

DIRECT PERFORMANCE-BASED SEISMIC DESIGN FOR LIQUID STORAGE TANKS

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Abstract: A performance-based design methodology has been developed for liquid storage tanks based on a surrogate, yet robust beam-element model. Following the identification of failure modes through Incremental Dynamic Analysis, appropriate performance levels are defined based on an existing seismic assessment methodology. The concept of Response Frequency Spectra (RFS) is proposed in view of offering a unique representation of the entire solution space for structural performance. RFS find an excellent application for the case of liquid storage tanks by adopting design parameters such as the tank wall thickness and the anchorage ratio. Although the wall thickness changes the strength capacity for the well-known Elephant's Foot Buckling failure mode, the corresponding probabilities of exceedance are not significantly modified. On the contrary, anchorage seems to be very important as the associated probabilities may be reduced even by 50% in some cases.

Introduction

Large-capacity cylindrical tanks are widely used to store a variety of liquids, such as petroleum and liquefied natural gas. The seismic risk of such industrial facilities is considerably higher compared to ordinary structures, since the damage induced by a strong ground motion may trigger uncontrollable consequences, not only on the actual facility but also on the environment. Recent earthquake events, e.g., Kocaeli (1999) and Tohoku (2011), have shown that heavy damage on tanks may lead to temporary loss of essential facilities, usually followed by leakage and/or fire. In order to meet a desired level of safety, the state-of-the-art Performance-Based-Earthquake-Engineering (PBEE) concept can be employed to account for any potential sources of uncertainty. However, recent codes of practice have not fully adopted the PBEE concept, and its application to industrial facilities is limited to simple code provisions.

Problem Definition

Optimal design procedures seek the structural member sizing and the associated properties in view of assuring a desired behaviour for a given seismic hazard. Current design codes and standards serve under a force-based framework that employs a q-factor lager than 1.0 to account for inelasticity. So far, existing methodologies aim to satisfy performance objectives such as Serviceability and Life Safety through analysis-design iterations that rely on an elastic (pseudo)spectral acceleration spectrum that represents a site hazard with 10% probability of exceedance in 50 years. Elastic static or dynamic analysis use the design spectrum as an appropriate loading input in view of providing a simplified design methodology for the majority of engineers. The reduction factor used for the nonlinear response is under a lot of question, while the associated uncertainties are taken into account on an input rather than an output basis using load and material safety factors. Although the aforementioned provisions apply to the majority of civil engineering structures, their suitability to structural systems with complex modes of failure such as liquid storage tanks, is under a lot of criticism, thus suggesting a strict revision of current practice towards a more direct design approach. Several attempts to simplify/optimise the design of structures have been performed to date (e.g., Priestley et al., 2007). Still the best way to capture a performance objective can be summarised in an iterative procedure that involves nonlinear analysis and linearized design.

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State of The Art

Vamvatsikos et al. (2014) developed a novel approach for the Recently. preliminary/conceptual design of structures. The concept is based on the so-called Yield Frequency Spectra (YFS) that provide a direct visual representation of a system's performance through the Mean Annual Frequency (MAF)-ductility (μ) relationship for a range of seismic coefficients (C_y). YFS employ the virtues of yielding parameters within a performance-based framework, which are proven to be very stable compared to the fundamental period (T) for example (Aschheim, 2002). A key-parameter to this procedure is the site hazard representation, which is not given through the traditional design response spectrum. Instead, a detailed view is provided using a three dimensional surface that employs the Mean Annual Frequency, the natural period and the corresponding spectral acceleration ($S_a(T)$). That represents the actual site hazard which allows the direct estimation of the loading input (i.e. response spectrum) for practically any desired performance level or MAF (Vamvatsikos et al., 2013). Obviously, the procedure outlined above does not fully account for the design of multi-degree-of-freedom (MDOF) structures, and as a result some level of iteration is going to be necessary to capture the desired performance level. On the other hand, it provides an excellent starting point for the final design using a SDOF system which forms the basis for almost every code of practice. Along these lines, the aforementioned methodology finds an excellent application for the case of liquid storage tanks, if a simplified modelling approach is adopted.

