Analytical Seismic Vulnerability Assessment for a Class of Modern Low-Rise Steel MRFs

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ABSTRACT: A set of guidelines was developed for the Global Earthquake Model (GEM), aiming to offer a practical, yet sufficiently accurate, analytical method for assessing the relationship between the ground shaking and the repair cost for a building class. The present work illustrates the methodology for a class of modern low-rise steel moment-resisting frames (SMRFs). The structural analysis is performed using Incremental Dynamic Analysis (IDA). The selection of a single Intensity Measure (IM) to parameterize IDA results and, eventually, vulnerability curves is being tackled through an extended IM comparison study across the entire structural response range considering both interstory drifts and peak floor accelerations. It is demonstrated that scalar IMs, defined as the geometric mean of spectral accelerations values $S_{agm}(T_i)$ estimated at several periods T_i can have an overall satisfactory performance. Once the uncertain structural response is determined, the methodology proceeds to the vulnerability estimation and consequently to loss assessment that is built upon a simplified component-based FEMA P-58 style methodology. The end product of this study is a high-quality set of vulnerability curves whose weighted moments are taken as the uncertain vulnerability function of the investigated building class.

1. INTRODUCTION

Given the lack of sufficient historical data on the seismic performance of a broad range of building classes worldwide, the value of an analytical model for performing a seismic vulnerability and consequently loss assessment is apparent. To this end, a set of guidelines was recently developed by Porter *et al.* (2014) aiming to offer a practical analytical method for assessing the relationship between the ground shaking and the repair cost for a building class. The term 'building class' refers to a set of index structures (Reitherman and Cobeen, 2003) which are appropriately selected, so as to account for variations of their key features (e.g. height, construction era etc) that are the most influential to seismic performance.

For assessing the structural response from elasticity up to global collapse, Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell, 2002) is employed. Furthermore, the important task of selecting a single Intensity Measure (IM) across the class will be addressed. Following the evaluation of the structural response, the study proceeds to the vulnerability and loss assessment of the low/mid-rise steel moment-resisting frame (SMRF) building class. This will be built upon the component-based FEMA P-58-1 approach (FEMA P-58-1, 2012) but the latter will be simplified in such a manner so as to minimize the invested effort.

2. CLASS DESCRIPTION & SAMPLING The test bed of the present work is a set of six (6) low/mid-rise SMRFs, built in the US in highseismicity regions. The main features differentiating the buildings within the class were considered to be: (a) the building height, defined as the number of stories and (b) the design base shear, as this was determined by the code-based value of spectral acceleration at 1sec, termed *SD*1 in US codes.

The first macroscopic characteristic was based on a sample of 3562 buildings in Memphis catalogued by Muthukumar (2008). For the SD1 distribution, used as a proxy for the base shear strength, we used county-level data from the high seismicity zones in the US (Kazantzi et al, 2014). The index buildings were defined by means of the class partitioning methodology (D'Ayala et al, 2014) and the corresponding representative structures were selected from a report issued by the National Institute of Standards and Technology (NIST) on the evaluation of the FEMA P695 methodology (NIST, 2010a). For more details on the definition of classes and index buildings, the interested reader should refer to Porter et al. (2014). The main features of the analyzed frames along with the resulted weights representing their contribution to the total sample are summarized in Table 1.

Table 1: Main features and weights of the index buildings.

Index	Feature X1 (No of stories)	Feature X2 (Code design level)	Weight
1ELF	1	0.6g	0.5503
2ELF	2	0.6g	0.1760
3ELF	4	0.6g	0.0337
5ELF	1	0.2g	0.1738
6ELF	2	0.2g	0.0556
7ELF	4	0.2g	0.0106

3. MODELING

Given that we are dealing with symmetric plan buildings, a 2D centerline idealization was adopted for modeling the Multi-Degree-of-Freedom (MDOF) index structures in the OpenSees (McKenna et al, 2000) analysis platform. The behavior of the structural members was depicted by lumped-plasticity elements having their properties (i.e. the properties of the beam and column end plastic hinges) evaluated by the regression equations suggested by Lignos Krawinkler (2011). Lumped-plasticity and elements as opposed to the more sophisticated distributed-plasticity fiber section elements was a conscious choice in favor of simplicity, speed of improved computation and numerical convergence (especially when approaching collapse in the sense of global dynamic instability). Geometric nonlinearities in the form of P- Δ effects were considered. Figure 1 depicts a typical plan view of the perimeter SMRFs.



