QUALITATIVE COMPARISON OF STATIC PUSHOVER VERSUS INCREMENTAL DYNAMIC ANALYSIS CAPACITY CURVES

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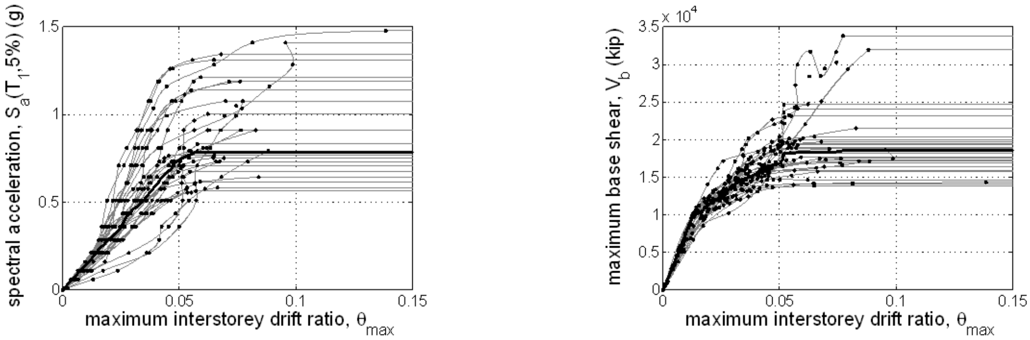
The Nonlinear Static Procedure (NSP), also known as ‘pushover’ analysis, is widely adopted in earthquake engineering practice. It is used for the estimation of various engineering demand parameters that provide a measure of the demand (and the capacity) of structures (e.g. displacements, storey drifts, forces, curvatures). Apart from element, or storey-level quantities, many NSPs provide the so-called ‘capacity curve’, i.e. the plot of the roof displacement against the total base shear applied on the building. Alternatively, local or global estimates of the building’s capacity can be obtained with Incremental Dynamic Analysis (IDA), which generates percentile ‘capacity curves’ in terms of seismic intensity versus the demand parameter of choice. The latter method is based on Nonlinear Response History Analysis (NRHA), and is thus more reliable and accurate compared to NSP. In this work, we study qualitatively the properties of the building capacity curves obtained with either NSP or IDA. We show that the comparison can be performed either in the framework of the static pushover or in that of IDA. When the static pushover setting is adopted, we show that pushover methods can be compared with the results of IDA by plotting the latter in the form of ‘dynamic capacity curves’ where the base shear instead of an intensity measure (e.g., spectral acceleration) is plotted on the ordinates. Alternatively, the comparison can be performed within the IDA setting if appropriate R-C_1-T relationships are adopted. Each setting shows the different qualitative characteristics of the two approaches and has different practical applications.

1. INTRODUCTION

This work compares Nonlinear Static Procedures (NSP) and Nonlinear Response History Analysis (NRHA) on a global level, visually expressed through the building’s capacity curve. NRHA is considered in the form of Incremental Dynamic Analysis (IDA) in order to obtain a measure of the “dynamic” capacity of the structure. The two methods can be directly compared in the local-member level, comparing the demand estimates on each of the buildings members. In this paper, we demonstrate that the comparison can be also performed on the global level, adopting either the framework of static methods or the framework of IDA. In the former case the comparison is possible if the IDA capacities are presented using an appropriate parameter on the ordinate, while in the latter case an R-C_1-T (or R-μ-T) relationship is necessary to bring the results of the NSP analysis to the setting of IDA.

2. COMPARISON IN PUSHOVER SPACE

Nonlinear response history analysis (NRHA) can be used for the seismic performance assessment of structures by adopting the Incremental Dynamic Analysis (IDA) method [1]. IDA involves repeatedly running NRHAs using a suite of ground motions scaled to different factors such that the response to each ground motion is obtained at many different intensities. Specifically, for any Engineering Demand Parameter (EDP) used to characterize structural response and an Intensity Measure (IM), e.g. the 5%-damped, first-mode spectral acceleration $S_a(T_1,5\%)$, we can generate IDA curves consisting of the EDP plotted as a function of the IM for each record (Figure 1a). Conventionally, the response EDP (dependent parameter) is plotted on the abscissa, and the IM (independent variable) is plotted on the ordinate. Given these IDA curves, the statistical distribution of response as a function of input can be summarized by curves that represent the 16%, 50% and 84% fractiles.



