A study on the correlation between dissipated hysteretic energy and seismic performance

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ABSTRACT:
The hysteretic energy dissipated by systems undergoing quasi-static or dynamic loading is often thought to represent a useful measure of their performance when subjected to earthquake excitation. In general, fuller hysteresis loops mean higher seismic energy removal from the structure, which is logically taken to imply better performance when comparing systems with similar strength. However, such observations are typically based on quasi-static loading tests. Dynamic loading conditions differ as energy input and energy dissipation are intimately related with the details of the system's hysteresis, in ways that often defy current intuition. Using incremental dynamic analysis on story-level oscillators with varying hysteresis characteristics, we can map this connection in detail. Structural response, as measured in terms of maximum or residual deformation, is shown to have little connection to the energy absorption. Therefore, hysteretic energy dissipation cannot quantitatively measure seismic performance but perhaps only serve as a general indicator.

Keywords: hysteretic energy dissipation, seismic performance, incremental dynamic analysis

1. INTRODUCTION

The hysteretic energy, absorbed by a structural system during a seismic event that is strong enough to induce a certain amount of nonlinearity to the system, has been recognized by several researchers as a potentially useful seismic performance indicator (e.g., Park et al. 1987; Bojorquez et al. 2011). In general, stable hysteretic loops with large energy dissipation capacity at a member level are thought to guarantee a better deformation performance of the system, implying that there is a good correlation between the dissipated hysteretic energy and the inelastic deformation demands. This notion is often founded on observations made in quasi-static cyclic tests, where it seems apparent that between two systems with similar strength, tested under the same cyclic loading protocol, the one with the higher energy absorption, i.e., "fuller" hysteresis loops, should exhibit superior performance. Thus, dissipated energy is a term that has become synonymous to performance and it is so pervasive as to become a key ingredient of modern seismic codes (e.g., EN1998). Hence, at the basis of seismic design, the definition of the behavior (reduction) factor q (or R) allows that the high strength of a linear elastic system having zero energy absorption can be substituted by the equally effective dissipating behavior of an elastoplastic system with a base shear strength that is q times lower (at least where the equal displacement rule holds). While there is no question about the need for ductility, the role of energy dissipation is still imperfectly understood.

Energy dissipation is typically understood as a proxy for viscous damping, a concept that was perhaps first introduced by Jacobsen (1960) through equivalent linearization techniques. Such methods provide an estimate of the (average) nonlinear displacement of elastoplastic oscillators by employing an equivalent linear single-degree-of-freedom (SDOF) system characterized by a longer period (estimated at a secant stiffness) and an increased value of viscous damping. Crucially, the increase in damping is provided as a direct function of the area under the force-deformation curve of the nonlinear oscillator, a quantity that is well correlated to the quasi-statically dissipated hysteretic energy. It is no wonder then that higher energy dissipation seems to be equivalent to higher damping, ergo better performance.

However, there exists evidence in recent literature that suggests otherwise. For example, Miranda and Ruiz-Garcia (2002) have shown that using the actual area under the backbone of an elastoplastic system to define equivalent damping yields worse results for maximum displacement estimation compared to other approaches. A number of recent studies have also explored the effect of the type of cyclic hysteresis on the seismic performance of structural systems. Rahnama and Krawinkler (1993), Foutch and Shi (1998), Huang and Foutch (2009) have observed that there is no clear correlation between the hysteresis type and the ductility demands. Ibarra et al. (2005) have shown that the hysteresis type becomes important mainly when the system approaches its global collapse state. Given that the hysteretic rules largely decide the amount of energy dissipation, questions may be easily raised. Similarly, when the connection of dissipated energy and performance is extrapolated from quasi-static tests to non-stationary loads, characteristic of actual earthquakes, current ideas about the important of hysteretic energy may not be generalizable.

