EVALUATION OF THE INFLUENCE OF VERTICAL STIFFNESS AND STRENGTH IRREGULARITIES ON THE SEISMIC RESPONSE OF A 9-STOREY STEEL FRAME

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ABSTRACT

A methodology based on Incremental Dynamic Analysis (IDA) is proposed for evaluating the response of structures with single-storey vertical irregularities. Using the well-known 9-storey LA9 steel frame, the objective is to study the effect of the irregular distribution of stiffness and strength along its height. This is achieved by means of IDA, i.e., by performing nonlinear time history analysis for a suite of twenty ground motion records scaled to several intensity levels. The reference and each modified structure are hence forced to show the complete spectrum of behavior from elasticity to final global instability, allowing the estimation of capacities for the full range of limit-states and enabling a straightforward comparison without needing to "tune" the structures to the same fundamental period and/or the yield base shear. Using the bootstrap method confidence intervals are calculated and hypothesis testing is performed for changes in the median and the dispersion of capacity for each limit-state. Thus, it becomes possible to isolate the effect of irregularities from any possible record-to-record variability. In conclusion, the proposed methodology enables a full-range performance evaluation using a highly accurate analysis method that pinpoints the effect of any source of irregularity for every limit-state.

INTRODUCTION

A large portion of building structures are in some sense vertically irregular. Some buildings have been initially designed so, e.g., in the case of a soft first-storey, or a large entrance lobby. Others have become so by accident, for example due to inconsistencies or even errors during the construction process, while the rest have been rendered irregular during their lifetime because of rehabilitation or change of use. Therefore, it is essential for structural engineers to obtain a better understanding of the seismic response of structures with vertical irregularities. This need has been recognized by current seismic guidelines (e.g. FEMA-356 [1], Eurocode standards [2]). Such guidelines contain a number of criteria in order to identify

vertically irregular buildings and determine whether their effect is "significant". Naturally, the question that arises is when should this effect be considered "significant"?

Several researchers have attempted to provide an answer, the most recent and comprehensive efforts including the work of Valmundsson and Nau [3], Al-Ali and Krawinkler [4] and Chintanpakdee and Chopra [5]. Valmundsson and Nau [3] evaluated the applicability of simplified elastic analysis methods suggested by seismic design code procedures on irregular structures. The definitions of seismic design codes for regular and irregular structures for mass, stiffness and strength were examined with respect to the response obtained from inelastic time-history analysis. Al-Ali and Krawinkler [4] followed by Chintanpakdee and Chopra [5] performed systematic investigations on the effect of vertical irregularities. Both studies were based on simple single-bay frames of 10 and 12 stories respectively. Al-Ali and Krawinkler [4] used a strong-beam-weak-column philosophy as opposed to Chintanpakdee and Chopra [5] who adopted a more realistic strong-column-weak-beam philosophy. In both studies time history analyses were performed with bins of 15 and 20 records, respectively. Despite the different approaches used, all efforts reached relatively compatible conclusions. However, several issues were left open.

The above mentioned studies focused mostly on the influence of irregularities to the seismic *demands* rather than *capacities*. This in fact constitutes a broadband comparison that encompasses several limit-states of the structure, rather than considering each limit-state, or level of structural response separately. Furthermore, in order to enable the comparison of different buildings, both Al-Ali and Krawinkler [4] and Chintanpakdee and Chopra [5] proposed to modify, or to "tune" the alternative designs, when necessary, in order to match the fundamental period and/or the yield base shear of the reference regular frame. This approach is clearly limited to relatively simple structural systems and cannot be extended to the comparison of realistic design alternatives. Finally, the influence of the record-to-record variability was not taken into account. Thus we have no evidence to determine how much of the observed changes can be attributed to the irregularities or to the natural randomness in the seismic loading.

To answer these questions we examine the LA9 9-storey steel frame (Figure 1) designed for a Los Angeles site (Foutch and Yun [6]) using Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell [7]). The effects of single-storey stiffness and strength vertical irregularities are compared and quantified against the response of the reference regular structure by means of IDA using a suite of twenty ground motion records. This enables us to perform a comparison across the full spectrum of structural performance without needing to "tune" our structures. In order to isolate the effect of irregularities from any possible record-to-record variability and to obtain evidence that show the influence of the modifications to the original structure the bootstrap method [8] is adopted. For each limit-state, confidence intervals are calculated and hypothesis testing is performed for the median and the dispersion of capacity, providing evidence of whether the differences in capacities are due to the vertical irregularities or the randomness in the seismic loading.

