

A photograph of a stone wall with a door, a light fixture, and a small window. The wall is made of light-colored, textured stone blocks. A dark door is partially open, revealing a stone wall behind it. A light fixture is mounted on the wall to the left of the door, and a small window is visible below it. The number 49 is overlaid on the wall.

49

Frequency-dependent  
LPMs for predicting  
seismic and mitigating  
non-seismic vibrations

Nikolaos Lesgidis  
Lukas Moschen  
Thomas Jaquet  
Anastasios Sextos

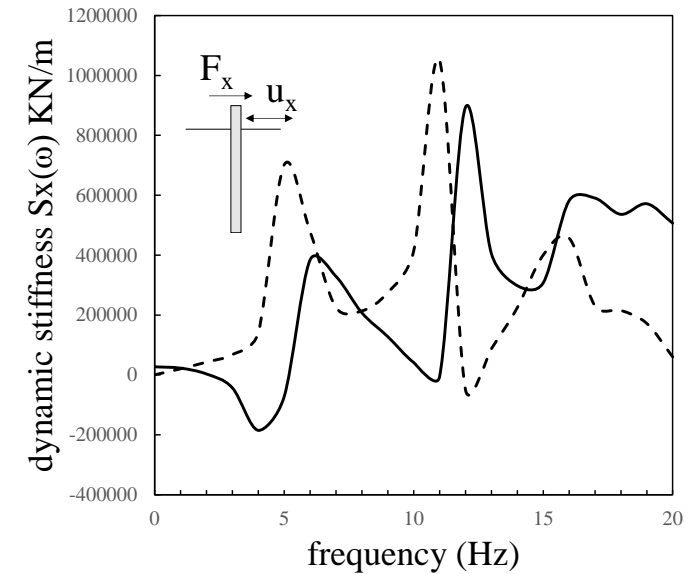
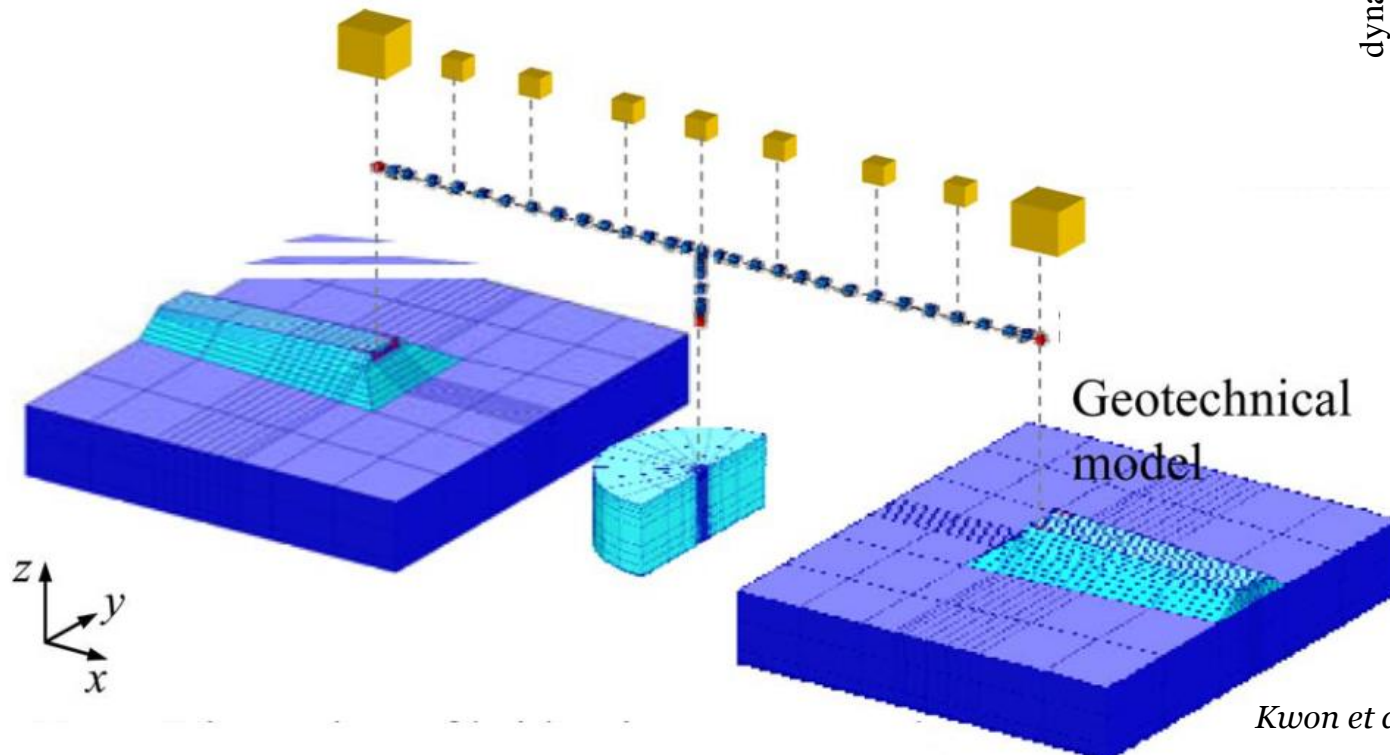


# Frequency-dependent SSI in time domain analysis?

SSI would require a large soil domain or the use of a dynamic stiffness matrix

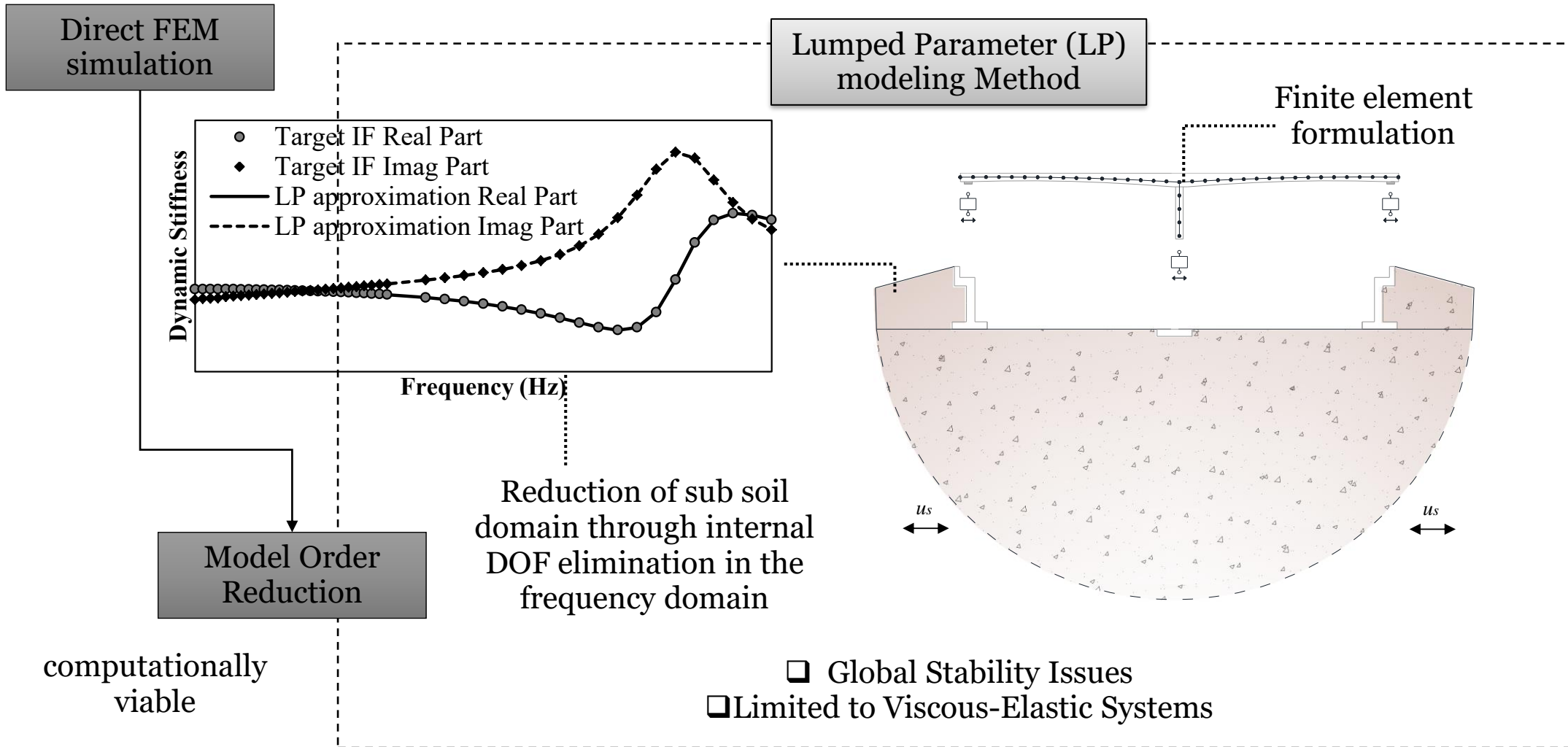
SSI is frequency dependent – analysis in the frequency domain

Several structural problems require response history analysis  
(single frequency springs and dashpots + cross stiffness terms can be negative)



*Kwon et al (2008)*

# SSI: Analytical LPM modelling



Lesgidis, N., Sextos, A.G. and Kwon, O.-S. (2018) "A frequency- and intensity-dependent macroelement for reduced order soil-structure interaction analysis", *Earthquake Engineering and Structural Dynamics*

# Stability issues in LP methods

LP model represented as a polynomial fraction in the frequency domain:

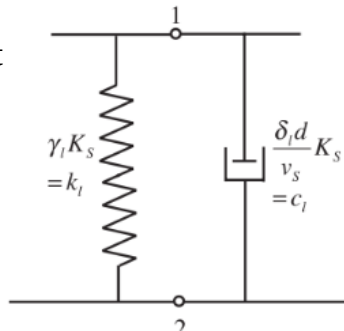
$$S_{LP}(s) = S_{LP,0} \cdot \frac{P(s)}{Q(s)} = S_{LP,0} \cdot \frac{p_0 + p_1 \cdot (s) + \dots + p_{N+1} \cdot (s)^{N+1}}{q_0 + q_1 \cdot (s) + \dots + q_N \cdot (s)^N}$$

Polynomial coefficients calibrated for:  $S_{LP}(s) = S_{tar}(s)$

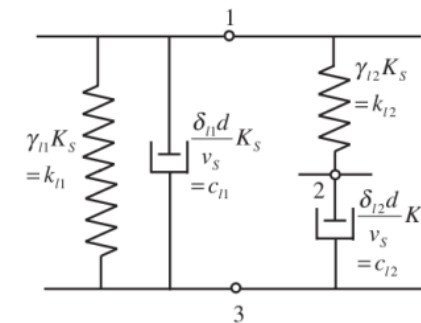
Translation to an ODE system (decomposition of polynomial fraction and term by term translation to physical components):

$$= \frac{P(s)}{Q(s)} = \sum_{i=1}^M \frac{q_i}{s - r_i} + \sum_{i=M}^{N+1-M/2} \frac{s - \alpha_i}{\left( (s - \alpha_i)^2 + \beta_i^2 \right)} + \sum_{i=M}^{N+1-M/2} \frac{\alpha_i - q_i}{\beta_i} \frac{\beta_i}{\left( (s - \alpha_i)^2 + \beta_i^2 \right)}$$

Example of 1<sup>st</sup> order term:



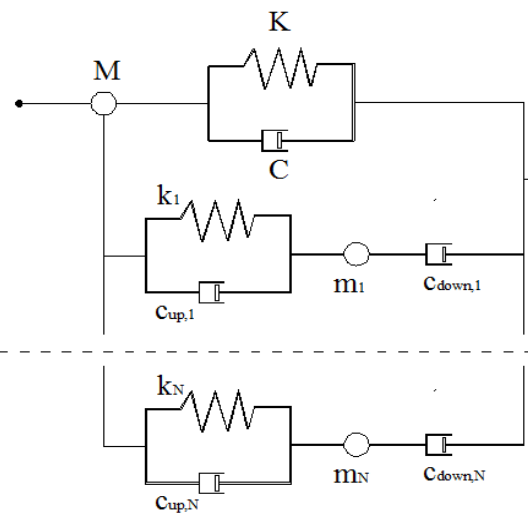
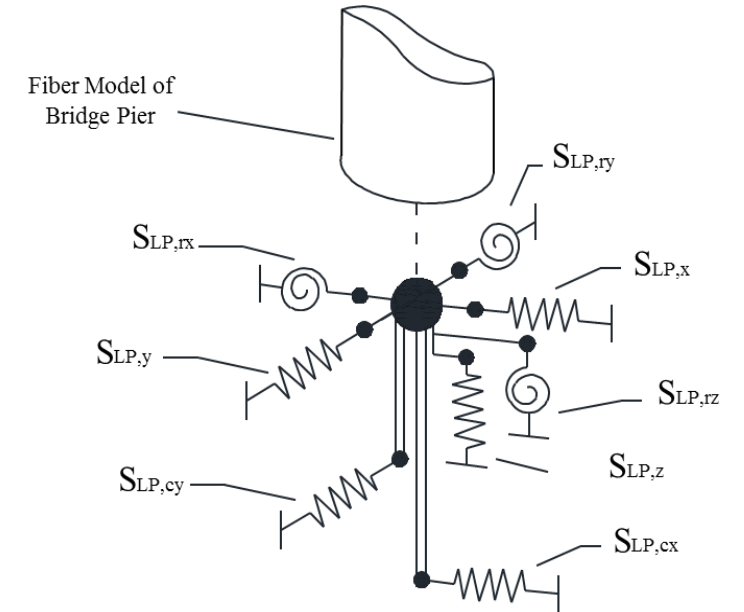
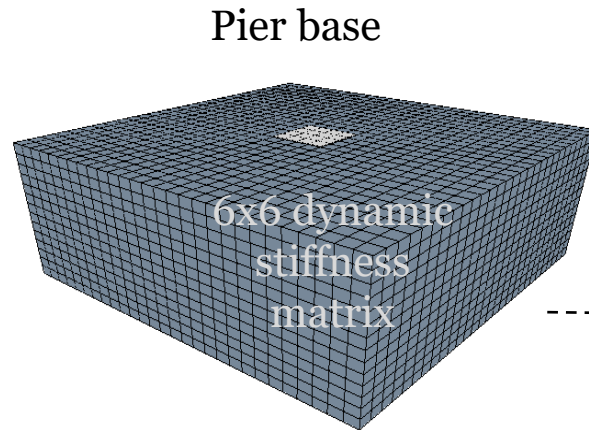
Example of 2<sup>nd</sup> order term:



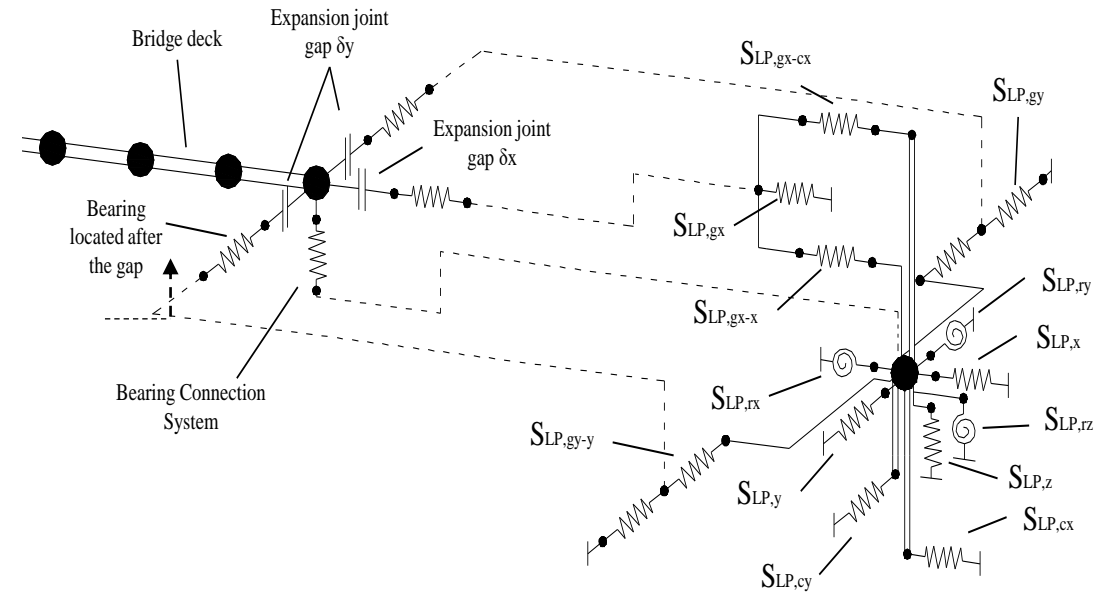
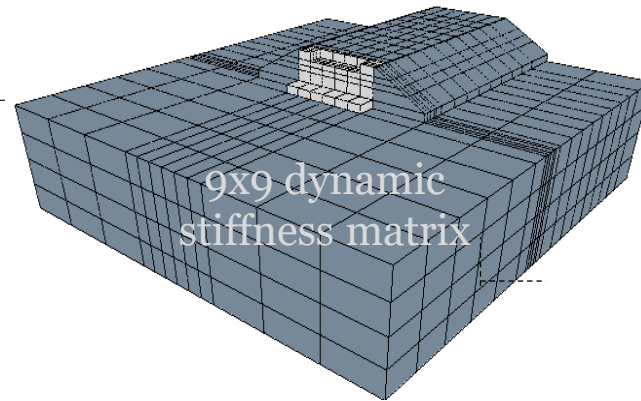
Sufficient condition to emulate the **Steady state** behavior. It will not necessarily lead to an ODE system with stable complementary solutions.

# Proposed Stable Lumped Parameter model

Restriction of LPM parameters to be positive through bound constraints in the calibration process.



