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A MODIFIED RESPONSE SPECTRUM METHOD FOR ESTIMATING PEAK FLOOR ACCELERATION DEMANDS IN ELASTIC REGULAR FRAME STRUCTURES

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ABSTRACT

In this paper an extended complete quadratic combination rule for quick assessment of peak floor acceleration demands (PFA) of elastic structures subjected to seismic excitation is proposed. The simplification from time history analysis to the response spectrum method is shown in detail. Based on a relative acceleration formulation combined with nonlinear optimization techniques cross correlation coefficients are determined to estimate relative and absolute PFA demands. For estimation of central tendency and dispersion of the seismic response, regression equations are derived to provide a simple implementation of the method in civil engineering design practice. Application of the proposed procedure to a 24-story moment resisting generic frame structure shows the improvement compared to common response spectrum methods.

INTRODUCTION

In seismic loss assessment there is a need for a simple but still sufficiently accurate method to predict peak floor acceleration (PFA) demands for reasons outlined in the following.

- Peak floor accelerations are used to define building content losses, typically accounting for a significant percentage of the total loss.
- Application of methods of nonlinear dynamics analysis such as incremental dynamic analysis (IDA) (Vamvatsikos and Cornell, 2002)) is still a challenge. Nonlinear dynamic analysis requires a high level of knowledge on issues such as modeling of nonlinear component behavior, viscous damping, and numerical procedures, etc., and interpretation of results needs experience. Furthermore, commercial software still needs further improvements to make nonlinear time response history analysis methods more appealing for the design engineer.
- Simpler methods of analysis may be approximate in nature, yet they can offer a wealth of information that can be exploited without incurring a large computational cost. Thus, they can make current assessment methods (e.g. ATC-58-1, 2012) accessible to practicing engineers.

In the present paper a modified response spectrum method is proposed to predict peak floor acceleration demands of elastic, regular, plane moment resisting frame structures. PFA demands are the basis of floor acceleration spectra used to assess acceleration sensitive contents and nonstructural components.

López-García et al. (2008) give an excellent overview of PFA demand assessment. From an evaluation of codified methods such as proposed in ASCE 7-10 (2010) and EN 1998-1 (2013) they

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come to the conclusion that these methods lead in many case to inappropriate PFA demand predictions. PFA demands are either over-predicted, resulting in uneconomic design, or underestimated with consequences such as structural/nonstructural damage and risk of life safety in a severe earthquake event. For instance, dividing the lateral story force obtained according to equivalent lateral force pattern method (ELFP) by the corresponding story mass leads to inappropriate PFA predictions (López-García et al., 2008). Methods based on response spectrum analysis using various modal combination rules such as SRSS and CQC also fail in many cases to predict the median PFA demand (López-García et al., 2008). Furthermore, Chaudhuri and Hutchinson (2004) and Miranda and Taghavi (2009) show that procedures based on seismic actions on rigid nonstructural components lead to unrealistic PFA estimations.

Other guidelines and recommendations, such as ATC-58-1 (2012), provide simplified equations for assessing peak response quantities of elastic and inelastic structures that lead in general to more adequate response predictions. However, each simplification comes along with restrictions in application. For instance, the procedure described in ATC-58-1 (2012) is limited to structures with 15 stories, and for each generic type of lateral load bearing system (shear wall, moment resisting frame or brace frame) a different formulation is provided.

In summary, most of the available procedures fail to provide realistic PFA demands, the physical interpretation of the outcomes is questionable, and/or the limitations for application are too narrow. Alternatively, Taghavi (2006) and Taghavi and Miranda (2008) propose a modified CQC rule in terms of the response spectrum method to predict absolute acceleration demands of elastic structures. The benefit of the modified CQC method is a clearly defined mathematical/physical model without any limitations, and the delivered response predictions are sufficiently accurate. In a more recent study on PFA demand prediction by Pozzi and der Kiureghian (2008) it is proposed to consider the contribution of truncated modes, equivalent to the static correction method (Chopra 2012).

In the present study an extended CQC rule for absolute PFA demand prediction, based on a relative acceleration formulation, is proposed. As an alternative to average power spectra density of ground motions, nonlinear optimization techniques and regression analysis are used to derive analytical expressions for cross correlation coefficients. As a showcase a 24-story moment resisting generic frame according to Medina (2003) that is subjected to the records of the far field ground motion set defined in FEMA P695 (2009) is considered.

FUNDAMENTALS

Consider a planar regular *N*-story generic frame structure subjected at its base to a random signal, such as a ground motion defined by the acceleration series $\ddot{w}_g(t)$. The coupled set of equations of motion of the multi-degree-of-freedom (MODF) system with the masses concentrated at the beam-column connection reads (Chopra, 2012):

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\mathbf{e}\ddot{w}_g(t) \tag{1}$$

In this equation $\mathbf{u}(t)$ is the vector of displacements relative to the base related to the dynamic degrees-of-freedom, **M** is the mass matrix, **K** is the stiffness matrix, and **e** denotes the quasi-static influence vector. It is assumed that the orthogonality properties of the mode shapes $\boldsymbol{\phi}_i$, $i = 1, ..., N$, of the undamped system also hold for damping matrix **C**. This can be achieved by defining the damping matrix **C** according to *Rayleigh* as a linear combination of matrices **M** and **K**, or alternatively, by assigning a damping ratio to each modal equation individually (Chopra, 2012). In the present study, each mode is damped equally with 5%.

Solving the equations of motion at each time instant, i.e., time history analysis, leads to the “most accurate” approximation of the real behavior of the earthquake excited structure. This requires a tool for dynamic structural analysis, and a set of recorded ground motions representative for the location of the building. In many cases, however, the earthquake excitation is simply characterized by a response spectrum. Thus, response spectrum analysis (RSA) (Chopra, 2012) is an attractive alternative method to predict seismic demands that is less elaborate since it can be conducted with static tools of structural analysis.