STRUCTURAL MODELS

The structure considered is a 9-storey steel moment resisting frame with a single-storey basement and a fundamental period of $T_1^* = 2.25 \text{ sec}$ (Figure 1). It has been designed for a Los

Angeles site, following the 1997 NEHRP (National Earthquake Hazard Reduction Program) provisions (Foutch and Yun [6]). A centerline model with fracturing connections was formed using the OpenSEES [9] platform. It allows for plastic hinge formation at the beam ends while the columns are assumed to remain elastic. The fracturing connections are modelled as rotational springs with 1% strain hardening and a strength drop to 60% of the plastic moment capacity at ten times the yield rotation. Similar behaviour was assumed for both positive and negative moments. Geometric nonlinearities in the form of P- Δ effects were considered while the effect of internal gravity frames has also been incorporated (Figure 1).



Figure 1: LA9 frame model.

Theoretically, an infinite number of vertically irregular designs can be obtained by selecting different properties and varying their distribution along the height. The irregularities considered are limited to the stiffness and the strength of the 9-storey frame which are modified separately for single stories, along the height of the building. Typical stiffness irregularity cases are the cases of soft stories or when elements of the lateral-force-resisting system such as braces are present on one storey but they are not present on adjacent stories. In many practical cases, strength changes occur together with stiffness, e.g., when the cross-section of a member is changed where both the moment of inertia and the plastic moment capacity are modified, however there are cases where strength only modifications may be encountered for example when changing the steel yield strength without modifying the cross-sections.

The storey properties are modified by upgrading or degrading the properties of all of the storey's members, i.e., the beams and the supporting columns, by a single modification factor. In our case one modification factor, equal to 2, was considered. Therefore, for the upgraded stiffness cases, the stiffness of all members of that storey is multiplied by 2, while for the degraded cases it is divided by 2. Stiffness irregularity cases are denoted as "KI", while strength irregularity cases as "SI". Each of the two types of irregularities was applied to each of the nine stories of the frame separately for a total of 18 cases: case "(n) KI" refers to the modification of the stiffness of the *n*-th storey and case "(n) SI" denotes a strength modification of the *n*-th storey.



Figure 2: (a) Twenty IDA curves and their median curve, (b) Profiles of median interstorey demands for different $S_a(T_1, 5\%)$ levels.

No	Event	Station	$\varphi^{\mathrm{o}\ 1}$	Soil ²	M^{3}	$R^{4}(km)$	PGA (g)
1	Loma Prieta, 1989	Agnews State Hospital	090	C,D	6.9	28.2	0.159
2	Northridge, 1994	LA, Baldwin Hills	090	B,B	6.7	31.3	0.239
3	Imperial Valley, 1979	Compuertas	285	C,D	6.5	32.6	0.147
4	Imperial Valley, 1979	Plaster City	135	C,D	6.5	31.7	0.057
5	Loma Prieta, 1989	Hollister Diff. Array	255	–,D	6.9	25.8	0.279
6	San Fernando, 1971	LA, Hollywood Stor. Lot	180	C,D	6.6	21.2	0.174
7	Loma Prieta, 1989	Anderson Dam Downstrm	270	B,D	6.9	21.4	0.244
8	Loma Prieta, 1989	Coyote Lake Dam Downstrm	285	B,D	6.9	22.3	0.179
9	Imperial Valley, 1979	El Centro Array #12	140	C,D	6.5	18.2	0.143
10	Imperial Valley, 1979	Cucapah	085	C,D	6.5	23.6	0.309
11	Northridge, 1994	LA Hollywood Storage FF	360	C,D	6.7	25.5	0.358
12	Loma Prieta, 1989	Sunnyvale Colton Ave	270	C,D	6.9	28.8	0.207
13	Loma Prieta, 1989	Anderson Dam Downstrm	360	B,D	6.9	21.4	0.24
14	Imperial Valley, 1979	Chihuahua	012	C,D	6.5	28.7	0.27
15	Imperial Valley, 1979	El Centro Array #13	140	C,D	6.5	21.9	0.117
16	Imperial Valley, 1979	Westmoreland Fire Station	090	C,D	6.5	15.1	0.074
17	Loma Prieta, 1989	Hollister South & Pine	000	–,D	6.9	28.8	0.371
18	Loma Prieta, 1989	Sunnyvale Colton Ave	360	C,D	6.9	28.8	0.209
19	Superstition Hills, 1987	Wildlife Liquefaction Array	090	C,D	6.7	24.4	0.180
20	Imperial Valley, 1979	Chihuahua	282	C,D	6.5	28.7	0.254