Figure 1: Typical plan view of the perimeter SMRFs (dimensions in ft, after NIST, 2010).

4. IDA FUNDAMENTALS

For evaluating the seismic performance of the index buildings, Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell, 2002) is proposed as the benchmark analysis methodology. IDA is a powerful tool of structural analysis that involves performing a series of nonlinear timehistory analyses for a suite of ground motion records, the latter scaled at increasing intensity levels. To define the IDA curves, two scalars are needed, these being an

Intensity Measure (IM) to represent the severity of the seismic input and an Engineering Demand Parameter (EDP) to monitor the structural response. For the present study, a number of different IMs were used for illustrating their efficiency whereas, two classes of EDPs were employed, these being the peak interstory drift ratio (IDR) at each story and the peak floor acceleration (PFA) at each floor. These two EDP types are deemed to be adequate for assessing the structural, non-structural and content losses (e.g., FEMA P-58).

The ground motion records needed for the IDAs come from the far-field record set of FEMA P695 (2009) which contains 22 natural ground motion records with two components each (i.e. 44 individual components). The records were selected from the PEER NGA database (PEER, 2006) and were recorded on firm soil sites. The records include ground motions that are recorded at least 10km far from the fault rupture without any discernible near-source pulse signal. Furthermore, no more than two strongest records were considered from any one earthquake event to prevent event bias.

5. IM SELECTION BACKGROUND THEORY

Despite the fact that several past studies were focused on the IM choice, to the authors' knowledge none of them has so far explicitly addressed the problem from a building class point of view. The best known option for an IM is the $S_a(T)$, i.e. the 5% damped spectral acceleration at the period of interest (usually the structure's first-mode period, T_1). It is relatively efficient, yet it has often been criticized for lack in sufficiency wherever large scale factors (higher than, say 3.0) are employed (Luco and Bazzurro, 2007). This is mainly the case for modern structures that need considerably intense ground motions to experience collapse. It should be noted here that the majority of the ground motion databases contain mostly small to moderate records and hence significant scaling is an inevitable process for assessing the collapse capacity of well-designed structures (e.g., Luco

and Bazzurro, 2007). On the other hand, this is rarely the case for older and deficient buildings. Furthermore, due to the dependence on the firstmode period, $S_a(T_1)$ does not satisfy the prerequisite for adopting a common IM across the building class, so as to uniformly parameterize the IDA results and consequently the vulnerability functions of the index buildings. A simple remedy is to choose a common period T that can be considered representative of the class. Two potential candidates are $S_a(1sec)$, for moderate-to-long period structures, and $S_a(T_{1m})$, where T_{1m} is the central value (mean or median) of the first-mode periods of all index buildings within the class. For the case at hand, the mean period was found to be $T_{1m} = 1.3$ sec. It's worth noting that $S_a(1 \text{ sec})$ is an IM that has seen much use in existing vulnerability/fragility studies for highrise buildings but its efficiency is highly questionable.

