Estimation of Uncertain Parameters using Static Pushover Methods

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ABSTRACT: Following recent guidelines (e.g. FEMA-350) seismic performance uncertainty is an essential ingredient for performance-based earthquake engineering. Uncertainty refers to both aleatory uncertainty, raised by the random record-to-record variability, and also to epistemic uncertainty primarily introduced by modeling assumptions or errors. A methodology for the performance-based estimation of the dispersion introduced by parameter uncertainties is developed. The methodology proposed provides an inexpensive alternative to the use of tabulated values, or to performing a series of time-consuming nonlinear response history analyses to obtain parameter uncertainty. As a testbed, the well-known 9-storey LA9 2D steel frame is employed using beam-hinges with uncertain backbone properties. The monotonic backbone is fully described by six parameters, which are considered as random variables with given mean and standard deviation. Using point-estimate methods, first-order-second-moment techniques and latin hypercube sampling with Monte Carlo simulation, the pushover curve is shown to be a powerful tool that can help accurately estimating the uncertainty in the seismic capacity. Coupled with SPO2IDA, a powerful R-μ-T relationship, such estimates can be applied at the level of the results of nonlinear dynamic analysis, allowing the evaluation of seismic capacity uncertainty even close to global dynamic instability.

1 INTRODUCTION

Analysis of structures under seismic loading is plagued by the natural randomness due to ground motion record variability, and also by epistemic parameter uncertainty, stemming from modeling assumptions or errors. Unfortunately little data is available on the seismic demand and capacity uncertainty, an issue that current design codes and guidelines deal with tabulated values. Even less information is available on epistemic parameter uncertainty that usually receives less attention due to the general consensus that the randomness in seismic loading is the primary source of response variability. Still, recent research has expanded the estimation range of analysis methods to large deformations that lie close to the global dynamic instability region, where parameter uncertainty may become equally important.

Perhaps the first systematic effort to produce a simplified methodology to treat uncertainties in seismic design is that of Cornell *et al.* (2002) who formed the background for the SAC/FEMA approach (FEMA-350 2000) for the probabilistic assessment of steel frames. This approach allows including epistemic uncertainties simply by considering the dispersion they cause on the median demand and capacity. These concepts have been further advanced, among others, by Baker and Cornell (2003), Lupoi *et al.* (2002) and Krawinkler *et al.* (2006).

For earthquake engineering applications, perhaps the most comprehensive method to handle the aleatory uncertainty introduced by seismic loading is the Incremental Dynamic Analysis (IDA) method (Vamvatsikos & Cornell 2002). IDA essentially requires multiple nonlinear dynamic analyses with a suite of ground motion records to provide a full-range performance assessment, from the early elastic limit-states to the onset of collapse. To account for other sources of uncertainty, the IDA approach can be combined with reliability analysis methods such as Monte-Carlo simulation. Such a process has been proposed by Vamvatsikos & Fragiadakis (2009) and although it is extremely powerful, it nevertheless necessitates the execution of thousands or millions of nonlinear response history analyses and therefore is beyond the scope of practical application. Clearly the engineering practice needs analysis methods that provide estimates of the response statistics within an affordable computational effort.

In search for a compromise, we turn to the novel methodologies for estimating response statistics using static pushover methods. During the past few years, static pushover analysis methods (SPO) have become essential for the earthquake engineering practice and therefore this analysis approach lies in the core of the methods we are proposing. Furthermore, a single static pushover analysis requires considerably less computational resources compared to the hundreds response history analyses of a full IDA run, which is practically beyond the capability of most professional engineers. Similarly to IDA, our method maintains a full-range performance evaluation capability using as link between SPO and IDA a powerful *R*-μ-*T* (reduction factor - ductility - period) relationship, known as Static Pushover to Incremental Dynamic Analysis (SPO2IDA) (Vamvatsikos & Cornell 2006). SPO2IDA is an improved *R*-μ-*T* relationship that offers accurate prediction capabilities even close to collapse using, instead of bilinear elastoplastic, a quadrilinear approximation of the static pushover envelope. Having such a tool at our disposal we can quickly perform all the necessary simulations and actually obtain an accurate estimate of the effect of uncertainty on the demand and capacity of structures. Using a nine-storey steel frame as a reference structure we will employ Monte Carlo simulation and moment-estimation techniques together with static pushover and SPO2IDA to achieve rapid evaluation of the seismic performance variability due to epistemic uncertainty in the model parameters.