Table-1: Twenty Gr	ound Motion Records
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¹Component ²USGS, Geomatrix soil class ³Moment magnitude ⁴Closest distance to fault rupture

METHODOLOGY

Incremental dynamic analysis (IDA) [7] is regarded as one of the most powerful analysis methods available, since it can provide accurate estimates of the complete range of the model's response, from elastic to yielding, then to nonlinear inelastic and finally to global dynamic instability. IDA involves performing a series of nonlinear dynamic analyses for each record by scaling it to several levels of intensity. Each dynamic analysis is characterized by two scalars, an Intensity Measure, IM, which represents the scaling factor of the record and an Engineering Demand Parameter, EDP (according to current Pacific Earthquake Engineering Research Center terminology), which monitors the structural response of the model. An appropriate choice for the IM for moderate period structures with no near fault activity is the 5%-damped first-mode spectral acceleration $S_a(T_1,5\%)$ while a good candidate for the EDP is the maximum interstorey drift θ_{max} of the structure. Limit-states (e.g., immediate occupancy or collapse prevention in FEMA-350 [10]) can be defined on each IDA curve and summarized to produce the probability of exceeding a specified limit-state given the IM level.

To perform IDA we used a suite of twenty records (Table-1) representing a scenario earthquake. These records belong to a bin of relatively large magnitudes of 6.5–6.9 and moderate distances, all recorded on firm soil and bearing no marks of directivity. Each record was scaled to a number of intensity levels appropriately chosen in order to cover the entire range of structural response for each irregular case. Scaling was performed by means of the adopted IM, while at each scaling level a nonlinear dynamic analysis was performed and a single scalar, the EDP, was used to describe seismic demand. Using the hunt-and-fill algorithm [11] to select the scaling levels allows the use of only fourteen runs per record to capture each IDA curve with accuracy. Figure 2(a) shows the IDA curves obtained for the base case. Appropriate interpolation techniques [11] were applied in order to approximate each IDA curve from the discrete points obtained from scaling each record to the selected intensity levels. Finally, all 20 curves where summarized to produce the median IDA curve, shown in Figure 2(a), and the median storey drift demands along the height of the frame for a number of $S_a(T_1,5\%)$ levels, shown in Figure 2(b).

In order to compare the performance of the modified versus the reference frame, a continuum of limit-states was defined, each at a given value of θ_{max} , spanning all the structural response range from elasticity to global dynamic instability. For each limit-state (i.e., each value of θ_{max}) the corresponding $S_a(T_1,5\%)$ values of capacity were obtained, one for each record [11] and they were appropriately summarized into their median value and dispersion. The standard deviation of the natural logarithms of the capacity values was used as the dispersion measure. By comparing the median and the dispersion of the capacities of the reference versus the modified frame for each limit-state (or value of θ_{max}) we gain an accurate and closely focused image of the effect of the modification on the structure's performance at each level of response.

