Seismic Displacement as an Acceleration Couple

A beam analog in earthquake engineering

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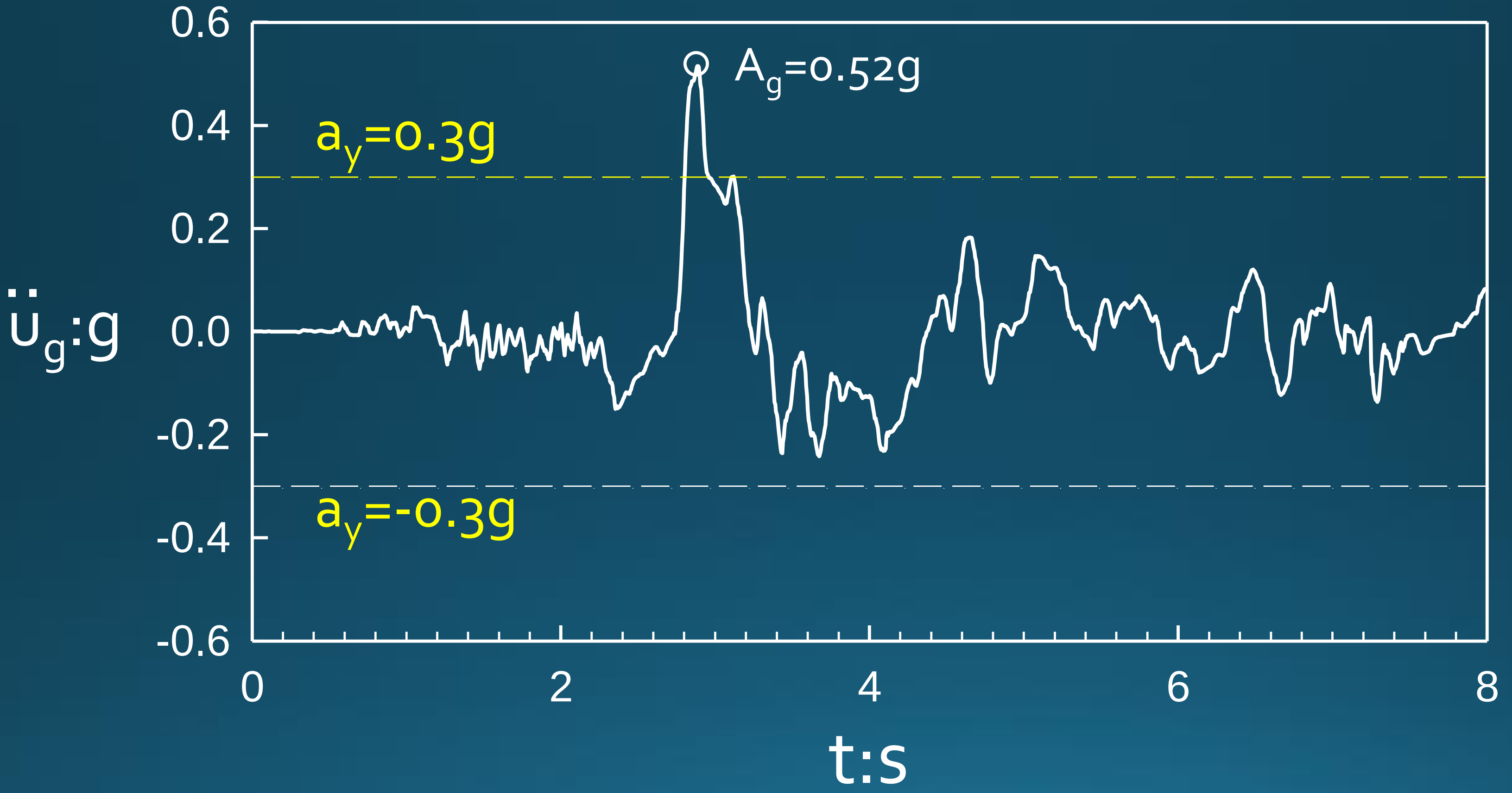
List of Contents

- ✓Problem definition & review of available models (Newmark sliding block)
- ✓Development of a new methodology for determining seismic / sliding displacement using a beam analog
- ✓Extension of the methodology for the inverse problem of earthquake motion
- ✓Application Examples
- ✓Conclusions