Therefore, we will investigate whether hysteretic energy dissipation is a fundamental quality of system performance. In other words, when comparing two systems having similar (or the same) backbone, we are asking whether the one with the "fuller" hysteresis loops (as evidenced from classic quasi-static cyclic tests) or, more generally, the one dissipating more energy via hysteresis in dynamic loading, is the one having the better seismic performance. Issues related to material or member failure criteria and whether these should be based on dissipated hysteretic energy or not will not be discussed here. Such questions can only be answered unambiguously by experiments and not via computational studies, like the one that we are going to embark upon. Hence, the conclusions of this study may be considered applicable to cases where catastrophic failure is not reached. Nevertheless, the latter is a rather common scenario of several experimental studies, where competing hysteretic systems (members, bearings, assemblies etc) are subjected to a given quasi–static cyclic loading protocol up to a certain displacement, which is not necessarily associated to the system's collapse state. In such cases, the resulting force-deformation cyclic curves are often being judged in terms of their perceived hysteretic energy dissipation.

2. ENERGY BALANCE EQUATION AND HYSTERETIC ENERGY

The equation of motion for a damped SDOF system subjected to a horizontal ground motion record can be written as

$$m\ddot{u} + c\dot{u} + f_s = -m\ddot{u}_g \tag{2.1}$$

where m is the mass of the system, c is the viscous damping coefficient, f_s is the restoring force, $\ddot{u}_g$ is the ground acceleration and $u, \dot{u}, \ddot{u}$ are the relative displacement, velocity and acceleration, respectively, of the mass with respect to the ground. The absorbed energy is evaluated according to the energy balance equation (e.g., Uang and Bertero 1990), derived from integrating over time the equation of motion (Eqn. 2.1), representing the equilibrium of forces, multiplied by the instantaneous displacement $du = \dot{u}dt$:

$$\int m\ddot{u}\dot{u}\,dt + \int c\dot{u}^2 dt + \int f_s \dot{u}\,dt = -\int m\ddot{u}_g \dot{u}\,dt \tag{2.2}$$

The energy balance equation is valid throughout the duration of the motion. The first term depicts the "relative" kinetic energy of the system, as measured with respect to the ground, representing energy temporarily stored in the kinematics of the system. The second is the damping energy dissipated by viscous damping, and the third is termed the absorbed energy, consisting of the irrecoverable hysteretic energy and the recoverable strain energy. Despite the presence of the recoverable part, the name "absorbed energy" is perfectly valid when integration is carried out until the system comes to rest, where strain energy essentially vanishes. The final term is the relative input energy imparted by the ground motion to the system, as measured relative to the ground, excluding any rigid body translation. Still, if integration is carried out to the time when the system comes to rest this is essentially equivalent to the absolute input energy (Uang and Bertero 1990). The actual input energy induced to a system during an earthquake event is thus dissipated in its entirety by means of viscous damping and hysteretically absorbed energies.

It is worth pointing out here, that the nature and connection to the system behaviour of the hysteretic and the damping energies is fundamentally different. The hysteretic energy is the energy dissipated through inelastic excursions during the seismic excitation whereas, the damping energy is related to the work done by the damping force. In a simplistic interpretation of the equation of motion these two energies may be considered together in a single damping energy term. Still, the distinction between the hysteretic and the damping energy is rather important when considering the damage potential of a structural system on account of its energy dissipation capacity, as these two mechanisms of energy dissipation operate on a fundamentally different level. Most importantly, increasing the damping has a straightforward effect towards reducing the seismic demands, as viscous damping has an ever-present dissipating effect regardless of the sign of the velocity vector, due to the square on the velocity term. On the other hand, for input energy, the ground motion acceleration is multiplied by the oscillator velocity at each time instant, resulting to either a positive or a negative energy increment. The same is true for the hysteretic energy as well, where the sign of the restoring force f_s and the velocity may become opposite. In other words, hysteretic energy and input energy are closely connected, where changing the hysteretic characteristics of a system causes fundamental changes to both. Therefore, while the beneficial effect of increasing the damping energy capabilities of a system is perfectly straightforward, the correlation of hysteresis to the damage induced to the system is neither obvious nor thoroughly examined, thus rendering conclusion-drawing a difficult task.