Abutment-Embankment



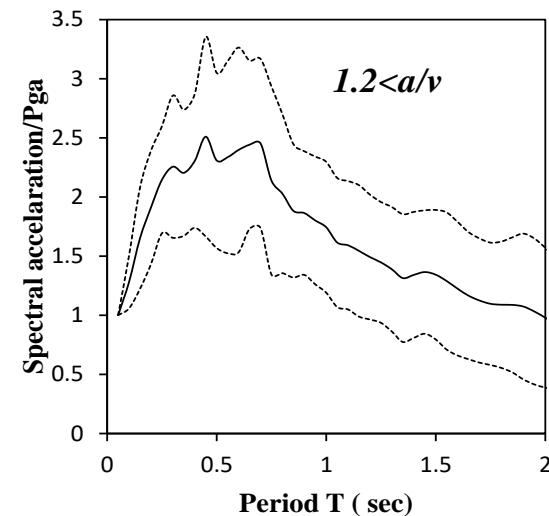
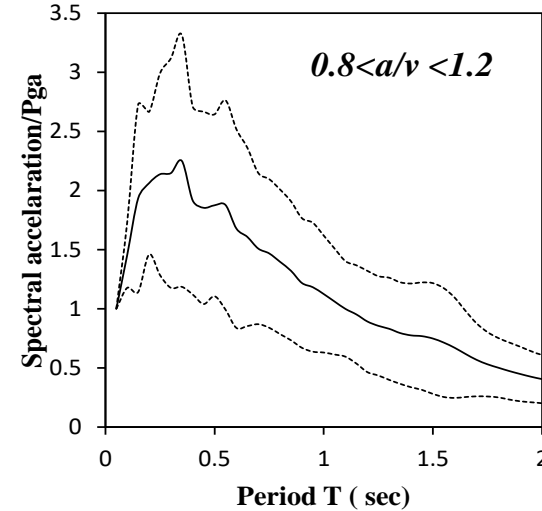
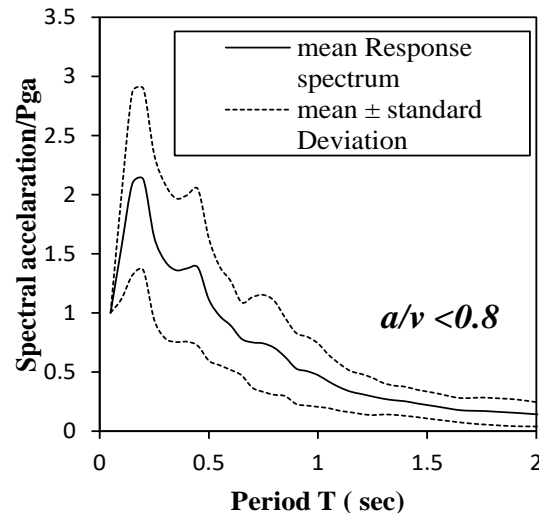
# Frequency-dependent SSI effect on system fragility



Pedini Overpass: Bridge Properties	
Overall length	78
Span Arrangement	19.00m -32.00m -19.00m
Deck Section	box girder, non prismatic
Piers(P1,P2)	solid circular section, height : 8.50 m
Abutments ( A1,A2)	laying on PTFE pot bearings

Different soil	Soil type
Scenarios	
S1	Soft Clay
S2	Loose Sand
S3	Medium Clay
S4	Medium Sand
S5	Hard Clay

Excitation sets based on frequency content





# FE Modeling



**Seismic Isolation:**  
Non linear Gap Model

**Pot Bearing:**  
Velocity dependent friction model

**Pier:** Reinforced  
concrete Fiber Model

**Abutment interface Region:**  
(a) Kelvin Voigt Assembly  
(b) LP model Assembly

**Pier interface Region:**  
(a) Kelvin Voigt Assembly  
(b) LP model Assembly

**Interface  
node A4**

**interface  
node P1**

**Interface  
node A2**

**Interface  
node A3**

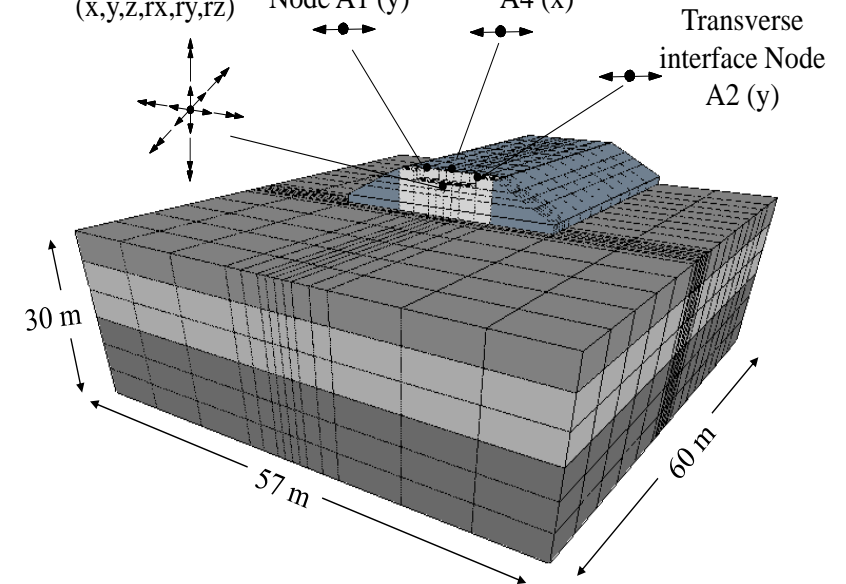
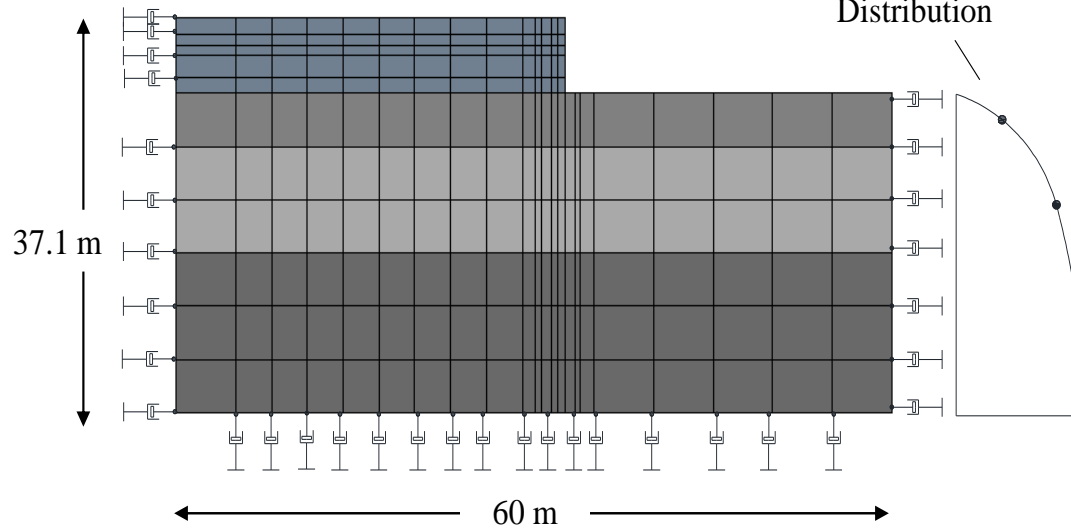
interface Node  
A3  
(x,y,z,rx,ry,rz)

Transverse  
interface  
Node A1 (y)

longitudinal  
interface Node  
A4 (x)

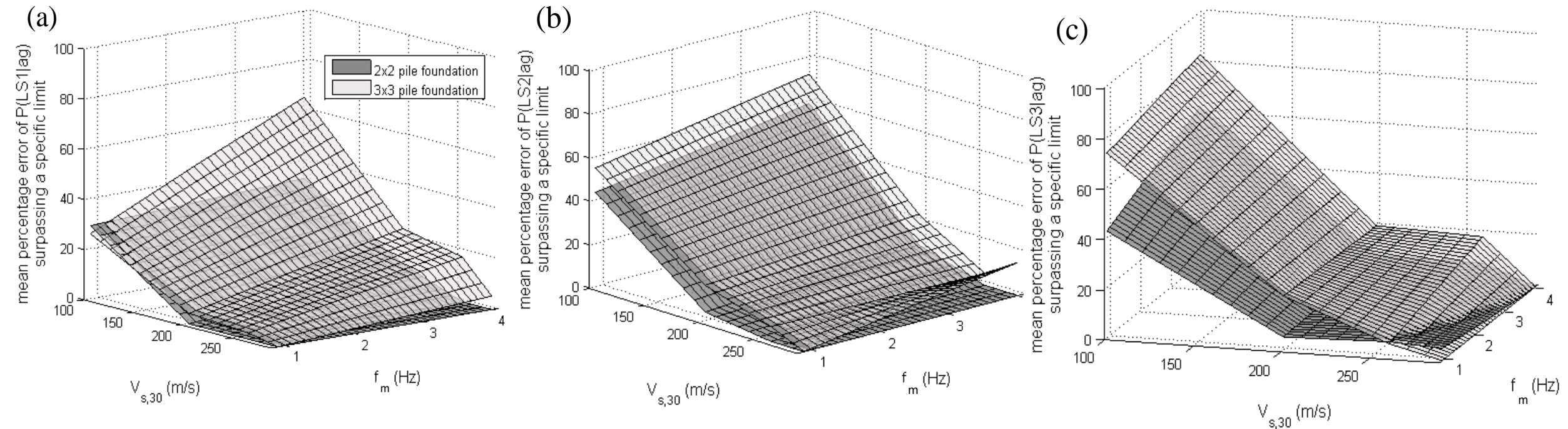
Transverse  
interface Node  
A2 (y)

Extraction of  
dynamic  
Impedance  
functions from  
FEM model  
formulation.



# Error induced when neglecting frequency-dependence

Percentage error in probability of reaching threshold limit states



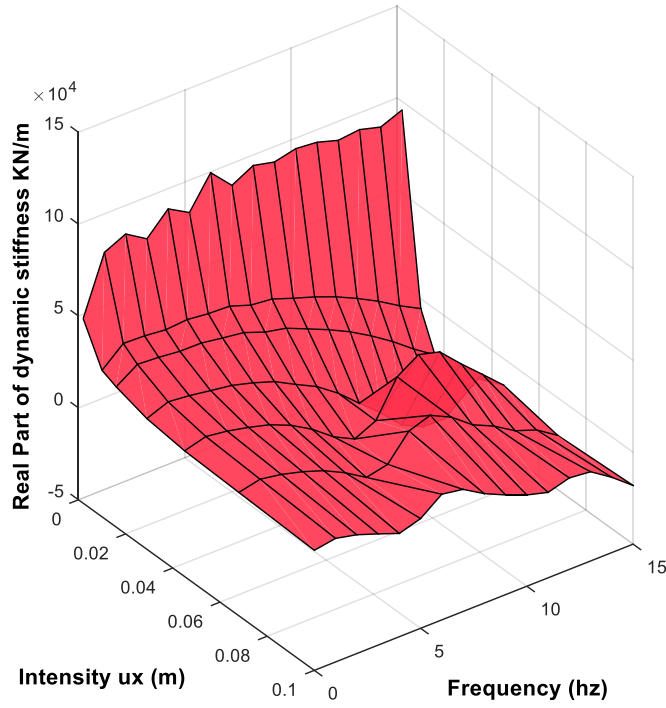
LS1 - Serviceability

LS2 - Damage control

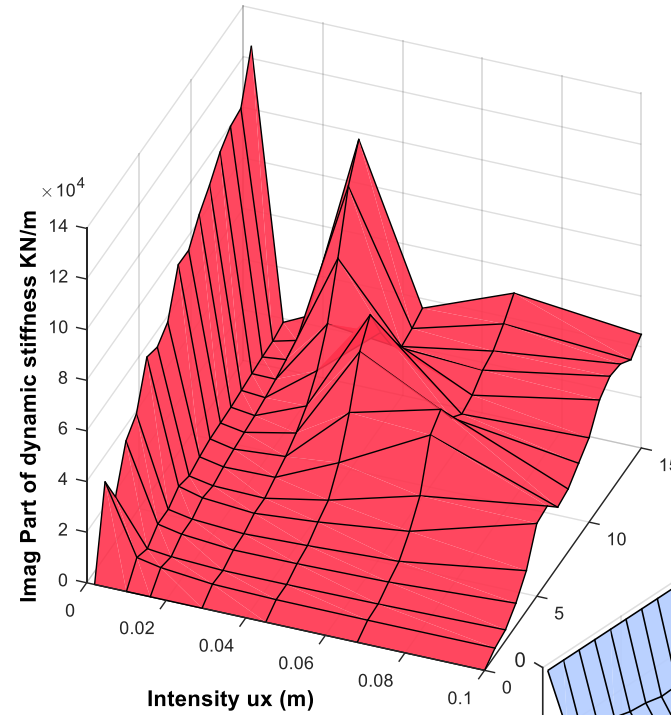
LS3 - Collapse prevention



# Inelastic LP modeling method

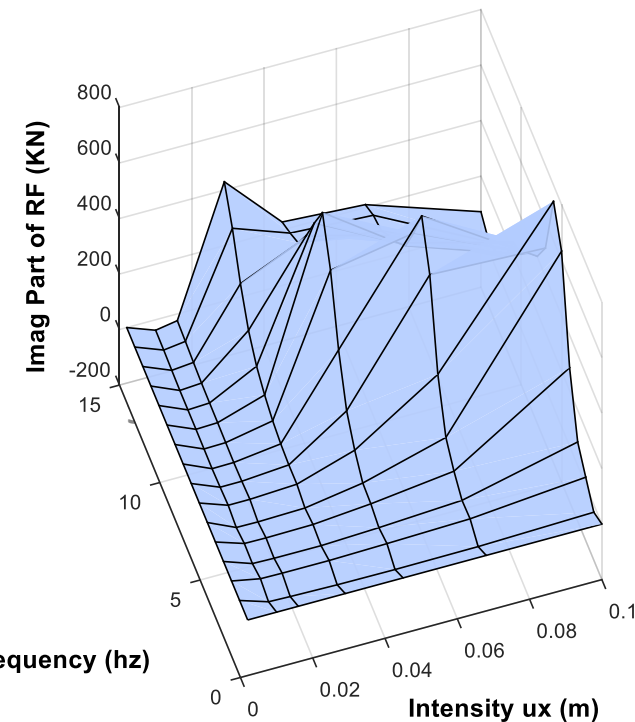
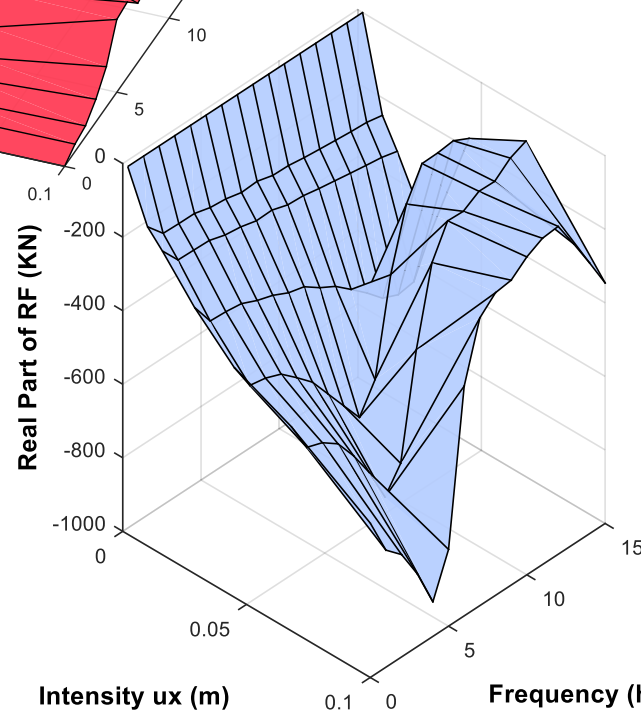


Extraction of Dynamic stiffness matrix for selected variable states in the inelastic range

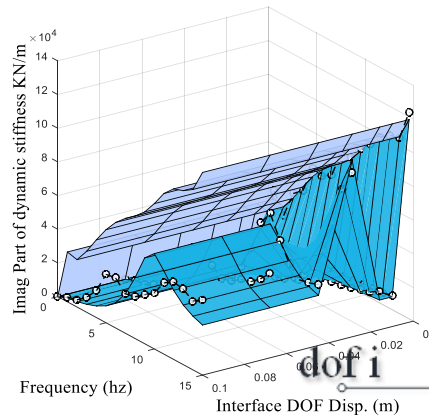
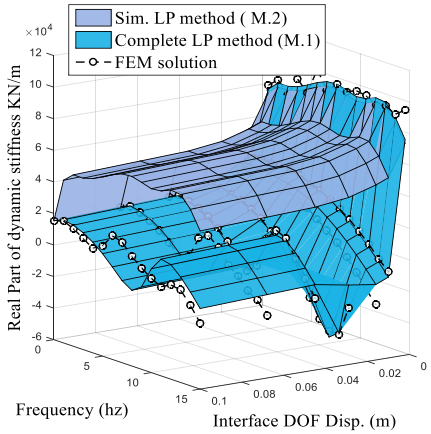


Selection and extraction of representative properties of target dynamic system

Extraction of dynamic returning force matrix for selected variable states in the inelastic range



# Frequency- and intensity-dependent LPMs

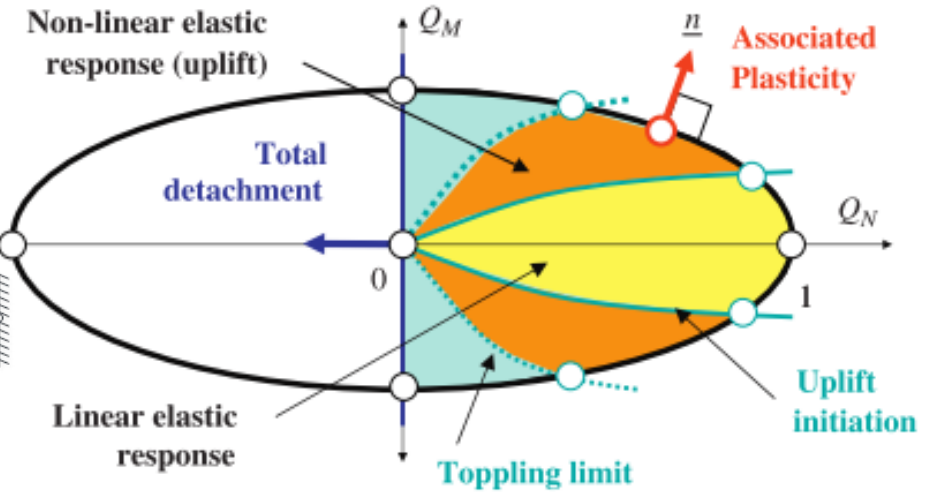
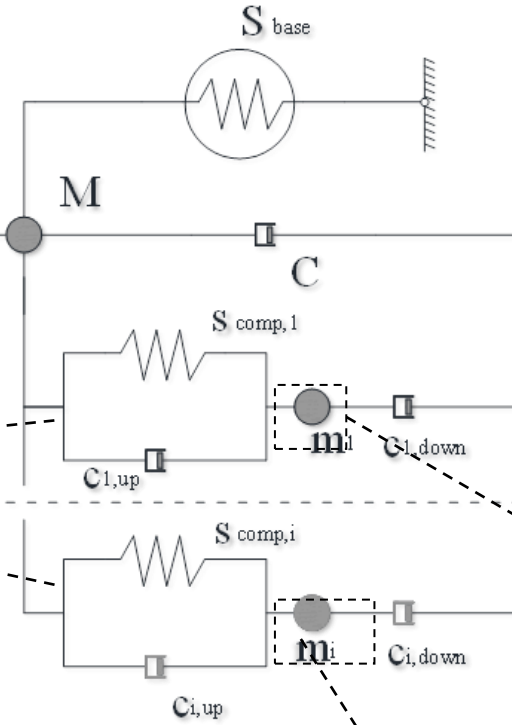


Base Component:  
*Macroelement*

Complementary Component:  
(M.1): *Externally controlled Spring*  
(M.2): *Conventional spring*

$$\dot{F}_{comp} = k_{comp}(\mathbf{a}_{base}, \mathbf{u}_{base}) \cdot \dot{\mathbf{u}}_{comp}$$

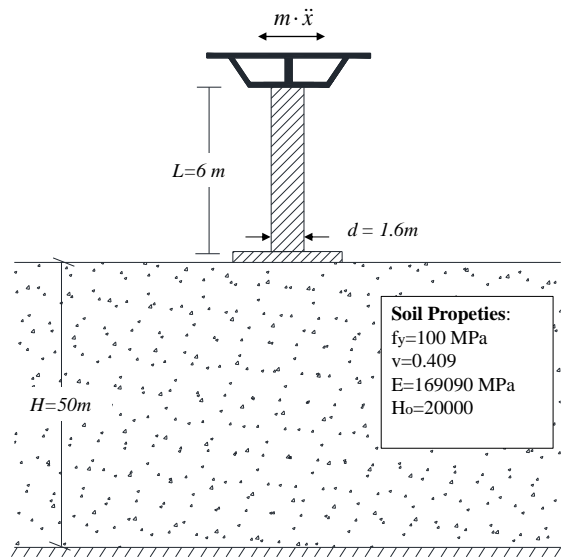
$$k_{comp}(\mathbf{a}_{base}, \mathbf{u}_{base}) = \begin{cases} k_o & \text{if } [\mathbf{a}_{base}, \mathbf{u}_{base}]^T < \mathbf{c}_o \text{ or Unloading} \\ \vdots & \\ k_n & \text{if } \mathbf{c}_{n-1} < [\mathbf{a}_{base}, \mathbf{u}_{base}]^T < \mathbf{c}_n \end{cases}$$