Ground acceleration recording



Rigid Block Sliding

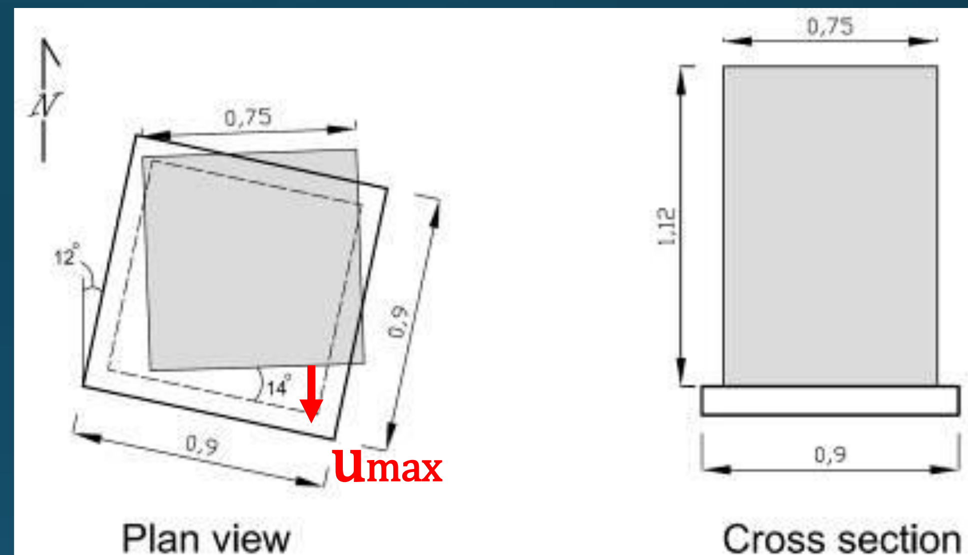
L'Aquila cemetery 2009

Lefkada island 2003

Geotech effects

Back-calculation of earthquake motion

using field observations of residual sliding



A_g = ?

V_g = ?

T_d = ?

L'Aquila 2009

Difficulties-uncertainties in back-calculating Ag, Vg .....

- ✓Unknown earthquake time history (waveform, duration)
- ✓Excitation along multiple axes
- ✓Different response modes (e.g. simple sliding, sliding with rocking, sliding with torsion)
- ✓Compliance of structural system
- ✓Interface properties

Need for an approximate-rational engineering approach…

Available formulae

$$u_{max} = \chi_1 \frac{V_g^2}{A_g} \eta^{-b_1}$$

Newmark (1965)
Richards & Elms (1979)

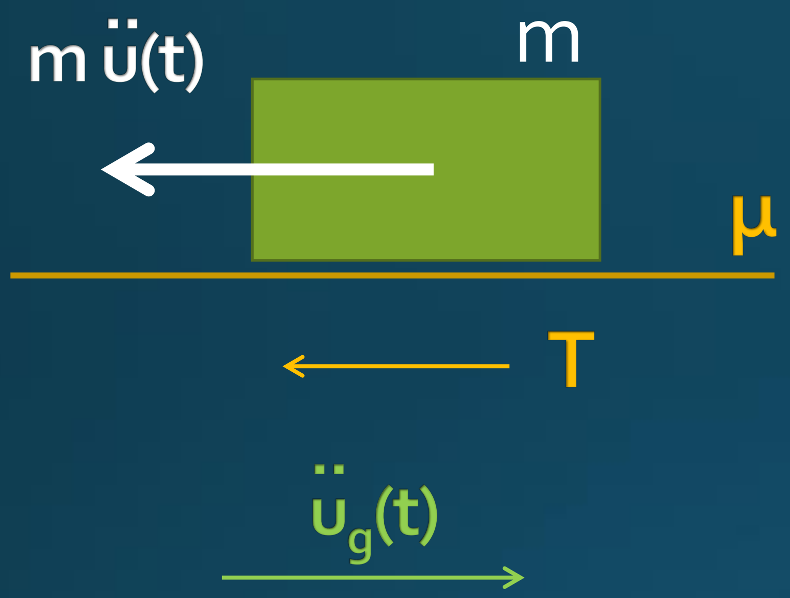
$$u_{max} = \chi_2 \frac{V_g^2}{A_g} e^{-b_2 \eta}$$