At a different level, it is equally troublesome to try to derive conclusions regarding system performance based not on dynamically-absorbed energy but on quasi-statically absorbed instead. Such tests are typically performed under a displacement-controlled loading protocol that not only imposes certain displacements but, given the hysteretic model, essentially also prescribes the input energy. By virtue of removing any influence of damping, this also ensures that all the energy will be dissipated via hysteresis only. Clearly this is something that can never happen in dynamic tests therefore any connections would be difficult to justify.

3. METHODOLOGY

To investigate the correlation between hysteretic energy and seismic performance, a number of SDOF systems will be used, each having different force-deformation characteristics. To evaluate their seismic performance Incremental Dynamic Analysis (IDA) is employed (Vamvatsikos and Cornell 2002). IDA is a powerful tool of structural analysis that involves performing a series of nonlinear time history analyses for a suite of ground motion records, the latter scaled at increasing intensity levels. To define an IDA curve two scalars are needed, these being an intensity measure (IM) and an engineering demand parameter (EDP) to record the structural response.

The 5% damped spectral acceleration at the vibration period of the SDOF systems, $S_a(T)$ is adopted as the IM, since it is considered to be an efficient intensity measure especially for SDOF systems (Shome et al. 1998). Moreover, to allow comparisons between the different models and periods investigated, the elastic spectral acceleration S_a is normalized by its value S_{ay} at yield to provide the dimensionless ratio $R = S_a / S_{ay}$, which is akin to the strength reduction factor R. Regarding the demand parameter EDP, in addition to the total absorbed hysteretic energy E_{hyst} recorded at the end of the dynamic time history analyses, the maximum displacement d_{max} is also employed as a measure for the peak seismic demands. Furthermore, residual displacements d_{res} will be monitored as a useful indicator of whether a damaged building should be retrofitted or demolished (e.g., Ruiz-Garcia and Miranda 2008). In all analyses, at the end of each record, the system is allowed to undergo several free vibration cycles in order to return to rest and permit an accurate measurement of the total hysteretic energies and residual displacements. For the IDAs a suite of sixty ground motion records is used. The records are assumed to be ‘ordinary’ in the sense that they do not raise any concerns regarding soft soil or near source directivity. The accelerograms are selected from the PEER Strong Motion database (PEER 2011).