The basis of RSA is an expansion of vector $\mathbf{u}$ in terms of modal contributions (Chopra, 2012),

$$\mathbf{u}(t) = \sum_{i=1}^{N} \boldsymbol{\phi}_i q_i(t) \tag{2}$$

where $\boldsymbol{\phi}_i$, $i = 1,...,N$, are the mode shapes, and q_i are the generalized or normal coordinates. Using this equation, the coupled set of equations of motion represented by Eq. (1) are transformed to a set of N uncoupled equations with modal coordinates D_i (Chopra, 2012),

$$D_i(t) = \frac{q_i(t)}{\Gamma_i}, \quad \Gamma_i = \frac{\boldsymbol{\phi}_i^T \mathbf{Me}}{\boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\phi}_i} \tag{3}$$

each uncoupled equation being in the form of the equation of motion of an SDOF system,

$$\ddot{D}_i(t) + 2\zeta_i \omega_i \dot{D}_i(t) + \omega_i^2 D_i(t) = -\ddot{w}_g(t) \quad , \; i = 1,...,N \tag{4}$$

These equations can be solved for D_i with standard methods of dynamic analysis. Then, the physical coordinates are determined through Eqs. (3) and (2). For instance, the absolute acceleration demand is determined according to

$$\ddot{\mathbf{u}}^{(tot)}(t) = \sum_{i=1}^{n \le N} \boldsymbol{\phi}_i \Gamma_i \ddot{D}_i(t) + \sum_{i=1}^{n \le N} \boldsymbol{\phi}_i \Gamma_i \ddot{w}_g(t) \tag{5}$$

The first sum of Eq. (5) is the contribution due to relative acceleration, and the second sum is the participation of the ground acceleration to the absolute acceleration. When combining the modal responses to the total response the number of considered modal contributions n is usually much smaller than N. If all modes are included (n = N), the second term of Eq. (5) is equal to $\mathbf{e}\ddot{w}_g(t)$.

MODIFIED MODAL RESPONSE SPECTRUM ANALYSIS

In modal response analysis, the peak value of each modal coordinate D_i is determined through a response spectrum. Modal combination of the single modal contributions to the total peak response in physical coordinates is, however, not straightforward, since a response spectrum does not contain information about sign and the phase shift of modal peak responses. Consequently, traditional modal superposition rules, such as the SRSS rule (Rosenblueth and Elorduy, 1969) and the CQC method (Wilson et al., 1981), were developed to assess relative peak response quantities such as the maximum displacement relative to the base.

When estimating the profile of PFA demands (i.e., the *absolute* acceleration) in terms of a modal superposition procedure, the peak ground acceleration (PGA) must be taken into account (Taghavi, 2006; Taghavi and Miranda, 2008; Pozzi and der Kiureghian, 2008). This becomes obvious when applying to Eq. (5) an extended complete quadratic combination rule (Taghavi, 2006):

$$\max\left(\left|\ddot{\mathbf{u}}^{(tot)}(t)\right|\right) = \mathbf{PFA} \approx \left[\sum_{i=1}^{n}\sum_{j=1}^{n} \phi_i \phi_j \Gamma_i \Gamma_j S_{a,i}^{(rel)} S_{a,j}^{(rel)} p_i p_j \rho_{ij} + 2PGA\sum_{i=1}^{n} \phi_i \Gamma_i S_{a,i}^{(rel)} p_i p_g \rho_{ig} + \mathbf{e}PGA^2 p_g^2\right]^{\frac{1}{2}} \tag{6}$$

In this equation the first term is the modal combination of the relative modal peak floor acceleration demands, where $S_{a,i}^{(rel)} = S_a^{(rel)}(T_i)$ is the spectral relative acceleration at the *i*th structural period T_i, and p_i denotes the peak factor ratio, respectively, corresponding to the *i*th mode. Coefficient ρ_{ij} quantifies the cross correlation between *i*th and *j*th modal contribution. The second term of Eq. (6) is the contribution from the correlation of the PGA with the relative modal spectral accelerations $i, i = 1,...,n$. The corresponding cross correlation coefficients and peak factors ratios are denoted as ρ_{ig}, p_{ig}, and p_g, respectively. Subscript g refers to quantities of the PGA. The PGA may be considered as a rigid mode, and thus, cross correlation coefficients ρ_{ig} are vector elements, whereas coefficients ρ_{ij} are the elements of a matrix. The last term of Eq. (6) expresses the correlation of the PGA with itself. However, for a physical interpretation peak factor ratios may be seen as conversation factors between spectral- and total acceleration and they depend on relative height of a floor in the structure as discussed later.

Approximation of the relative acceleration response spectrum

Eq. (6) is based on the response spectrum of *relative* acceleration (relative with respect to the base). Codified spectra are, however, given in terms of (absolute) spectral pseudo-acceleration demands, $S_a^{(abs)}(T)$. Assuming that $f(T)$ is a known adjustment function, the relation between the absolute and relative spectral peak acceleration demands is given by

$$S_a^{(rel)}(T) = S_a^{(abs)}(T) - f(T) \tag{7}$$

Subsequently, as an example the adjustment functions $f(T)$ are derived for the 44 records of the FEMA P695 ground motion set (FEMA P695, 2009) as the difference of absolute and relative spectral peak accelerations. In Fig. 1 gray lines show the adjustment functions for the 44 records plotted against period T. The corresponding statistical quantities, i.e., median and 16th and 84th percentiles are plotted in red. For practical application it desirable to have for $f(T)$ an analytical expression available. Inspection of the empirical data leads to the conclusion that the statistical quantities of the adjustment function can be approximated by a linear combination of two exponential terms,

$$f(T) \approx \alpha_0 + \sum_{k=1}^{2} \alpha_{k,1} e^{-\alpha_{k,2}T} \tag{8}$$

Subsequent regression analysis yields the appropriate coefficients α_0, $\alpha_{k,1}$ and $\alpha_{k,2}$ ($k = 1,2$) depending on the statistical quantity as listed in Table 1. Figure 2 depicts in blue these smooth analytical fits of the adjustment functions.