Modelling

Modelling of complex systems constitutes a key-parameter to the successful design of structures. The simulation procedure of a structural system should provide structural models that not only offer valid analysis results, but also minimise the estimated computational time. Even though the view that a few runs may be enough to optimise the design of a structure is widespread among engineers, determining a performance level with respect either to design or the assessment of an existing facility, requires a fair amount of scenarios to be considered, which may in turn affect not only delivery times but also the quality of the study. In that sense, the response of liquid storage tanks can be idealised using a two-degree-offreedom (2DOF) system, where the two masses (impulsive and convective) are considered decoupled (Calvi and Nascimbene, 2011; Malhotra et al., 2000; Priestley et al., 1986). The geometric and modal characteristics of the hydrodynamic problem may be determined using equivalent parameters for the impulsive and convective masses. For the purpose of this study, the recommendations of Eurocode 8 (CEN, 2004) are adopted featuring Part 4 (CEN, 2006), where the design of tanks is discussed in detail. Under the assumption that the impulsive pressure is acting on the tank walls only, one may obtain estimates for parameters such as the natural period coefficients (C_i and C_c), the masses (m_i and m_c) and the effective height components (h_i and h_c). These parameters are distinguished with the aid of subscripts "i" and "c" denoting "impulsive" and "convective" respectively. Other studies, however, have shown that the contribution of the convective mass to the overall response of the structure can be ignored, as the impulsive mass is held responsible for the majority of the damage that tanks suffer during a strong ground motion event (Malhotra, 1997; Vathi et al., 2013).

A simplified modelling methodology for liquid storage tanks was recently developed by Bakalis et al. (2014b). The surrogate modelling approach offers a balanced "computational efficiency versus accuracy" compromise for the nonlinear static or dynamic analysis. It is based on the work of Malhotra and Veletsos (1994) for liquid storage systems, where the uplifting mechanism of unanchored tanks is modelled in detail. A brief summary of the modelling procedure adopted is presented below. The base plate is divided into an even number of strips and one of them is subjected to an incremental uplifting static load (V) in order to determine the associated resistance (Fig. 1). The strip model consists of force-based fibre beam column elements with an element length of the order of approximately 15 times

the base plate thickness, t_b. A uniaxial elastoplastic material is assigned to the fibres in order to capture the inelastic behaviour of the base plate during uplift. Geometric nonlinearities are also taken into account through a co-rotational formulation. The foundation of the tank is modelled using Winkler springs to account for the soil or concrete slab stiffness beneath the plate. The Winkler springs are also assigned an elastic-no-tension material which is suitable for the simulation of the base plate uplifting. The base plate itself is modelled using rigid beams that are supported on elastic multilinear springs. The latter simulate the nonlinear uplifting resistance for each of the 'N' beam-spokes, representing equal-area sectors of the circular base plate. One may notice that the convective component of the fluid is not considered in the model. This decision is twofold. Obviously, the contribution of the longperiod convective component to the rigid-impulsive response of a broad tank may be deemed negligible (Vathi et al., 2013). That essentially provides a single rather than a double degree-of-freedom (DOF) system, the efficiency of which can only be appreciated within a probabilistic framework. As a result, the impulsive mass (m) of the system is connected to the base using the elastic element shown in Fig. 2(a). The deflected shape shown in Fig. 2(b) presents the uplifting mechanism of the tank, where the base shear (V_b) induces a certain amount of uplift (w) on the beam-spokes of the model. The aforementioned mechanism offers the ability to estimate all major modes of failure, when they are expressed as a function of uplift. Sloshing response on the other hand is only affected by parameters such as the convective mass of the fluid and the available freeboard, and may be calculated though a simple response spectrum analysis for the convective component, following the CEN (2006) provisions.



Fig. 1: Strip model explained



Fig. 2: (a) Tank model and (b) its deflected shape

Case Study Description

In an attempt to capture the response of liquid storage tanks, a structural system with the following geometric characteristics is adopted. The tank considered has a radius (*R*) equal to 13.9m and a total height (*h*_i) of 16.5m. The bottom course wall (*t*_w) is 17.7mm thick, while the corresponding base plate (*t*_b) and annular ring (*t*_a) thickness are 6.4mm and 8.0mm respectively. The fluid stored in the tank is assumed to reach the maximum allowable fluid height of *h*=14m, resulting to a 'fluid height over radius' ratio (*h*_t/*R*) equal to 1.01. The details of the liquid storage system adopted are summarised on Table 1.

