Applicability of Nonlinear Static Procedures to RC Moment-Resisting Frames

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ABSTRACT

The applicability of nonlinear static procedures for estimating seismic demands in a variety of building types was evaluated within the recently concluded ATC-76-6 project. Results are reported herein for several pushover methods applied to three RC moment frame buildings, relative to baseline data provided by nonlinear response history analysis. Response quantities include peak interstory drifts, floor accelerations, story shears, and floor overturning moments. The single-mode pushover methods that were evaluated include the N2 and ASCE 41 coefficient methods. Multi-modal pushover methods include the modal pushover with elastic higher modes, and the consecutive modal pushover. Target displacements were estimated using typical R-C_1-T relationships.

Results indicate that the good performance of the single-mode methods for low-rise buildings rapidly deteriorates as the number of stories increases. Multi-mode techniques generally can extend the range of applicability of static pushover methods, but at the cost of significant additional computation and with uncertainty about the reliability of the results.

INTRODUCTION

Nonlinear Response History Analysis (NRHA) is recognized as the most rigorous method available for the analysis of structures undergoing inelastic response to earthquake excitation, but its use in practice is limited because of doubts regarding the selection and scaling of ground motions, the interpretation of dispersion in the results, the effort required to develop valid models where cyclic degradation is modeled, and the time required for analysis of complex models. In contrast, Nonlinear Static Procedures (NSPs) have been attractive because of their relative simplicity; their use of smoothed design spectra as input (which dispenses with questions about ground motion selection) and their output of point estimates of response quantities. Nevertheless, questions have been raised about their validity, given their inability or limited ability to consider dynamic interaction of the "modes" of response. Recognition of potential limitations was made in ATC-40 (1996) and demonstrated clearly in FEMA 440 (2005). This resulted in the use of single-mode pushovers being removed from the main body of the *NEHRP Provisions and Commentary* and being restricted for the structural design of new buildings to buildings less than 40 ft. in height in a non-binding Resource Paper in Part 3 of the *Provisions and Commentary* (BSSC 2009). A more precise quantification of the potential and limitations of pushover methods was sought in the ATC 76-6 project.

This project utilized building designs and models developed in the previous ATC projects. Only relatively simple single-mode and multimode pushover methods were considered, because NRHA generally will be preferred over pushover methods that impose complex analysis requirements on the user and those for which analysis results may have poor or uncertain reliability. Since the applicability of the simple pushover methods to one-story buildings that can be modeled using lumped mass as single-degree-of-freedom (SDOF) systems is not in question, one may view the study as evaluating the point at which additional stories cause the pushover methods to introduce unacceptable error. Evaluation of error in any response parameter may be considered relative to the mean and dispersion obtained in the response parameter obtained in NRHAs of the model for a large number of ground motions considered representative of the hazard level of interest.

BUILDINGS AND MODELS

Although a variety of building types were considered within the ATC 76-6 project, only results obtained for three reinforced concrete (RC) moment-resistant frame (MRF) buildings are reported in this paper. The 2-, 4-, and 8-story RCMRF buildings each have three bays and are regular and symmetric. Each was designed according to the 2003 IBC as described in the FEMA P695 (2009) report. Column sizes were determined to satisfy flexural strength requirements at beam-column joints as well as joint shear requirements.

Models of the building frames reported in FEMA P695 were used in the analyses reported herein. Seismic framing was modeled along with a leaning column to represent P-Δ effects, using planar (2D) models containing one-dimensional line-type elements. Gravity loads were considered using a load combination of 1.05(Dead Load) + 0.25(Live Load) for P-Δ effects as well as for determining the flexural strengths of columns; the effect of potential variation of axial load on column strength was not represented. Nonlinear in-cycle degrading response of beams, columns and joints was simulated, e.g., for beam-column plastic hinges as shown in Figure 1. Typical models have an initial elastic branch, a strain hardening branch, and a descending branch that terminates at an ultimate chord rotation equal to 0.1 rad. An initial uncracked stiffness as well as the potential for shear failures were not modeled. Rayleigh damping amounting to 5% of critical damping was assigned to the 1st and the 3rd mode of vibration, at all beams and all columns, but not at the joints. To compensate for the absence of damping in the joints, the stiffness proportional damping coefficients were increased by 10%. First mode periods based on cracked section stiffness were 0.625, 0.855, and 1.80 sec for the 2-, 4-, and 8-story frames, respectively.