2 STRUCTURAL MODELS

The structure considered is the nine-storey steel moment-resisting frame with a singe-storey basement. The frame has been designed according to 1997 NEHRP (National Earthquake Hazard Reduction Program) provisions for a Los Angeles site. We use a centerline model with fracturing connections created with OpenSees (McKenna & Fenves 2001). The beams are modeled with lumped-plasticity elements, thus allowing the formation of plastic hinges at two ends, while the columns are considered elastic. Geometric nonlinearities have been incorporated in the form of *P-Δ* effects, while the internal gravity frames have been explicitly modeled. In effect, this is a first-mode dominated structure with a fundamental period of T_1=2.35s and a modal mass equal to 84% of the total that still allows for a significant sensitivity to higher modes.

The fracturing connections are modelled as rotational springs with moderately pinching hysteresis and a quadrilinear moment-rotation backbone, shown in normalized coordinates in Figure 1 (see also Ibarra *et al.* 2005). The random parameters considered refer to the properties of the quadrilinear backbone curve, which initially allows for elastic behaviour up to a_{M_y} times the nominal yield moment M_y, then hardens at a non-negative normalized slope of a_h that terminates at a rotational ductility μ_c. Beyond this point, a negative stiffness segment starts having normalized slope a_c. The residual plateau appears at a normalized height *r*, delaying the failure of the connection until the ultimate rotational ductility μ_u. Thus to completely describe the backbone of the monotonic envelope of the hinge moment-rotation relationship, six parameters are necessary: a_{M_y}, a_h, μ_c, a_c, *r* and μ_u, where similar behaviour is assumed for both positive and negative moments. This is essentially a complex backbone that is versatile enough to simulate the behaviour of numerous moment-connections.

Table 1. Random parameters and their statistics

	Mean	c.o.v	Lower bound	Upper bound
My	1.0	0.20	0.70	1.30
ah	0.1	0.40	0.04	0.16
μu	3.0	0.40	1.20	4.80
ac	-0.5	0.40	-0.80	-0.20
r	0.5	0.40	0.20	0.80
μ_u	6.0	0.40	2.40	9.60

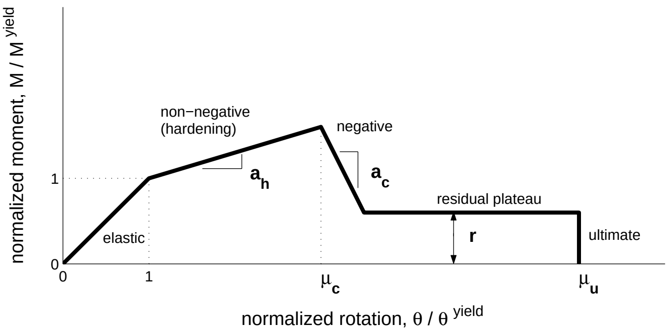

Figure 1 The moment-rotation relationship of the beam point-hinge in normalized coordinates.

The backbone properties of the plastic hinges are considered as random variables and hence are the only source of epistemic uncertainty. The parameters are modelled to be independently normally distributed with mean and c.o.v as shown in Table 1. The mean values represent best estimates of the fracture parameters, while the c.o.v was assumed equal to 40% for all the parameters except for the yield moment where 20% was used instead. To avoid assigning the random parameters with values with no physical meaning, e.g. a_h>1, or *r*<0, their distribution is appropriately truncated within 1.5 standard deviations as shown in Table 1. The resulting connection model is flexible enough to range from fully-ductile, nearly elastoplastic connections (e.g. *r*=0.80, μ_u=9) down to outright brittle cases that suddenly fracture at low ductilities (e.g. μ=2.4).

To account for parameter uncertainty stemming from the properties of the connections of a steel moment frame the plastic hinge properties can be varied simultaneously for the whole structure, or individually by applying local changes to several connections. In the later case the precise locations of the connections are randomly assigned thus assessing the global performance when the capacity of a number of connections is uncertain e.g. due to poor manufacturing or localized phenomena. Luco & Cornell (2000) studied the response of moment-resisting frames with random fracturing connections typical to those of pre- and post-Northridge steel buildings. On the other end, varying together the properties of every frame connection is expected to have a more pronounced effect on the response, pinpointing the influence of each of the six parameters on the global capacity. This scenario is the one that we are going to adopt for our present study as it is consistent with the case where the engineer does not have sufficient data for the moment-rotation backbone and therefore his/her model relies on empirical values and his judgment or experience.