(a) (b)

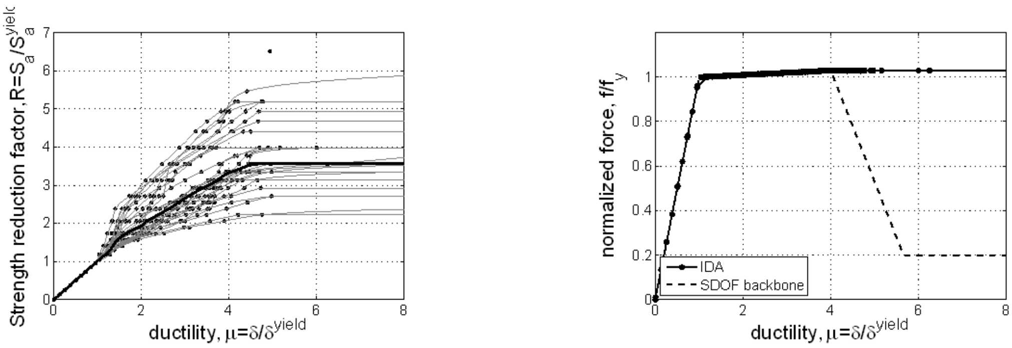
Fig. 1 IDA results for a nine-storey steel building: (a) IDA curves plotted as a function of $S_a(T_1,5\%)$, (b) dynamic pushover curves plotted as a function of the peak base shear V_b.

Alternatively, the results of IDA can be plotted using the same coordinates as in the NSP, resulting in the so-called “dynamic capacity curve”, calculated for each ground motion [2,3]. The dynamic capacity curves can plot roof displacement, roof drift (θ_{roof}) (i.e. roof displacement normalized by the building’s height) or any other EDP as a function of base shear. The effect of plotting on the ordinate the base shear (V_b) instead of $S_a(T_1,5\%)$ is shown in Fig. 1b, where the common EDP is $\theta_{\max}$, the maximum interstorey drift over the height of the building. It should be noted that base shear is a structural response parameter, covariate with the drift, and therefore is an EDP, which should not be considered to be an IM. In order to consider base shear as the IM, it would be necessary to develop a building-specific ground motion prediction equation to derive appropriate seismic hazard curves for base shear, thereby eliminating the benefit of using a general-purpose IM. Thus, we will differentiate between the two different approaches by using the terms *IDA setting* and *IDA curves* when having an IM on the ordinate, versus *pushover setting* and *dynamic pushover/capacity curves* (or *DPO curves*) when plotting the base shear instead.

Due to the large variability observed in the single-record IDAs (Fig. 1a), it is customary to summarize them with their median curve, plotted here with a solid dark line and the 16%, 84% fractiles that denote the dispersion. The median IDA can be calculated either as the median of the EDP given IM, or as the median IM given EDP. Both approaches will yield approximately the same results. The dispersion around the median, or in other words the variability of the IDAs, can be measured with beta, β, i.e. the standard deviation of the natural logarithms of the IM values for a given EDP value. If the data follow a lognormal distribution, β is equal to half the difference of the 16 and 84% fractile IM values. The dispersion can be used to obtain an estimate of the likelihood of a single-record IDA curve being close or away from the median curve. Similar percentile DPO curves can be generated by taking percentile values of base shear given EDP (Fig 1b).

The practice of plotting IDA results in a pushover setting can be found in several publications [2,3] and has its roots in nonlinear mechanics where it is customary to visualize the nonlinear response of the structure with a force-displacement plot. Furthermore, when $S_a(T_1,5\%)$ is used in IDA, the independent variable is plotted on the ordinates although it is customary to have it on the abscissas. This practice is preferable, since it results to curves that are more familiar to engineers, having a clear initial elastic branch and terminating at a horizontal flatline that indicates the seismic intensity that the building collapses.