Whitman & Liao (1985)

$$u_{max} = A_g T_g^2 N_{eq} e^{\sum_{m=1,2,3} c_m \eta^m}$$

Yegian *et al*. (1991)

Rigid block sliding



$$m\ddot{u} + \mathrm{T} - m\ddot{u}_g = 0$$

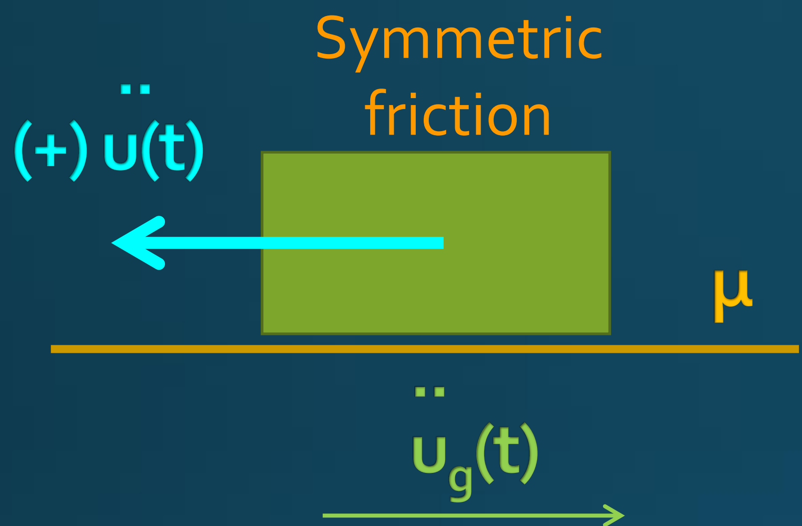
ή

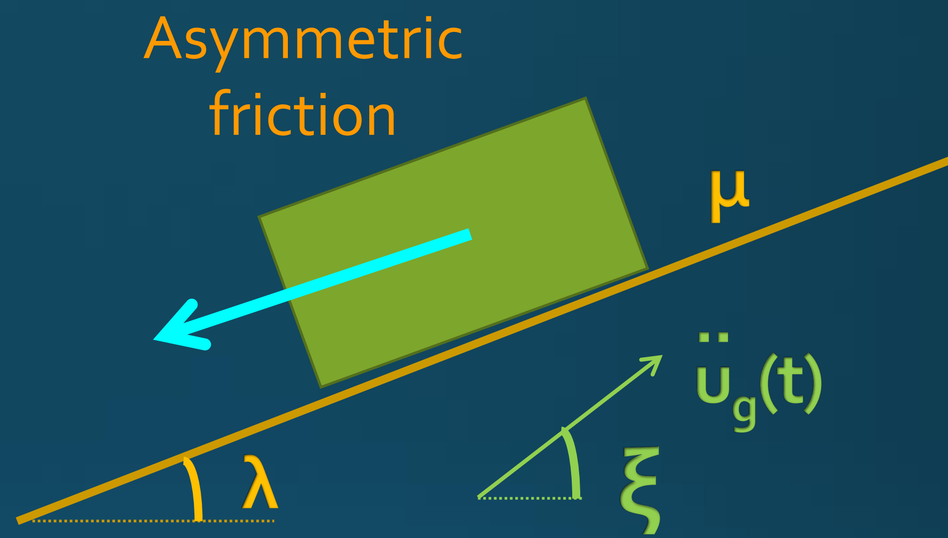
$$+m\ddot{u}_g - \mathrm{T} = m\ddot{u}$$