4. HYSTERETIC ENERGY VERSUS SEISMIC PERFORMANCE

4.1. Hysteretic models

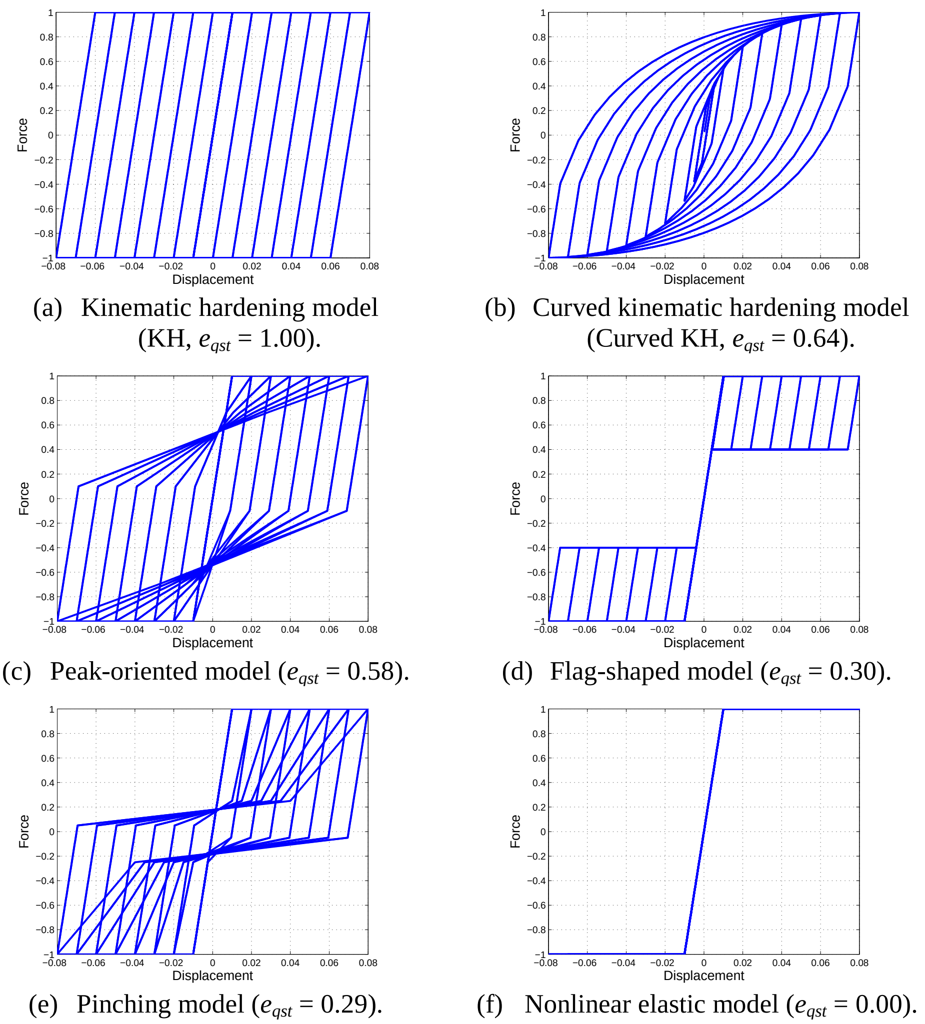
The main question to be answered is whether the hysteretic energy absorbed, as estimated by the area of the force-deformation loops in dynamic or quasi-static loading conditions, is well correlated to seismic displacement demands. In recent years, several hysteretic models have been developed in order to simulate as realistically as possible the performance of a structural system under seismic excitation. To this end, we have considered a series of single-degree-of-freedom (SDOF) oscillators, all sharing practically the same elastic-plastic force-deformation backbone (allowing for some curved transition in one case) but with varying hysteretic characteristics to depict a wide spectrum of force-deformation behaviors representative of different components, materials and structures.

The six systems considered are presented in Fig. 1, arranged in order of decreasing quasi-statically absorbed hysteretic energy. At the very top of the group, a classic elastoplastic system with kinematic strain hardening (Fig. 1a) was chosen to serve as the benchmark for comparing the performance of the more elaborate systems to follow. Adding a curved transition between the elastic segment and the plastic plateau while maintaining kinematic strain hardening defines the “curved” system of Fig. 1b. Cyclic stiffness degradation is introduced to different degrees of severity by the peak-oriented and the pinching systems (Ibarra et al. 2005) in Figs 1c and 1e. Recent advances in self-centering systems (Christopoulos et al. 2002) are represented by the flag-shaped hysteretic loops of Fig. 1d. Finally, the nonlinear-elastic oscillator of Fig. 1f lies at the opposite extreme end compared to the kinematic hardening, having the same backbone but no hysteretic energy dissipation capacity. The oscillators were assumed to have a 5% viscous damping and for each model a range of periods was employed, from 0.5 to 2.0sec, in 0.5sec increments. The dynamic analyses of the SDOF oscillators were carried out by means of the OpenSEES open-source analysis platform (McKenna et al. 2000).