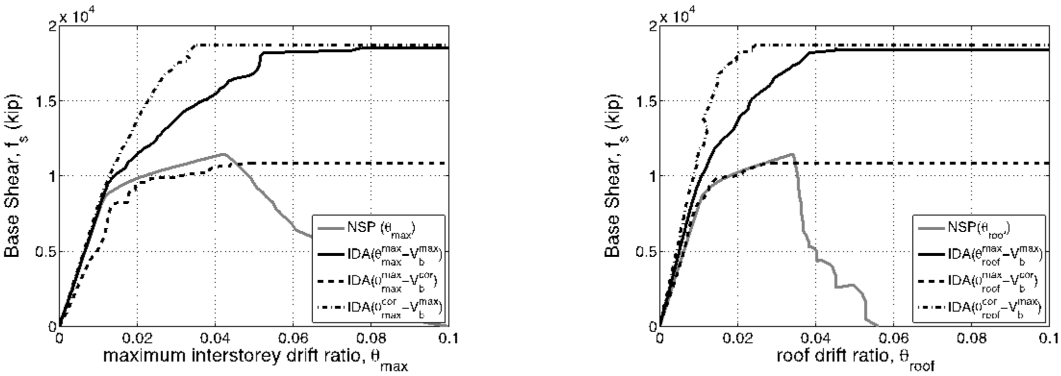
Looking at Fig. 1 it is clear that the dispersion around the median IDA is considerably larger than the dispersion around the median DPO curve. To further investigate the difference of the two plots of Fig. 1, we obtain similar results for a much simper, single-degree-of-freedom (SDOF) system, shown in Fig. 2. Again significant dispersion exists in the IDA setting, where the strength reduction factor $(R = S_a(T_1,5\%)/S_a^{yield}(T_1,5\%))$ is used as the IM. On the other hand, in a pushover setting, no dispersion is evident. In this, case the dynamic curves all lie on the backbone of the oscillator (dashed line in Fig. 2b), as long as the backbone has a positive slope. Then, if one plots maximum (over time) displacement versus the corresponding base shear, the curves will still follow the backbone. If instead the maximum base shear is plotted together with the corresponding displacement, the curves will become horizontal when the backbone descends.



(a) *(b)*

Fig. 2 IDA results of a nonlinear SDOF system with a backbone having a negative stiffness segment. Different intensity measures are plotted on the vertical axis: (a) strength reduction factor R, (b) peak oscillator's force f_s.

From the qualitative comparison of Fig. 1 and Fig. 2 useful conclusions can be drawn. When plotting either in IDA or pushover setting, the source that produces the dispersions observed in IDA analysis is related to the different damage patterns, or collapse mechanisms, activated by the ground motion record, which depend on the building's design and the ground motion characteristics. If we look at an advanced stage of the inelastic response, e.g. θ_{max} values beyond 0.07, in the first case the dispersion that appears in the plots on the left (Fig. 1a and Fig. 2a) is caused by the variability in the $S_a(T_1,5\%)$ value of each ground motion. To investigate the dispersion of the plots on the right (Fig. 1b and Fig. 2b), the variability of the records is no longer revealed in the case of the SDOF oscillator (Fig. 2b), where only one damage pattern is possible. Therefore, the IDAs show the usual record-to-record variability, while the dynamic pushover curves are all the same, coinciding with the backbone, i.e. with the force-deformation relationship that describes the capacity of the oscillator. Similarly, in the MDOF case, records that trigger similar damage mechanisms, or, in general, damage mechanisms that reach similar values of base shear, result in dynamic curves that are closer. Thus, we get the impression of a reduced dispersion around the median since much of the ground motion variability is now hidden. In the MDOF case, variability is also observed in the elastic domain; for the same base shear value, the seismic forces arise from the interaction of the various modes over time, with peak value resulting from different load patterns that vary with the ground motion record. No such elastic-level dispersion is observed for the SDOF oscillator, where only one degree-of-freedom exists. Thus all IDA and DPO curves coincide before yielding (Fig. 2a,b).



(a) (b)

Fig. 3 Median dynamic pushover curves versus the corresponding static pushover: (a) plotted against maximum interstorey drift, θ_{max}, (b) plotted against roof drift, θ_{roof}. The superscripts "max" and "cor" denote that the quantity is the maximum over the entire timehistory, or it corresponds to the time instant that the other parameter is maximized, respectively. For example θ_{max}^{cor} is the θ_{max} value at the instant that the base shear is maximum and θ_{max}^{max} is the maximum θ_{max} value during the timehistory.