Table 1. Regression coefficients for analytical approximation of the adjustment function

Percentile	α_0	$\alpha_{1,1}$	$\alpha_{1,2}$	$\alpha_{2,1}$	$\alpha_{2,2}$
16th	-2.28	3.55	1.87	2.40	1.87
50th	-3.35	1.04	0.53	6.36	2.73
84th	-4.72	4.48	0.52	3.59	4.58

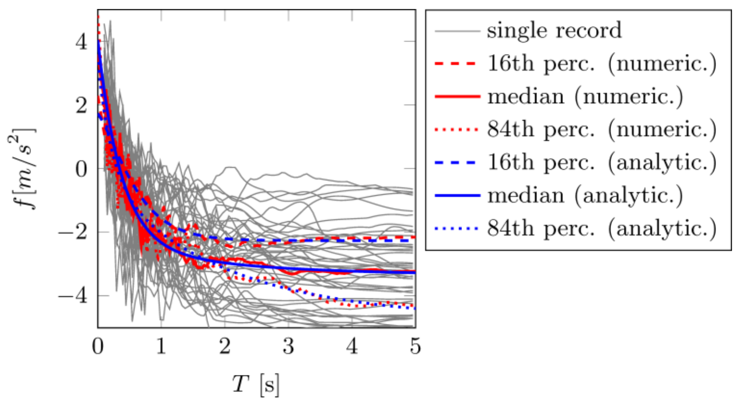

Figure 1. Adjustment function for single records (gray), statistics (red), and fitting (blue)

Identification of cross correlation coefficients

The cross correlation coefficients and the corresponding peak factors of Eq. (6) are a priori unknown variables. In the traditional CQC combination rule as proposed in Der Kiureghian (1981) cross correlation coefficients are defined as the ratio of cross-spectral moments of the power spectra. In this approach the peak factors are neglected. Der Kiureghian (1981) conducted a first order approximation of the cross-spectral moments, yielding the traditional CQC modal combination rule.

Taghavi (2006), however, shows that for the assessment of PFA demands peak factors cannot always be neglected. Therefore, Eq. (6) is re-written in terms of peak factor ratios:

$$\max\left(\left|\ddot{\mathbf{u}}^{(tot)}(t)\right|\right) = \mathbf{PFA} \approx \left[\sum_{i=1}^{n}\sum_{j=1}^{n} \phi_i \phi_j \Gamma_i \Gamma_j S_{a,i}^{(rel)} S_{a,j}^{(rel)} \frac{p_i p_j}{p_g^2} \rho_{ij} + 2PGA \sum_{i=1}^{n} \phi_i \Gamma_i S_{a,i}^{(rel)} \frac{p_i}{p_g} \rho_{ig} + \mathbf{e} PGA^2 \right]^{\frac{1}{2}} \tag{9}$$

While other researchers (Taghavi, 2006; Taghavi and Miranda, 2008; Pozzi and Der Kiureghian, 2012) use approximated power spectra of the ground motions to find analytical expressions for cross correlation coefficients, in the present study, alternatively, nonlinear optimization techniques in combination with regression methods are applied. Therein, in a first step the profile of relative peak floor acceleration demands ($\mathbf{PFA}^{(rel)}$) is estimated. Then, based on this estimate the absolute PFA demands ($\mathbf{PFA}$) are derived.

Correlation coefficients for the relative PFA demand

Upfront it is necessary to reduce the number of variables for the optimization process, and thus, in the first term of Eq. (9) representing relative peak floor acceleration demand, the cross correlation coefficients and the peak factors are condensed to a single variable ρ_{ij}^*,

$$\rho_{ij}^* = \frac{p_i p_j}{p_g^2} \rho_{ij} \tag{10}$$

Then, the relative peak floor acceleration demand reads:

$$\max\left(\left|\ddot{\mathbf{u}}^{(rel)}(t)\right|\right) = \mathbf{PFA}^{(rel)} \approx \left[\sum_{i=1}^{n}\sum_{j=1}^{n} \phi_i \phi_j \Gamma_i \Gamma_j S_{a,i}^{(rel)} S_{a,j}^{(rel)} \rho_{ij}^* \right]^{\frac{1}{2}} \tag{11}$$

Subsequently, using nonlinear optimization analysis (i.e., the function *fminunc* of MatLab, 2013) cross-correlation coefficients ρ_{ij}^{*} of Eq. (11) are identified. In this analysis, the profiles of relative acceleration demand $\mathbf{PFA}^{(rel)}$ are given quantities that are determined upfront for several test structures through time history analysis. For instance, taking into account n modes of an N-story structure (exhibiting with N dynamic degrees-of-freedom) n^2 combinations of ρ_{ij}^{*} can be identified.