The hysteretic energy dissipation capacity under quasi-static cyclic loads, for each one of the hysteretic models examined, was evaluated on the premise of the energy ratio e_{qst}, which is defined by the following equation:

$$e_{qst} = \frac{E_{hyst,i}}{E_{hyst,KH}} \tag{4.1}$$

where $E_{hyst,i}$ is the hysteretic energy absorbed by the hysteretic model i and $E_{hyst,KH}$ is the hysteretic energy absorbed by the kinematic hardening hysteretic model, when both are subjected to the same cyclic loading protocol. The corresponding e_{qst} ratios are reported in Fig. 1 for each system and they vary from one to zero, with one representing the kinematic hardening hysteretic model (Fig. 1a) and zero associated with the nonlinear elastic model (Fig. 1f). It is worth noting that the e_{qst} ratios for the moderately pinching (Fig. 1e) and the flag-shaped (Fig. 1d) models are almost identical, at least for this example.



(a) Kinematic hardening model (KH, $e_{qst} = 1.00$).

(b) Curved kinematic hardening model (Curved KH, $e_{qst} = 0.64$).

(c) Peak-oriented model ($e_{qst} = 0.58$).

(d) Flag-shaped model ($e_{qst} = 0.30$).

(e) Pinching model ($e_{qst} = 0.29$).

(f) Nonlinear elastic model ($e_{qst} = 0.00$).

Figure 1. Backbone and hysteretic loops of the considered nonlinear oscillators.

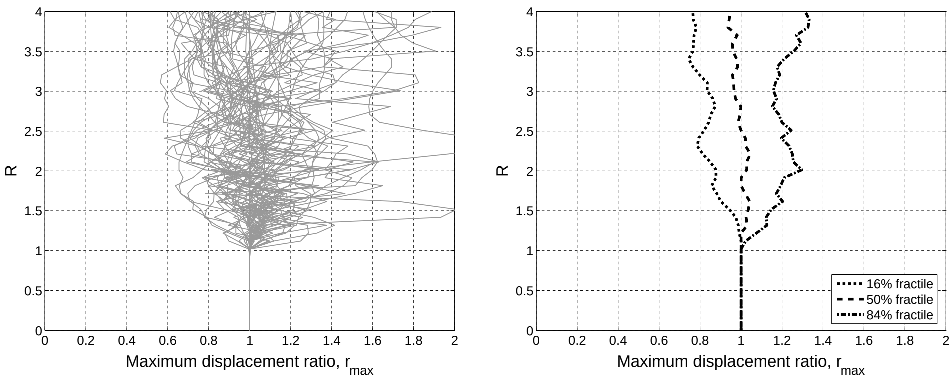
To evaluate the systems' performance under dynamic loads, we also need to assess their behavior in terms of energy and displacement. To allow for simplicity in the comparisons, these are performed in terms of the x% fractile values, over all records, of the hysteretic energy ratio e_{dyn}, the maximum displacement ratio r_{max} and the residual displacement ratio r_{res}, defined for each system i with respect to the kinematic hardening hysteresis:

$$e_{dyn,x\%} = \left[\frac{E_{hyst,i}}{E_{hyst,KH}} \right]_{x\%} , \tag{4.2}$$

$$r_{max,x\%} = \left[\frac{d_{max,i}}{d_{max,KH}} \right]_{x\%} , \tag{4.3}$$

$$r_{res,x\%} = \left[\frac{d_{res,i}}{d_{res,KH}} \right]_{x\%} . \tag{4.4}$$

Due to the high record-to-record variability in the dynamic results, it is important to quantify both the central value and the dispersion of their distribution to fully capture the range of behavior (e.g., Vamvatsikos and Cornell 2002; Kazantzi et al. 2008). Hence, typical values for x include 50%, i.e., the median as a central value, and 16, 84% to evaluate the associated dispersion. Figs 2a and 2b illustrate the way fractile values of the response ratios were computed by means of IDA curves. Fig. 2a in particular, presents the "spaghetti plot" of 60 IDA curves in terms of R and r_{max} scalars. The r_{max} dimensionless demand parameter was evaluated as the ratio of the maximum displacement response computed for the peak-oriented model over the maximum displacement response computed for the kinematic hardening model. The SDOF systems under consideration were assumed to have a natural period of T = 1.0sec whereas for each one of them a total of 2400 (i.e., 60 ground motion records x 40 intensity increments) nonlinear time history analyses were performed. The record-to-record variability is evidently significant. This renders the response predictions for any one record highly random. The randomness may be quantified by the 16, 50, 84% fractile ratios. These corresponding ratios appear in the Fig. 2b, where the peak-oriented and the kinematic hardening system are shown to have the same median d_{max} response (i.e., a ratio of practically 1.0) at all levels of intensity, in agreement with past studies (Rahnama and Krawinkler 1993; Vamvatsikos and Cornell 2006).