Table 1. Properties of the tank examined						
Variable description		Notation (units)	Numerical values			
	Radius	R_t (m)	13.9			
	Height	<i>h</i> _t (m)	16.5			
	Wall thickness per course	t _w (mm)	17.7/15.7/13.7/11.7/9.7/7.8/6.4/6.4/6.4			
Tank properties	Base plate thickness	t₀ (mm)	6.4			
	Annular ring thickness	<i>t</i> a (mm)	8.0			
	Roof mass	mr (ton)	35			
	Yield strength	fy (MPa)	235			
	Steel Young's Modulus	<i>E</i> s (GPa)	210			
Fluid	Height	<i>h</i> f (m)	14.0			
properties	Density	<i>p</i> f (kg/m³)	1,000			

Performance Objectives As Seen Through an Assessment Point of View

Field investigations after major earthquakes have revealed a variety of failure modes on atmospheric tanks. They may be summarised to shell buckling, base sliding and sloshing damage to the upper tank shell and roof. EC8-part 4 (CEN 2006) provides special provisions for these modes of failure, as shown by Vathi et al. (2013). For instance, when partial uplift is allowed, either for design purposes or due to poor detailing of the anchors, the rotation of the plastic hinge developed on the base plate of the tank should not exceed a certain rotational capacity, specified in EC8. Moreover, the excitation of the long period convective mass may cause sloshing of the contained liquid, which can in turn damage the upper parts of the tank (roof, upper wall course). During strong ground motion events, hydrostatic and hydrodynamic effects may lead to high internal pressure on the tank walls. Overturning for those thin shell structures is resisted by compressive meridional stresses on the wall. Although high pressure may increase the capacity against buckling by introducing high hoop stress, local yielding may trigger an elastic-plastic buckling failure around the lower course of the tank's perimeter, known as the "Elephant's Foot Buckling".

The most damaging failure modes are mainly associated with plate/shell rupture, as they may result in loss of the contained liquid. Rupturing either the bottom layers of the tank wall

or the base plate is expected to trigger uncontrolled loss of the stored material, with all the associated consequences considered. Recently, Bakalis et al. (2014a) developed a performance-based assessment methodology for atmospheric steel liquid storage tanks. The PBEE framework considers three damage states of increasing severity, namely minor (DS1), severe without leakage (DS2) and loss of containment (DS3). Although this classification may seem reasonable for roughly understanding the extent of damage, the accurate assessment of loss may become tricky as, for example, the different mechanisms involved in a single damage state may be associated with varying degrees of component damage. For instance, the sloshing height response represents relatively easy-to-repair damage at the top of the tank, compared to an exceedance of a plastic rotation limit at the base. Thus, it becomes more informative to also classify damage based on the actual component that has failed. Fig. 3 presents the associated failure modes on the median Incremental Dynamic Analysis (IDA) curve (Vamvatsikos and Cornell, 2002) for the unanchored liquid storage tank considered. The FEMA P695 (FEMA, 2009) far field ground motion set is used for the nonlinear dynamic analysis. The base uplift is adopted as one of the Engineering Demand Parameters (EDP) and the impulsive period spectral acceleration $S_a(T_{imp})$ or the peak ground acceleration PGA (similar due to low T_{imp}) are employed as suitable Intensity Measures (IM) that adequately capture the response of a liquid storage system. It is evident that a component-based classification of damage is guite informative, where the upper course of the tank (SL=sloshing), its lower course (EFB), the base plate (θ_{pl} =plastic rotation), and the anchors (AN=yielding/fracture of anchors) are individually examined. Failure modes such as buckling and plastic rotation are revealed during the nonlinear time-history analysis. Sloshing damage at the top of the tank wall is also considered. Still, as shown in Fig. 3, this may only appear at excessive spectral acceleration values for large tanks due to the ultra-long convective period (T_{con}).



Fig. 3: Single record and median IDA curves for the unanchored tank examined. Sloshing damage appears well beyond the limits of the graph.

The local performance objectives summarised in Table 2 may indeed offer a comprehensive procedure for understanding the extent of damage on a liquid storage unit, even offering the potential for assigning detailed repair cost estimates. However, it may often be the case that a global classification is required, suitable for characterising one or more tanks without specific reference to the component that has been damaged. In that sense, DS1 represents minor damage induced by a sloshing wave height of the contained liquid equal to the freeboard. DS2 refers to severe damage at any component of the tank without leakage, where the exceedance of either a sloshing wave height equal to 1.4 times the available freeboard or a plastic rotation of 0.2 rad at the base plate triggers the damage state violation. DS3, finally, provides information on the loss of containment through the exceedance of

either the axial EFB capacity (N_{EFB}) or the base plate plastic rotation of 0.4 rad. As far as anchored systems are concerned, the yielding of the anchors is considered for DS1, while the fracture of the connection for DS2. Global performance objectives are presented on Table 3.



