68

Table 15 Evaluation of the behavior factor for AvgSa and $\beta_{TOT}=0.525$

Building	T ₁	μ_T	S _{MT}	S _{a50%}	CMR	SSF	ACMR	β_{TOT}	Acc.ACMR	Check	q
3story	0.54	5.41	0.91	2.43	2.66	1.00	2.66	0.525	1.96	Pass	5.43
6story	1.14	4.83	0.54	1.06	1.97	1.00	1.97			Pass	4.03
12story	1.80	2.77	0.34	0.62	1.85	1.00	1.85			Fail	3.78
12story (str)	1.70	2.94	0.34	0.72	2.15	1.00	2.15			Pass	4.39

Table 16 Evaluation of the behavior factor for AvgSa and β_{TOT} estimated from IDA

Building	T ₁	μ_T	S _{MT}	S _{a50%}	CMR	SSF	ACMR	β_{TOT}	Acc.ACMR	Check	q
3story	0.54	5.41	0.91	2.43	2.66	1.00	2.66	0.505	1.90	Pass	5.60
6story	1.14	4.83	0.54	1.06	1.97	1.00	1.97	0.475	1.84	Pass	4.29
12story	1.80	2.77	0.34	0.62	1.85	1.00	1.85	0.427	1.72	Pass	4.31
12story (str)	1.70	2.94	0.34	0.72	2.15	1.00	2.15	0.464	1.81	Pass	4.75

6 COLLAPSE MARGIN AND BEHAVIOR FACTOR EVALUATION

An investigation is presented on the evaluation of behavior factor used for seismic code design. Different potential approaches were compared, starting from the basis of FEMA P695, arguably the only standard available. Different directions were studied, using alternative intensity measures and a better integration of uncertainty. When using as an intensity measure the first mode spectral acceleration, $S_a(T_1)$ the resultant uncertainty was similar or lower in comparison with the uncertainty obtained by the Tables of FEMAP695 for a good model and design quality. Using the geometric mean of spectral accelerations lower dispersions were achieved, offering more reliable estimates without the need for an empirical spectral shape correction factor.

In addition, the results indicate that the allowable behavior factor differs with the period (and number of stories) of the structure. Longer periods seem to be associated with lower behavior factors, indicating that the use of a single period-independent value for q may be unfairly biasing the design of low/mid-rise structures to be more conservative than needed.

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