On account of single buildings, an option that has appeared lately is use of a geometric mean of spectral acceleration values at different periods:

$$S_{agm}(T_i) = \left[\prod_{i=1}^n S_a(T_i)\right]^{1/n} \tag{1}$$

This was introduced by Cordova et al. (2000) as the geometric mean of two S_a components evaluated at two period levels, these being the fundamental period (T_1) and a period that is two times the fundamental period $(2 \cdot T_1)$. The latter period level was introduced at the IM estimation so as to account for the period elongation associated with the structural damage. This choice was proven to significantly improve both the efficiency and the sufficiency of the estimation, compared to $S_a(T)$. It also remains practical, as a Ground Motion Prediction Equation (GMPE) for $S_{agm}(T_i)$ is easily estimated existing $S_a(T)$ GMPEs, from given the correlation of spectral ordinates (Cordova et al, 2000). Since $S_{agm}(T_i)$ offers a considerable extension to the applicability of scaling (Vamvatsikos and Cornell, 2005; Bianchini et al, 2009), it is for the time-being by far the recommended approach whenever undertaking nonlinear dynamic analysis without careful record selection (e.g., IDA).

More recently, Tsantaki and Adam (2013) showed that the improvement achieved over the resulted record-to-record variability at collapse, by means of adopting a geometric mean of spectral accelerations, may be further increased by espousing an enhanced period range. For the latter, they proposed a simple analytical expression that links the initial period T_1 to an elongated period and illustrated that the geometric mean of the spectral accelerations evaluated across that period band, leads to a notable reduction in the seismic collapse capacity dispersion. A similar IM was also used by Shakib and Pirizadeh (2014) who studied the probabilistic seismic performance of a ten-story steel building at various degrees of vertical irregularity (i.e. setback ratios). The adopted $S_{agm}(T_i)$ was evaluated over a period range from $0.5T_1$ to $1.5T_1$ in increments of $0.05T_1$. However, in order to end up with a common IM for all structural configurations to allow the comparisons between buildings, instead of using the fundamental period of each individual model structure the mean first-mode period T_1 of the building set was adopted. A similar scalar IM based on $S_a(T_1)$ and a parameter N_p that accounts for the spectral shape in a period range, was proposed by Bojorquez and Iervolino (2011) and found to have improved efficiency was (Bojorquez and Iervolino, 2011; Modica and Stafford, 2014).

In view of the aforementioned findings, this study will compare the effectiveness of eight IMs, namely $S_a(T_1)$, $S_a(1 \sec)$ and $S_a(T_{1m})$ together with five $S_{agm}(T_i)$ choices, each employing a different selection of common periods T_i :

- a. Five logarithmically-equally-spaced periods $(T_i)_1$ within $[T_{2m}, 1.5 \cdot T_{1m}]$, where T_{2m} and T_{1m} are the mean T_2 and T_1 periods, respectively of the index buildings,
- b. Seven logarithmically-equally-spaced periods $(T_i)_2$ within $[\min T_2, 1.5 \cdot \max T_1]$,

- c. Five linearly-equally-spaced periods $(T_i)_3$ in $[T_{2m}, 1.5 \cdot T_{1m}]$,
- d. Four periods defined as $(T_i)_4 = [T_{2m}, \min [(T_{2m}+T_{1m})/2, 1.5 \cdot T_{2m}], T_{1m}, 1.5 \cdot T_{1m}],$
- e. Five periods defined as $(T_i)_5 = [T_{2m}, \min [(T_{2m} + T_{1m})/2, 1.5 \cdot T_{2m}], T_{1m}, 1.5 \cdot T_{1m}, 2 \cdot T_{1m}]$

Of the eight IMs, only $S_a(T_1)$ cannot be used for class-level vulnerability assessment, as previously mentioned. Yet, it will be examined alongside the others to establish a baseline for comparison with current practice. It should also be noted that, the actual definition of the individual $S_a(T)$ components to be employed in the determination of the eight candidate IMs depends on the GMPE used for the hazard (Baker and Cornell, 2006). GMPEs may be defined for the arbitrary $S_a(T)$ horizontal component or the geometric mean of the two $S_{a}(T)$ horizontal components per recording. The latter is the case for most GMPEs produced lately (e.g., PEER-NGA project) and it is the paradigm that we shall adopt in the examples that follow. Thus, for instance, the fifth choice for $S_{agm}(T_i)$ above, termed $S_{agm}(T_i)_5$, becomes a geomean combination of 10 different S_a -values, two for each of the five periods.