Initially, for three different generic frames with different number of stories (i.e., N = 6, 12, and 24, respectively, exhibiting a fundamental period of $T_1 = 0.2N$ and a linear fundamental mode shape subjected to two earthquake records of the FEMA P695 ground motion set cross correlation coefficients ρ_{ij}^{*} are identified. In Fig. 2 and Fig. 3 the outcomes ρ_{ij}^{*} are plotted in two dimensions against the inverse of the corresponding periods T_i and T_j, i.e. the linear frequencies $1/T_i$ and $1/T_j$. The following characteristics are observed:

- The magnitude and distribution of the cross correlation coefficients with respect to the frequency is independent of the number of stories. With increasing number of stories the contour plot becomes smoother because more modes are involved. However, the overall relation between the cross correlation coefficients, and frequencies $1/T_i$ and $1/T_j$ becomes already visible if only six modes are included.
- As expected the cross correlation coefficients are symmetric with respect to the diagonal defined by $1/T_i = 1/T_j$, i.e., $\rho_{ij}^{*} = \rho_{ji}^{*}$.
- Comparison of Fig. 2 and Fig. 3 reveals that the cross correlation coefficients based on different earthquake records show the same overall trend. In the frequency domain of about 5 Hz the cross-correlation coefficients exhibit a global minimum. Furthermore, along the diagonal $1/T_i = 1/T_j$ the cross-correlation coefficients are constant if $1/T_i = 1/T_j \geq 10$ Hz, independently of the underlying ground motion record.
- In contrast to the traditional CQC rule, here the cross correlation coefficient ρ_{ii}^{*} is not equal to one, and negative values are possible, because ρ_{ij}^{*} is a combination of the actual cross correlation coefficient and peak factor ratios, see Eq. (10). Furthermore, the approximation introduced with Eq. (10) leads to the assumption that peak factor ratios do not depend on the relative height of a floor in the structure, which is inconsistent with empirical observations (Taghavi, 2006). An improvement of the proposed method will be presented in future.

According to the results of Fig. 2 and 3 it is reasonable to determine ρ_{ij}^{*} for each record of the FEMA P695 ground motion set. Subsequently analytical expressions for the median and dispersions of the cross correlation coefficients ρ_{ij}^{*} are derived through regression analysis.

The regression analysis is conducted in two steps. In the first step only the diagonal members ρ_{ii}^{*} are approximated by means of a one-dimensional relation. In Fig. 4 markers represent the statistically evaluated diagonal cross correlation coefficients with respect to period T. Inspection of these numerical outcomes may lead to the conclusion that a linear combination of two hyperbolic tangent functions,

$$\rho_{ii}^{*} \approx \beta_0 + \sum_{k=1}^{2} \left[\beta_{k,1} \tanh\left(\left(\frac{1}{T_i} + \beta_{k,2} \right) \beta_{k,3} \right) \right] \tag{12}$$

yields an adequate fit for ρ_{ii}^{*}. The regression coefficients for this equation are listed in Table 2.

Table 2. Regression coefficients for cross correlation coefficients ρ_{ii}^{*}

Percentile	β_0	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{1,3}$	$\beta_{2,1}$	$\beta_{2,2}$	$\beta_{2,3}$
16th	2.220	2.054	0	-0.298	0.852	-12.010	0.645
50th	2.070	1.673	0	-0.254	0.619	-11.670	0.572
84th	2.090	1.441	0	-0.317	0.385	-11.070	0.405

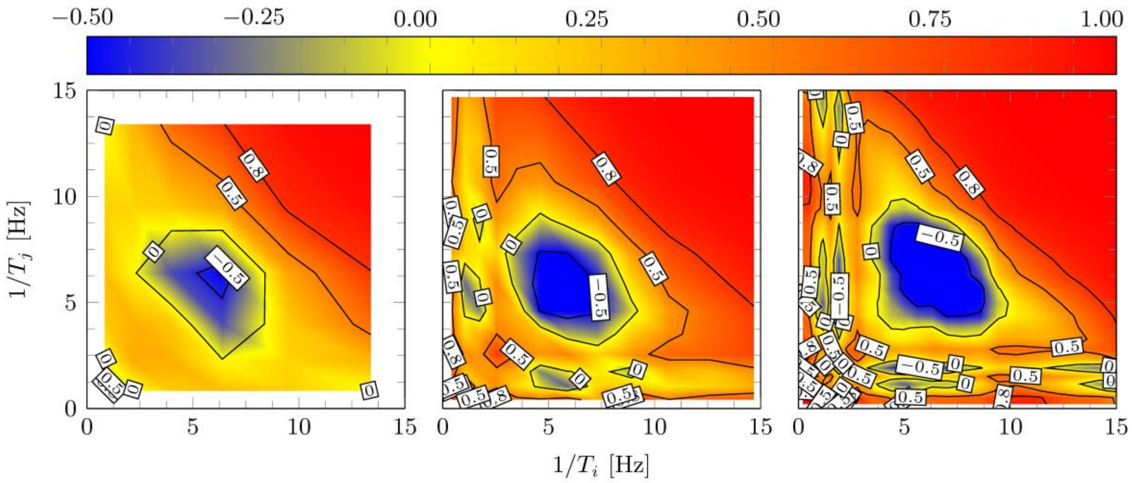

Figure 2. Cross correlation coefficients ρ_{ii}^{*} for 6- (left), 12- (center) and 24-story frame (right) for the Northridge earthquake record (1994, Beverly Hills – Mulhol Station, NGA sequence 953)