GROUND MOTIONS

NRHAs were performed using the suite of 44 far-field ground motion records identified in FEMA P-695 (2009). The records were normalized as discussed in FEMA P-695. In the present study, the amplitudes of these ground motions were further scaled by scale factors (SF) equal to 0.5, 1 and 2. These scale factors correspond to ground motion at a Los Angeles, California site with mean recurrence intervals of approximately 100, 400, and 2475 years. Figure 2 shows the mean and the median design spectra for a 400-year mean recurrence interval (SF = 1.0).

NONLINEAR STATIC PROCEDURES (NSPs)

Estimates of response parameter values were determined according to the following five analysis procedures:

- *ASCE/SEI 41-06*: Buildings are "pushed" with a first-mode lateral load pattern according to ASCE 41 until the roof reaches a target displacement defined as:

$$d_t = C_0 C_1 C_2 C_3 S_a(T_e) \frac{T_e^2}{4\pi^2} g \qquad (1)$$

  where S_a is the elastic spectral acceleration value at effective period T_e, C_0 is the modal participation factor, C_1 is obtained with the improved R-C_1-T relationship developed in FEMA 440 (2005):

$$C_1 = 1 + \frac{R-1}{aT_e^2} \qquad (2)$$

  C_2 is calculated with the formula suggested in ASCE/SEI 41-06:

$$C_2 = 1 + \frac{1}{800}\left(\frac{R-1}{T_e^2}\right)^2 \qquad (3)$$

  and C_3 is assumed to be equal to 1. In the preceding, R is the strength reduction factor ($R = S_a C_m W / (V_y g) \geq 1$), and C_m is the modal mass participation ratio (M_n^* / W). Site Class B soils were assumed, and therefore a = 130 in Equation (2).

- *Eurocode/N2 method:* The N2 method proposed by Fajfar and Fischinger (1988) has been codified in Eurocode 8 (2004). A pushover analysis is done and the resulting capacity curve is idealized using an elasto-perfectly plastic relationship to determine the period T_e of the equivalent SDOF (ESDOF) system. Different expressions are suggested for short and for medium-to-long period ranges; for the latter case d_t^* is equal to the displacement of the corresponding elastic SDOF system, calculated as:

$$d_{et}^* = S_a(T_e)\left[\frac{T_e}{2\pi}\right]^2 \qquad (4)$$

  where $S_a(T_e)$ is the elastic acceleration response spectrum at the period T_e.

- *Modal Pushover Analysis (MPA)*: This procedure, initially proposed by Chopra and Goel (2002), determines response parameter values for independent modal pushover analyses in the 1st, 2nd, and possibly 3rd elastic modes of vibration. Although subsequently Goel and Chopra (2005) recommended deformations determined by MPA be imposed on the structure in a second pushover analysis phase to determine member forces, the more common practice of determining all

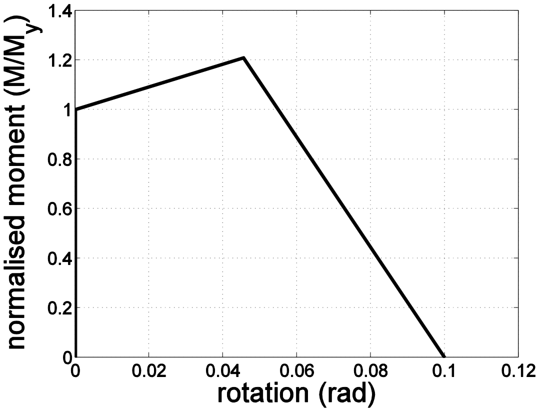


Figure 1. Typical moment-rotation relationship used for the plastic hinge part of component models.

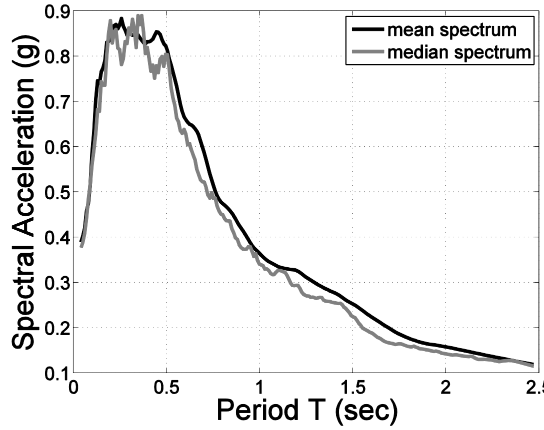


Figure 2. 5%-damped unscaled mean and median response spectra of the 44 ground motion records used in this study.

response quantities by SRSS combination of independent modal analyses (Chopra and Goel (2002)) was followed in this work. Target displacements δ_t for each mode were determined using the ASCE/SEI 41-06 R-C_1-T relationship. Each pushover analysis was done with gravity loads present; the effects of gravity loads, as determined in a separate gravity load analysis without imposed lateral displacements were then subtracted from the modal response quantities, SRSS combinations were taken, and the gravity load effects were then added to the result. Two modes were used for the 2-story RCMRF and three for the 4- and 8-story RCMRFs.