3 STATIC PUSHOVER TO INCREMENTAL DYNAMIC ANALYSIS (SPO2IDA)

A fast and accurate approximation has been recently proposed for Incremental Dynamic Analysis (IDA), both for single and multi-degree-of-freedom systems utilizing information from the force-deformation envelope (or backbone) of the static pushover to generate the summarized 16%, 50% and 84% IDA curves (Vamvatsikos & Cornell 2005, 2006). The prediction is based on the study of numerous SDOF systems having a wide range of periods, moderately pinching hysteresis and 5% viscous damping, while they feature backbones ranging from simple bilinear to complex quadrilinear with an elastic, a hardening and a negative-stiffness segment plus a final residual plateau that terminates with a drop to zero strength as shown in Figure 1 (Ibarra *et al.* 2003). Having compiled the results into the SPO2IDA tool, we can get an approximate estimate of the performance of virtually any SDOF oscillator without having to perform the costly analyses, and almost instantaneously recreate the fractile IDAs in normalized coordinates of R versus ductility μ. SPO2IDA is in essence a powerful R-μ-T relationship that will provide not only central values (mean or median) but also the dispersion, due to record-to-record aleatory randomness, using a multilinear approximation of the static pushover.

The SPO2IDA tool has been extended to first-mode dominated MDOF structures (Vamvatsikos & Cornell 2005), enabling an accurate estimation of the fractile IDA curves even close to collapse without needing any nonlinear dynamic analysis. The application of the SPO2IDA tool on the LA9 moment-resisting steel structure is schematically shown in Figures 2 and 3. The process involves approximating the static pushover curve with a multilinear envelope to allow extracting the properties of the backbone curve (Fig. 1). Typically for first-mode dominated structures instead of a multilinear approximation of the static pushover, as shown in Figure 1, a trilinear approximation will suffice (Fig. 2). Moreover, Vamvatsikos & Cornell 2005 report that the choice of the lateral load pattern has a significant effect on the SPO envelope and therefore the application of SPO2IDA on MDOF structures entails the identification of the most-damaging pattern. While it is advocated to search for the most damaging load pattern, for this model of the LA9 steel frame it was found that a triangular or a first-mode lateral load pattern will provide sufficiently accurate results. Although SPO2IDA provides all three 16%, 50% and 84% fractiles, we need to work only with the median (50% fractile) when studying the effect of parameter uncertainty.

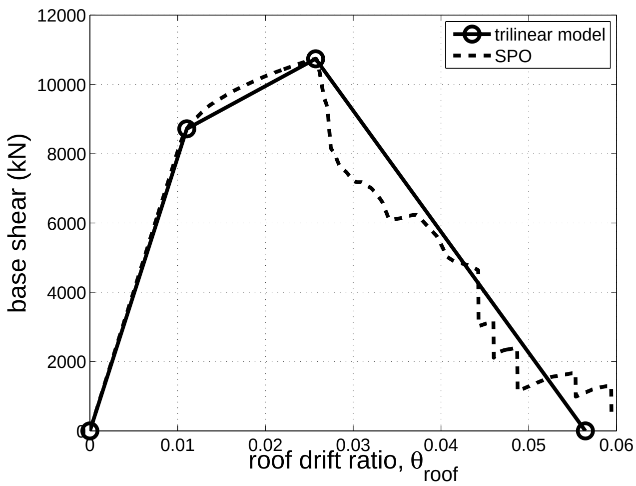

Figure 2 The SPO curve for the LA9 nine-storey steel structure and its approximation with a trilinear model

Having approximated the SPO capacity curve with a trilinear model, the backbone parameters can be easily extracted. Following the terminology of SPO2IDA (for a backbone description similar to that of Fig.1), for a trilinear approximation the extracted parameters will be: F_y^{SPO} (instead of M_y in Fig. 1), a_h^{SPO}, μ_h^{SPO}, a_c^{SPO}, while r^{SPO} is set equal to zero and μ_u^{SPO} is defined as the intersection of the horizontal axis with the descending branch of the trilinear model. The four parameters are given as input to SPO2IDA to produce the median capacities (Fig. 3). Since the capacities of SPO2IDA are in dimensionless *R*-μ coordinates, they need to be scaled to another pair of IM, EDP coordinates, more appropriate for MDOF systems, such as the $S_a(T_1,5\%)$- and the maximum interstorey drift ratio θ_{max}.