Stick: $\ddot{u}_g(t) < \mu m g$: $T = m\ddot{u}_g(t)\, sgn(\dot{u}_g)$

Slip: $\ddot{u}_g(t) > \mu\, m g = T_{sl}$: $T = \mu m g\, sgn(\dot{u})$

Sliding modes





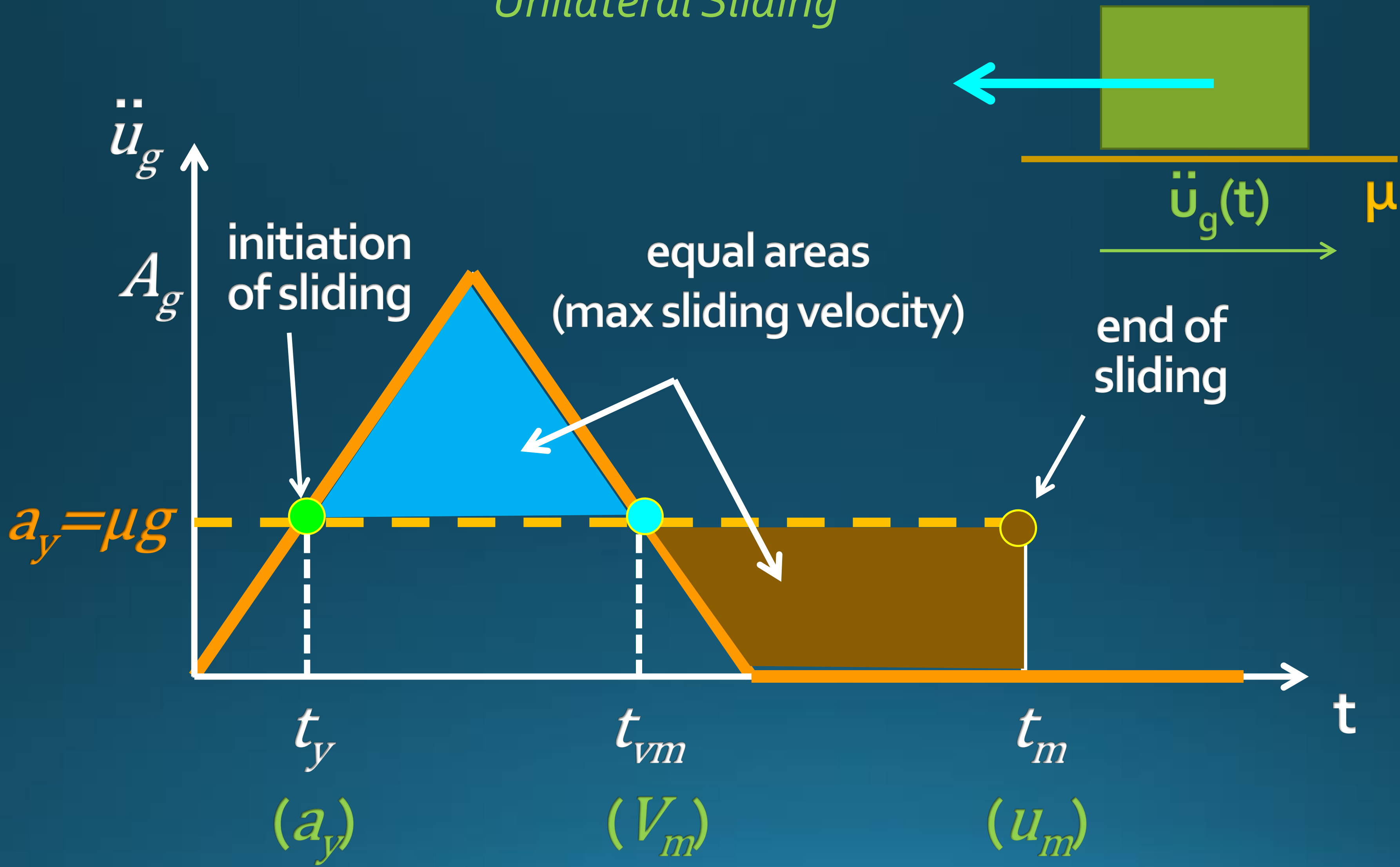
Unilateral / Bilateral excitation

$$\mu_{1,2} = \mu \frac{\cos\lambda}{\cos(\lambda-\xi)}\left[\frac{1\mp\mu^{-1}\tan\lambda}{1\pm\mu\tan(\lambda-\xi)}\right]$$

Nate Newmark
(1910-1981)