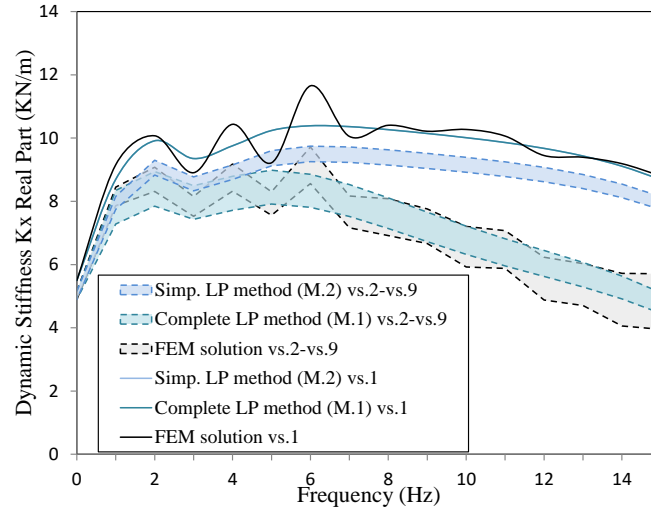
Complementary Component:  
*Conventional Mass*

Complementary Component:  
*Conventional Dashpot*

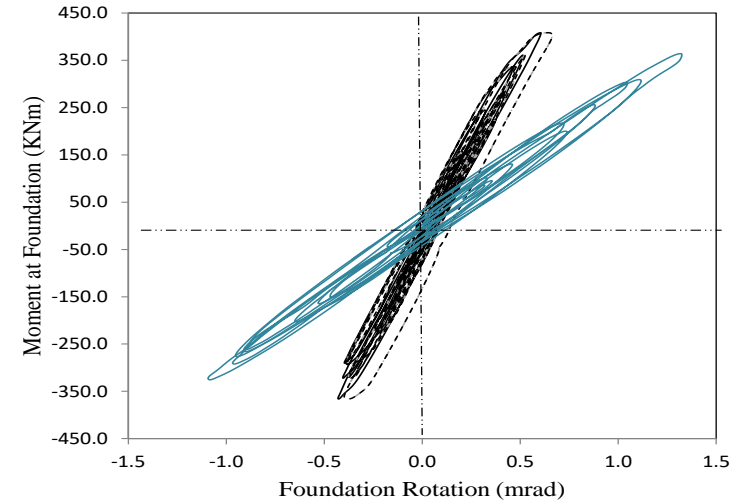
# Verification example



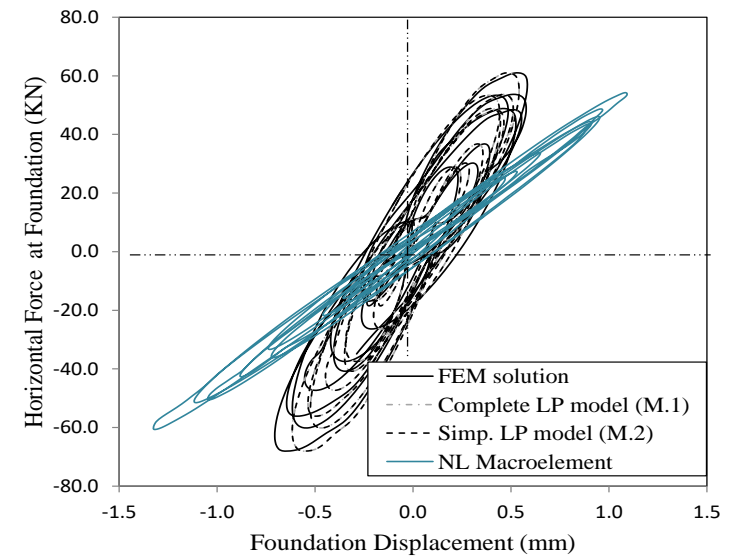
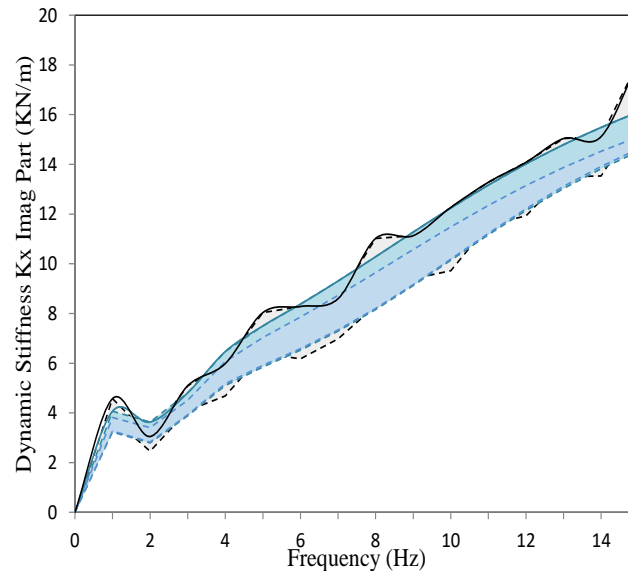
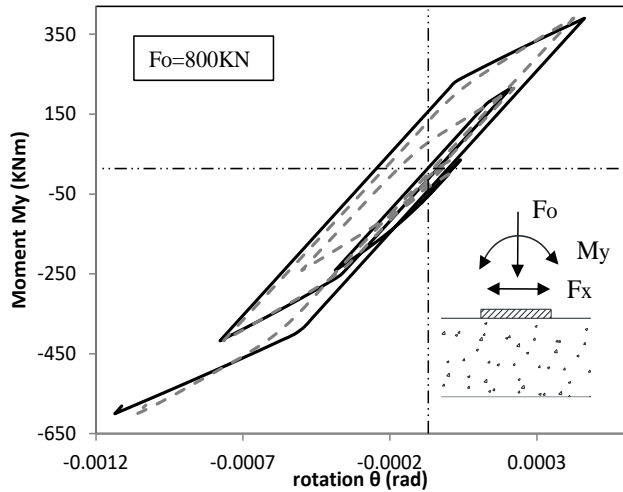
## Sample of Dynamic Properties in the Frequency domain:



## Model verification results:

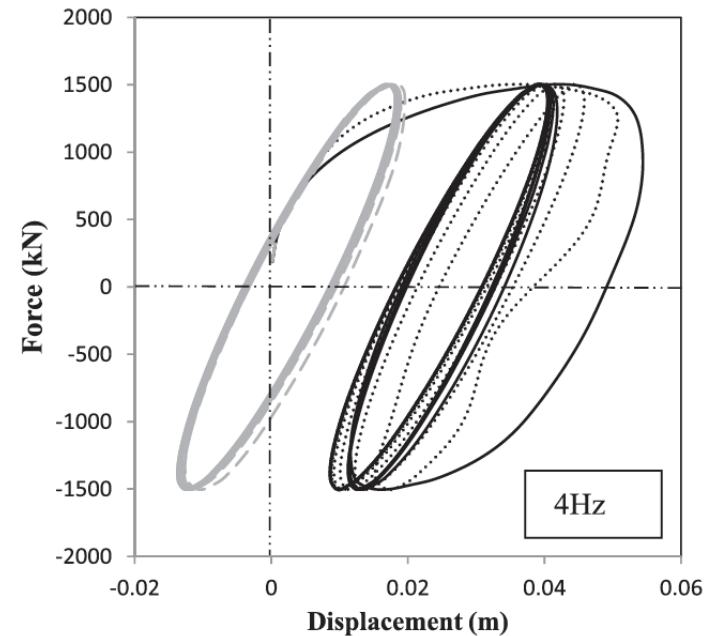
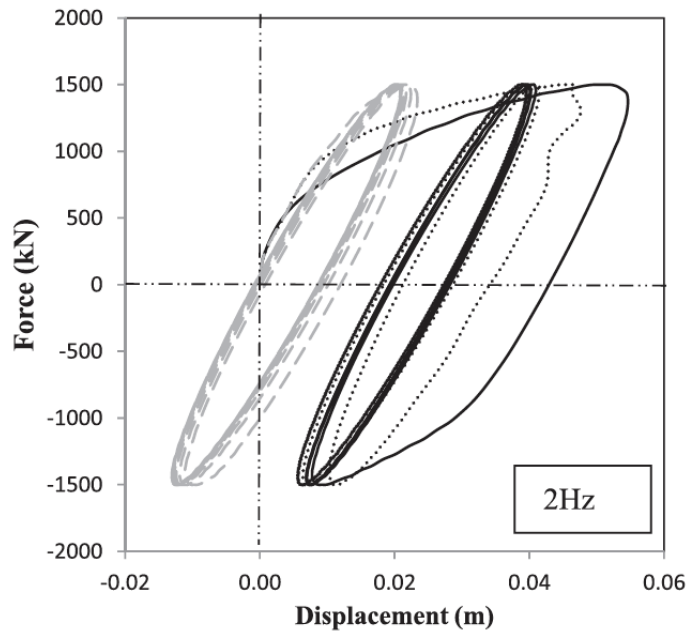
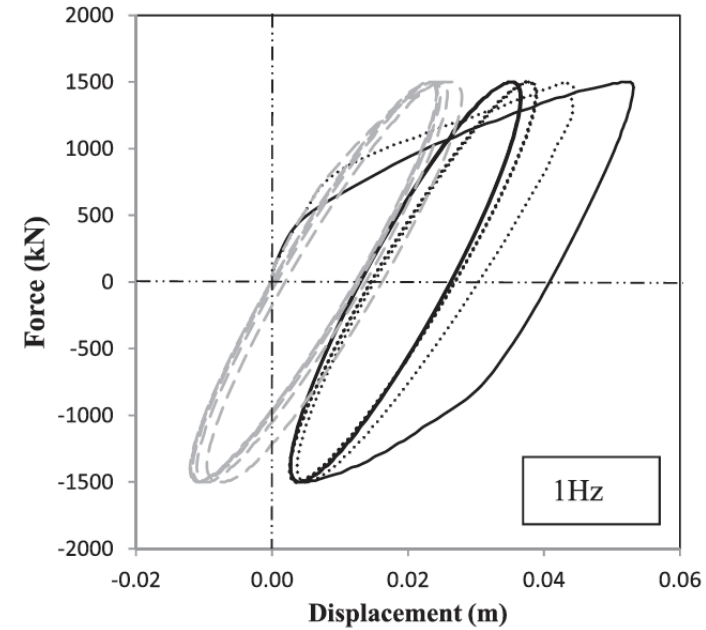
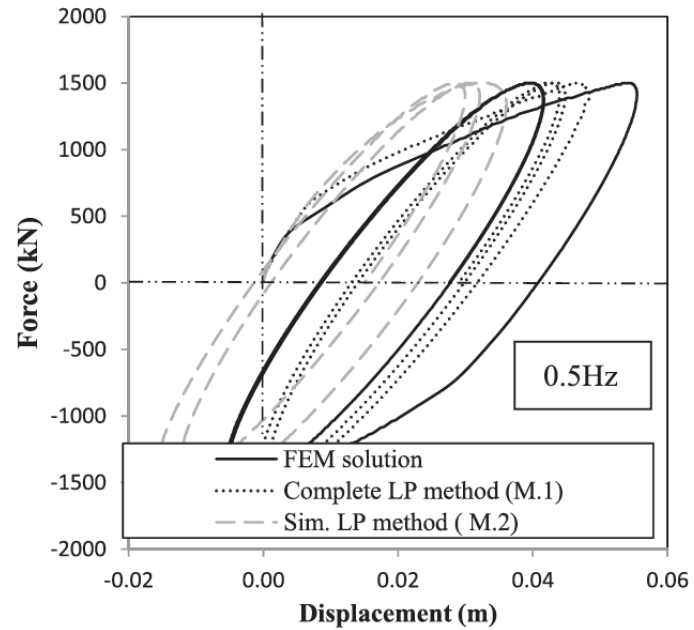


## Static Properties Calibration:

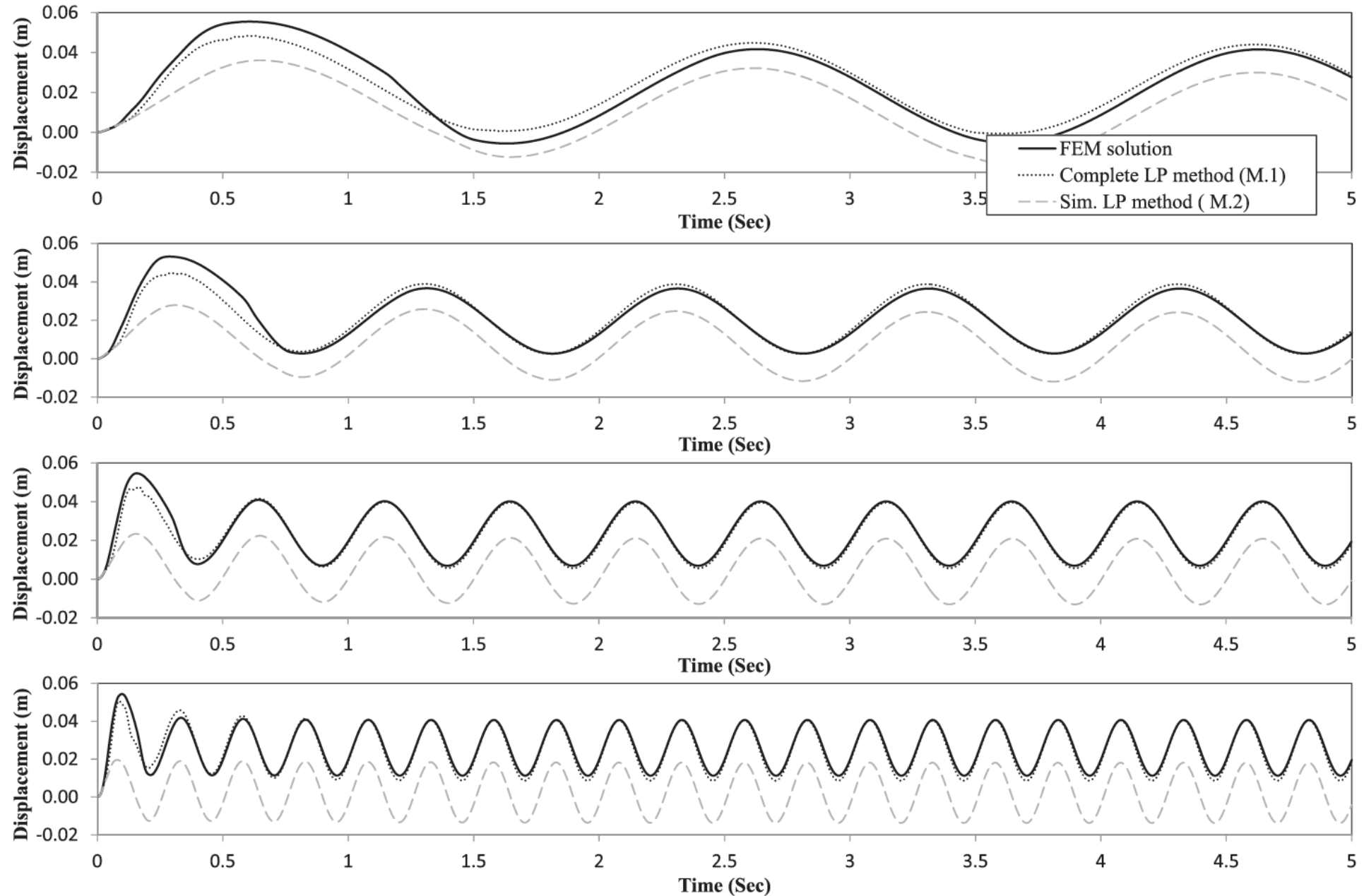




# High intensity dynamic response (F-d & d(t))

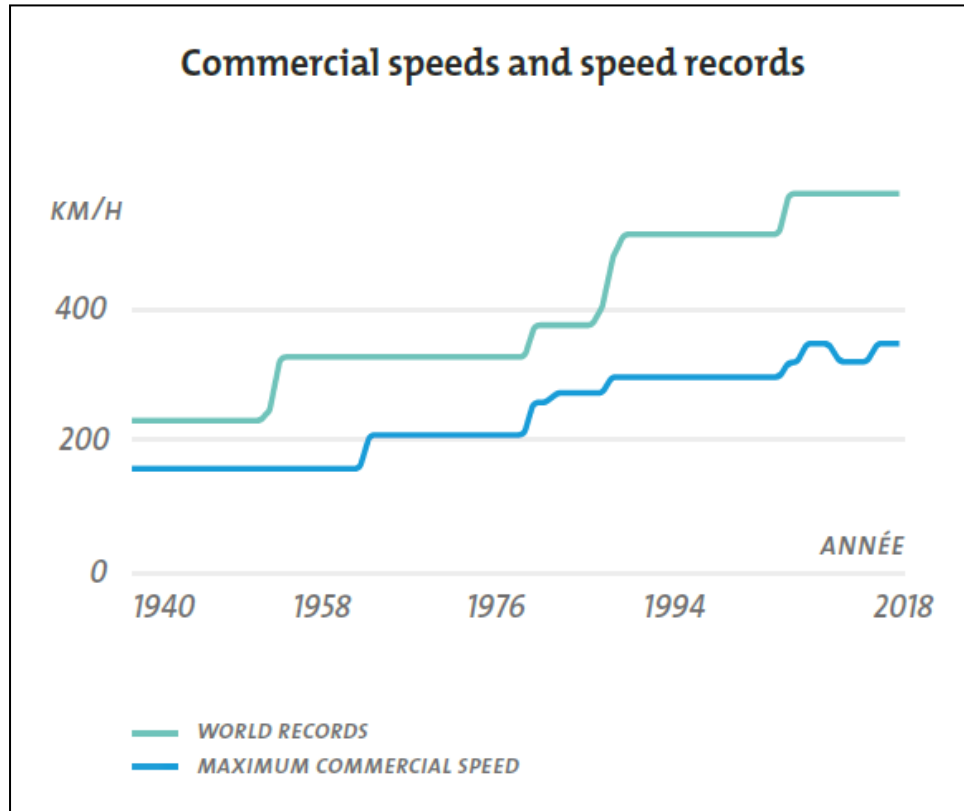


# High intensity dynamic response (F-d & d(t))



# Application in railway engineering

Key goal of railway industry: Increase of the **speed** and allowable carried load of railway trains.