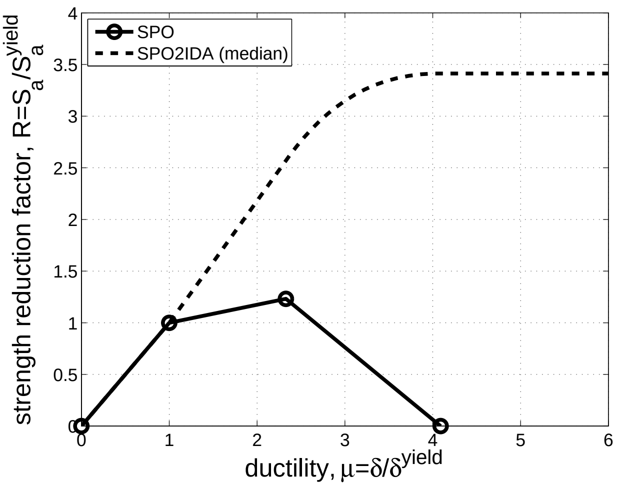

Figure 3 The trilinear approximation of the SPO curve and the corresponding IDA produced with the SPO2IDA tool in *R*-μ coordinates.

The scaling from *R*-μ to $S_a(T_1,5\%)$-θ_{max} can be performed with simple algebraic calculations:

$$\begin{aligned}\mathbf{S_a}(T_1,5\%) &= \mathbf{R}\,S_a^{yield}(T_1,5\%)\\ \boldsymbol{\theta}_{roof} &= \boldsymbol{\mu}\,\theta_{roof}^{yield}\end{aligned} \qquad (1)$$

Once $\boldsymbol{\theta}_{roof}$ is known, $\boldsymbol{\theta}_{max}$ can be extracted from the results of the SPO, since for every load increment the correspondence between the two EDPs is always available. Prior to applying Equation (1) we have to determine $S_a(T_1,5\%)$ and θ_{roof} at yield. This task is trivial for SDOF systems, but it is not straightforward for MDOF structures mainly due to the effect of higher modes. Some records will force the structure to yield earlier and others later, thus yielding will always occur at different levels of $S_a(T_1,5\%)$ and θ_{roof}. Driven by our trilinear approximation to the SPO curve, we let the yield roof drift, θ_{roof}^{yield}, be defined as the apparent yield point of the approximation. This assumption is not strictly true for MDOF structures and it strictly becomes accurate only if the first mode is dominant, but it is enough for our purpose. Therefore, the accurate estimation of $S_a^{yield}(T_1,5\%)$ comes down to approximating the elastic "stiffnesses" (or slopes) of the median IDA curves plotted with θ_{roof} as the EDPs. The stiffness, denoted as k_{roof} is the median value obtained using elastic response history analysis with a few ground motion records, or simply by using standard response spectrum analysis. Finally, $S_a^{yield}(T_1,5\%)$ will be:

$$S_a^{yield}(T_1,5\%) = k_{roof}\theta_{roof}^{yield} \qquad (2)$$

In summary, the process of producing an approximate IDA curve from a single static pushover run involves the following steps. Initially perform a static pushover analysis with a first-mode lateral load pattern and then approximate it with a trilinear model. Next SPO2IDA will provide the normalized IDA curves. The final step is scaling the IDA curves to the $S_a(T_1,5\%)$ and θ_{max} coordinates. This requires the elastic slope of the actual IDA, k_{roof} when θ_{roof} is used for the EDP. Therefore, a few linear elastic response history or response spectrum runs are need. Note that k_{roof} has to be calculated only once for all instances of the model since none of the uncertain parameters influences the elastic response. With the aid of Equations (1) and (2) we obtain the IDAs in $S_a(T_1,5\%)$-θ_{roof} coordinates. The final IDAs are reached using the mapping between θ_{roof} and θ_{max}, available from the results of the SPO. For the SPO curve of Figure 1, the median IDA obtained with SPO2IDA and the actual IDA curve using thirty ordinary ground motion records (Vamvatsikos & Fragiadakis 2008) are shown in Figure 4. For our model, the error in the procedure is typically 10-20%, while the computing time comes down from 2-3 hours required for a single IDA to just a couple of minutes for SPO2IDA, approximately two orders of magnitude less.

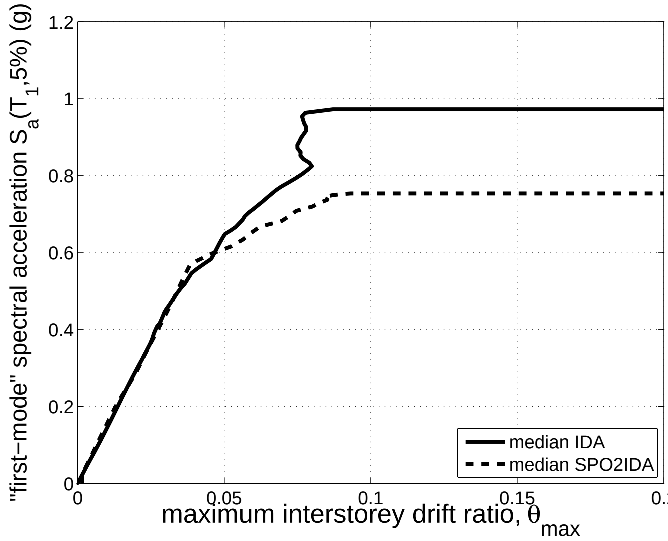

Figure 4 The median SPO2IDA capacity curve of Figure 3, plotted against the actual median IDA.