(a) 60 IDA curve ratios (b) 16, 50, 84% fractile ratios

Figure 2. Estimation of fractile values of maximum displacement response ratios, r_{max} of the peak-oriented and the KH systems (T = 1.0sec) via IDA.

4.2. Performance comparisons

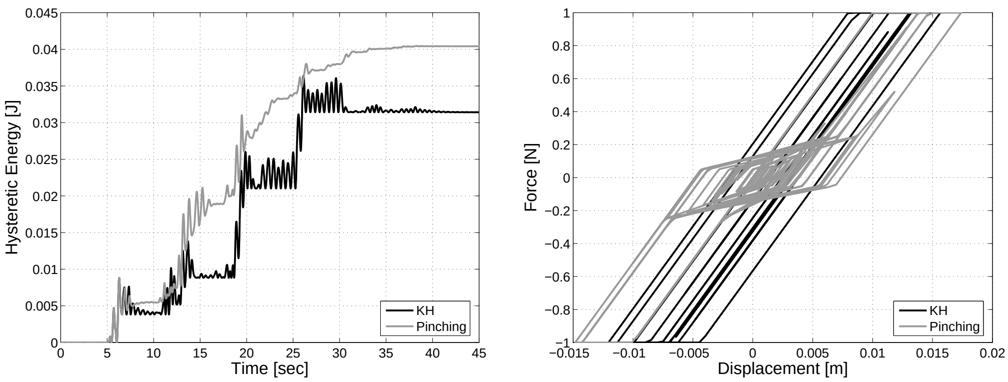
The results in terms of the median response ratios of the five alternative hysteretic models (Fig. 1b-f) versus the kinematic strain hardening (KH) reference system for three representative R values (1.5, 2.0 and 4.0), associated with near and post-yield regions, are summarized in Table 1. Despite the fact that a range of periods was considered, the tabulated results only refer to T = 1.0sec for brevity. Results at longer periods are practically the same, while shorter periods introduce only marginal differences.

Firstly, it is apparent that dynamically and statically absorbed energies are not well correlated. For example, at R = 1.5, all the systems, except the two self-centering flag-shaped and nonlinear elastic models, absorb more energy than the KH (in a median sense). The curved system actually dissipates nearly twice the hysteretic energy of the KH model, despite having almost 40% lower quasi-static energy absorption. At the same intensity, the pinching system, which absorbs only half the energy of the peak-oriented system in quasi-static loading, has almost 30% higher dissipation for dynamic loading. These energy ratios are slowly evened out at higher intensities, all reaching nearly unity at R = 4.0, excluding the two self-centering systems.