In Fig. 3, it is shown that the dynamic pushover curves do not descend as the NSP curves do (gray lines). Moreover, since base shear and drift ($\theta_{\max}$ or θ_{roof}) do not take their maximum values at the same instants in time, three variations for generating the dynamic capacity curves are investigated in Fig. 3. The first corresponds to the case where the maximum drift and maximum base shear (V_{b}) are plotted, while in the second case the V_{b} values considered are the values when the drift is maximized. In the third case, we use the drift values at the instant of maximum V_{b}. Another practice, not examined here, is considering the maximum drift and the peak base shear of a time window, e.g. ±0.5 seconds, around the instant that the maximum drift occurs [2]. Since $\theta_{\max}$ is the EDP most commonly used in IDA while θ_{roof} is customarily plotted on the horizontal axis of the NSP curves, we perform the comparison for both EDPs. According to Fig. 3, median DPO and NSP curves will always have distinct differences, which may look smaller or larger when we show single records instead of the medians. For both EDPs the elastic slopes of the DPO and the NSP capacity curves (Fig. 3) practically coincide. However, significant differences are observed in the maximum base shear capacity for both EDPs, when we plot the maximum base shear versus the maximum or the corresponding drift value. Good prediction of the maximum base shear capacity is observed only when the base shear corresponds to the time instant that the drift is maximized. Furthermore, the DPO curves are not able to follow the negative slope of the NSP curves. This is a data-processing issue, since in DPO plotting we consider maximum force and/or displacement values over the timehistory.

3. COMPARISON IN IDA SPACE

Another approach to perform the comparison between NSP and IDA is to express the NSP curve in the IM and EDP coordinates chosen for the IDA. To facilitate a direct comparison, we divide the base shear force by the building's mass and we adjust the "elastic stiffness" (or slope) of the NSP to that of the IDA, i.e. by matching their elastic segments. The results of such a procedure are shown in Fig. 4 where we plot the NSP curve, obtained using a first-mode lateral load pattern, against the median IDA for a 20-storey steel moment-resisting frame having ductile connections. Qualitatively, we can make some general observations, which permit inference of the approximate shape of the median IDA simply from the characteristics of the NSP curve [1]. More specifically:

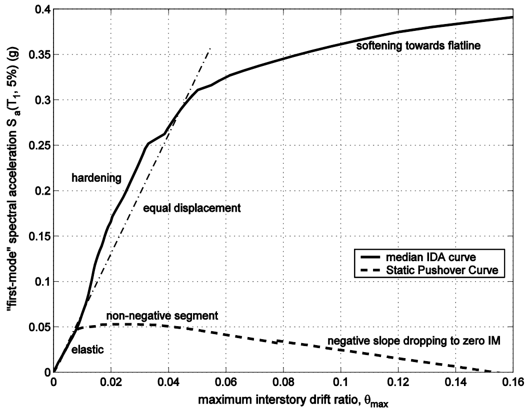
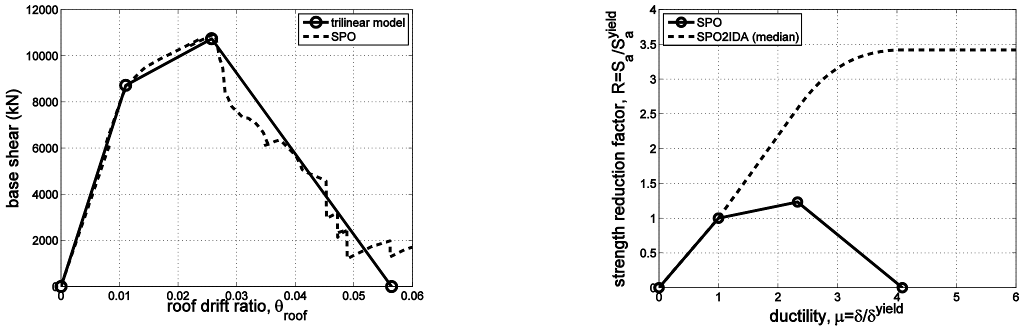


Fig. 4 Comparison of NSP and IDA curves for a 20-storey steel frame [1].

- By construction, the elastic region of the NSP curve matches well the IDA, including the first sign of non-linearity appearing at the same values of IM and EDP for both.
- A subsequent reduced, but still non-negative stiffness region of the NSP curve correlates on the IDA with the approximate 'equal-displacement' rule (for moderate-period structures), i.e. a near continuation of the elastic regime slope; in fact, this near-elastic part of the IDA is often preceded by a hardening portion.
- A negative slope on the NSP curve translates to a "softening" region of the IDA, which can lead to collapse (indicated by flattening of the IDA to horizontal), unless it is arrested by a non-negative segment of the NSP curve before it reaches zero in IM terms.
- A non-negative region of the NSP curve that follows after a negative slope that has caused a significant IM drop, apparently presents itself in the IDA as a new, modified secant-stiffness rule (i.e. an near-linear segment that lies on a secant) that has lower "stiffness" than the elastic.