6. IM COMPARISON STUDY

The efficiency of the different IMs defined in section 5 was tested, in an attempt to identify the optimal IM across the structural response range. The latter is monitored by means of two structural response measures, these being the interstory drift ratio (IDR) and the peak floor acceleration (PFA). The proposed methodology differs from similar studies that have appeared in the literature, (e.g. Bianchini et al, 2009; Tothong and Luco, 2007), in two important aspects, namely (a) using an IM given EDP (IM|EDP) basis and (b) employing all IDR and PFA values at each story or floor, respectively. Working on an IM|EDP basis essentially translates to using "vertical stripes" of points in Figure 2, produced as cross sections of the 44 IDA curves for each index building with a vertical line signifying a given EDP value.

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Figure 2: 44 IDA curves for the 3ELF 4-story (SD1_{max}) *index building and a "vertical stripe" of IM "capacity values" at an interstory drift level of 3%.*

Thus, for each EDP type and for any number of its values, one may estimate IM "capacity values" required by each corresponding record to reach each prescribed EDP target. This has the obvious advantage of allowing a detailed pointwise assessment of efficiency that can reach all the way up to collapse (Vamvatsikos and Cornell, 2005). On the contrary, past studies have often relied on processing directly EDP values, typically for numerous levels of the IM at the same time, thus being forced to stay away from global collapse where response becomes essentially infinite or undefined. Finally, testing efficiency for local IDR and PFA values, rather than just the maximum interstory drift, is essential for IMs that are geared towards loss assessment. This idea was first tested in a rudimentary form by Aslani and Miranda (2005), Bradley et al. (2010) as well as in NIST (2010b) and is put forward herein as a rigorous test for selecting IMs for loss assessment. Thus, for each N-story building (N = 1, 2, 4), the candidate IMs will be checked for 2N+3 different EDPs: One IDR per story (N total), one PFA per floor (N+1total to include the ground level) plus the overall maximum interstory drift and the roof drift.

Figure 3 illustrates for the 3ELF 4-story index buildings and the eight considered IMs, the "maximum over height" dispersions of the IM capacities β_{IM} given IDR or PFA. The range of both EDP types has been selected to span from

the elastic up to the inelastic level of structural response. The maximum signifies the worst performance along the height of the building. The "average β_{IM} over height" of the building is also plotted to indicate the average efficiency of each IM at each level of deformation. Although showing the results from a single index structure, the same trends persist in each of the six lowrise index buildings.

Considering the overall results, some characteristic features appear. The maxima of dispersion given IDR (Figure 3a), show some high values in the early elastic region. However, as the average dispersion plots of IDR given IM in Figure 3b show, the aforementioned crest is indeed an isolated local effect, rather than a feature over the entire structure. A somewhat similar hump appears for PFAs, only now shifted from elasticity and close to the away (nominal/global) yield region of the structure (see Figure 3d). It seems that regardless of the IM, there is significant difficulty in capturing the complex interaction of modes that happens in the yield and post-yield region as well as the gradual transition to an elongated first-mode period that is characteristic of large deformations. This is generating dispersions that can grow from 30% up to 70%. Furthermore, this is not a localized effect, but rather widespread throughout the height of the structure, as it appears both in the maximum (Figure 3c) and average dispersions (Figure 3d).

The IM ranking for the lowrise buildings is not clear across the entire range of IDR values (Figures 3a and 3b), with $S_a(T_{1m})$ showing the best performance in the elastic region and $S_{agm}(T_{i},5\%)_5$ taking advantage in regions where the spread of inelasticity results in substantially elongated periods. The dispersion associated with the $S_{agm}(T_{i},5\%)_5$ is in the order of 30% for the examined building class (see Figure 3b). Regarding the IM ranking for the PFAs, the performance of each IM eventually stabilizes but at high PFAs.