This observation, which is in contrast to the conventional view that hysteretic models with fuller loops guarantee higher hysteretic energy absorption under non-stationary loads, can be explained by looking into the details of the hysteretic behavior and the energy dissipation histories. For example, the handicap of the KH system at low intensities is a consequence of its purely elastic unloading-reloading behavior. The KH system can only dissipate energy when it deforms along the yield plateau. However, a ground motion record is likely to induce only a few reversing cycles to the KH system that are not sufficient to give rise to its hypothetically superior hysteretic behaviour. The weak ground motion acceleration spikes, whose effect on the system is limited below the nonlinear range, are wasted to temporarily stored recoverable strain energy. Fig. 3 compares the hysteretic energy time histories as well as the force-displacement hysteretic responses of the KH and the pinching SDOFs with a period of T = 1.0sec. Both oscillators were subjected to the same ground motion record, which was scaled to R = 2.0. As evident from the presented graphs, the hysteretic energy absorbed by the pinching model is higher (see Fig. 3a). This outcome holds ground on the fact that for low R values, the systems undergo only a limited number of nonlinear excursions (see Fig. 3b) and thus the advantage of the fuller KH loops towards hysteretic energy dissipation is not completely utilized. On the contrary, the pinching system displays hysteretic energy absorption even when cycling below the plastic plateau, thus steadily dissipating energy (rather than temporarily storing it) as seen in Fig. 3b. It is worth pointing out that, for this particular scaled ground motion, the pinching system, despite its superior energy dissipation capacity exhibits a higher peak deformation (see Fig. 3b) compared to the KH system. Eventually, the fuller loops of the KH model will assert themselves in terms of energy dissipation at higher intensities, as shown, e.g., by the shape of the fractile IDA results in Fig. 4a, and should provide it with the presumed advantage suggested by quasi-static tests.

Table 1. Summarized comparison of the quasi-statically and dynamically dissipated hysteretic energy versus the displacement response for the considered hysteretic models (T = 1.0sec). All quantities are shown as median values of response ratios normalized by the kinematic strain hardening system.

Loading	response	curved KH	peak-oriented	flag-shaped	pinching	nonlinear elastic
Quasi-static	e_{qst}	0.64	0.58	0.30	0.29	0.00
Dynamic R = 1.5	$e_{dyn,50\%}$	2.35	1.39	0.79	1.83	~ 0
	$r_{max,50\%}$	0.90	1.04	1.04	1.03	1.17
	$r_{res,50\%}$	0.22	1.34	~ 0	0.76	~ 0
Dynamic R = 2.0	$e_{dyn,50\%}$	1.52	1.22	0.81	1.36	~ 0
	$r_{max,50\%}$	0.92	1.02	1.07	1.04	1.33
	$r_{res,50\%}$	0.18	1.48	~ 0	0.81	~ 0
Dynamic R = 4.0	$e_{dyn,50\%}$	0.96	1.02	0.69	0.86	~ 0
	$r_{dis,50\%}$	0.99	0.95	1.14	1.06	1.50
	$r_{res,50\%}$	0.29	1.28	~ 0	0.70	~ 0

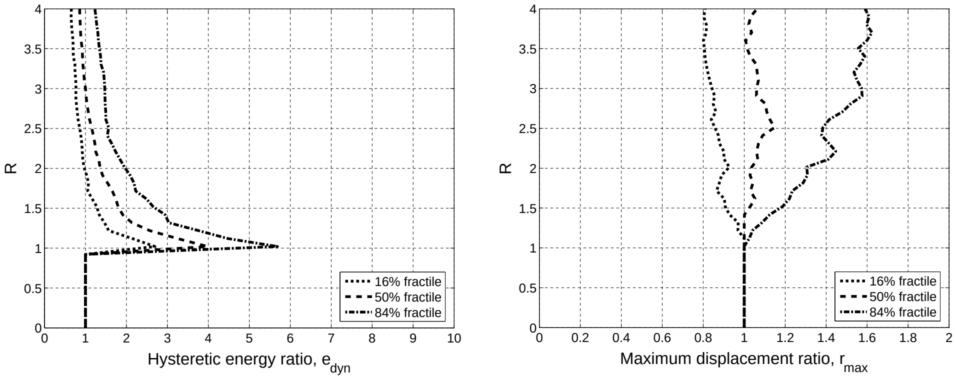


(a) hysteretic energy (b) force-displacement loops

Figure 3. Hysteretic energy time histories and force-displacement hysteretic loops for kinematic hardening and pinching models (T = 1.0sec), for a single record scaled to R = 2.0.

However, Table 1 shows that any superior hysteretic energy dissipation performance does not seem to be reflected in the evaluated maximum or residual displacement demands. Across the whole intensity range, KH, peak-oriented, pinching and curved KH systems share practically the same d_{max} response.