RSSB 2413 – T1073 “Loading Requirements for Track Systems”

Michel Leboeuf, UIC high speed rail report (2018)

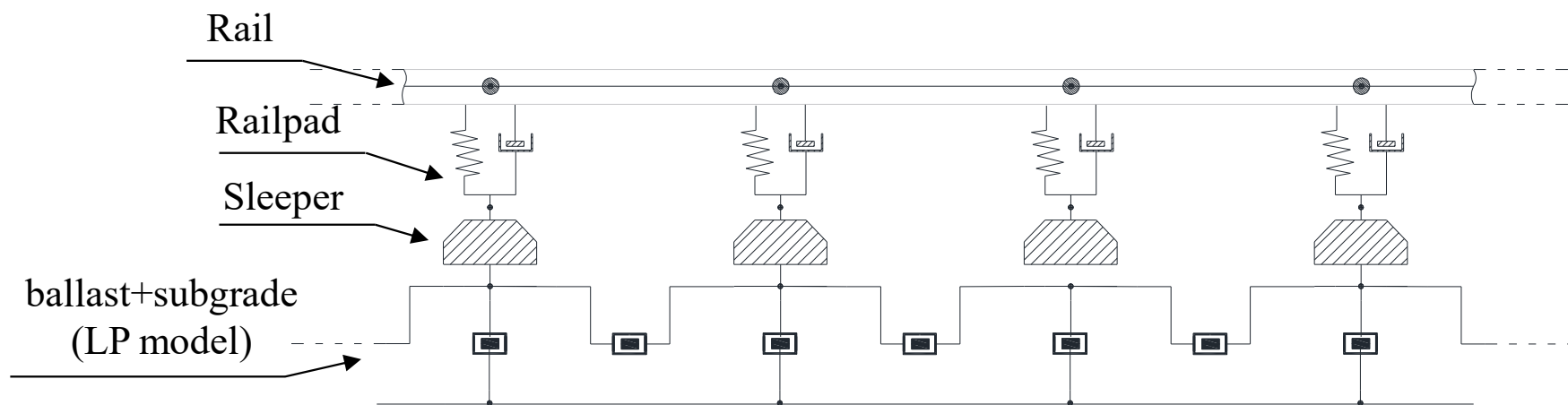




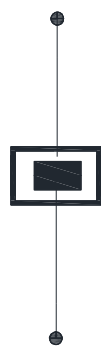
# Order reduction method



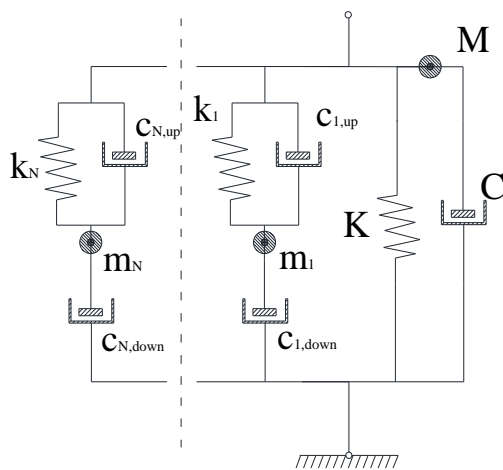
Infinite series of LP model assembly



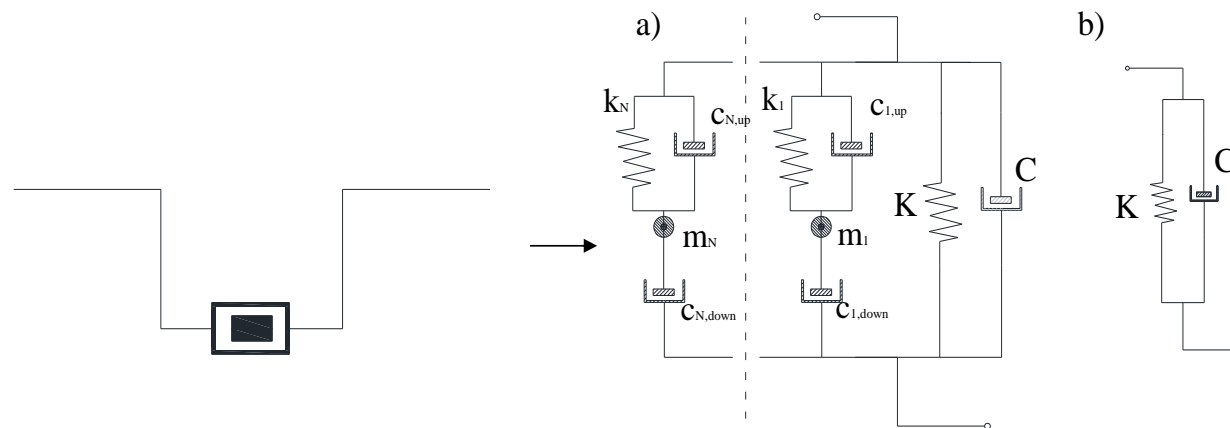
Finite set of LP subcomponents:



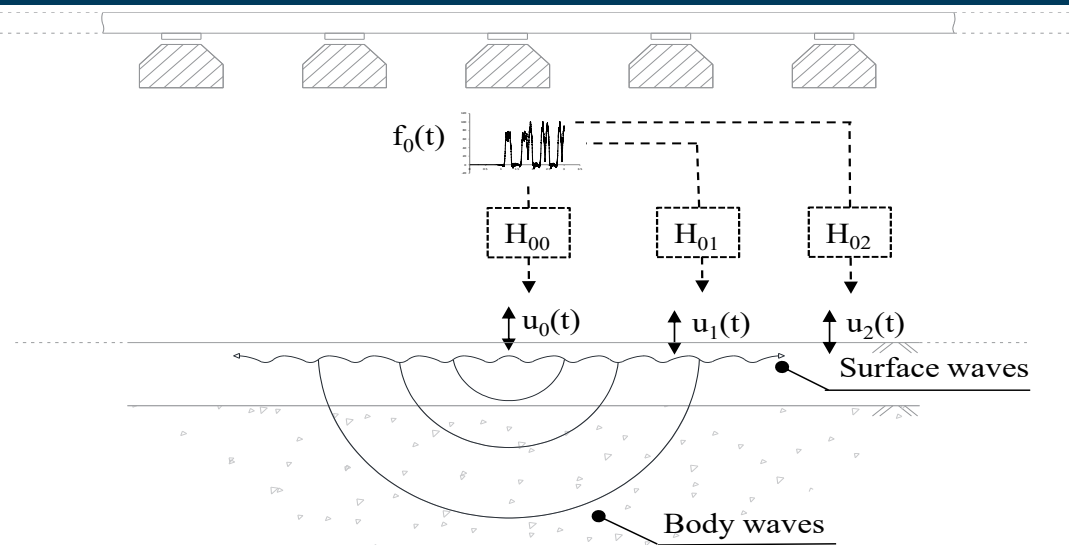
**Main LP component**



**Cross LP component**



# Order reduction method



$$\begin{bmatrix} H_{00} & H_{01} & \cdots & H_{0N} \\ H_{10} & H_{11} & & \vdots \\ \vdots & & \ddots & \\ H_{N0} & \cdots & & H_{NN} \end{bmatrix}$$

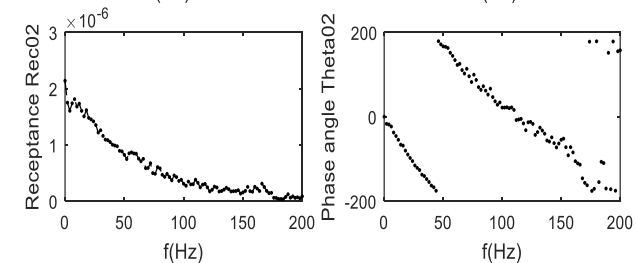
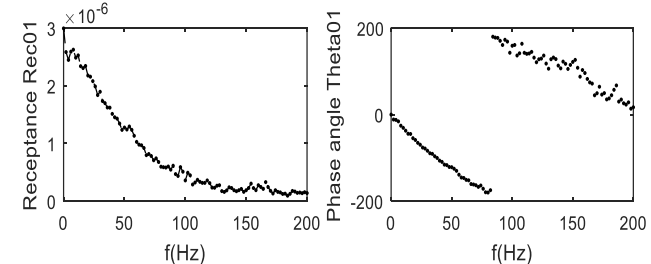
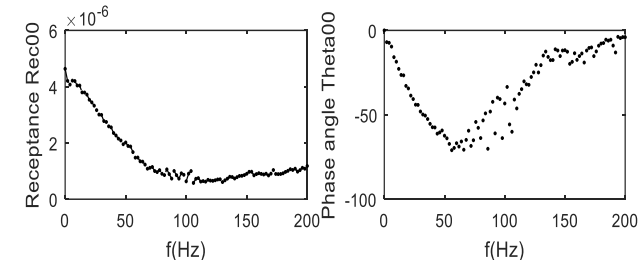
Where

$$H_{00} = H_{11} = H_{NN} \longrightarrow$$

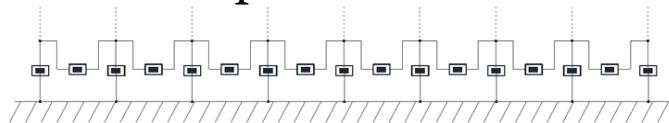
$$H_{01} \longrightarrow$$

$$H_{02} \longrightarrow$$

Dynamic Flexibility matrix  
(Frequency domain)



Calibration through optimization



⋮

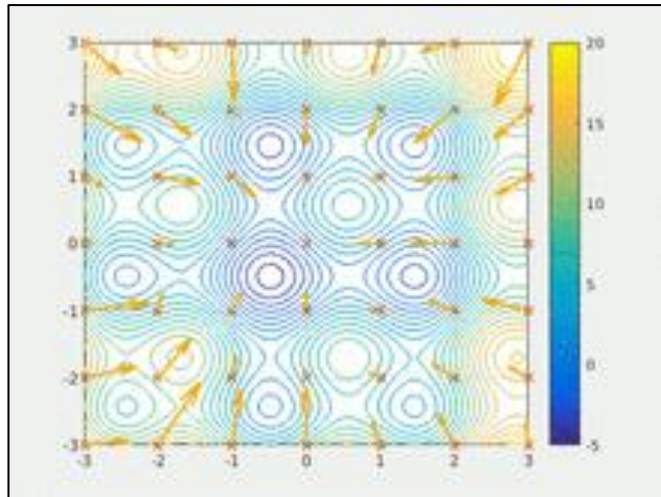
# Calibration through multi-objective optimization

$$\min_{\mathbf{x}_{com}} g_{com}(\mathbf{x}_{com}) = \sum_{j=1} w_j \cdot f_j(\mathbf{x}_{com})$$

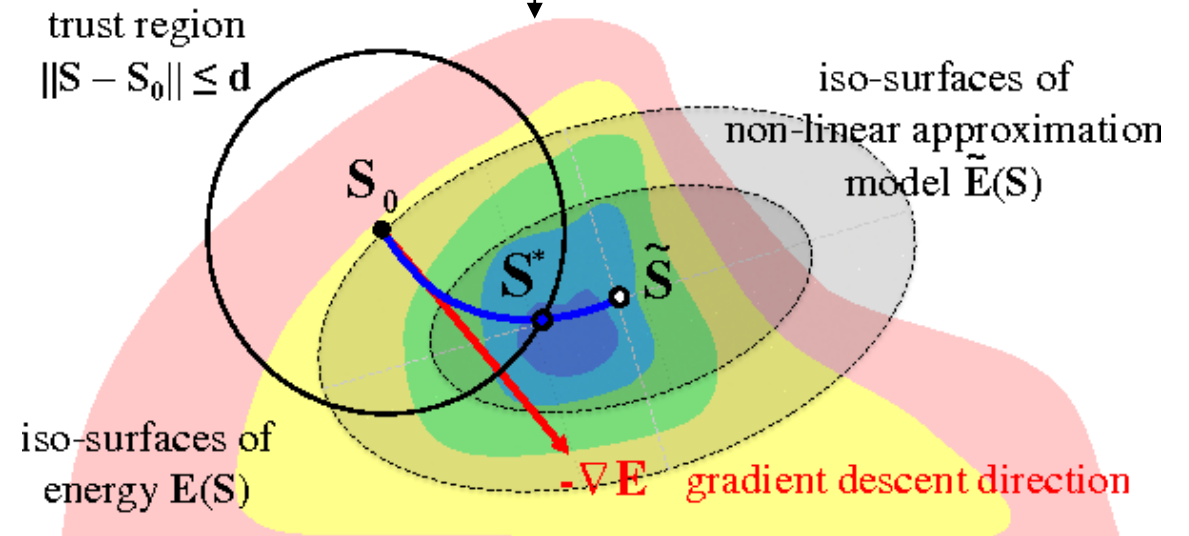
subject to  $\mathbf{x} \geq 0$

Global level

Local level



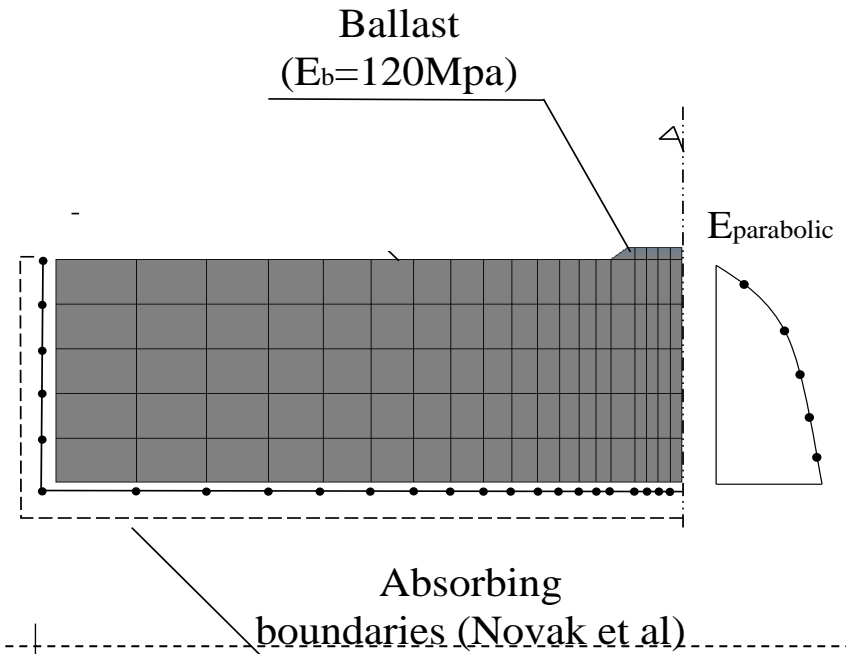
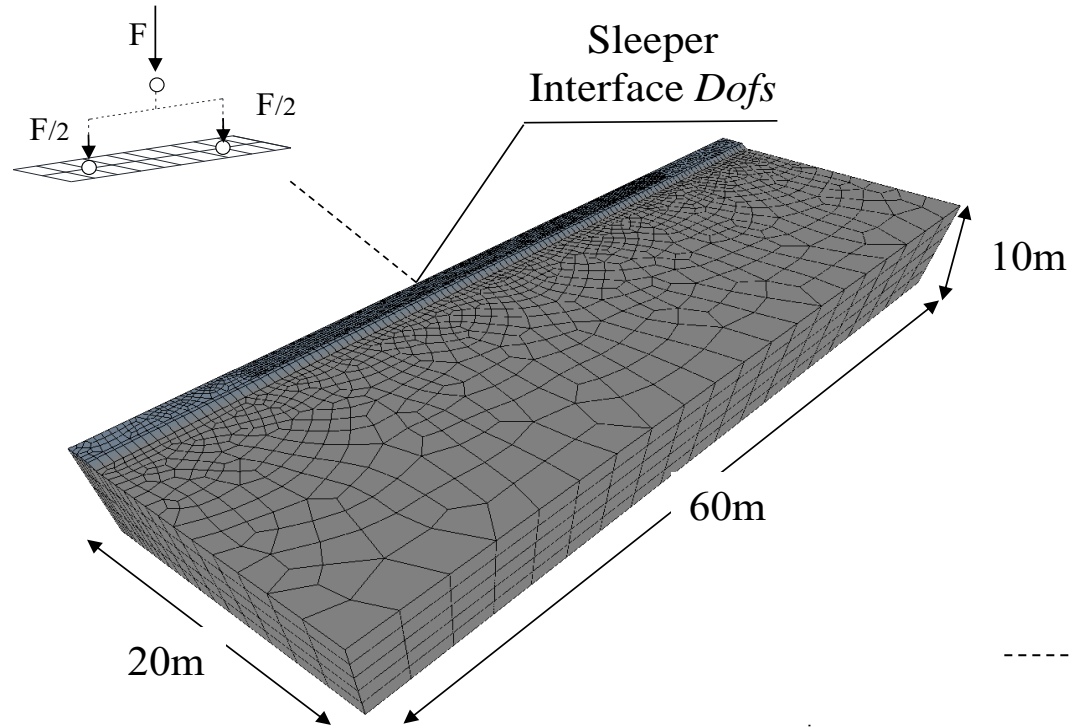
Particle swarm optimization



Interior point trust region



# Numerical verification case study



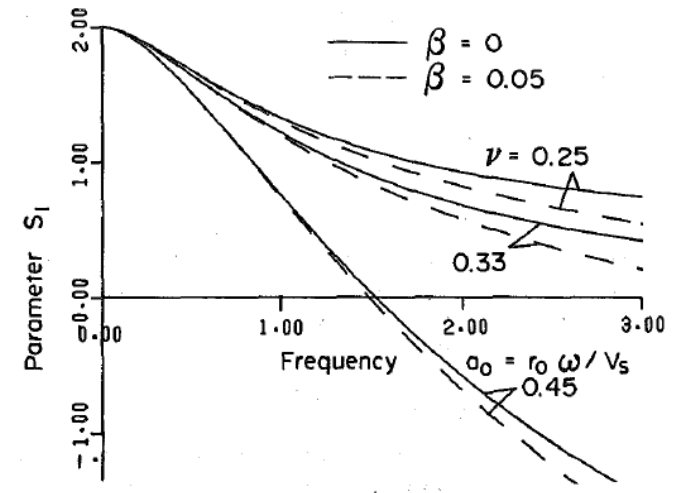
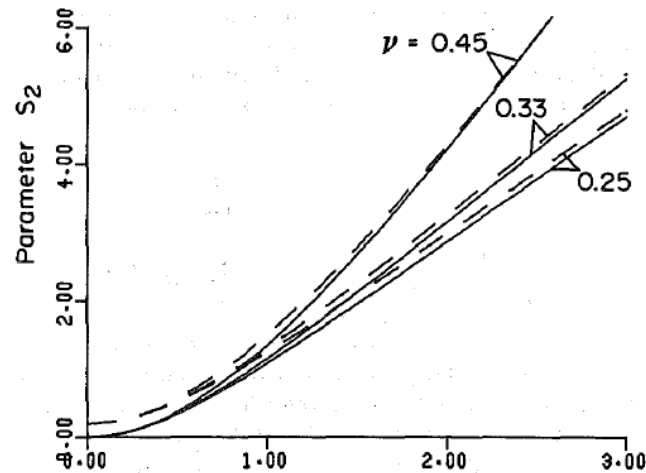
## Novak Boundaries

Fundamental Solution of wave equation in infinite disc:

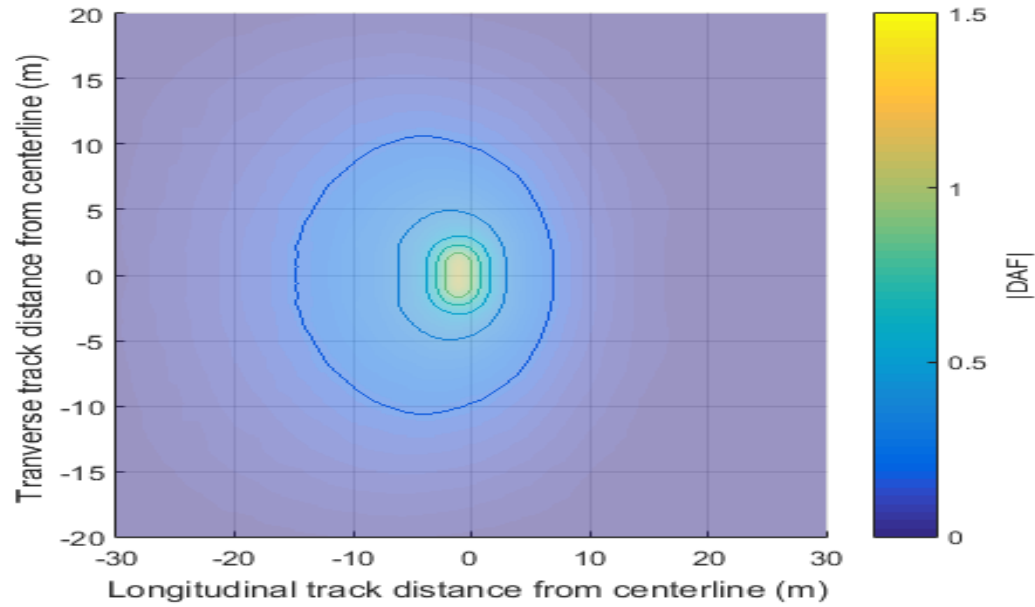
$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu) \frac{\partial \bar{\epsilon}}{\partial x} + \mu \nabla^2 u$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu) \frac{\partial \bar{\epsilon}}{\partial y} + \mu \nabla^2 v$$