Fig. 4: Global versus Local performance objectives for unanchored tanks

Table 2: Local performance objectives				
Local DSi	DS Capacities			
DS1 _{SL}	freeboard			
DS1 _{AN}	Anchorage yielding (δ _y)			
DS2 _{SL}	1.4* freeboard			
DS2 _{AN}	Anchorage fracture (δ _u)			
DS2 _{9pl}	0.2rad			
DS3 _{0pl}	0.4rad			
DS3efb	EFB			

Table 2: Local parformance objectives

Table 3: Global	performance	objectives
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Tank Description	Global (DSi)	DS Capacities
	DS1	DS1 _{SL}
Unanchored	DS2	DS2 _{SL} or DS2 _{8pl}
	DS3	DS3 _{0pl} or DS3 _{EFB}
	DS1	DS1 _{SL} or DS1 _{AN}
Anchored	DS2	DS2 _{SL} or DS2 _{AN} or DS2 _{8pl}
	DS3	DS3 _{0pl} or DS3 _{EFB}

Response Frequency Spectra

Several simplified modelling approaches have been proposed for the design of liquid storage tanks to date (CEN, 2006; Malhotra et al., 2000). Still, a direct performance-based design has not been realised so far. Yield Frequency Spectra present a unique opportunity for a generous revision on current codes and standards in view of capturing the desired performance level for a SDOF system. Liquid storage tanks form an excellent case study when a modelling procedure similar to the one developed by Bakalis et al. (2014a, 2014b) is adopted. The 'yield'-parameter concept may be applicable here, however, other parameters that represent the system strength in a more design-oriented manner exist. Consequently, the YFS concept can be further developed to a response-driven approach, hereafter called Response Frequency Spectra (RFS), in view of estimating various design parameters for the desired level of performance. A simple static pushover analysis is more than enough to obtain the capacity curves for the engineering design parameters of interest.

One may suggest a variety of variables to represent the strength of a liquid storage tank. Eurocode 8 (CEN, 2006) provides a limiting stress through Eq. (1)-(3) for the Elephant's Foot Buckling failure mode, where 'p' is the maximum internal pressure for the seismic design scenario considered. That essentially forms the background for the local DS3_{EFB} performance objective prescribed above. According to Eurocode 8, the internal pressure results from the sum of hydrostatic and hydrodynamic (i.e. impulsive) actions shown in Eq. (4). The cylindrical non-dimensional coordinate system (ξ , ζ , θ) adopted for the impulsive pressure estimation, combined with the linear (decreasing with height) pattern that is normally adopted for the design of the tank shell, suggests that EFB checks should not be limited to the lower course of the tank. In that sense, buckling checks should be extended to other parts of the tank where the 'wall thickness-maximum internal pressure' combination is equally important. It appears that the buckling stress is strongly tied to the tank wall thickness, and as a result it is reasonable to assume that the response is governed by the lower course wall thickness of the tank.

$$\sigma_m = \sigma_{c1} \left[1 - \left(\frac{pR}{t_w f_y} \right)^2 \right] \left(1 - \frac{1}{1.12 + r^{1.15}} \right) \left[\frac{r + f_y/250}{r + 1} \right]$$
(1)

$$\sigma_{c1} = 0.6E \frac{t_W}{R} \tag{2}$$

$$r = \frac{R/t_w}{400} \tag{3}$$

$$p_i(\xi,\zeta,\theta,t) = C_i(\xi,\zeta)\rho H \cos\theta A(t)$$
(4)

Fig. 5 presents the Response Frequency Spectra with respect to the lower course wall thickness (t_w) . A variety of design scenarios are considered and the strength parameter is assigned values ranging from 12-30mm. Local performance objectives with respect to plastic rotation and EFB modes of failure are presented on the RFS curves in order to highlight the importance of the methodology within a performance-based framework. The output may be presented either on a MAF (Fig. 5a) or a Probability of exceedance basis (Fig. 5b). One may notice the different distributions among the various performance objectives. It is evident that the EFB mean annual frequency or probability of exceedance is reduced as the tank wall thickness is increased. At the same time, the associated overturning moment capacities are shifted to higher estimates, which makes perfect sense as the strength of the system is fully aligned to the wall thickness. Contrary to EFB, plastic rotation develops higher probabilities of exceedance for larger t_w values, which may seem like a paradox on the first place. A closer look at the plastic rotation capacities suggests that they remain unaffected. The base plate plastic rotation is a function of the uplifting resistance of the structure and an increase at the order of millimetres on the wall may be deemed negligible to the entire weight of the tank. Naturally, the plastic rotation probability of exceedance is expected to be higher given the increase on the corresponding EFB capacities.