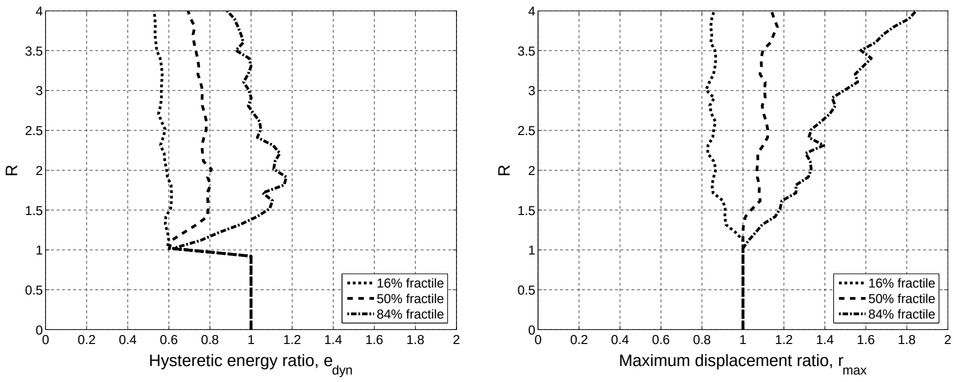
This trend is illustrated for the pinching system in Fig. 4b. Furthermore, the pinching and curved KH models clearly have superior performance in terms of residual displacement compared to the KH at all levels of intensity.



(a) total hysteretic energy ratio, e_{dyn} (b) maximum displacement ratio, r_{max}

Figure 4. Hysteretic energy and maximum displacement ratios of the pinching normalized by the KH model for T = 1.0sec.

Regarding the flag-shaped system, although it was found to dissipate in the median sense 20-30% less energy than the KH model (see Table 1) the reported values for the peak displacements are only marginally higher. Even so, the wide dispersions appearing in Figs 5a and 5b suggest that there are several records that combine both lower dissipation and lower peak displacement compared to the KH system.



(a) total hysteretic energy ratio, e_{dyn} (b) maximum displacement ratio, r_{max}

Figure 5. The 16, 50, 84% fractiles of response ratios for the flag-shaped over the kinematic hardening model for T = 1.0sec.

For the boundary non-dissipating nonlinear elastic model the computed results suggest that the median maximum displacements are in general higher, reaching an approximately 40-50% higher median at R = 4.0 compared to that predicted by the KH benchmark model. Nevertheless, it should be taken into account that this increase in the maximum deformation demands may well counterbalanced by the fact that the two self-centering systems together with the curved KH one, display the lowest values of r_{res}, aided by their unloading behavior that, as shown in Fig. 1, tends to relieve some or all of the maximum displacement when returning to rest. This may be considered to be a clear advancement against the other examined models.

In conclusion, it can be inferred from the presented results that observations in cyclic loading tests regarding the hysteretic behavior of a system are neither representative nor stable indicators for its potential dynamic energy dissipation. Furthermore, the superior hysteretic energy dissipation performance does not seem to be reflected in the evaluated maximum or residual displacement demands. Thus, it is not the area of the loops themselves that seems to matter but rather the finer details of the hysteretic rules.

5. CONCLUSIONS

Considering a range of story-level oscillators with varying hysteretic characteristics it was examined by means of incremental dynamic analyses whether the dissipated hysteretic energy can serve as a useful seismic performance indicator. The study reveals that, hysteretic dissipated energy is not consistently well-correlated to seismic performance. Other force-deformation characteristics, such as the shape and curvature of the backbone and whether it shows self-centering behavior or not, have a more profound influence on the response of structural systems. Therefore we propose that some care should be exercised whenever discussing the energy-dissipation characteristics of different systems, since the reliability of this measure for comparison in terms of seismic performance is questionable.

AKCNOWLEDGEMENT
The authors gratefully acknowledge the support of the European Research Executive Agency via Marie Curie grant PCIG09-GA-2011-293855.

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