- *Consecutive Modal Pushover (CMP):* This procedure applies lateral loads approximately corresponding to the first, second, and possibly third elastic modes in sequence, all as part of a single pushover analysis. Thus, interaction of multiple modes is considered in a way that may cause different inelastic mechanisms to form. Also, the consecutive application of modal responses may come closer to representing the higher mode responses that take place when the peak displacement response is realized dynamically. Finally, member forces resulting from the analysis are consistent with member capacity limits (e.g. beam shears do not exceed the shears associated with development of a plastic mechanism). As presented by Poursha et al. (2009), the first pushover analysis uses an inverted triangular load pattern for medium-rise buildings and a uniform force distribution for high-rise buildings. A second pushover analysis is done, consisting of a sequence of first and second mode forces. The first mode forces are applied until the roof displacement equals $a_1\delta_t$, where a_1 is the first mode modal mass ratio and δ_t is the target displacement determined for the first mode. Upon reaching $a_1\delta_t$, incremental forces are applied that follow a second mode pattern. Thus, the incremental displacement used for this analysis stage is $(1-a_1)\delta_t$. A third pushover analysis is required for buildings with periods of 2.2 seconds or higher. The peak value of any demand parameter in each separate pushover analyses is compared to identify the largest (or envelope) value.
- *Modal Response Spectrum Analysis (MRSA):* For buildings responding inelastically whose deformed shapes resemble their elastic mode shapes, perhaps the simplest multimodal approach to estimate deformations is to extrapolate linear

modal response spectrum analysis results into the inelastic regime. In this case, lateral load patterns, matching those used in the MPA procedure, are used in a linear elastic analysis. Target displacements are calculated using the C_1 and C_2 relationships of ASCE/SEI 41-06; the coefficient C_3 was taken equal to 1.0. SRSS combinations of modal demand parameter values are taken to obtain the final response estimate.

RESULTS

Nonlinear Response History Analysis. Figure 3 presents selected results from nonlinear response history analysis. Median results are plotted for different scale factors. Dispersion in response values (e.g. Figure 8) to some degree reflects the method used to scale ground motion records in FEMA P695, which is based on the geomean of peak ground velocities of orthogonal ground motion components. Results for these structures indicate:

- Maximum story drift demand is higher in the lower stories while the minimum values are always at the top story. Story drift tends to concentrate in the lower stories with increasing scale factor. Higher modes appear to have an appreciable effect on story drift and story shears.
- Overturning moments tend to follow the concave pattern associated with first mode or code equivalent lateral force patterns. Dispersion in overturning moments is relatively small, attributable to the contributions of higher mode lateral forces to the overturning moment at any level tending to cancel out. Overturning moments tend to "saturate" as the scale factor increases.
- Story shears in the upper stories are significantly larger than would be expected from first mode or code equivalent lateral force patterns. Base shears developed in the dynamic analyses are larger than the capacities indicated by a first-mode pushover analysis. Story shears tend to "saturate" as the scale factor increases.