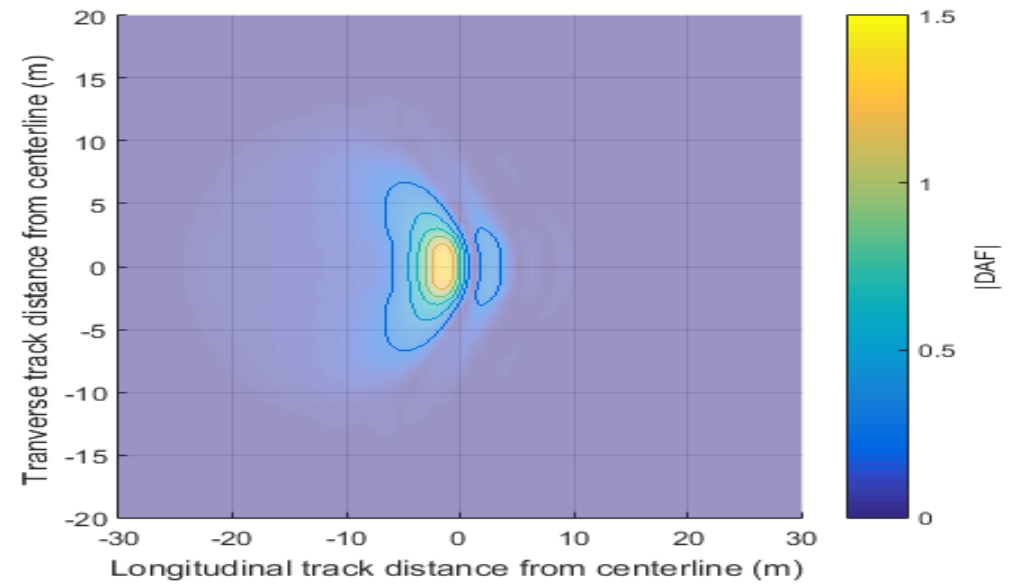
$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu) \frac{\partial \bar{\epsilon}}{\partial z} + \mu \nabla^2 w$$



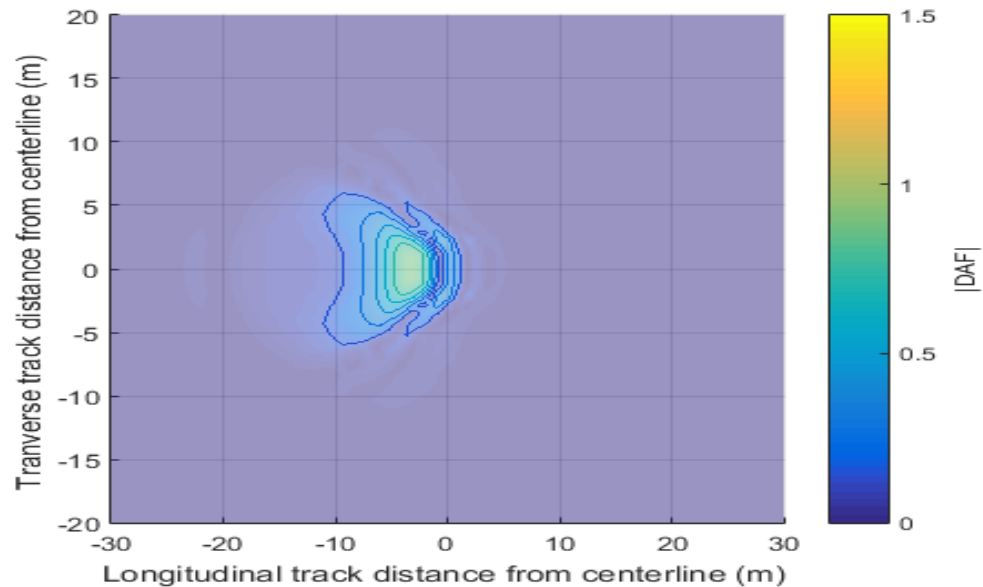
# Numerical verification case study



$V_{load} = 68 \text{ km/h}$

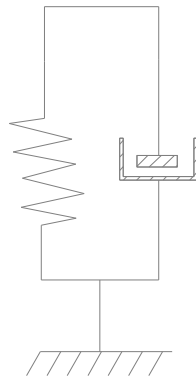
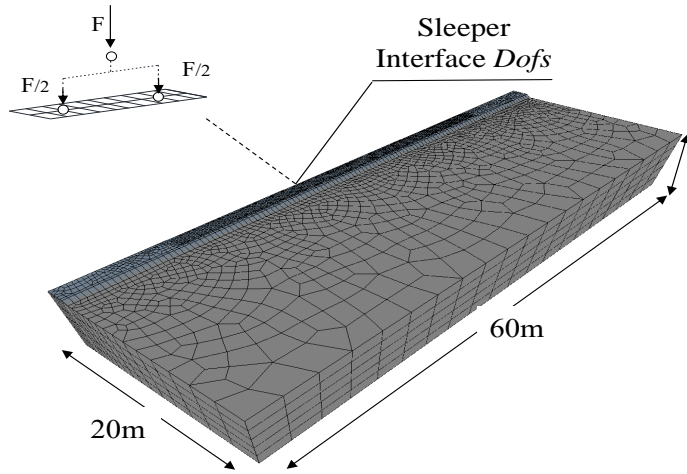


$V_{load} = 347 \text{ km/h}$

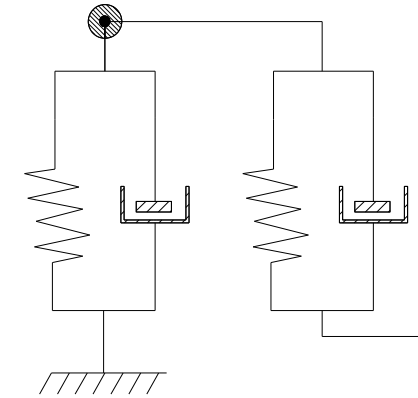


$V_{load} = 470 \text{ km/h}$

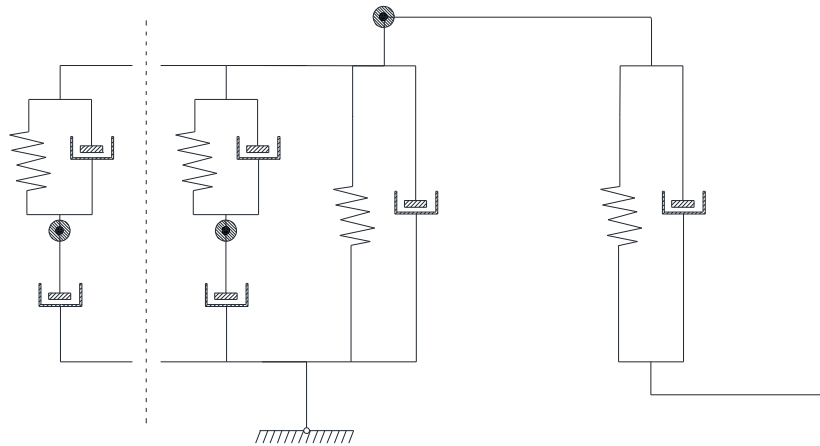
# 4 different methodologies for the subgrade system to compare



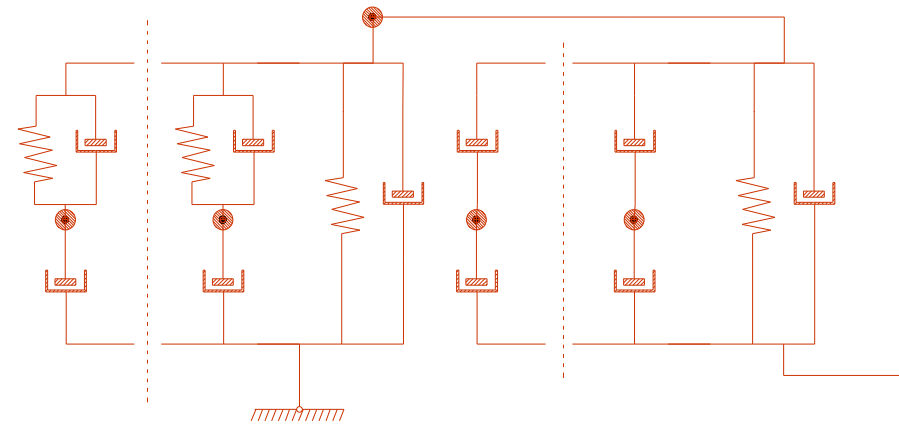
**Model:** Kelvin - Voigt  
**Calibration:** targeting direct Receptance at predominant frequency



**Model:** Kouroussis et al  
**Calibration:** targeting direct Receptance and use proposed closed form for cross terms.



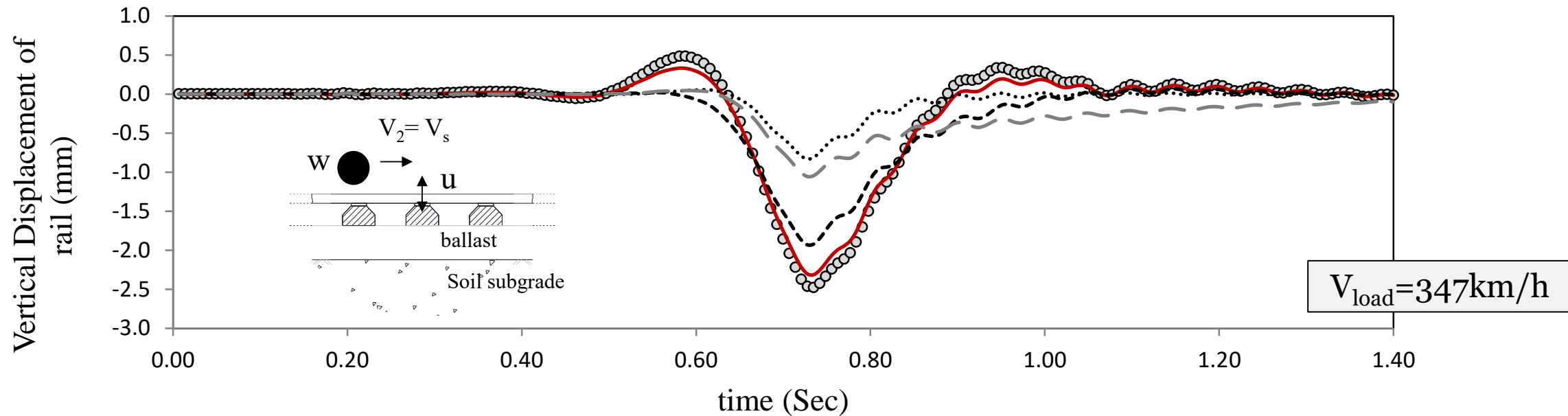
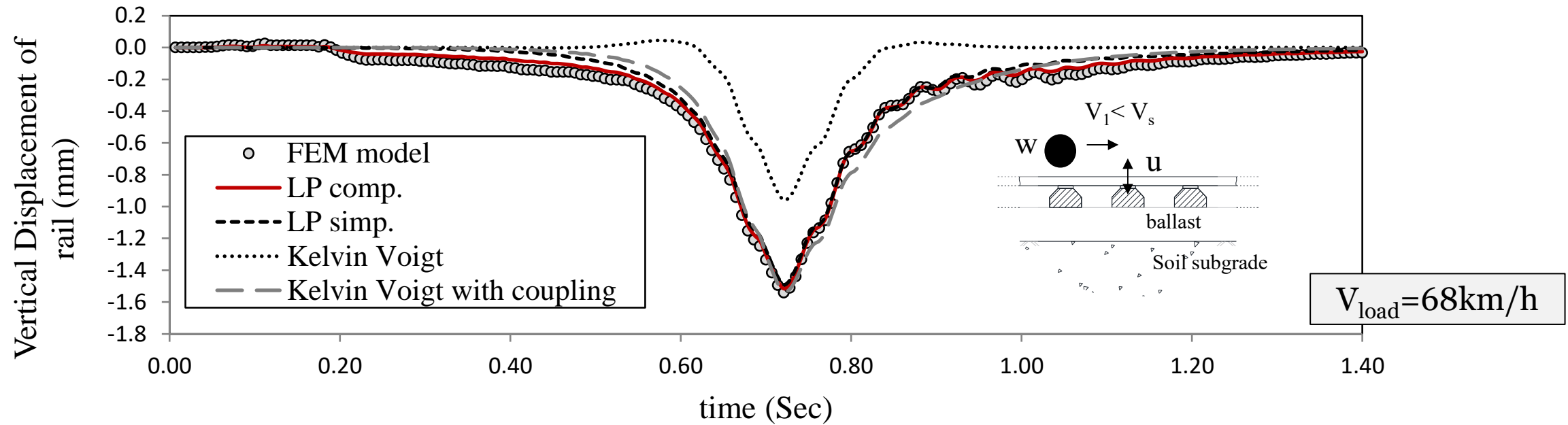
**Model:** simplified LP  
**Calibration:** PSO + interior point trust region



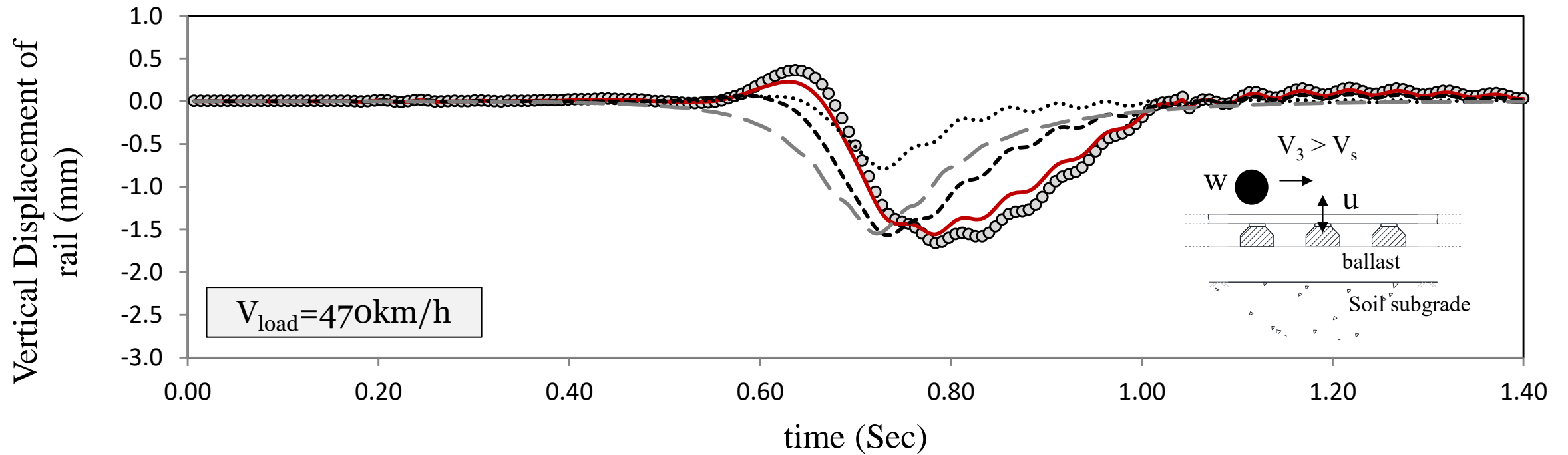
**Model:** complete LP  
**Calibration:** PSO + interior point trust region



# Four different methodologies for the subgrade system to compare



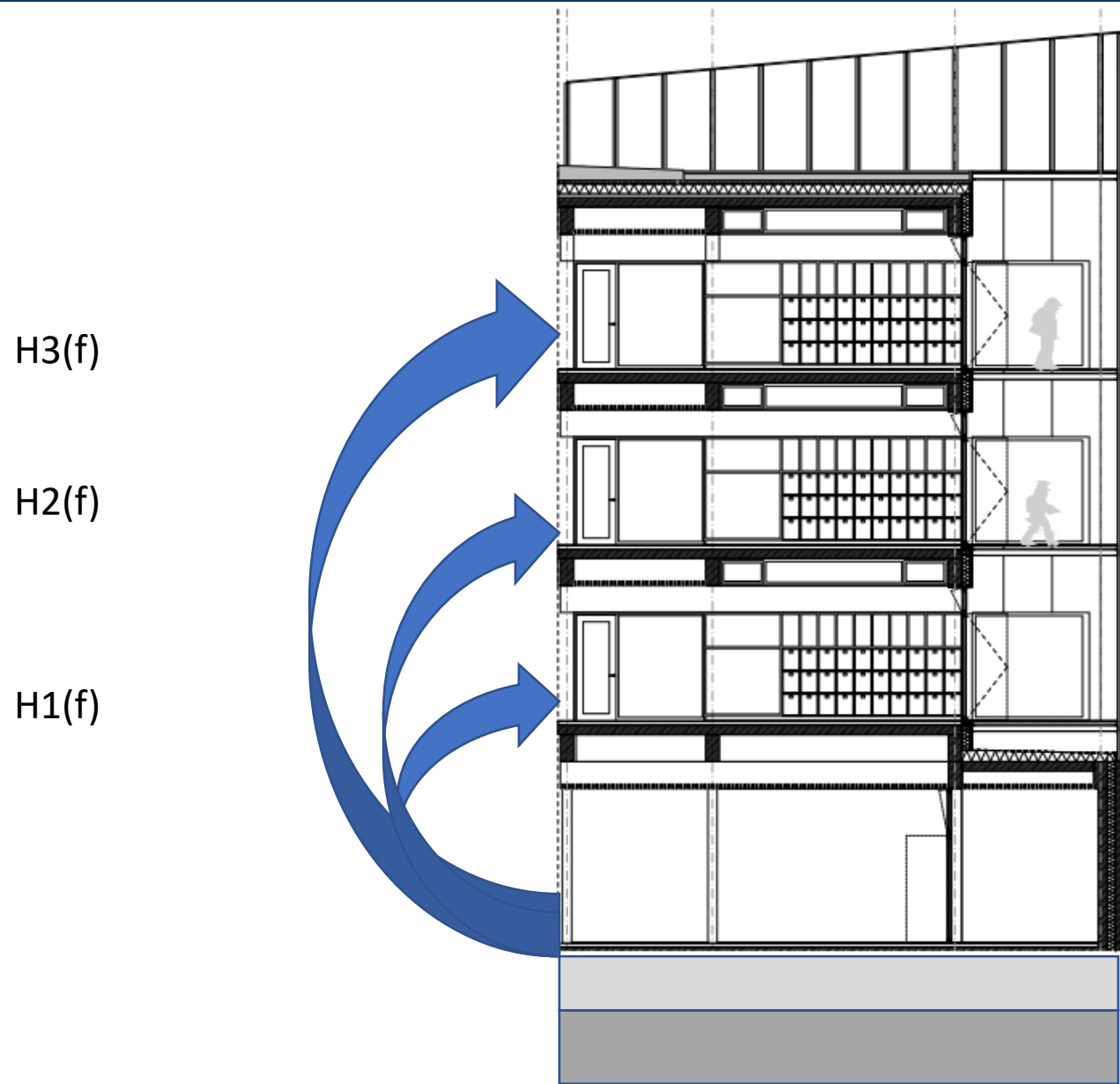
# Four different methodologies for the subgrade system to compare