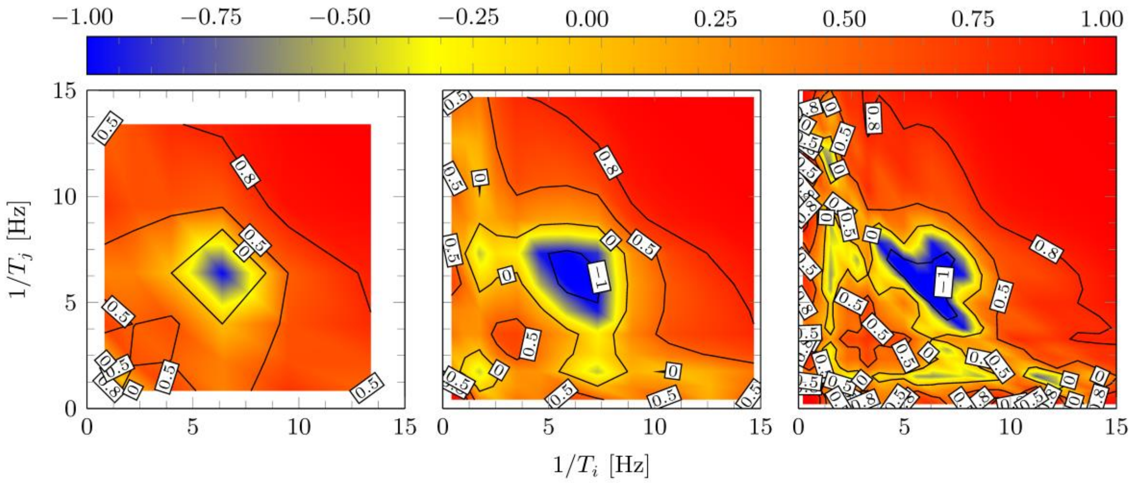

Figure 3. Cross correlation coefficients ρ_{ii}^{*} for 6- (left), 12- (center) and 24-story frame (right) for the Kobe earthquake record (1995, Shin – Osaka Station, NGA sequence 1116)

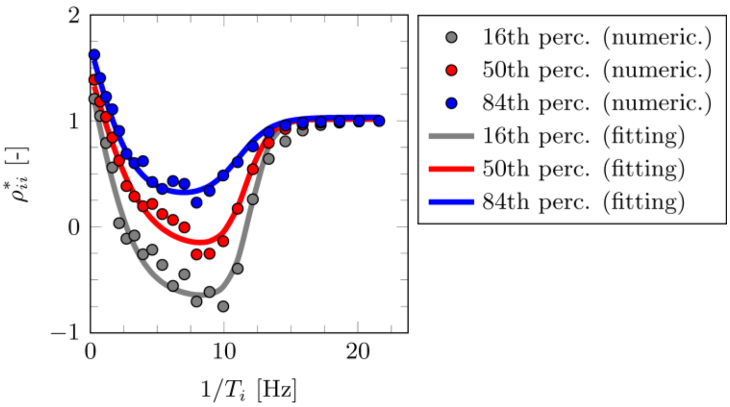

Figure 4. Fitting of cross correlation coefficients ρ_{ii}^{*}

Then, in a separate step, Eq. (12) is expanded to cover also non-diagonal cross correlation coefficients ρ_{ij}^{*}, $i \neq j$ in the complete frequency space,

$$\rho_{ij}^{*} \approx \beta_0 + \sum_{k=1}^{2}\left[\beta_{k,1}\tanh\left(\left(\frac{1}{\bar{T}_i}+\beta_{k,2}\right)\beta_{k,3}+\sum_{l=0}^{4}\left(\frac{\gamma_{k,l}}{\left|\bar{T}_j^{l}\right|}\right)\right)\right] \tag{13}$$

This equation contains the absolute value of the rotated period $\left|\bar{T}_j^{l}\right|$,

$$\begin{pmatrix}\bar{T}_i\\ \bar{T}_j\end{pmatrix}=\begin{bmatrix}\cos(\pi/4) & \sin(\pi/4)\\ -\sin(\pi/4) & \cos(\pi/4)\end{bmatrix}\begin{pmatrix}T_i\\ T_j\end{pmatrix} \tag{14}$$

The second term of Eq. (13) including a hyperbolic function governs the shape of coefficients ρ_{ij}^{*} along the $\bar{T}_j$ axis, and thus, Eq. (13) covers diagonal as well off-diagonal cross correlation coefficients. Additionally, a polynomial of 4th order is added. Note that for high frequency modes the proposed polynomial approximation leads to inaccurate results. Thus, in a future study the regression equation needs to be improved. The required regression coefficients are listed in Tables 2 and 3.

Table 3. Coefficients for cross correlation coefficients ρ_{ij}^{*} (in [‰])

Perc.	$\gamma_{1,0}$	$\gamma_{1,1}$	$\gamma_{1,2}$	$\gamma_{1,3}$	$\gamma_{1,4}$	$\gamma_{2,0}$	$\gamma_{2,1}$	$\gamma_{2,2}$	$\gamma_{2,3}$	$\gamma_{2,4}$
16th	-36.11	48.24	84.29	-11.03	0.46	-205.90	304.00	-68.58	3.99	-0.01
50th	-4.11	-104.90	115.50	-14.10	0.56	-167.00	160.70	-65.28	7.22	-0.31
84th	39.92	-246.50	182.40	-22.78	0.95	-149.10	-185.90	22.14	0.39	-0.19

In Figs 5, 6 and 7 the analytical approximation, $\rho_{ij,fit}^{*}$, of regression coefficients ρ_{ij}^{*} according to Eq. (13) (left subplot), the corresponding outcome, $\rho_{ij,num}^{*}$, of the optimization analysis (central subplot), and the relative error between the analytical fit and the empirical data,

$$err=\frac{\rho_{ij,fit}^{*}-\rho_{ij,num}^{*}}{\rho_{ij,num}^{*}}100 \quad [\%] \tag{15}$$

is depicted. Fig. 5 shows the outcome for the 16th percentile, Fig. 6 the median, and Fig. 7 the 84th percentile. In the right subplot of Figs. 5, 6, and 7 a positive error indicates that the analytical regression coefficient over-estimates the corresponding quantity from the optimization analysis. The error plots of Figs. 5 and 6 exhibit errors of more than ±200%, however, only locally concentrated in certain domains. This can be attributed to the fact that in these domains the denominator of Eq. (15) is close to 0. In contrast, the relative error of the 84% percentile value depicted in Fig. 7 does not show local peak error values.