It should be noted that such comparisons are made possible by expressing all capacities in a common IM, in our case the 5%-damped spectral acceleration at the first mode of the reference structure $S_a(T_1^*, 5\%)$. While this would pose no problem when dealing with strength modifications, it may not appear suitable for stiffness irregularities which result in modified building periods. Objections may be raised regarding the fact that using each structure's $S_a(T_1, 5\%)$ may seem a better choice. In reality, previous research has shown that the choice of the IM is only a matter of efficiency, i.e., how many records are needed to achieve a given

confidence in the results (Luco [12]). Furthermore, Vamvatsikos and Cornell [13] have shown that in the inelastic range (which actually is the range of interest) and for a relatively wide range of periods around T_1 , similar values of efficiency are achieved. In earlier research (Al-Ali and Krawinkler [4], Chintanpakdee and Chopra [5]) comparisons between different designs were made possible by "tuning" the irregular frames to have the same first mode period and/or yield base shear as the base case. While this practice enables the comparison of idealized structures using only a few timehistory analyses, it is not appropriate for the direct comparison of realistic design alternatives. Thus, the proposed IDA-based methodology allows performing comparisons in a much more realistic way that does not involve modifying the base or the alternate structure in any way since the selection of the IM is simply a post-processing issue. As an example, Figure 3 shows how the median IDA curves and the corresponding $S_a(T_1^*, 5\%)$ as the common IM.



Figure 3: Using a common IM to compare the median IDA curve of the base case versus those produced when upgrading or degrading a structural property.

RESPONSE STATISTICS

Figures 4 up to 6 demonstrate the effect of vertical irregularities on the performance of the LA9 frame for single-storey stiffness and strength vertical irregularities. Figure 4 shows the median $S_a(T_1^*, 5\%)$ capacities for all limit-states (i.e., values of θ_{max}) considered, for both the upgraded and the degraded cases, normalized by the corresponding median capacities of the base case. For brevity results are shown only for the odd-numbered stories, i.e., stories 1, 3, 5, 7 and 9.

The first column of Figure 4 refers to stiffness irregularities and the second to strength irregularities, while each of the five rows refers to a single storey whose properties where modified. Therefore, Figure 4 shows the values of the ratio of $S_a(T_1^*, 5\%)$ capacities of the irregular case over the capacities of the base case for each single-storey modification as they appear when we scan an IDA plot, such as that of Figure 3, along the horizontal axis. Figures 5 and 6 show the normalized median interstorey drift demands over the height of the frame for four intensity levels, namely $S_a(T_1^*, 5\%) = 0.25g$, 0.5g, 0.75g and 1.0g. The drift values of the irregular case are again normalized by the corresponding drift values of the base case.



Figure 4: The median $S_a(T_1^*, 5\%) - \theta_{\text{max}}$ capacities for the upgraded and the degraded cases, normalized by the corresponding base case values.



Figure 5: Normalized median interstorey drift demands for single-storey stiffness irregular (KI) cases for four $S_a(T_1^*, 5\%)$ -levels.

Effects of Stiffness Irregularities

The effects of single-storey stiffness irregularities on the median $S_a(T_1^*, 5\%)$ -capacities are shown in the first column of Figure 4. When the irregularity is at the lower stories a variation up to 10% is observed at the limit-states near collapse, while for stiffness irregularities on higher stories mostly the lower limit-states are affected where capacity variations up to 30% are observed. A modified first storey inversely influences some early inelastic limit-states, but for higher θ_{max} values the picture is changed and a stiffer first storey increases the capacity, while a softer storey reduces it. On the other hand when the irregularity appears at top stories, significant changes are seen at the early limit-states. For θ_{max} up to 4%, stiffer stories offer up to a 30% increase in capacity, while softer stories offer a similar decrease up to $\theta_{max} = 10\%$. For mid-height stories the influence seems to be negligible (less than 5%) regardless of the limit-state. In Figure 5 we can see the distribution of median interstorey drift demands over the height of the building for given levels of $S_a(T_1^*,5\%)$. It becomes apparent that single-storey modifications cause wide-spread changes all over the building. This is a direct consequence of the redistribution of the seismic actions due to the beam-hinge model adopted. At the lower intensity levels these effects are concentrated close to the modified storey (0.25-0.5g), while at higher intensity levels they seem to have a more uniform distribution where the maximum values of the ratio of demands often migrates to stories other than the modified. It should be noted that the large variations in demand that may be observed at the lower stories usually are variations of rather small quantities that do not influence θ_{max} since the maximum drift demands occur at the top stories (Figure 2(b)). Interestingly enough, when close to collapse the storey demand profile becomes practically the same in all cases leading to 50% reduced demands regardless of the position of the modification or whether it is upgrading or degrading (4th column, Figure 5). This is indicative of a robust collapse mechanism that is not affected by the initial differences in stiffness, while for almost all irregular cases collapse takes place at the same intensity level.