# The problem of structural vibrations



# Prediction Model



## Vibration:

$$V(f) = \text{Soil}(f) \times \text{SSI}(F) \times \text{SS}(f) \times \text{Hi}(f)$$

Conversion.  $V(f)$  to KB (FTM) according to RIL

$$KB_{FTm} = \sqrt{\frac{1}{N} \sum_{i=1}^N KB_{FTi}^2}$$

➔

$$KB_{FTr} = \sqrt{\frac{1}{T_r} \sum_j T_{e,j} \cdot KB_{FTm,j}^2}$$

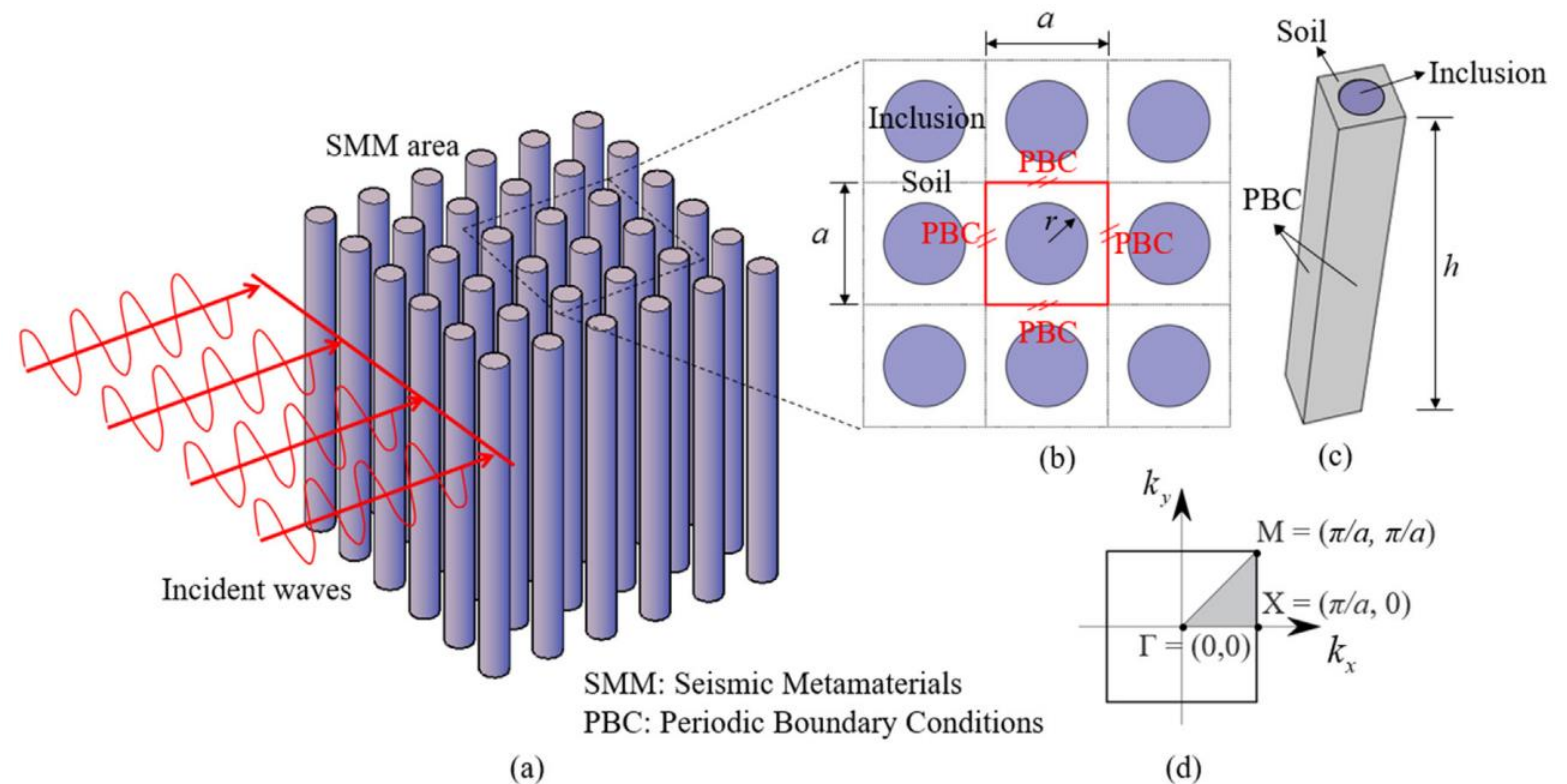
$$KB_{FTR} < A_r \text{ (DIN 4150)}$$

## Ground Borne Noise:

$$L_pA = L_vA + 10 \log(4S/A) + 10 \log s$$

# Vibration Mitigation below the perception level

- Seismic isolation / Springs ?
- Meta-surfaces? (seismic soil-metamaterials, buried mass-resonators, above-surface resonators, auxetic materials)
- Soft Barriers?
- Pile Group foundation
- Deterministic vs. Probabilistic



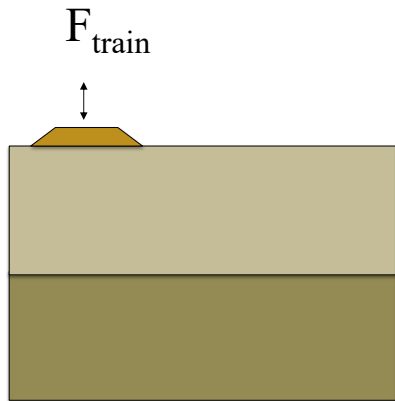
Li, T., Su, Q., Kaewunruen, S. (2020) Seismic metamaterial barriers for ground vibration mitigation in railways considering the train-track-soil dynamic interactions, Construction and Building Materials

Prediction

# Pile foundation to mitigate vibrations

Soil report,  
on-site tests

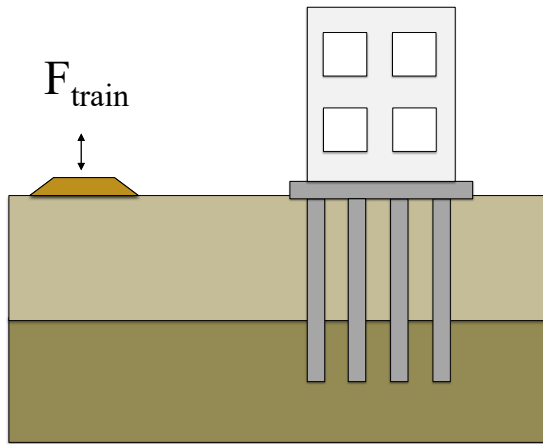
Free field model A:  
vibration source and soil medium



**System Identification**

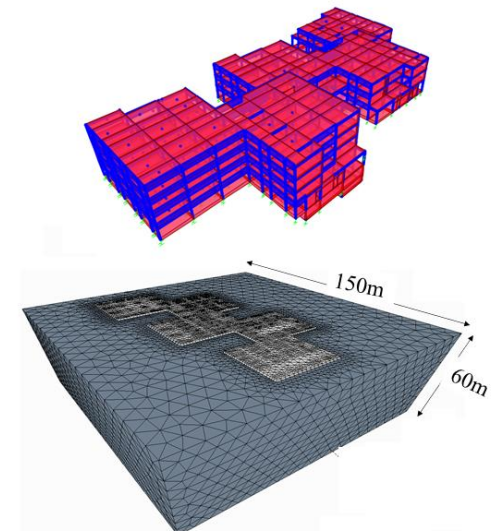
Soil, track, train properties,  
simulation options

Archetype model B:  
pilegroup foundation with  
archetype building



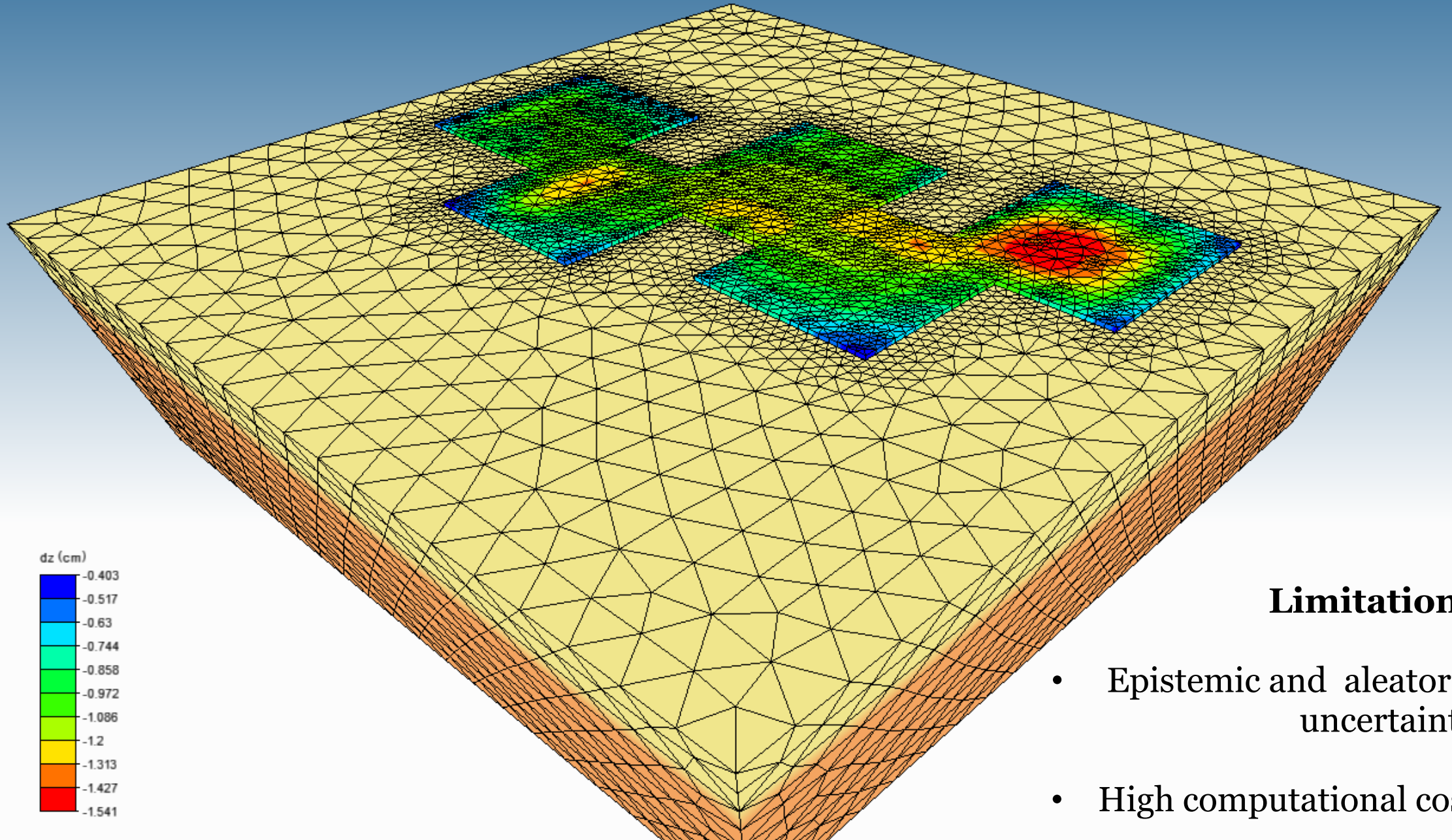
Pilegroup  
efficiency

Complete model C:  
Detailed foundation and building





# Holistic FE model

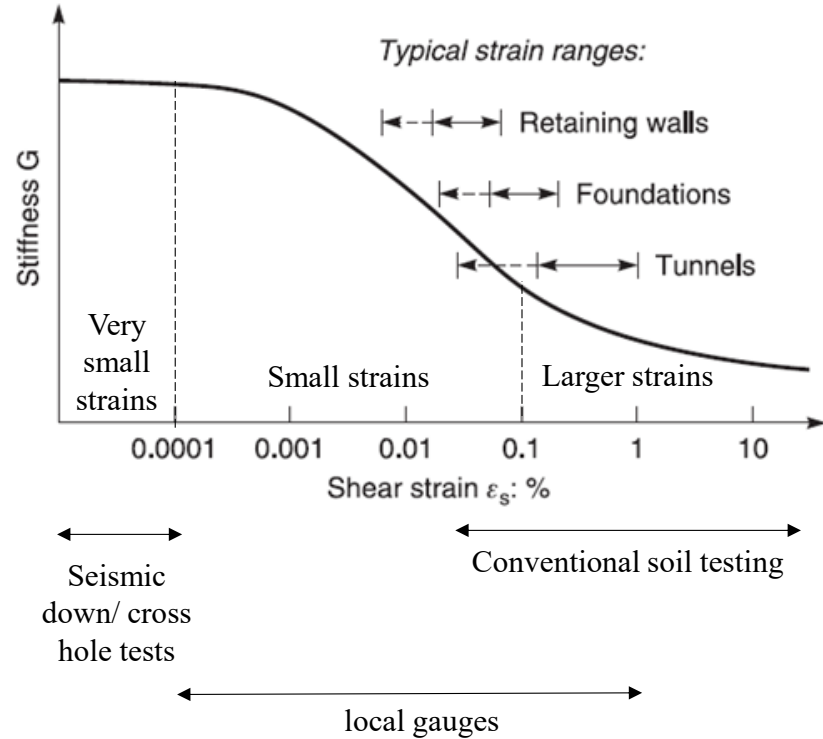


## Limitations

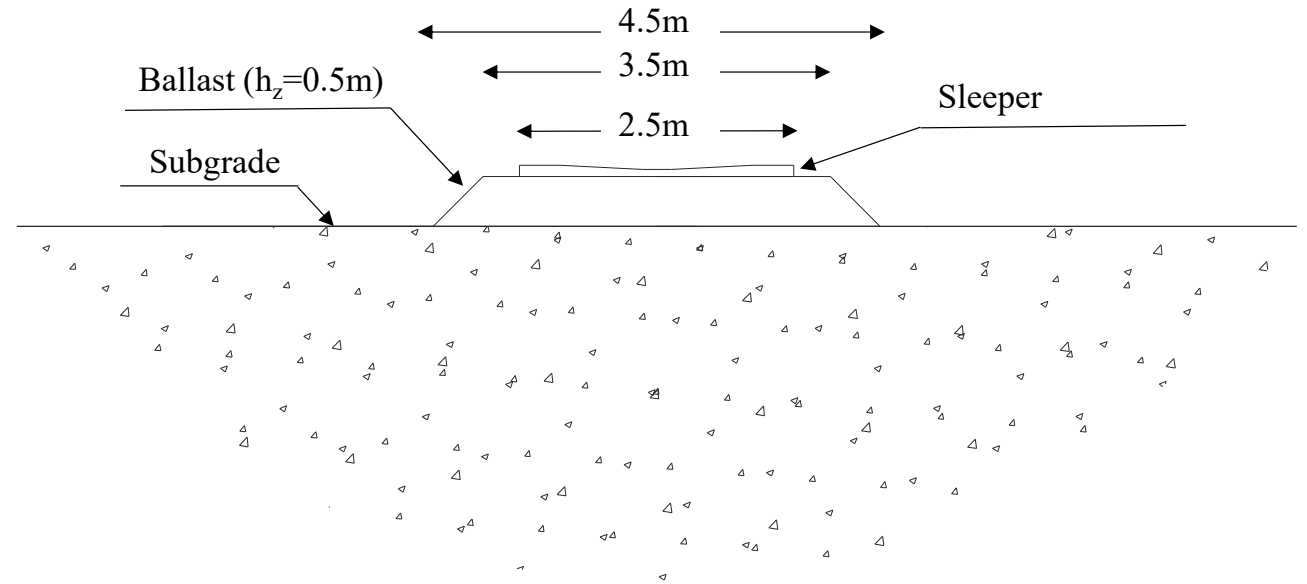
- Epistemic and aleatoric uncertainty
- High computational cost

# Uncertainty treatment

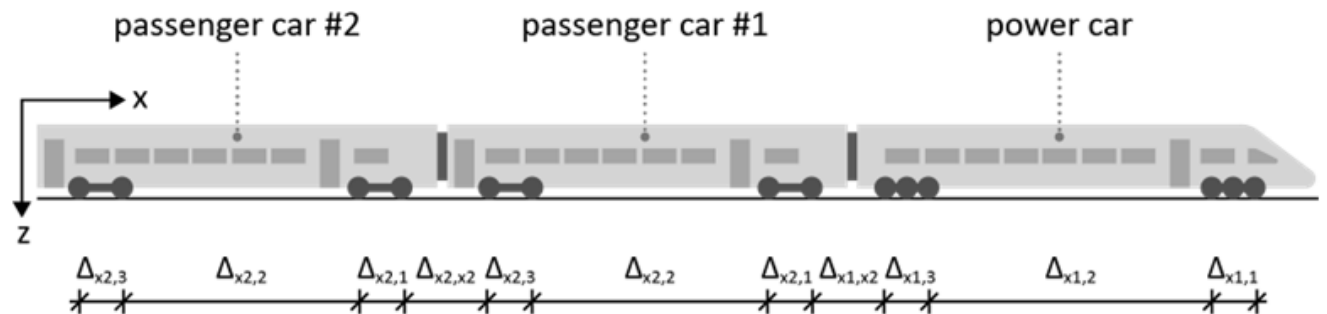
## Soil properties (stiffness and damping)