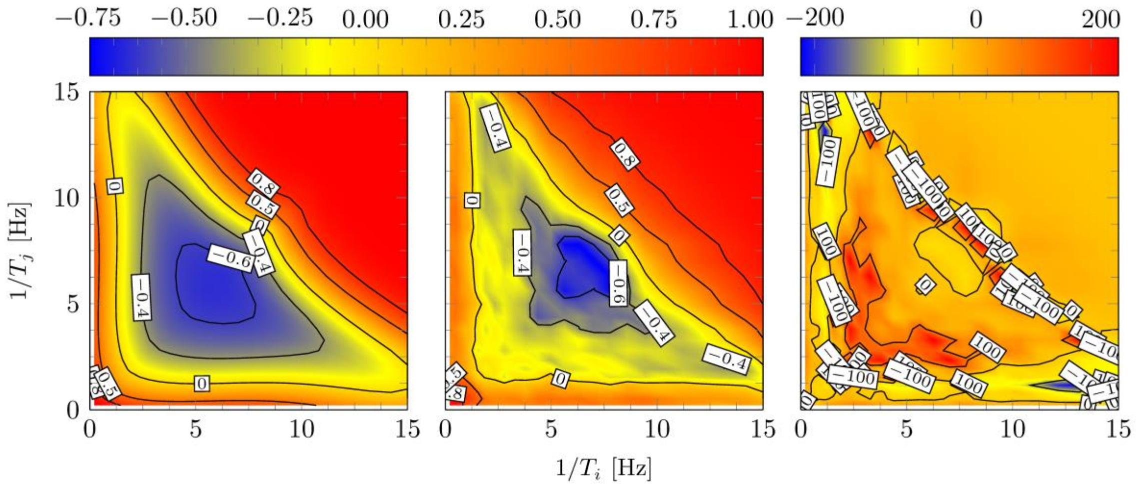

Figure 5. Cross correlation coefficients ρ_{ij}^{*} for the 16th percentile. Analytical approximation (left), numerical solution from optimization analysis (center), and relative error (right)

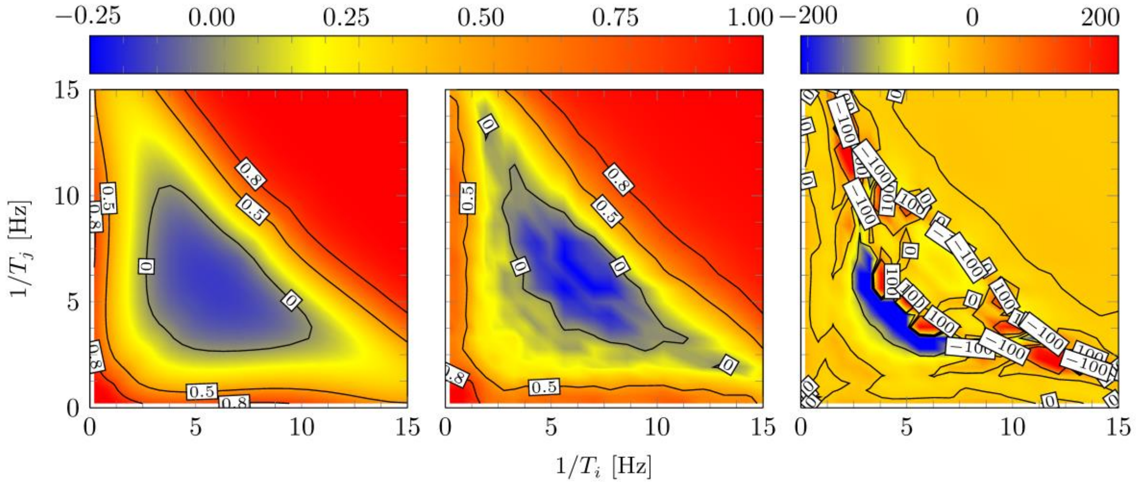

Figure 6. Cross correlation coefficients ρ_{ij}^{*} for the median. Analytical approximation (left), numerical solution from optimization analysis (center), and relative error (right)

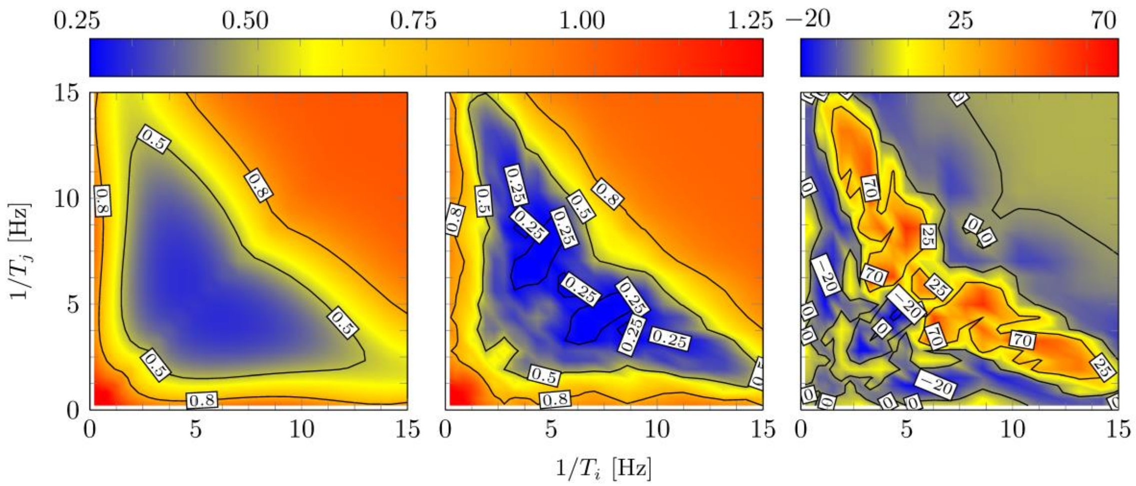

Figure 7. Cross correlation coefficients ρ_{ij}^{*} for the 84th percentile. Analytical approximation (left), numerical solution from optimization analysis (center), and relative error (right)