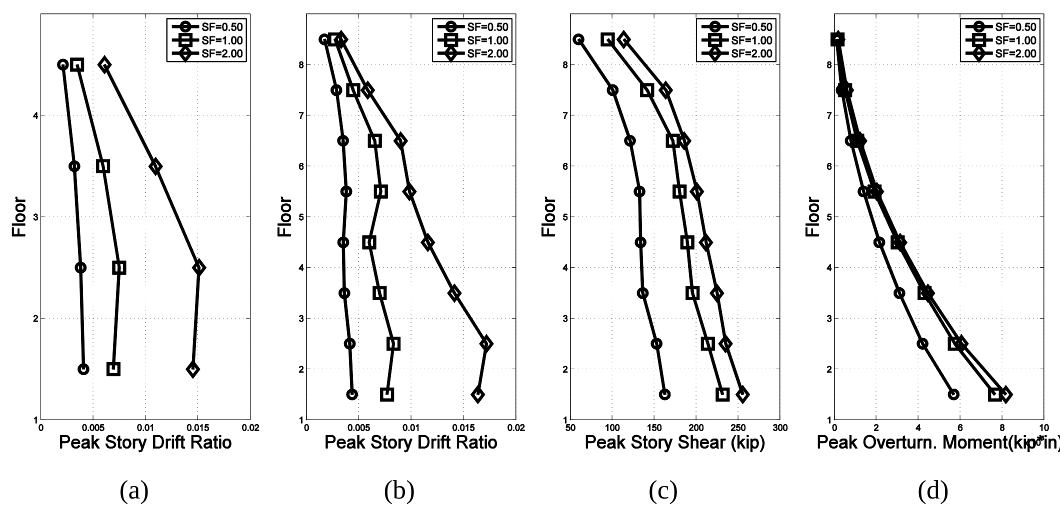
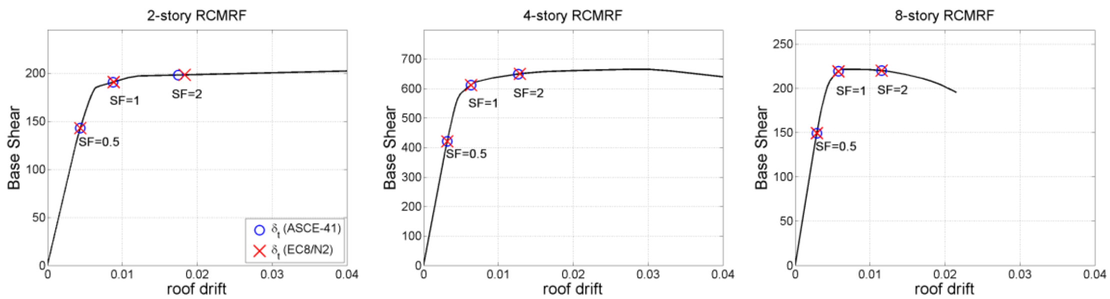


Figure 3. Profiles of median peak response quantities for SF=0.5, 1.0, and 2.0: (a) story drifts for the 4-story RCMRF, and (b) story drifts, (c) story shears, and (d) overturning moments for the 8-story RCMRF.

First Mode Nonlinear Static Analysis. Figure 4 shows the capacity curves obtained in first-mode pushover analyses of the three moment frames and the target displacements determined using the formulas of ASCE-41 and Eurocode 8 for the mean spectra scaled by SF=0.5, 1.0, and 2.0. Both methods produce nearly identical target displacement estimates for the three moment frames; for this reason only ASCE-41 target displacements were used for estimating response quantities in first-mode pushover analyses. Roof displacements at yield are about 0.5 - 0.6% of the height. Target displacements for a scale factor of 0.5 are in the elastic regime, those for the records scaled by 1.0 are nearly elastic, and those for a scale factor of 2.0 cause moderate inelastic response, developing system ductilities of 2-3.

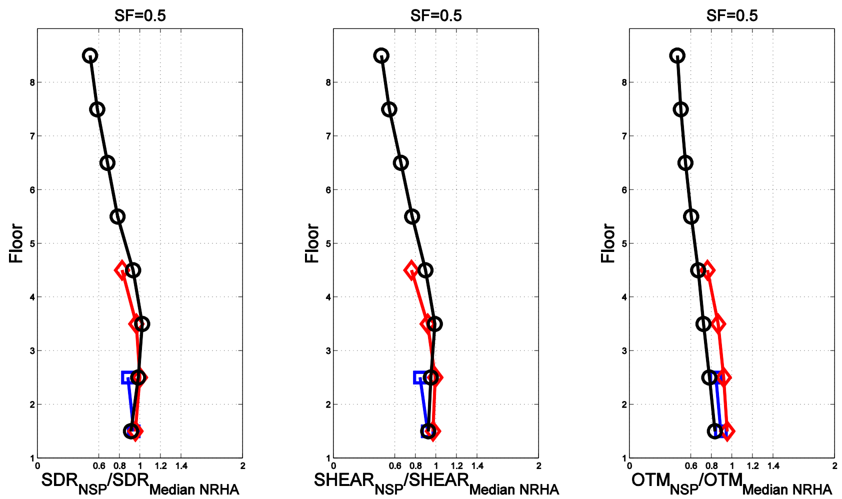


(a) (b) (c)

Figure 4. First mode static pushover curves and target displacement estimates for the 2-, 4-, and 8-story reinforced concrete moment frames, for ground motion scale factors of 0.5, 1.0, and 2.0.

Ratios of demand parameter values determined by first mode pushover analysis and medians of the peak values obtained in NRHA are plotted over the height of each frame in Figure 5 (SF=0.5) and Figure 6 (SF=2.0). Accuracy clearly varies with scale factor (e.g. Figures 5a and 6a), response quantity (Figures 6a and 6b), and number of stories. Accuracy in the estimation of one response quantity does not signify accuracy in the estimation of other response quantities even for the same scale factor and structural model. Reasonable accuracy was obtained for the three demand parameters considered for the 2-story RCMRF, although the error in shear at the upper story slightly exceeded 20%. For the 4-story frame, the accuracy of story drifts, story shears and overturning moments degraded as the scale factor increased from 0.5 to 2.0. Errors in story drift, story shear, and floor overturning moments exceeded 20% at some locations and some scale factors at some locations in the 4-story frame. The degradation in accuracy with increase in scale factor was more pronounced for story drifts. The accuracy of NSP estimates also degraded with increase in scale factor for the 8-story frame, where the error in the top stories of story shears and overturning moments was of the order of 60%. In general, larger errors were encountered in the buildings with more stories.