## Track properties (height of ballast)

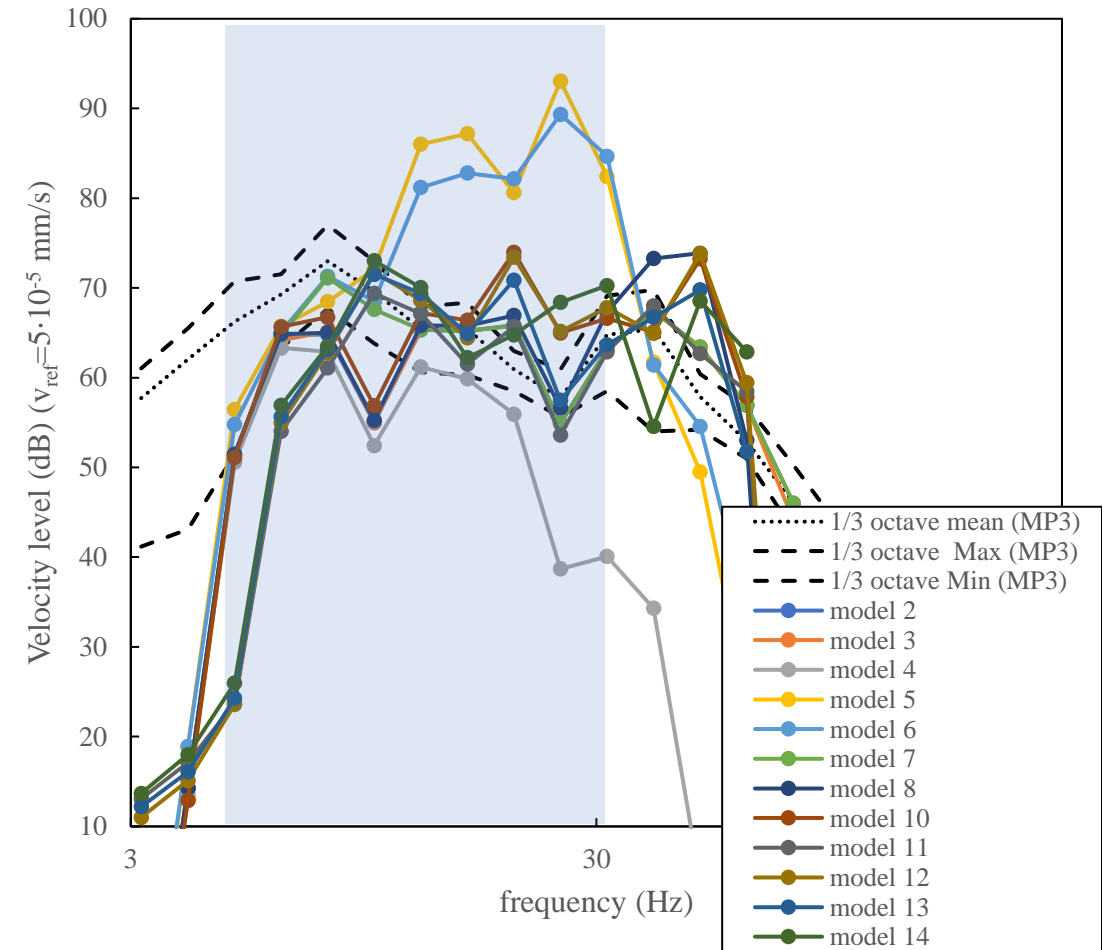


## Train properties (speed)

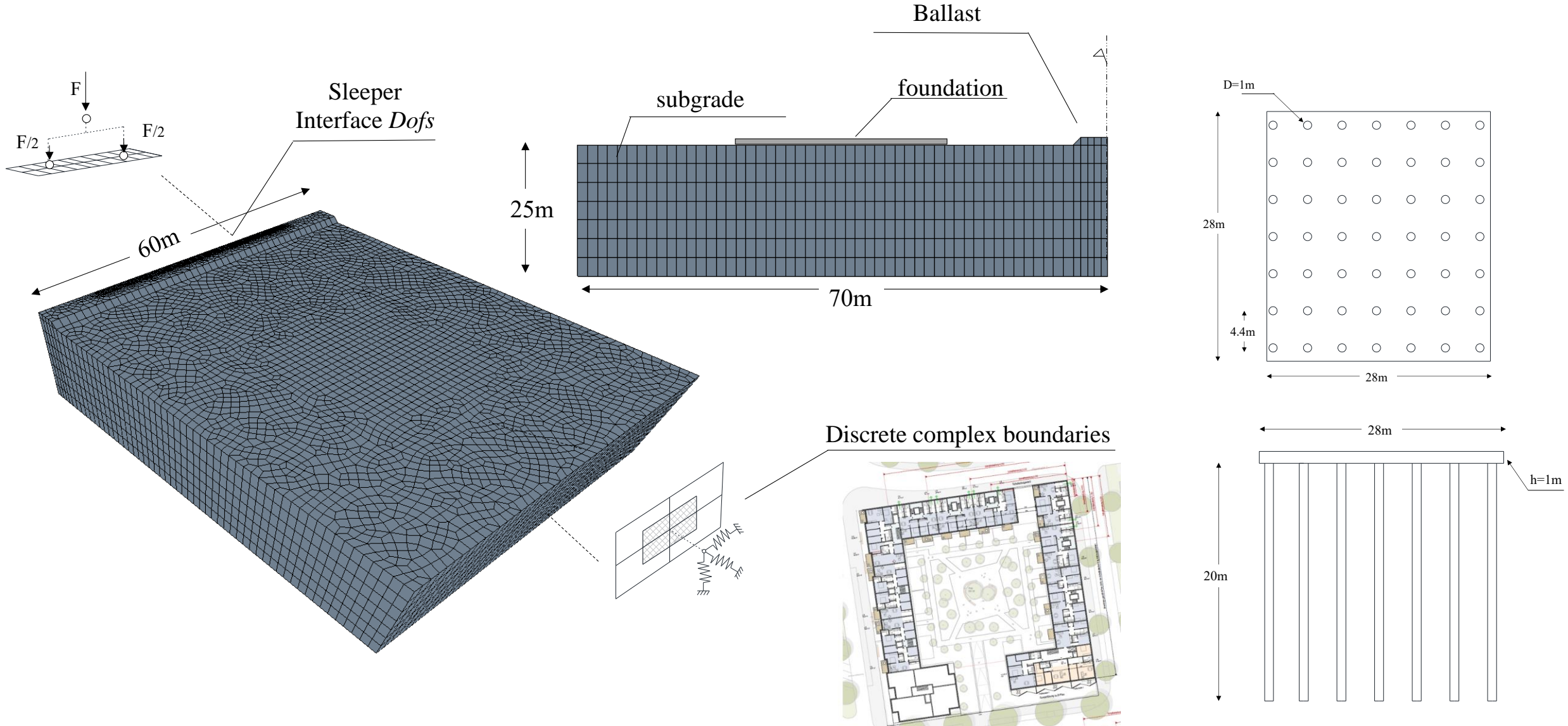


# System Identification

Model id	Dimensions (Lx, Ly)	depth	mes h ver.	mes h hor.	Damping f=10Hz	Half-space Boundaries	Train model	Rail pad stiffness	
model 1	80x20	15.1	2.1m	3m	0%	frequency dependent LP	Locomotive Class 66 Train+ Y25 Bogie Wagons  adjusted X2000 power car + Y25 Bogie Wagons	255MN/m	
model 2	80x40	15.1	2.1m	1.3m		Lysmer			
model 3	80x40	15.1	2.1m	1.3m					
model 4	80x40	15.1	2.1m	1.3m	2% Rayleigh (1Hz,12Hz)	frequency dependent LP			
model 5	80x40	15.1	2.1m	1.3m	0%				50MN/m
model 6	80x40	15.1	1.5m	1.3m					100MN/m
model 7	80x40	15.1	1.5m	1.3m					
model 8	80x40	15.1	2.1m	1.3m					
model 9	80x40	15.1	2.1m	1.3m					
model 10	80x40	15.1	2.1m	1.3m					
model 11	80x40	15.1	2.1m	1.3m					
model 12	80x40	15.1	2.1m	1.3m	255MN/m				
model 13	80x40	15.1	2.1m	1.3m					
model 14	80x40	15.1	1.5m	1.3m					
model 15	140x80	15.1	1.5m	1.3m					

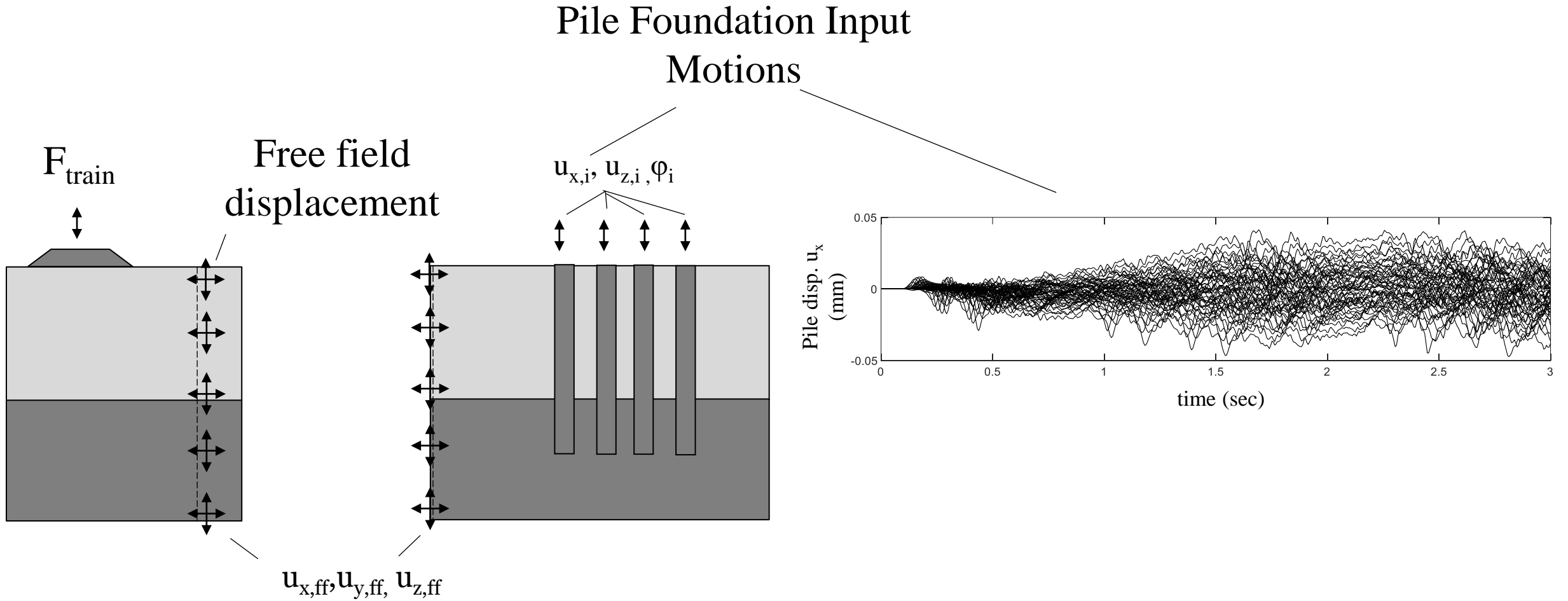


# Pile group configuration

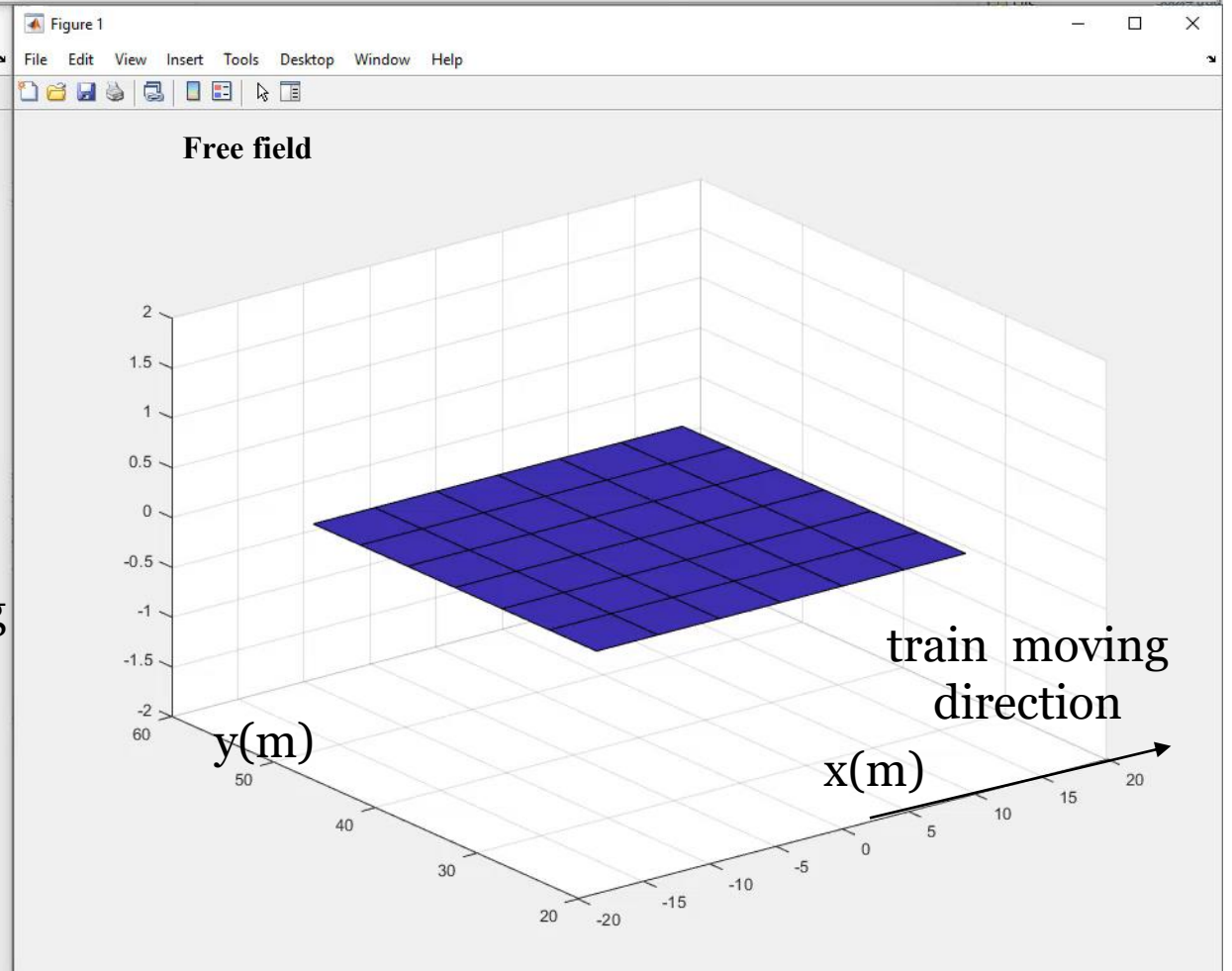
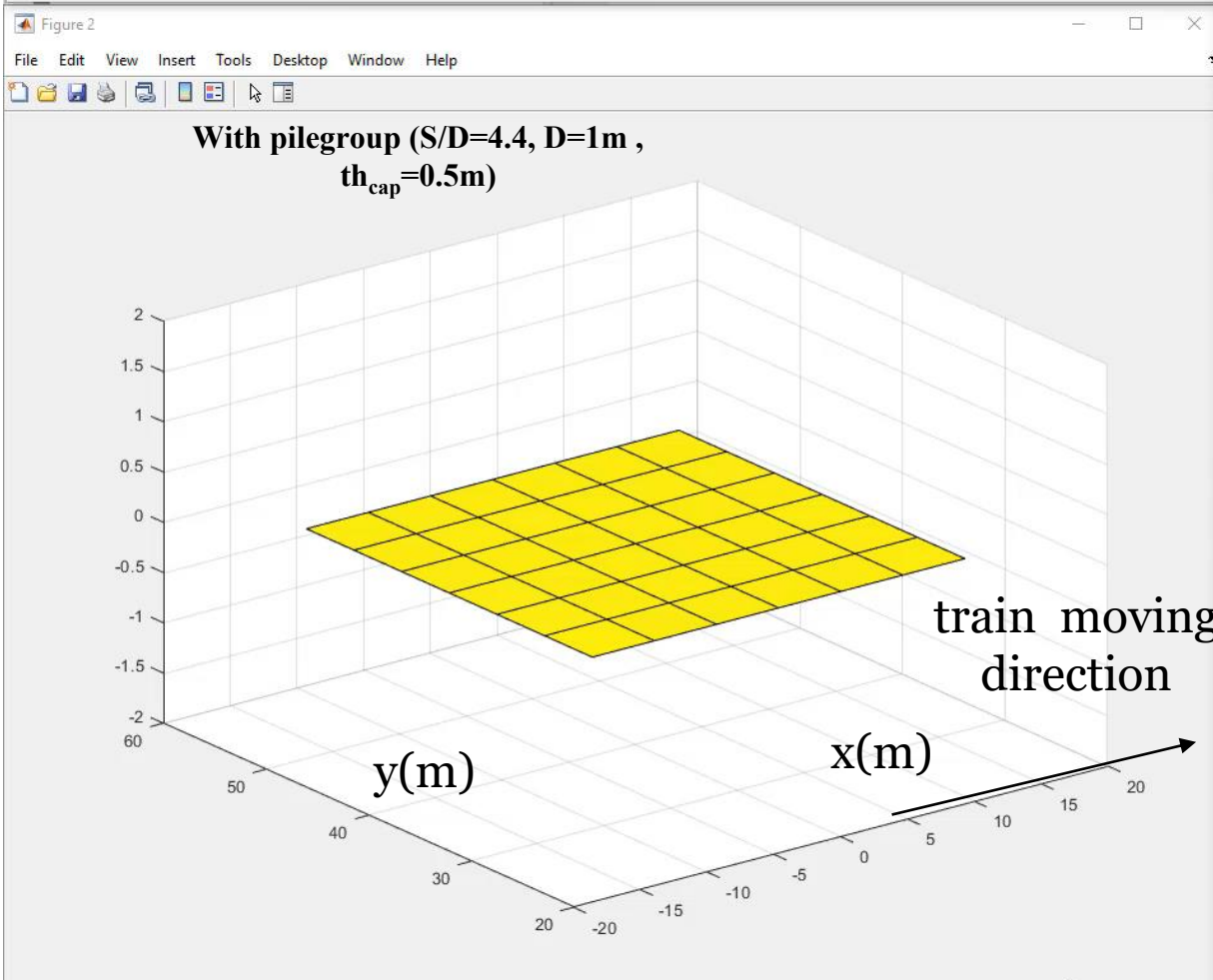
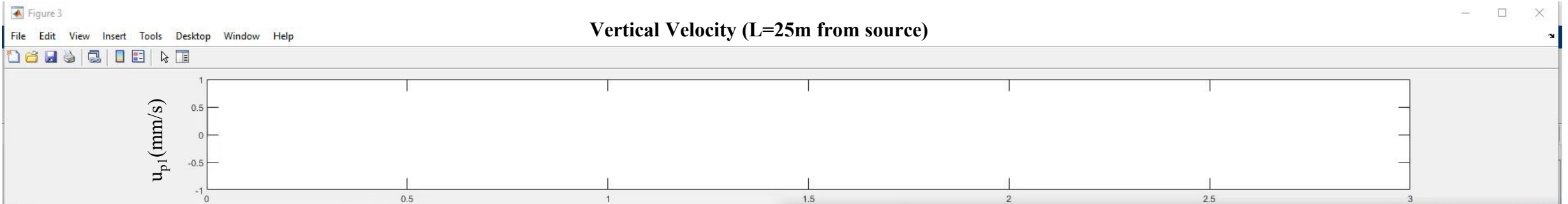