Fig. 5: Response Frequency Spectra using *t*_w as the strength parameter. (a) MAF and (b) Probability of exceedance in 50 years versus the normalised overturning moment.

Setting the tank wall thickness as an appropriate design parameter, obviously provides various levels of strength. Still the t_w increase is not accompanied by the desired drop in the probability of exceedance. In other words, from a performance point of view, it is rather uneconomical to propose a generous increase in wall thickness, as the differences both in the MAF and the probability of exceedance are generally small. Anchorage may serve as a better design target. API-650 (2007) determines anchorage requirements through Eq. (5)-(8), where G_e is the modified gravity accounting for vertical acceleration effects (A_v), W_s the total weight of the tank wall and w_{rs} the roof loading acting on the walls. According to the set of equations given below, the anchorage ratio 'J' seems to be a rather deterministic design parameter that is determined by a single seismic intensity level.

$$J = \frac{M_{ovt}}{D^2 \left[w_t (1 - 0.4A_v) + w_a - 0.4w_{int} \right]}$$
(5)

$$w_t = \left[\frac{W_s}{\pi D} + w_{rs}\right] \tag{6}$$

$$w_a = 99t_a \sqrt{f_y HG_e} \le 201.1 HDG_e \tag{7}$$

$$w_{\text{int}} = 0.25Dp \tag{8}$$

A more intuitive representation for the anchorage requirements is used herein. Following the ideas of Malhotra (2000), partial anchorage may be introduced to the tank examined in view of providing additional stability to the system. The concept can be summarised to a simple procedure where the overturning moment demand (M_{ovt}) is offset by an increasing overturning resistance capacity. Obviously, associated displacement values must be shifted too, as raising the value of M_{ovt} that triggers uplift means that the surrogate elastic beam supporting the impulsive mass deflects more before we actually reach this point. This kind of analysis is valid assuming that the anchors and their connection to the tank have sufficient ductility to assist the tank throughout its deformation range. RFS are presented in Fig. 6 using partial anchorage ratios that range from 0% (i.e. unanchored) to 20% of *WH*. Local performance objectives are displayed on RFS, featuring DS3_{epl} (Fig. 6a) and DS3_{EFB} (Fig. 6b) respectively. A significant reduction in probability of exceedance is evident following the increase in anchorage requirements, both for the plastic rotation and the EFB failure modes. The effect of anchors is strong enough to cause a 50% reduction in probability of

exceedance between the unanchored and the 5% *WH* anchored system. It appears that RFS provide an alternative point of view to the design of liquid storage tanks. Anchorage requirements may be determined following the desired level of performance (i.e. MAF or probability of exceedance) instead of the single-intensity approach suggested by API-650, where the tank response should be evaluated in a post-design process that leads to an endless iteration between design and assessment. On the contrary, through Fig.6a,b, one can easily select the required anchorage ratio to achieve any performance objective before engaging in more detailed analysis or design. For example, if one requires a maximum probability of exceedance of 5% in 50 years, for the tank of the given dimensions and site, an anchorage capacity in the order of 0.18*WH* is required, with base plate damage being the decisive limit-state.



Fig. 6: Response Frequency Spectra using *anchorage* as the strength parameter. Probability of exceedance in 50 years versus the normalised overturning moment for (a) the $\theta_{pl}=0.4$ rad and (b) the EFB local performance objective.

Conclusions

Response Frequency Spectra have been introduced as a practical approach towards the performance-based design of liquid storage tanks. The concept is simple enough to enable a range of performance targets to be considered that can be connected to the global or local response of the single-degree-of-freedom oscillator. Design parameters such as the lower course wall thickness and the anchorage strength are examined in an attempt to highlight the importance of the proposed design methodology. They only serve as an example, and as a result other parameters such as the base plate and annular ring thickness can also be considered. Although limitations exist, this approach may deliver conceptual designs for liquid storage systems that are very close to the prescribed performance objectives, thus minimising the endless analysis-design iterations.

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