4 METHODOLOGY

In order to accurately calculate the performance statistics of a structure using static pushover methods, we introduce an approach based on the classic Monte-Carlo (MC) simulation, and as simpler and less resource-demanding alternatives we propose methods that rely on moment-estimating techniques such as Rosenblueth's $2K$+1 point estimate method (PEM) (Rosenblueth 1981) and the first-order, second-moment method (FOSM) (Pinto *et al.* 2004). In the sections to follow we discuss the application of the three methods.

4.1 *Monte Carlo Simulation*

Using the Monte-Carlo (MC) simulation on top of SPO2IDA we are able to obtain a sufficient sample of IDA curves which can be easily post-processed to get meaningful response statistics. Since we are interested primarily on the mean and the dispersion of the capacity, the MC method can be combined with the Latin Hypercube Sampling (LHS) method (McKay *et al.* 1979) to ensure improved accuracy within as few simulations as possible. As previously discussed, the random variables are the six parameters that fully describe the backbone of the plastic hinge moment-rotation relationship (Fig. 1) and they are varied concurrently throughout the structure. The properties of the random parameters are shown in Table 1. Using their distribution properties we perform latin hypercube sampling to obtain N_{LHS} Monte-Carlo samples using the algorithm of Iman & Conover (1982) to ensure zero correlation among the variables. Each realization of the LA9 frame is subjected to an SPO analysis and then the SPO2IDA tool is utilized to finally obtain N_{LHS} median IDA realizations, as discussed in the previous section.

In our development we are interested in estimating a central value and a dispersion for the S_{a}-values of capacity for a given limit-state defined at a specific value of $\theta_{\max}$. As a central value we will use the median of the $S_{\mathrm{a}}(T_1,5\%)$-capacities given $\theta_{\max}$, $\Delta_{S_{\mathrm{a}}|\theta_{\max}}$, while the dispersion caused by the uncertainty in the median capacity will be characterized by its β-value, (Cornell *et al.* 2002), i.e. the standard deviation of the natural logarithm of the median S_{a}-capacities conditioned on $\theta_{\max}$: $\beta_{\mathrm{U}}=\sigma_{\ln S_{\mathrm{a}}|\theta_{\max}}$. In terms of the work of Jalayer (2003) we will be essentially adopting the IM-based method of estimating the mean annual frequency of limit-state exceedance. Thus, while we will be implicitly using all S_{a}-related quantities as conditioned on $\theta_{\max}$, we will simplify the notation by dropping the conditioning "…$|\theta_{\max}$" from all formulas.

Thus, if $S^{j}_{a,50\%}$, (j=1,…,N_{LHS}) are the median S_{a}-capacities for a given value of $\theta_{\max}$ and $\overline{\ln S_{a,50\%}}$ is the mean of their natural logarithm, then we can obtain the overall median and dispersion β_{U} as:

$$\Delta_{S_a} = \mathrm{med}_j\left(S^{j}_{a,50\%}\right) \tag{3}$$

$$\beta_U = \sqrt{\frac{\left(\sum_j \ln S^{j}_{a,50\%} - \overline{\ln S^{j}_{a,50\%}}\right)^2}{N_{LHS}-1}} \tag{4}$$

where "med$_j$" is the median operator over all indices j.

4.2 *Point estimate methods*

Point estimate methods (PEM) can be used to calculate the first few moments of a function in terms of the first moments of the random variables. Rosenblueth's $2K$+1 method (Rosenblueth 1981) is based on a finite difference concept and is one of the easiest point estimate methods to implement. In essence it is a simulation technique that requires $N_{\mathrm{PEM}}=2K+1$ simulations, where K is the number of random variables. The advantage of the method is that it does not require knowledge of the distribution of the random variables since only the first two moments are sufficient. To apply PEM for our purpose, the log of the median S_{a}-capacities given $\theta_{\max}$ is considered a function of the six random parameters:

$$\ln S^{j}_{a,50\%} = f\left(\mathbf{X}\right) = f\left(a_{My}, a_h, \mu_c, a_c, r, \mu_u\right) \tag{5}$$

where f is a function (with unknown analytical form) of the random variables for the given limit-state (i.e. value of $\theta_{\max}$ considered) and $\mathbf{X}$= [a_{My}, a_{h}, μ_{c}, a_{c}, r, μ_{u}] is the vector of the random modeling parameters.