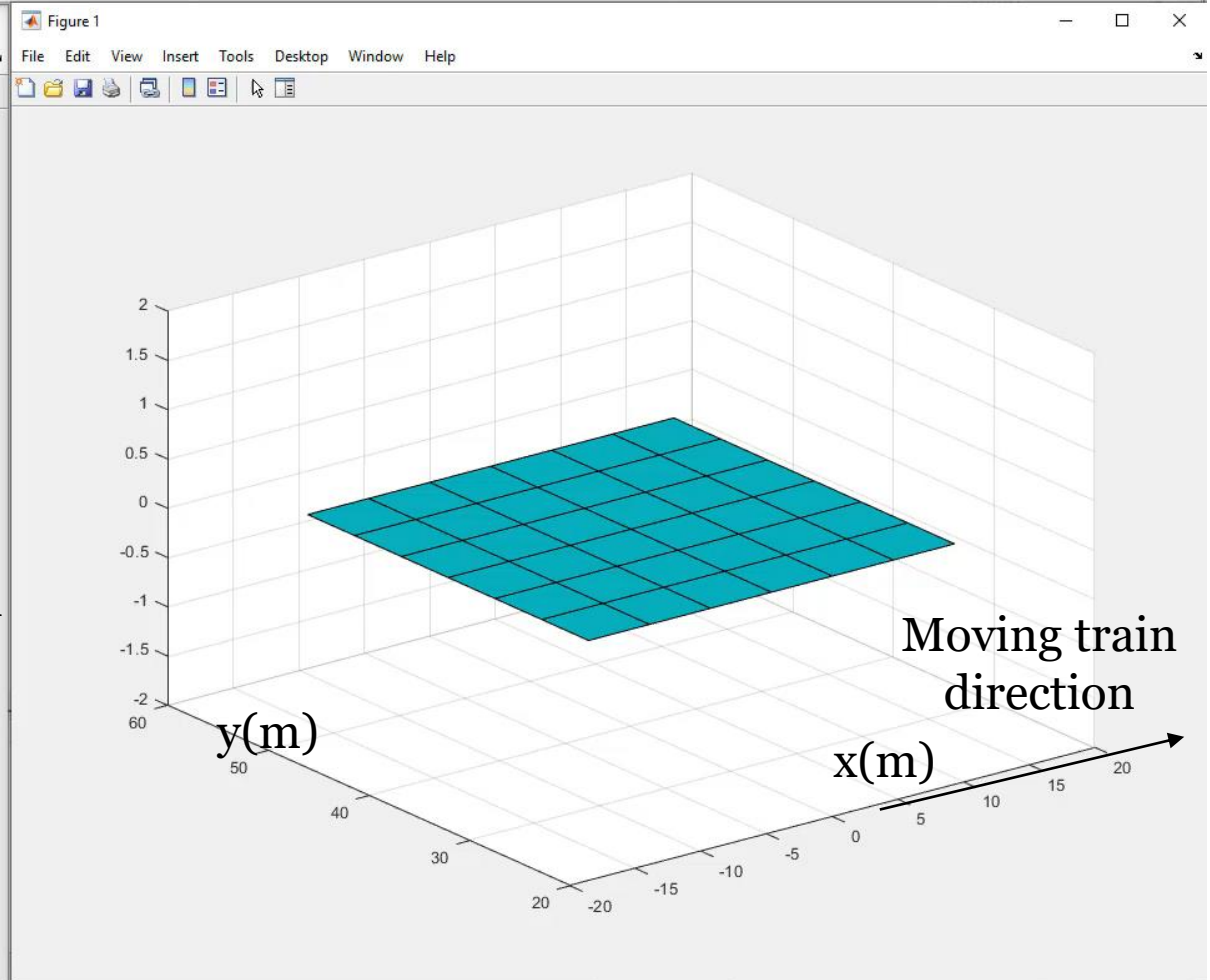
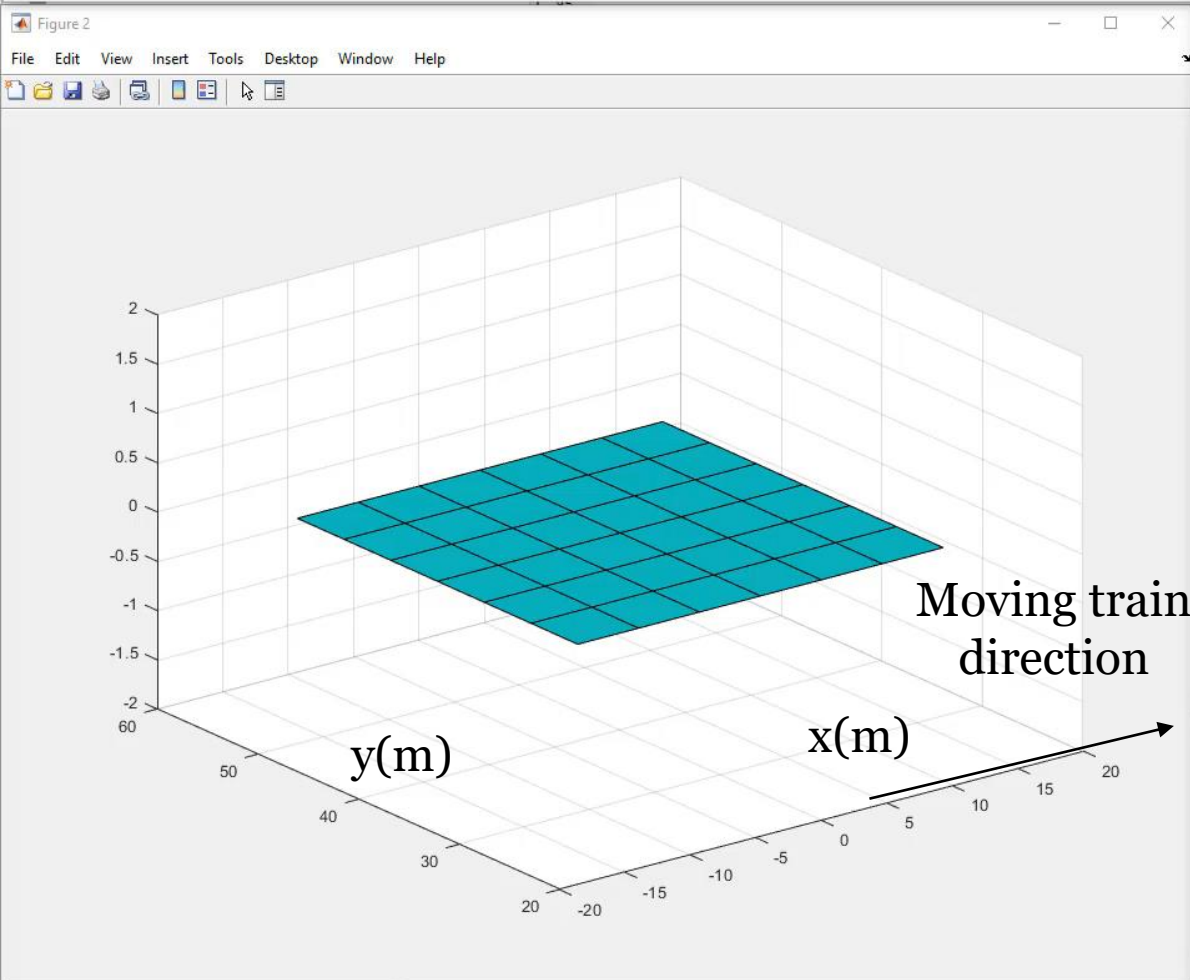
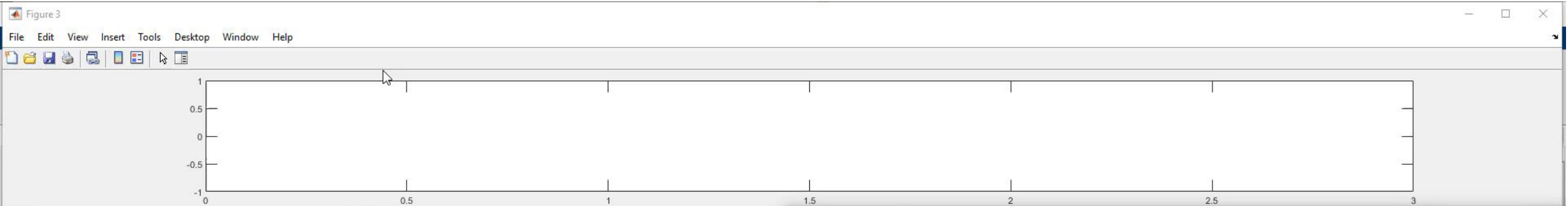
# Kinematic interaction



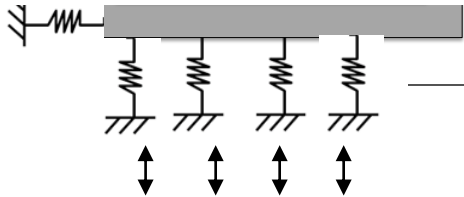
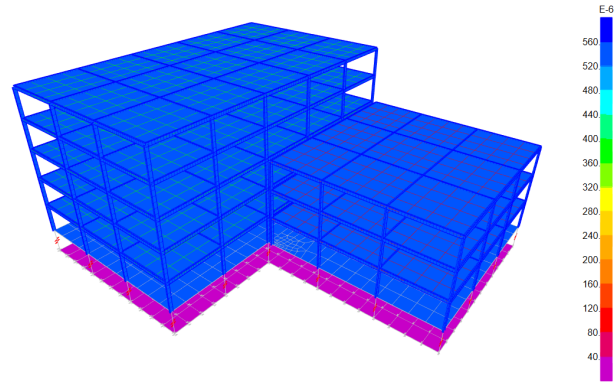
# Vibration mitigation by means of a pile group foundation



# Vibration mitigation by means of a pile group foundation

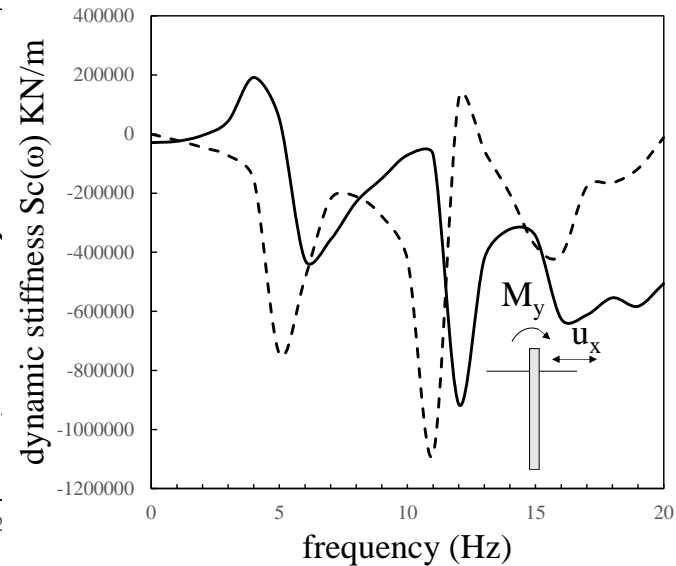
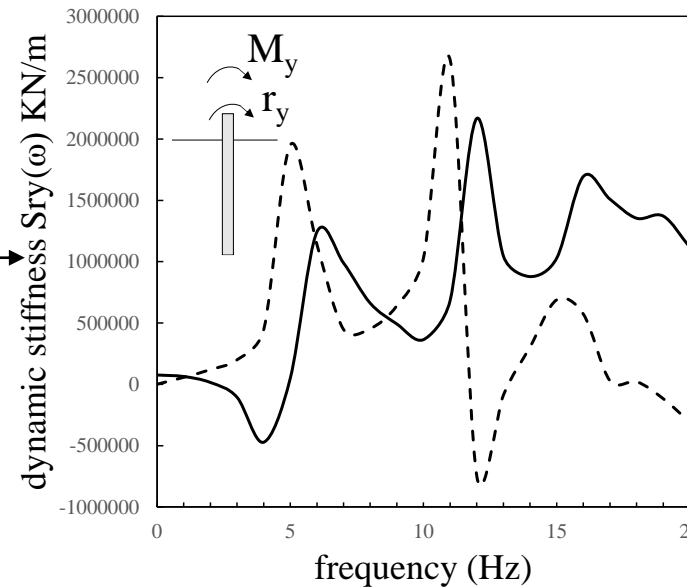
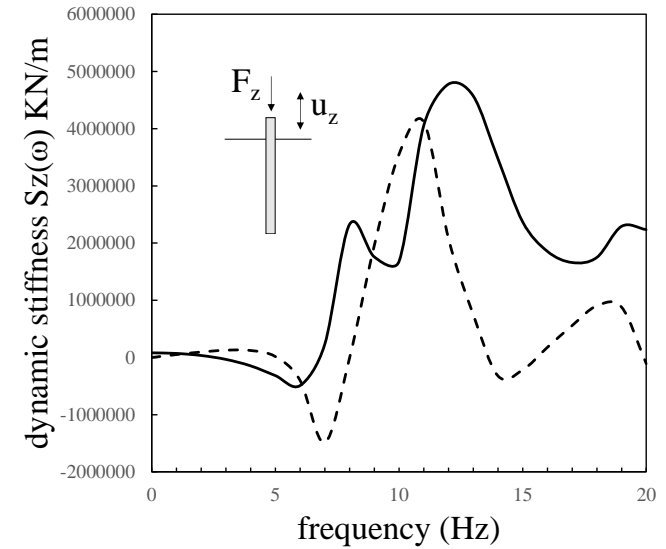
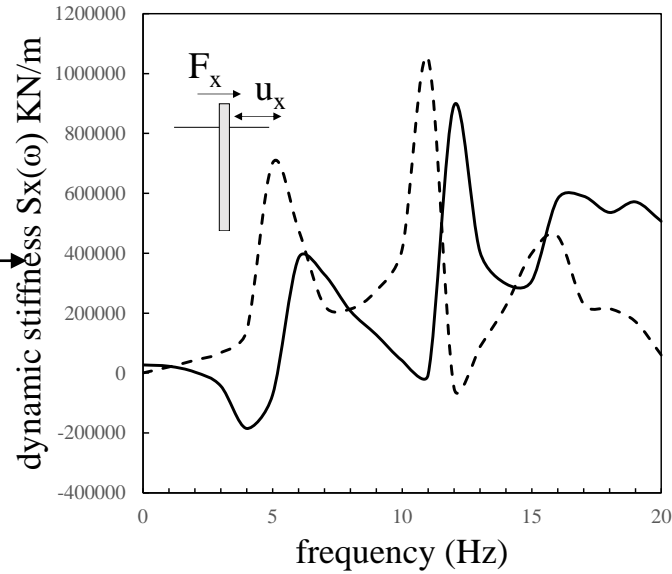


# Inertial interaction: use of frequency-dependent LPMs



Dynamic Stiffness matrix

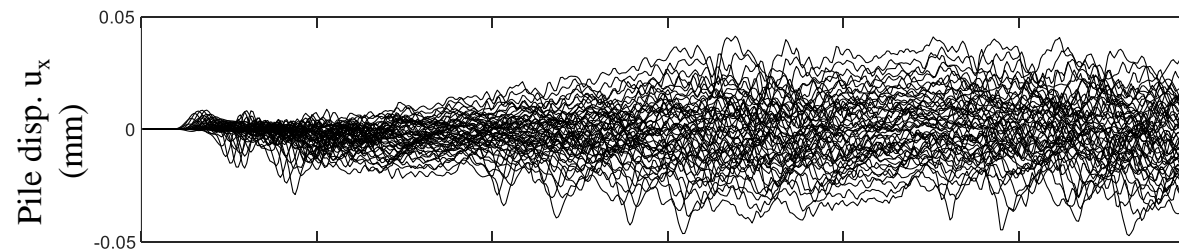
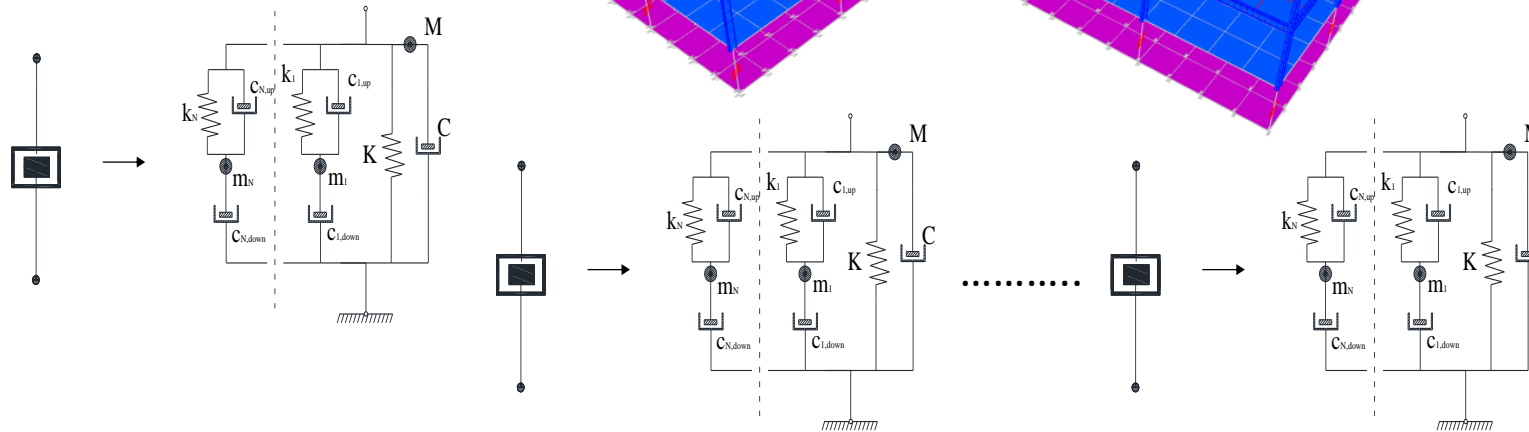
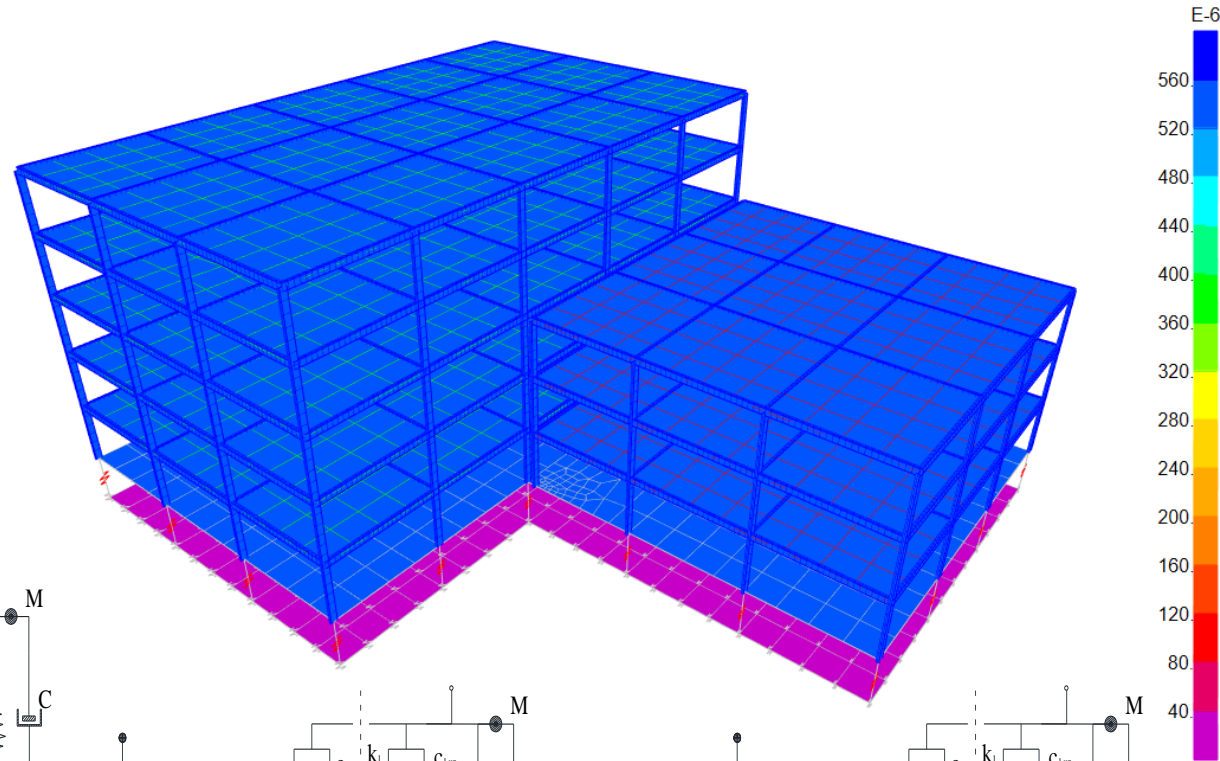
$$\mathbf{S}_{IB}(\omega) = \begin{bmatrix} \mathbf{S}_{11}(\omega) & \cdots & \mathbf{S}_{1N}(\omega) \\ \vdots & \ddots & \vdots \\ \mathbf{S}_{N1}(\omega) & \cdots & \mathbf{S}_{NN}(\omega) \end{bmatrix}$$





# Inertial interaction: use of frequency-dependent LPMs

Structural response in the time domain using frequency-dependent LPMs and spatially variable F.I.M.



Kinematic and inertial interaction sub-structuring just like in earthquake engineering application + coupled terms (for the surface foundation case) and surface wave induced F.I.M. at an angle  $\theta$