Correlation coefficients for the absolute PFA demand

The last two terms in Eq. (9) acknowledge the contribution of the PGA to the absolute PFA demand. Specifically, the second term of Eq. (9) represents the cross correlation between the PGA and the relative spectral peak acceleration $S_{a,i}^{(rel)}$ in terms of the extended CQC rule. Therefore, the corresponding cross correlation coefficients need to be identified. In the same manner as for cross correlation coefficients ρ_{ij}^{*} describing the modal interaction of the relative PFA demand according to Eq. (10) also for the combination of $S_{a,i}^{(rel)}$ and PGA a modified cross correlation coefficient ρ_{ig}^{*},

$$\rho_{ig}^{*} = \frac{p_{ig}}{p_{g}} \rho_{ig} \tag{16}$$

is defined. In an optimization analysis, analogous to the one to identify coefficients ρ_{ij}^{*}, coefficients ρ_{ig}^{*} are obtained. Like ρ_{ij}^{*}, they tend to be independent of structural and modal properties. Fig. 8 shows with discrete markers the outcomes for median, 16th and 84th percentile of the response. These discrete values are subsequently analytically approximated according to exponential function:

$$\rho_{ig}^{*} = \sum_{k=1}^{2} \delta_{k,1}\, e^{\delta_{k,2}/T_i} \tag{17}$$

The regression coefficients are listed in Table 4. Fig. 8 shows also these analytical approximations of ρ_{ig}^{*} proving that Eq. (17) is sufficient accurate to reflect these cross correlation coefficients.

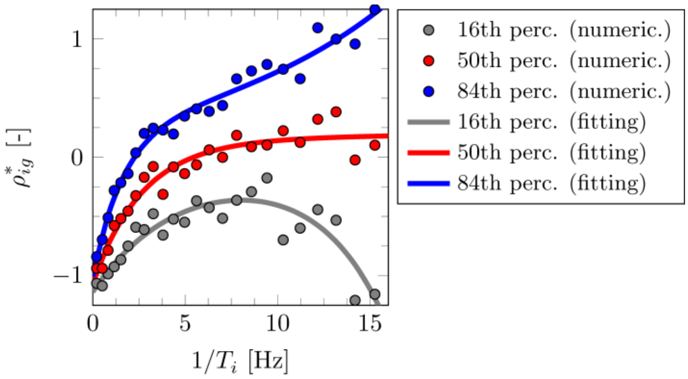

Figure 8. Fitting of cross correlation coefficients ρ_{ig}^{*}

Table 4. Coefficients for cross correlation coefficients ρ_{ig}^{*} (in [‰])

Percentile	$\delta_{1,1}$	$\delta_{1,2}$	$\delta_{2,1}$	$\delta_{2,2}$
16th	-1119.00	-210.40	-17.25	274.70
50th	150.70	12.16	-1214.00	-370.10
84th	272.60	98.00	-1279.07	-586.40

STEPS OF THE PROPOSED MODIFIED RESPONSE SPECTRUM METHOD

The steps for predicting the PFA demands of an elastic multi-story frame structure according to the proposed modified response spectrum method can be summarized as follows.

1. Create a model of the structure and perform modal analysis.
2. Select the appropriate design pseudo-acceleration response spectrum $S_a(T)$. In the present study the pseudo-acceleration spectra for the 5% damped SDOF oscillator subjected to the 44 records of the FEMA P695 far-field record set (FEMA P695, 2009) are utilized.

3. Estimate from the pseudo-acceleration spectra the relative acceleration response spectra using the adjustment function defined by Eqs. (7) and (8).
4. Determine cross correlation coefficients ρ_{ij}^{*} according to Eq. (13). The profile of relative peak floor acceleration demands, $\mathbf{PFA}^{(rel)}$, is approximated according to Eq. (11).
5. Estimate the cross correlation coefficients ρ_{ig}^{*} with Eq. (17) to account for the cross correlation between the spectral relative accelerations and the PGA.
6. Use Eq. (9) to estimate the profile of absolute peak floor acceleration demands, **PFA**.

APPLICATION

The proposed method is applied to a very flexible 24-story generic frame structure with a fundamental period of $T_1 = 4.80$ s. This structures exhibits periods in a broad range. 5% viscous damping is assigned to each mode.

Figure Fig. 9 shows the profiles of relative (left subplot) and the absolute (right subplot) peak floor acceleration demands normalized with respect to the PGA. Gray lines correspond to the PFA profiles of time history analysis for the single records of the 44 records of the FEMA P695 ground motion set. As reference serves the median of these individual outcomes that is depicted in red. Black lines represent the outcomes of the proposed extended CQC method. i.e. the median peak floor acceleration profiles of relative and absolute acceleration demands. The results of the traditional SRSS modal combination rule are shown in blue.

It is readily observed that the relative and absolute PFA demand prediction of the proposed modified CQC procedure is much closer to the actual response obtained from time history analysis than the outcomes of the traditional SRSS method without increase of the computation cost. While the SRSS method leads to a biased approximation of relative PFA demands, for the proposed CQC method only a few modes are required for a sufficiently accurate assessment. Particularly from the base up to 80 % of relative height the SRSS method underestimates the absolute PFA demands by more than 50%, which is unacceptable even for gross seismic loss assessment. In contrast, the modified CQC method leads to distinct improved prediction of this quantity.