Modal Pushover Analysis. Figure 7 shows the modal pushover curves for the 2-, 4-, and 8-story reinforced concrete moment frames. Target displacements determined using mean elastic response spectra for scale factors of 0.5, 1.0, and 2.0 are shown. The higher mode pushover curves did not display reversals (e.g. Hernández-Montes et al. 2004). Target displacements were such that elastic response was predicted in the independent 2nd and 3rd mode pushover analyses.



(a) (b) (c)

Figure 5. Ratios of first mode NSP and NRHA response estimates for SF=0.5: (a) peak story drifts, (b) peak story shears, and (c) peak overturning moments.

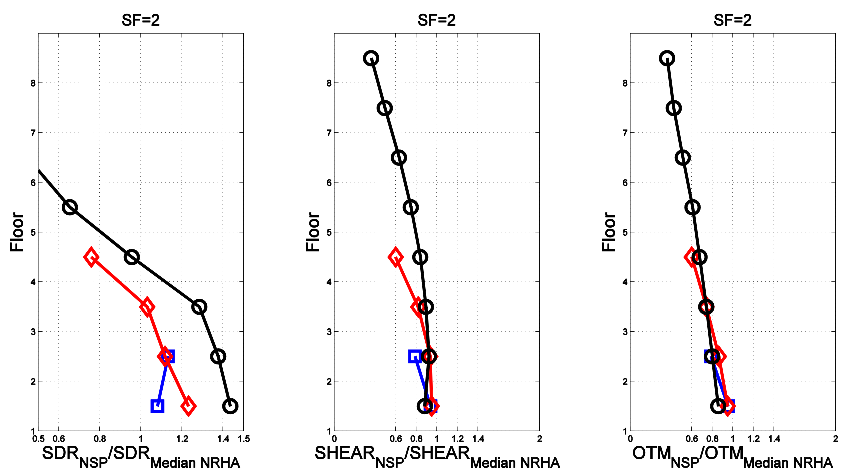


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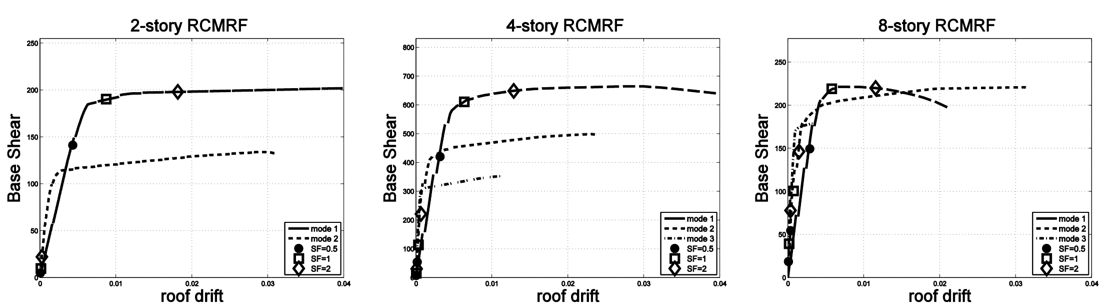
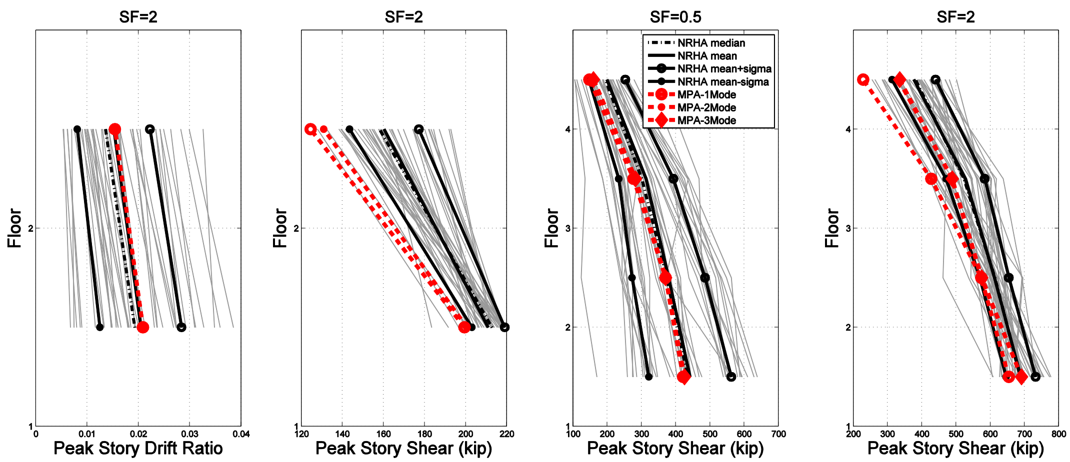
Figure 6. Ratios of first mode NSP and NRHA response estimates for SF=2.0: (a) peak story drifts, (b) peak story shears, and (c) peak overturning moments.