3.1 IDA capacity of SDOF systems using approximate methods

Quantitatively, it is possible to approximate the results of IDA using *R*-C_1-*T* (or *R*-μ-*T*) relationships available in the literature. Among such relationships, the SPO2IDA tool [4] and the IN2 method [5] can approximate the median IDAs over the entire range of response for single-degree-of-freedom systems, utilizing information from the force-deformation envelope (or backbone) of the static pushover. In the discussion that follows we adopt SPO2IDA for our calculations. Because the SPO2IDA set of equations [4] incorporates fairly sophisticated routines to fit response data for the particular oscillator parameters of interest, estimates have greater accuracy than the closed-form relationships used in R-C_1-T (or R-μ-T) relationships over a large range of oscillator parameter. The response data used for the SPO2IDA estimates are 5%-damped SDOF systems featuring backbones that range from simple bilinear to complex quadrilinear. This data allows the SPO2IDA tool to provide estimates of response statistics (median and 16th and 84th percentile) considering record-to-record (aleatory) randomness. In the case of static pushover analysis, estimates of the global response (θ_{max} or θ_{roof}) on an SDOF system can be obtained using a multilinear approximation of the static pushover curve.



(a) (b)

Fig. 5 (a) The static pushover curve for a nine-storey steel structure and its trilinear approximation, and (b) the SPO2IDA prediction in normalized R-μ coordinates.

For SDOF structures, IDA curves can be represented in normalized coordinates of the strength reduction factor, R and ductility μ. The strength reduction factor R is defined as the ratio $S_a(T_1,5\%)/S_a^{yield}(T_1,5\%)$, where $S_a^{yield}(T_1,5\%)$ is the $S_a(T_1,5\%)$ value to cause yield (equal to the base shear force at yield divided by the oscillator mass), while the ductility, μ, is the peak displacement of the oscillator, δ, normalized by the yield displacement, δ^y. Thus, once the period and the properties of the force-displacement relationship are known for the SDOF system, SPO2IDA directly provides estimates of the 16th, 50th, and 84th fractile demand and capacity in normalized R, μ coordinates. The application of the method requires a multilinear approximation of the static pushover curve to determine the properties of the backbone curve (Fig. 5a).

3.2 Capacity of MDOF systems

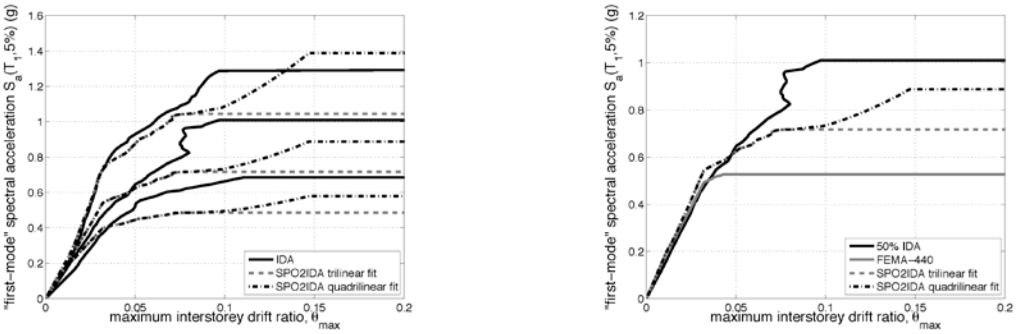
Once the approximation of the IDA curve is available in R, μ coordinates, a set of algebraic calculations is carried out to characterize the IDA capacities of the corresponding MDOF structure. In the discussion that follows, we will use SPO2IDA as the R-C_1-T relationship of choice. A thorough discussion on the procedure suggested can be also found in references [6,7].