First of all, PEM requires the evaluation of $\ln S^{0}_{a,50\%}$, the base-case value of f that corresponds to all random variables being set equal to their mean m_{X_k}. The remaining $2K$ simulations are obtained by shifting each parameter X_{k} (k = 1,…,6) from its mean by $\pm\ \sigma_{X_k}$ while all other variables remain equal to their mean m_{X_k}. The logs of the median S_{a}-capacities when the X_{k} parameter is perturbed are denoted as $\ln S^{k+}_a$ and $\ln S^{k-}_a$, where the sign indicates the direction of the shift. For example if the 4$^{\mathrm{th}}$ parameter, $X_4=a_{\mathrm{c}}$, is perturbed, then the capacities $S^{k\pm}_a$ are calculated as:

$$\ln S^{4+}_a = f\left(m_{a_{My}}, m_{a_h}, m_{\mu_c}, m_{a_c}+\sigma_{a_c}, m_r, m_{\mu_u}\right) \tag{6}$$

$$\ln S^{4-}_a = f\left(m_{a_{My}}, m_{a_h}, m_{\mu_c}, m_{a_c}-\sigma_{a_c}, m_r, m_{\mu_u}\right)$$

Based on Eq. (6), for each random variable we calculate the quantities, Y_{k} and v_{k}, where:

$$Y_k = \frac{\ln S^{k+}_a + \ln S^{k-}_a}{2} \tag{7}$$

$$v_k = \frac{\ln S^{k+}_a - \ln S^{k-}_a}{\ln S^{k+}_a + \ln S^{k-}_a} \tag{8}$$

Finally, the estimated conditional mean and coefficient of variation (c.o.v) of S_{a} are obtained as follows:

$$m_{\ln S_a} = \ln S^0_{a,50\%} \prod_{k=1}^{K} \frac{Y_k}{\ln S^0_{a,50\%}} \tag{9}$$

$$V_{\ln S_a} = \sqrt{\left\{ \prod_{k=1}^{K} \left(1+v_k^2\right) \right\} - 1} \tag{10}$$

Then, assuming lognormality, the median $S_a|\theta_{max}$ and the dispersion β_U can be estimated as:

$$\Delta_{\ln S_a} \approx \exp\left(m_{\ln S_a}\right) \tag{11}$$

$$\beta_U \approx m_{\ln S_a} V_{\ln S_a} \tag{12}$$

For every limit-state, Eqs (6)-(8) are used to estimate the median IDA curve and the β-dispersion values with only $2K$+1 static pushover runs. Still some attention is needed in applying this method, since the natural logarithm of a number that is less than one is negative and therefore the PEM may yield unpredictable results for low S_a-values. We can easily overcome this problem if the $\ln S_a$ values are shifted by adding a positive constant c, thus replacing $\ln S_a$ by $\ln S_a + c$ in all the above equations. Then, we only need to remove c from $m_{\ln S_a|\theta_{max}}$ to get the median. The process was found to be insensitive to the choice of c, as long as its value is kept relatively low. A value of c=10 was used in this study but no significant differences were found for c-values up to 100.

4.3 *First-Order Second-Moment method*

The first-order, second-moment (FOSM) method is another easy-to-implement approximating method that can be used to calculate the first moments of a nonlinear function. The number of simulations required is $2K$+1, equal to that of the PEM, while this method also does not require prior knowledge of the distribution of the random parameters. According to FOSM the unknown, in closed form, nonlinear function f can be approximated through the use of a Taylor expansion to obtain its first and second moments. Following the notation of Eq. (5), the function f=$\ln S_a$ is approximated using Taylor series expansion around the mean $\overline{\mathbf{X}}$=[$m_{a_{My}}$, m_{a_h}, m_{μ_c}, m_{a_c}, m_r, m_{μ_u}]. Assuming that the random variables are not correlated the approximation takes the form (Pinto *et al.* 2002):

$$f(\mathbf{X}) \approx f\left(\overline{\mathbf{X}}\right) + \sum_{k=1}^{K} \left(X_k - m_{X_k}\right) \left.\frac{\partial f}{\partial X_k}\right|_{\overline{\mathbf{X}}} + \frac{1}{2} \sum_{k=1}^{K} \left(X_k - m_{X_k}\right)^2 \left.\frac{\partial^2 f}{\partial X_k^2}\right|_{\overline{\mathbf{X}}} \tag{13}$$