(a) (b) (c)

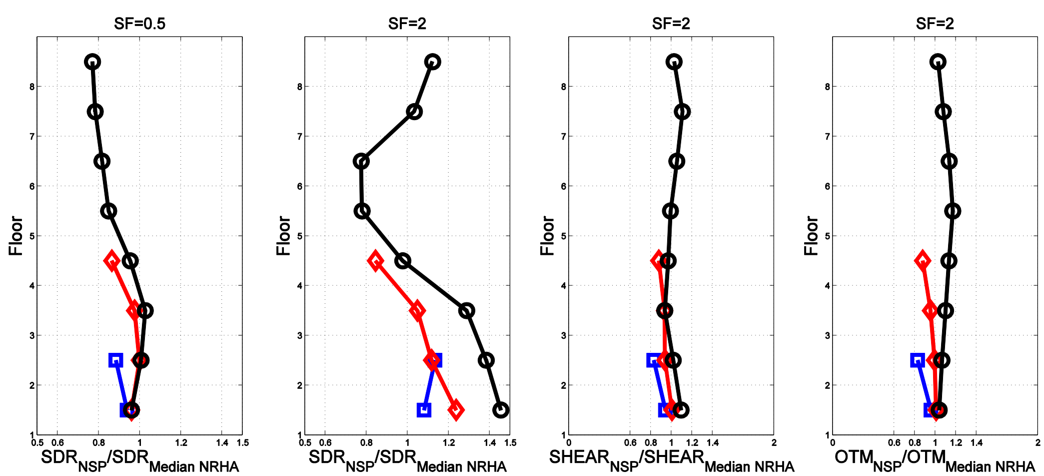
Figure 7. Static pushover curves and target displacement estimates for the 2-, 4-, and 8-story reinforced concrete moment frames, for ground motion scale factors of 0.5, 1.0, and 2.0.

For the 2-story frame, second mode contributions to floor displacements (not shown), story drifts (Figure 8a), and overturning moments (not shown) were negligible; reasonably accurate estimates of these quantities were obtained with the first mode estimates. While story shears were estimated with reasonable accuracy for a scale factor of 0.5 even with single mode pushover analysis (Figure 5b), the inclusion of second mode contributions in the modal pushover analysis procedure, while beneficial, did not sufficiently increase the story shear estimates to result in an accurate estimate of story shear at a scale factor of 2.0 (Figure 8b). This suggests a relatively severe constraint on the reliable application of both single mode and simple multimodal pushover methods.



(a) (b) (c) (d)

Figure 8. Estimates made with modal pushover analysis and profiles of (a) story drifts and (b) story shears for the 2-story RCMRF at SF=2, and profiles of story shears for the 4-story RCMRF at (c) SF=0.5 and (d) SF=2.



(a) (b) (c) (d)

Figure 9. Ratio of modal pushover analysis results and nonlinear response history analysis results for the reinforced concrete moment resisting frames: (a) peak story drift (SF=0.5), (b) peak story drift (SF=2.0), (c) peak story shears (SF=2.0), and (c) peak overturning moments (SF=2.0).

For the 4-story frame, second and third mode contributions to floor displacements (not shown) and story drifts (Figure 9b) were negligible; reasonably accurate estimates of these quantities were obtained with first mode estimates at a scale factor of 0.5 (Figure 5a and Figure 6a). In contrast to the 2-story frame, the accuracy of modal pushover analysis estimates of story shears improved as the scale factor increased from 0.5 to 2.0 (Figure 8c and Figure 8d). While second mode shears had a significant and beneficial effect, third mode story shears, while beneficial, had a relatively small contribution. Relatively accurate estimates of overturning moments were made, and these benefited from inclusion of the second mode contribution (Figure 9d compared with Figure 6c).