Since the capacities of SPO2IDA are in dimensionless R-μ coordinates, they need to be scaled to another pair of IM, EDP coordinates, more appropriate for MDOF systems, such as the $S_a(T_1,5\%)$ and the maximum interstorey drift ratio θ_{max}. The scaling from R-μ to $S_a(T_1,5\%)$-θ_{max} is performed with simple algebraic calculations:

$$\begin{aligned} \boldsymbol{S}_a(T_1,5\%) &= \mathbf{R}\, S_a^{yield}(T_1,5\%) \\ \boldsymbol{\theta}_{roof} &= \boldsymbol{\mu}\, \theta_{roof}^{yield} \end{aligned} \tag{1}$$

The bold font denotes a vector quantity, thus **R** and **μ** are the ordinates and abscissas of the

IDAs in normalized coordinates, respectively. Once $\boldsymbol{\theta}_{\text{roof}}$ is known, $\boldsymbol{\theta}_{\max}$ can be extracted from the results of the static pushover, since for every load increment the correspondence between the two EDPs is always available. Prior to applying Eq. (1) we have to determine the values of $S_a(T_1,5\%)$ and θ_{roof} at yield. This task is trivial for SDOF systems, but it is not straightforward for MDOF structures. Due to the effect of higher modes, some records will force the structure to yield earlier and others later; thus, yielding will always occur at different levels of $S_a(T_1,5\%)$ and θ_{roof}. Taking advantage of the approximation of the static pushover curve, we assume that the yield roof drift is approximately equal to the yield point of the multilinear approximation. This assumption is not strictly true for MDOF structures. It is precise only if the first mode is dominant, but it is sufficient for our purpose. Therefore, the accurate estimation of $S_a^{yield}(T_1,5\%)$ comes down to approximating the elastic "slopes" of the median IDA curves plotted with θ_{roof} as the EDP. The slope, denoted as k_{roof}, is the median value obtained using elastic response history analysis with a few ground motion records, or simply by using standard response spectrum analysis. For first-mode dominated systems, a quick estimate can also be obtained by employing a first-mode approximation via the roof displacement participation factor, e.g. the C_0 factor defined in ASCE/SEI 41-06 [8].



(a) (b)

Fig. 6 (a) IDA median and 16, 84% capacity curves and its corresponding approximation using the proposed approach. The difference between the trilinear and the quadrilinear approximation is demonstrated. (b) Approximation of the median IDA when different R-C_1-T approximations are followed (taken from [7]).

For example, using the target displacement equation of ASCE/SEI 41-06 and given that in the elastic range the coefficients C_1, C_2, C_3 are equal to one, the roof drift and the IDA slope k_{roof} are obtained as:

$$\theta_{\text{roof}} = \frac{\delta_{\text{roof}}}{H} = C_0 S_a \frac{T_1^2}{4\pi^2 H} g \tag{2}$$

$$k_{\text{roof}} = \frac{S_a(T_1,5\%)}{\theta_{\text{roof}}} = \frac{4\pi^2 H}{C_0 T_1^2 g} \tag{3}$$

where H is the height of the building and g the surface gravity acceleration in appropriate units. Finally, we obtain:

$$S_a^{yield}(T_1,5\%) = k_{\text{roof}}\, \theta_{\text{roof}}^{yield} \tag{4}$$

For the SPO curve of Fig. 5a the median IDA obtained with SPO2IDA and the actual IDA curve using thirty ordinary ground motion records are shown in Fig. 6b. For our model, the error in the conditional $S_a(T_1,5\%)$ capacities is typically 10-20%. In addition, while a preliminary design must have been established prior to the NRHAs required for the IDAs, SPO2IDA can be used to establish constraints required to ensure the preliminary design will have acceptable seismic performance. It is worthwhile to note that compared to the quadrilinear pushover approximation, for the nine-storey steel frame, the trilinear curve slightly biases our IDA results towards lower S_a-capacities. For comparison purposes we also show the capacities obtained with the FEMA-440 expression. This formula is accurate enough only for low elastic and nearly-elastic $S_a(T_1,5\%)$ intensities, since it has been suggested for a different purpose and therefore is deliberately conservative.

4. CONCLUSIONS

A qualitative comparison between NSP and NRHA has been presented. NSP can be used to provide insight to the building's capacity, helping the engineer to understand how the system will respond from a global perspective, and also can be used as in aid in preliminary design in performance-based earthquake engineering. To this cause we investigate the relationship between the global results of static pushover and IDA. The direct comparison is possible and can be performed either in the setting of IDA or in that of NSP. Distinct similarities and differences appear in each setting, offering different insight into the structural behavior.

5. ACKNOWLEDGMENTS

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