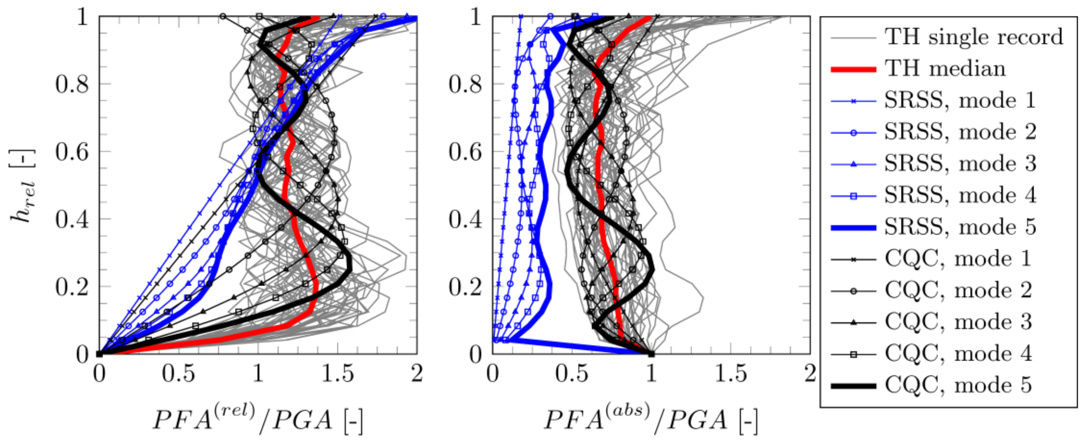

Figure 9. Profiles of normalized median peak floor acceleration demands, relative (left) and absolute (right) peak floor accelerations demands

SUMMARY AND CONCLUSIONS

Based on an extended CQC modal combination rule, a modified responses spectrum method for estimation of peak floor acceleration demands in elastic structures has been proposed. The absolute acceleration demands are determined from the acceleration demands relative to the base, the PGA, and the cross correlation between the two latter quantities. For the records of the ground motion set cross correlation coefficients, which consider the cross correlation both between the modal contributions of relative acceleration and between the modal relative accelerations and the peak ground acceleration, are identified through optimization analysis. The outcomes are subjected to a regression analysis to yield analytical expressions of these coefficients. In an application to a 24-story frame the basic idea as well as the accuracy of this method is demonstrated. Compared to other empirical formulations and codified methods this method exhibits a clear physical interpretation, and it is not restricted to a specific type of lateral load bearing structure and/or to specific construction materials. A planned comprehensive study of real frame structures serves to evaluate the limits of the proposed procedure in combination with a static pushover estimation method also developed by the authors (Moschen et al., 2013). The benefit is a more accurate assessment of PFA demands of *inelastic* structures.

REFERENCES

ASCE/SEI 7-10 (2010) Minimum design loads for buildings and other structures, American Society of Civil Engineers, Reston VA

ATC-58-1 (2012) Seismic performance assessment of buildings, Volume 1 – Methodology. 100% draft, Applied Technology Council, Redwood City CA

Charney FA (2008) “Unintended consequences of modeling damping in structures”, *Journal of Structural Engineering*, 134(4): 581-592

Chaudhuri SR and Hutchinson CT (2004) “Distribution of peak horizontal floor acceleration for estimating nonstructural element vulnerability”, *Proceedings of the 13th World Conference on Earthquake Engineering*”, Vancouver, Canada, 1-6 August

Chopra AK (2012) Dynamics of structures, 4th ed., Prentice-Hall, Boston MA

Der Kiureghian A (1981) “A response spectrum method for random vibration analysis of MDF systems”, *Earthquake Engineering & Structural Dynamics*, 9(5): 419-435

EN 1998-1 (2013) Design of structures for earthquake resistance – Part 1: General rules, seismic actions and rules for buildings, European Committee for Standardization, Brussels, Belgium

FEMA P695 (2009) Quantification of building seismic performance factors, Applied Technology Council, Redwood City CA

López-García D, Price P, Torrejón E, Vial F and Wichmann P (2008) “Peak floor accelerations in multistory buildings subjected to earthquakes”, *Proceedings of the 14th World Conference on Earthquake Engineering*, Beijing, China, 12-17 October

MATLAB (2013), The Mathworks Inc.

Medina RA (2003), Seismic demands for nondeteriorating frame structures and their dependence on ground motions, Ph.D. Thesis, The John A. Blume Earthquake Engineering Center, Department of Civil and Environmental Engineering, Stanford CA

Miranda E and Taghavi S (2009) “A comprehensive study of floor acceleration demands in multi-story buildings”, *Proceedings of ATC-SEI Conference on Improving the Seismic Performance of Existing Buildings and Other Structures*, San Francisco CA, 9-11 December, 616-626

Moschen L, Vamvatsikos D and Adam C (2013) “Towards a static pushover approximation of peak floor accelerations”, *Proceedings of Vienna Congress on Recent Advances in Earthquake Engineering and Structural Dynamics*, (Adam C, Heuer R, Lenhardt W, & Schranz C, eds) , Vienna, Austria, 28-30 August, paper no. 94

Pozzi M and Der Kiureghian A (2012) “Response spectrum analysis for floor acceleration”, *Proceedings of the 15th World Conference on Earthquake Engineering*, Lisbon, Portugal, 24-28 September

Rosenblueth E and Elorduy J (1969) “Response of linear systems to certain transient disturbances”, *Proceedings of the 4th World Conference on Earthquake Engineering*, Santiago, Chile, 13-18 January, 185-196

Taghavi S (2006) Probabilistic seismic assessment of floor acceleration demands in multi-story buildings, Ph.D. Thesis, The John A. Blume Earthquake Engineering Center, Department of Civil and Environmental Engineering, Stanford CA

Taghavi S and Miranda E (2008) “Response spectrum method for estimation of peak floor acceleration demand”, *Proceedings of the 14th World Conference on Earthquake Engineering*, Beijing, China, 12-17 October

Vamvatsikos D and Cornell CA (2002) “Incremental dynamic analysis”, *Earthquake Engineering & Structural Dynamics*, 31(3): 491-514

Wilson EL, Der Kiureghian A and Bayo, EP (1981) “A replacement for the SRSS method in seismic analysis”, *Earthquake Engineering & Structural Dynamics*, 9(2): 187-192