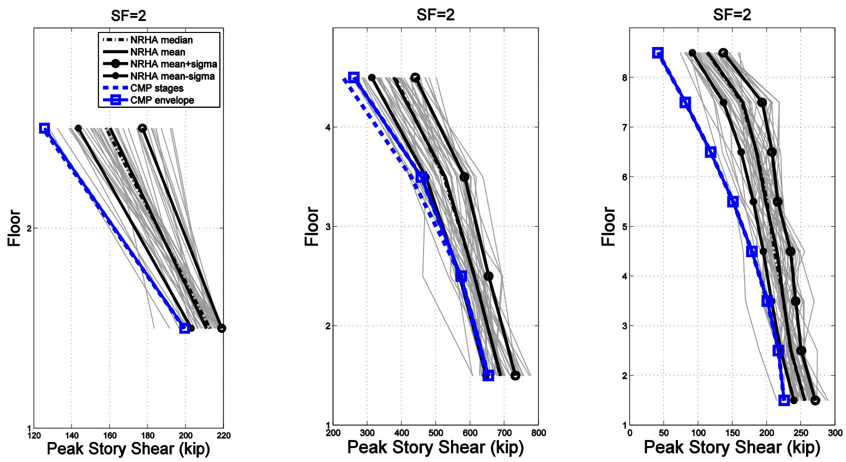
For the 8-story frame, second mode contributions to story drift were not negligible and improved the story drift estimates (Figure 9 compared with Figure 5a and Figure 6a), but accuracy varied with location and scale factor. In particular, the accuracy of story drift estimates improved at the upper stories at a scale factor of 2.0. The accuracy of story shear estimates also varied with location and scale factor. Both second and third mode contributions to story shears were appreciable (Figure 9c compared with Figure 6b); inclusion of higher mode story shears improved story shear estimates at the upper stories. Second mode contributions to overturning moments were not negligible and improved the estimates; overturning moments tended to be underestimated at a scale factor of 0.5 (not shown), and were slightly overestimated at a scale factor of 2.0 (Figure 9d).

Consecutive Modal Pushover. Selected results obtained by application of Consecutive Modal Pushover (CMP) analysis to the three RCMRFs are shown in Figure 10 and Figure 11. Figure 10 shows story shear results with dashed lines for the two CMP stages and with solid lines for their envelope, in addition to the single-record and NRHA curves.

For the 2-story frame, story drifts were estimated with reasonable accuracy (not shown). However, story shears were accurately estimated at a scale factor of 1.0, were significantly overestimated at a scale factor of 0.5, and were significantly underestimated at a scale factor of 2.0. Thus, relatively severe limitations on the reliable applicability of the CMP are apparent.

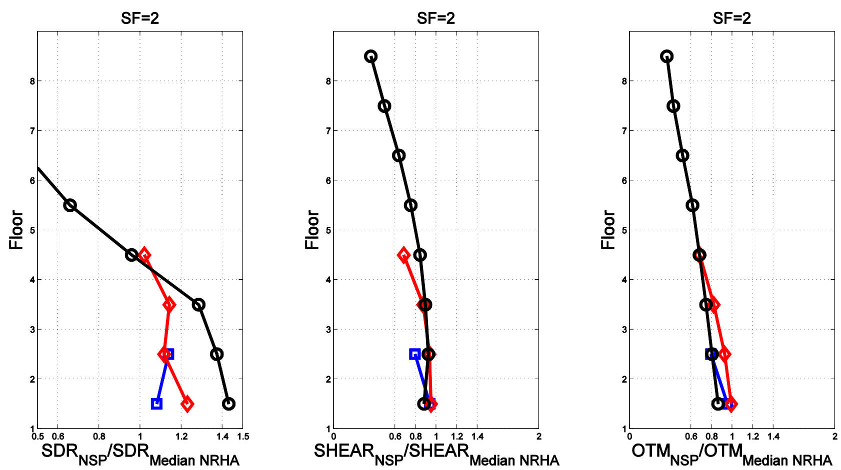
For the 4-story frame, the accuracy of estimates of story drift, story shear, and floor overturning moments varied with location and scale factor. For example, story shears in the upper stories were significantly overestimated at a scale factor of 0.5, and story shears over the height of the building were significantly underestimated at a scale factor of 2.0 (Figure 10b).

For the 8-story frame, peak displacements were overestimated at a scale factor of 2.0, just as occurred with the first mode and multiple mode pushover analyses. As for the 4-story frame, the accuracy of estimates of story drift, story shear, and floor overturning moments varied with location and scale factor.