The gradient and the curvature of f can be approximated with a finite difference approach, which is why we need $2K$+1 simulations. The random parameters are set equal to their mean to obtain S_a^0 and then each random parameter is perturbed according to Eq. (6). Thus, the first and the second derivative of f with respect to X_k, will be:

$$\frac{\partial f}{\partial X_k} = \frac{\ln S_a^{k+} - \ln S_a^{k-}}{\sigma_{X_k}} \tag{14}$$

$$\frac{\partial^2 f}{\partial X_k^2} = \frac{\ln S_a^{k+} - 2\ln S_a^0 + \ln S_a^{k-}}{\sigma^2_{X_k}} \tag{15}$$

Truncating after the linear terms in Eq. (13) provides a first-order approximation for the limit-state mean capacities, where essentially they are assumed to be equal to the base-case values $\ln S_a^0$. A more refined estimate is the mean-centered, second-order approximation, which according to Eq. (13) can be estimated as:

$$m_{\ln S_a} \approx \ln S^0_{a,50\%} + \frac{1}{2} \sum_{k=1}^{K} \left.\frac{\partial^2 f}{\partial X_k^2}\right|_{\overline{\mathbf{X}}} \sigma_{X_k^2} \tag{16}$$

Thus the median S_a capacity, assuming lognormality, comes out to be:

$$\Delta_{Sa} \approx \exp\left(m_{Sa}\right) \tag{17}$$

Finally, we use the first-order approximation for the standard deviation that is calculated using the formula:

$$\beta_U \approx \sum_{k=1}^{K} \left(\left.\frac{\partial f}{\partial X_k}\right|_{\overline{\mathbf{X}}} \right)^2 \sigma^2_{X_K} \tag{18}$$

5 NUMERICAL RESULTS

In order to test the validity of the above-described approximating procedures we applied Monte Carlo simulation using full IDA with thirty ordinary ground motion records on the LA9 nine-storey steel moment-resisting frame (Vamvatsikos & Fragiadakis 2008). Therefore, we have the exact seismic performance metrics in IDA-terms of N_{LHS}=200 realizations of the nine-storey frame. The response statistics obtained with the actual IDA are considered as a reference solution that our approximate SPO-based estimations have to comply with.

The IDA and SPO2IDA curves of the Monte Carlo simulation are post-processed to provide the overall median IDA, Δ_{Sa}, and the β_U-dispersion, given θ_{max}. Figure 5 shows the median S_a-capacities conditional on the limit-state, θ_{max}. The black lines correspond to the capacities obtained using Monte Carlo simulation and the gray lines were derived with the approximating PEM and FOSM methods. When Monte Carlo simulation is used on top of SPO2IDA ($MC_{SPO2IDA}$) the conditional median, Δ_{Sa}, is very close to that of the actual IDA for the every limit-state, until θ_{max}=0.08, while for higher limit-states, beyond 0.1, the agreement is still remarkable. Approaching the collapse limit-states, the error on $\Delta_{S_a|\theta_{max}}$ remains constant and approximately equal to 16%. Combining PEM or FOSM with SPO2IDA seems to provide a very accurate prediction for the median capacities for limit-states with $\theta_{max}\geq0.1$. For early and intermediate limit-states, the moment-estimating methods provide estimates of the median practically identical to that of MC_{IDA} and $MC_{SPO2IDA}$. For θ_{max} values in the 0.04-0.08 range, the moment-estimating methods seem to slightly fluctuate implying that the estimations of the PEM and the FOSM are less stable compared to those of the Monte Carlo methods. However, their small error justifies their use over the resource-consuming Monte Carlo methods.

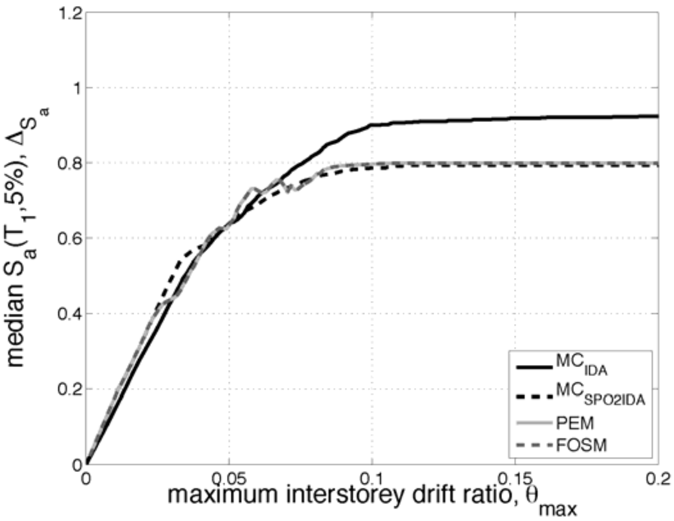

Figure 5 Conditional median of S_a given θ_{max} estimated by Monte Carlo on IDA and SPO2IDA versus PEM and FOSM on SPO2IDA.