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Figure 3: Maximum and average dispersions of the IM for given values of the IDR and PFA response of the 3ELF 4-story index buildings considering eight IMs.

Evidently, among the least efficient IMs across the examined EDP range, is the S_a (1sec,5%). The dispersions achieved by $S_a(T_1)$, which is only useful for single buildings, and $S_a(T_{1m})$ were also proven to be high, rendering their use relatively expensive: More ground motion records will need to be employed for a good estimate of the distribution of loss. Among the remaining IMs and considering their performance both in elastic and inelastic regions for IDR and PFA, the $S_{agm}(T_i)_5$, that employs five periods, the T_{1m} and the T_{2m} and their "elongated" versions, was proven to provide relatively stable dispersion estimates. In fact, it shows better efficiency practically everywhere with the exception of the PFA hump where it performs slightly worse than other IMs. Significant data also exists (Kazantzi et al, 2014) to show that the efficiency results are also mirrored in sufficiency: Bringing the response of records with different characteristics closer together, obviously reduces the expected bias (Vamvatsikos and Cornell, 2005; Bradley *et al*, 2010). In conclusion, it seems that a successful IM is created by specifying an appropriate period range that includes periods both above and below T_{1m} and combining the corresponding spectral accelerations in a geometric mean (see also Bianchini *et al*, 2009; Tsantaki and Adam, 2013).

7. VULNERABITY ESTIMATION

When estimating seismic losses, in order to inject the needed variability, one can define three variants of each index building: one variant with relatively rugged components, one with typical components, and one with relatively fragile components. Only the top six (6) or so nonstructural/content component types and the 12th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP12 Vancouver, Canada, July 12-15, 2015

top 1 or 2 structural component types are considered. By "top components" is meant the components that contribute most to construction cost new.

The values of peak floor accelerations at each floor or roof diaphragm and peak transient drift ratios at each story, captured via IDA, are input to fragility functions for each component at each floor (for acceleration-sensitive components) story (for drift-sensitive or components). One uses Monte Carlo methods to simulate ground motion time history, damage for each component, and repair costs per damaged component type and damage state. Total damage factor (DF, repair cost as a fraction of replacement cost new) in any simulation is given by Equation 2, in which V denotes the replacement cost new of the building, f denotes the fraction of V represented by the component types in the inventory, a is an index to floor level, N_a is the number of diaphragms, c is an index to component types, N_c is the number of component types considered, d is an index to damage states for a given component type, N_d is the number of possible damage states, n(a,c,d) is the number of damaged components at floor a, type c, in damage state d, and u(c,d) is the unit cost to repair a component of type c from damage state d.

$$DF = \frac{1}{V \cdot f} \sum_{a}^{N_a} \sum_{c}^{N_c} \sum_{d}^{N_d} n(a,c,d) \cdot u(c,d) \quad (2)$$

One calculates DF for each of many simulations for each combination of structural model and component set at each level of ground motion intensity, and captures mean damage factor (MDF) and coefficient of variation (COV) as a function of ground motion intensity. One equally weights the poor, typical, and superiorquality variants to estimate the MDF and COV for each index building and applies the class partitioning weights to calculate the MDF and COV for the class as a whole.

8. CONCLUSIONS

A practical methodology has been presented for performing analytical vulnerability assessment

for low/mid-rise steel building classes. Significant novel features of the proposed approach include: (a) Using class partitioning to select representative index buildings, (b) the use of simple structural models together with IDA for performing structural assessment, (c) the introduction of the geometric mean of spectral accelerations at adjacent periods as a sufficient and efficient intensity measure across an entire building class, (d) the use of a reduced list of "top components" that need to be taken into account for assessing the damage factor and (e) Monte Carlo simulation to propagate the uncertainty from different realizations of each index building to the class vulnerability results.

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