(a) (b) (c)

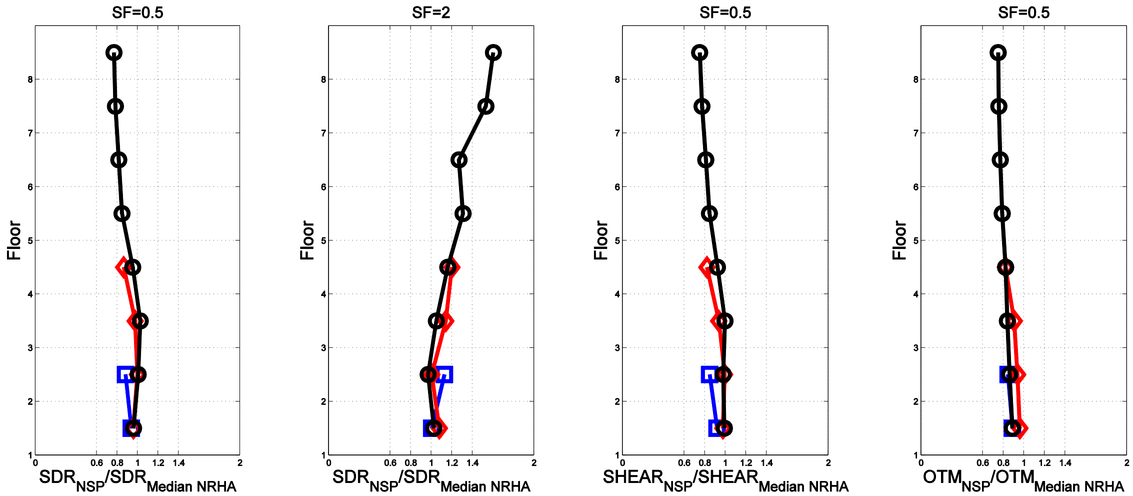
Figure 10. Story shear estimates using consecutive modal pushover analysis for the 2-, 4-, and 8-story reinforced concrete moment resisting frame



(a) (b) (c)

Figure 11. Ratio of Consecutive Modal Pushover Analysis results to NRHA results for the reinforced concrete moment resisting frames, for SF=2: (a) peak story drifts, (b) peak story shears, and (c) peak overturning moments.

Elastic Modal Response Spectrum Analysis. Selected results from elastic modal response spectrum analysis are shown in Figure 12 for selected response quantities at scale factors of 0.5 and 2.0. In Figure 12 it is apparent that the accuracy of story drift estimates, while a function of scale factor, is on par with that of the multimodal pushover analysis method (Figure 9a and Figure 9b), while relatively good shear (Figure 12c) and overturning moment (Figure 12d) estimates were obtained under elastic response (SF=0.5) for the 2-, 4-, and 8-story frames. However, even for elastic response (SF=0.5), story shears at the upper stories and overturning moments generally were underestimated for the 4- and 8-story RCMRFs.



(a) (b) (c) (d)

Figure 12. Ratio of elastic Modal Response Spectrum Analysis results to NRHA results for the reinforced concrete moment resisting frames, for SF=0.5: (a) peak story drifts, (b) peak story shears, and (c) peak overturning moments.

CONCLUDING REMARKS

Single-mode pushover analysis is valuable for its ability to characterize inelastic response of a building in the first mode and for illustrating the nature of the inelastic mechanism that develops in first mode response. Pushover methods often will identify, or even exaggerate, the potential for weak story response. However, neglect of higher mode contributions and interaction of “modes” in potentially causing different inelastic mechanisms to develop is a weakness in using NSP methods as a quantitative tool for evaluation of demands in multistory construction.

Accuracy of NSP estimates depends on the response quantity of interest, the intensity of inelastic response (or excitation), as well as the mechanism that develops. Acceptable error relative to median NRHA results depends on the consequences. In non-critical locations a large percentage error may be acceptable while small absolute errors may be critical in other cases. Alternatively, acceptable error might be judged relative to dispersion in NRHA results. In the authors’ judgment, without proper consideration of the response dispersion inherent in multi-record NRHA, the limit of reliable applicability of single-mode pushover analysis for local response assessment appears to be about 2 stories for moment-resistant frames, while multimode-pushover analysis appears to be limited to about 4 stories. Although these limits are suggested based on limiting errors relative to median NRHA results, they are not absolute. The development of different mechanisms and issues related to degradation of components and duration of ground motion, not considered in this work, may be important. For the regular frames explored in this study, linear elastic modal response spectrum analysis provided estimates of interstory drift on par with the best multimodal pushover methods. Potential improvements to the consecutive modal pushover analysis method should be explored before fully assessing this method.

With these limitations in mind, appropriate care is advised in all applications of nonlinear static methodologies when used for quantitative, rather than qualitative, estimation of seismic performance**.**

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