Figure 6 shows the dispersion β_U of the $S_a(T_1,5\%)$-capacities conditioned on θ_{max}. The agreement of the methods proposed is remarkable for the whole range of limit-states even when approaching collapse. More specifically the dispersion at the collapse was found close to 0.25 with the $MC_{SPO2IDA}$ approach and 0.29 with the actual IDA, thus resulting to a 15% error. The $MC_{SPO2IDA}$ slightly overestimates β_U for limit-states lower than 0.07, and underestimate it beyond 0.1. PEM and FOSM seem to provide very accurate estimation for limit-states until 0.07, while for higher drift values the β-dispersion estimates are similar to that of $MC_{SPO2IDA}$, underestimating the MC_{IDA} by almost 15%. It is important to note that a quadrilinear approximation to the pushover will slightly reduce such errors at the expense of more difficult fitting. We also need to point out that the moment-estimating methods, as in the case of the median IDAs, again exhibited numerical instabilities due to the post-processing of the IDA curves. To draw useful engineering conclusions for the β-values their effect has been slightly smoothed out for Figure 6 using a non-parametric locally weighted regression (LOESS) technique with a coarse span for the moving average (Cleveland 1979).

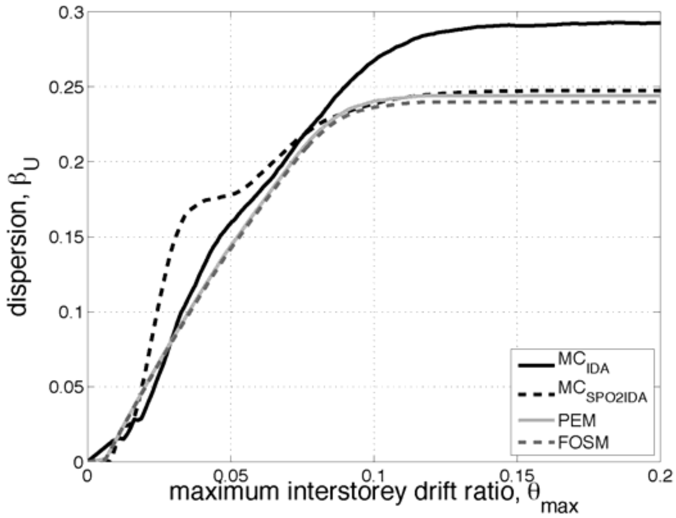

Figure 6 Conditional β-dispersion values of S_a given θ_{max} estimated by Monte Carlo on IDA and SPO2IDA versus PEM and FOSM on SPO2IDA.

The accuracy observed in Figure 6 can be partially explained by the small error on the prediction of the median IDAs (Fig. 4), which practically remains constant for every simulation and was found to be of the order of 10-20%. Since the error in the prediction of the median IDA is constant, or varies slightly, the methods proposed are particularly suitable for calculating the dispersion due to epistemic uncertainty, β_U, which normally necessitates more simulations than those required to estimate the median.

6 CONCLUSIONS

An innovative approach has been presented to propagate the epistemic uncertainty from the model parameters to the actual seismic performance of a structure, providing inexpensive estimates of the response parameters of the limit-state capacities. The methodology proposed has been applied on a nine-storey, steel moment-resisting frame with quadrilinear fracturing connections that are fully described by six non-deterministic parameters. Monte-Carlo simulation with latin hypercube sampling and moment-estimating techniques are adopted on top of the Static Pushover to Incremental Dynamic Analysis (SPO2IDA) tool. SPO2IDA provides computationally inexpensive full-range performance estimations, approximating the time-consuming full IDA. The overall accuracy of the proposed methods was found to be remarkable and comparable to the results of the full IDA, at only a fraction of the cost. All in all, the proposed tool is an excellent resource for accurate estimation of the seismic performance of structures having uncertain system properties, for the first time providing specific results for each limit-state that can be used in the place of the generic, code-prescribed values.

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