Effects of single-storey Strength Irregularities

Strength irregularities (SI) are shown in the second column of Figure 4. It is clear that strength irregularities have a more pronounced effect compared to stiffness irregularities. For early limit states (θ_{max} less than 2%), the response is not affected since little or no yielding has occurred yet. Other than that, the position of the modified storey plays a dominant role on the structure's capacity for each limit-state. When the modified storey is located close to the base (stories 1 and 3), the irregularity in strength seems to have an inverse effect on the capacities for all limit-states apart from those close to collapse. A weak first storey seems to isolate the stories above, thus reducing the drift demand and protecting the building, while a strong first storey appears to have the opposite effect. However, this effect gradually decays as we move towards higher limit-states. Modifications on mid-height stories have a milder effect, where stronger stories provide a 15% bonus in capacity. When one of the top stories is made stronger the ratio of capacities is increased for limit-states beyond 2% and remains constant until near collapse. Weaker top stories decrease the ratio for θ_{max} values between 2% and 7%, while approaching collapse this effect disappears.

The median interstorey drift demand profiles of Figure 5, show that similarly to the stiffness irregularities, strength modifications have a widespread effect all over the structure that depends highly on the intensity level. For $S_a(T_1^*, 5\%) = 0.25$ g, introducing a weaker/stronger storey at any level of the structure will correspondingly increase/decrease the drift demands in the neighbourhood of the modification but when this happens at a lower storey it seems to affect inversely the above stories. Regardless of the storey that is modified though, the most significant changes always appear at the top three stories and often are of the order of 50%. As $S_a(T_1^*, 5\%)$ increases these effects become less pronounced, especially for the lower and higher stories. On the other hand, for middle stories (5-7) the influence of the modifications changes character becoming more uniformly distributed along the height of the building as the level of intensity increases. At 1.0g though the picture changes again. No matter if it is weakened or strengthened, the structure gains a rather uniform 50% reduction in demands for all stories. The single exception, in agreement with the findings of the previous paragraph, is the softening of the upper stories which leads to demands that are equal to or higher than those of the reference structure. Again, for this building, we have a rather consistent collapse

mechanism since almost all single-storey strength modification cases do not affect the way that the building fails, only the $S_a(T_1^*, 5\%)$ -level that this happens.



Figure 6: Normalized median interstorey drift demands for single-storey strength irregular (SI) cases for four $S_a(T_1^*, 5\%)$ -levels.

BOOTSTRAP AND HYPOTHESIS TESTING

The IDA curves display significant record-to-record variability, as becomes obvious in Figure 2(a). It is thus necessary to investigate the accuracy of the results given the limited sample size of 20 records. Looking just at the median differences in Figure 4 can often be misleading: small differences in the elastic range, where the record-to-record variability is generally low, may be statistically more significant than larger differences in the highly variable near-collapse range. Thus, we need to calculate confidence intervals and perform some hypothesis testing on whether the results that we see are indeed an effect of the irregularities or simply an artefact of the record-to-record variability.

Analytical formulas are usually not available when percentile values are involved; hence we turn to the bootstrap method (Efron and Tibshirani [8]) to fill this gap. It offers a reliable way to compare the median capacities between the reference frame and any modified structure for each limit-state. In essence it allows a direct comparison of two fractile IDA curves on the basis of capacity, in our case the curve of the irregular case against the curve of the base case. Since in both cases the randomness is induced by the record suite, which for the sake of a fair comparison should be kept common, we have a classic case of paired samples (Rice [14]). By sampling with replacement from the original 20 accelerograms to generate alternate 20-record suites we can calculate the ratio of the median $S_a(T_1^*, 5\%)$ -capacities of the modified over the base case for every record suite and for every limit-state. Thus, if we repeat this process for numerous random samples (say 1000), confidence intervals can be generated for the ratio of the median capacities for each θ_{max} value. If we use the superscript "(x)" to denote the sample's x% fractile, then the (1-x)·100% confidence interval on the ratio of the capacities for a given limit-state can be formally calculated as:

$$\left[\left(\frac{\hat{S}_{\text{irreg}}}{\hat{S}_{\text{base}}} \right)^{(x/2)}, \left(\frac{\hat{S}_{\text{irreg}}}{\hat{S}_{\text{base}}} \right)^{(1-x/2)} \right]$$
(1)

Where \hat{S}_{irreg} and \hat{S}_{base} represent the sample of median $S_a(T_1^*, 5\%)$ capacities for a given θ_{max} . If for a given θ_{max} the interval contains the unity-axis, then we do not have significant evidence at the $(1-x)\cdot 100\%$ level to accept that the change in the median capacities, for the specific limit-state, is caused by the irregularity. There are two more features worth our attention in these plots, namely the width of the confidence intervals and the symmetry of the curves with respect to the unity-axis. The first provides a measure of the sensitivity of the results to the record selection and the second measures how many of the random record-suites cause increase or decrease of the median capacity. Obviously, if the unity-axis is outside the irregularity has caused the observed variation. Still, if the confidence interval contains the unity-axis we have relatively strong evidence if the axis is close to the bounds, but clearly little or no evidence if it passes through the middle of the interval.

Figure 7(a) shows how the bootstrap method can be applied to identify the cases where the ratio of the median capacities, observed in Figure 4, is strongly influenced by the randomness in the seismic loading, given the record variability and the limited sample size. In order to draw objective conclusions one must treat Figure 7(a) as complement to Figure 4. Most of our conclusions are verified, however there are cases where the confidence interval is equally divided above and below the unity-axis, such as case (3) SI⁻ for drift values beyond 10%. Figure 4 led us conclude that the ratio of capacities for that case was close to one and thus the influence of irregularity was insignificant. Figure 7(a) implies that in that case it is equally likely the ratio to be greater or smaller than one depending on the choice of records made. Furthermore, it is shown that the width of the intervals tends to become larger at higher limit-states. A more efficient IM that is better related to the high inelastic deformations developed at those limit-states would reduce this dispersion (e.g. Vamvatsikos and Cornell [13]).

In order to have the complete picture an estimate of the ratios of the achieved dispersions of the modified over the reference frame is also necessary. For this purpose bootstrap percentile confidence intervals are also calculated for the ratios of the standard deviation of the natural logarithm of the capacities. This is a natural dispersion measure for data that are approximately lognormally distributed (e.g. Benjamin and Cornell [15]). In Figure 7(b) the confidence intervals on the dispersion around the median are shown. For many cases the width of the intervals is small and contains the unity-axis. However, there are cases (e.g., (1) KI and (9) KI) where large intervals that do not contain the unity-axis are observed, especially at the limit-states near collapse. In general, larger sensitivity in the dispersions is observed for the stiffness irregularity cases where the irregular frame undergoes a period change that can probably account for most of these effects. For these cases, even for mild changes in the median capacities, we may have to be alert for large variations in the dispersion.



Figure 7: (a) Bootstrap 90% confidence intervals on the ratio of the median $S_a(T_1^*, 5\%)$ capacities given θ_{max} of the modified over the base frame, (b) Bootstrap 90%
confidence intervals on the ratio of the dispersion of $S_a(T_1^*, 5\%)$ -capacities given θ_{max} of the modified over the base frame. Light-coloured lines are used for the
lower bound and darker ones for the upper bound.

CONCLUSIONS

A methodology based on Incremental Dynamic Analysis (IDA) for comparing the capacities of different structural designs has been proposed and applied in order to study the effect of vertical irregularities on a multi-storey building. The proposed methodology achieves a more focused view by examining the effects on each limit-state separately and can be used for realistic problems since it does not require "tuning" the structures to the same fundamental period and/or yield base shear in order to compare them. The effects of single-storey stiffness and strength modifications along the height of the building have been examined. The bootstrap method was shown to provide an efficient and reliable sanity check for our results. This was necessary because of the large variation which is often introduced by the choice of ground motion records made. In conclusion, vertical irregularities have been shown to produce different effects that depend on the type of irregularity, the storey where it happens and most importantly, the intensity of the earthquake, or equivalently the response level or damaged state of the structure. While some consistent trends have been identified these only hold for the summarized values of many records. Individual records will often go against the "median" behaviour, something that designers should keep in mind.

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