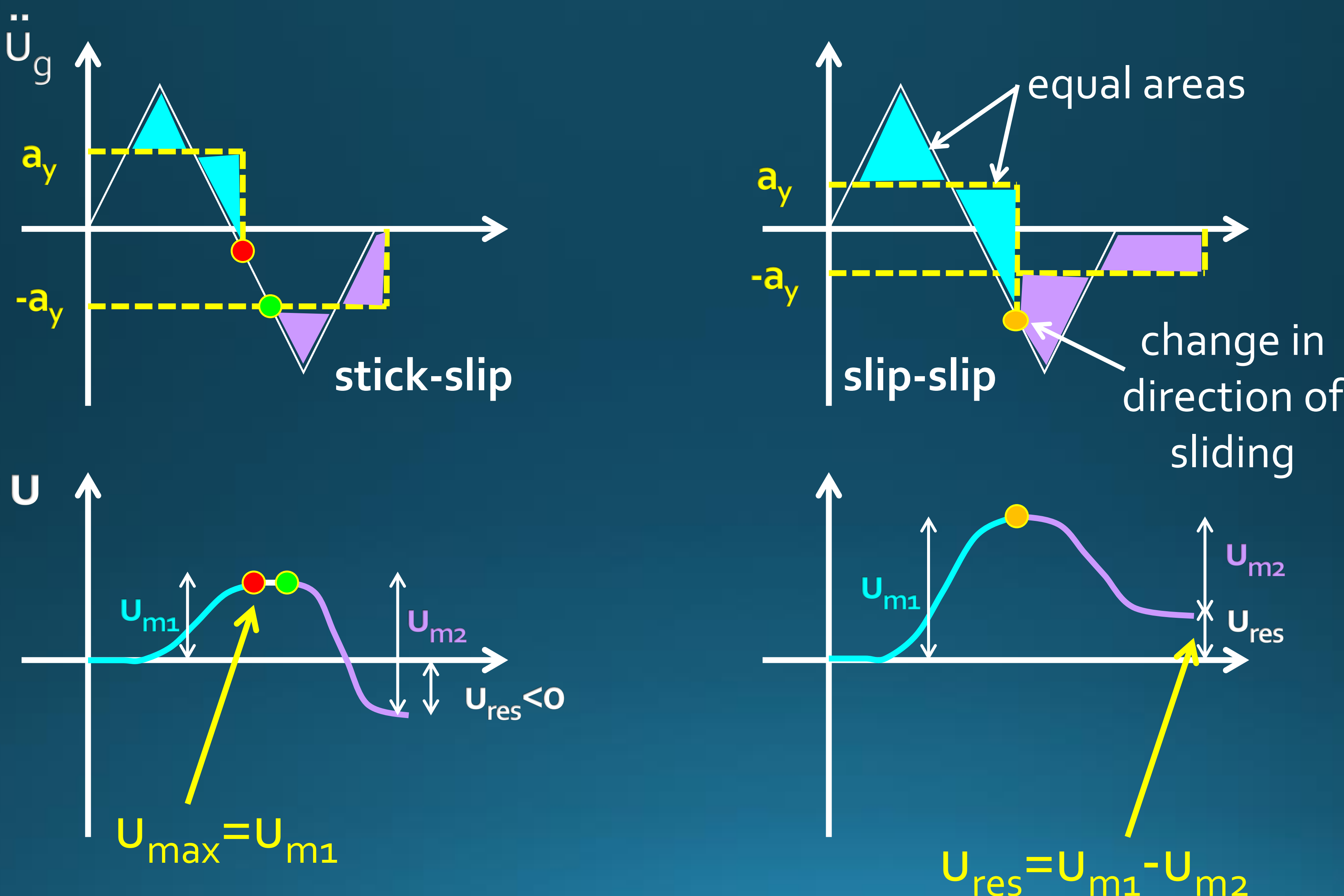
Graphical solution of Newton's equation

Unilateral Sliding

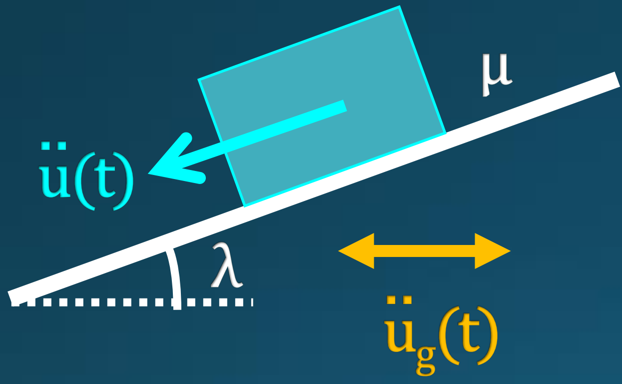


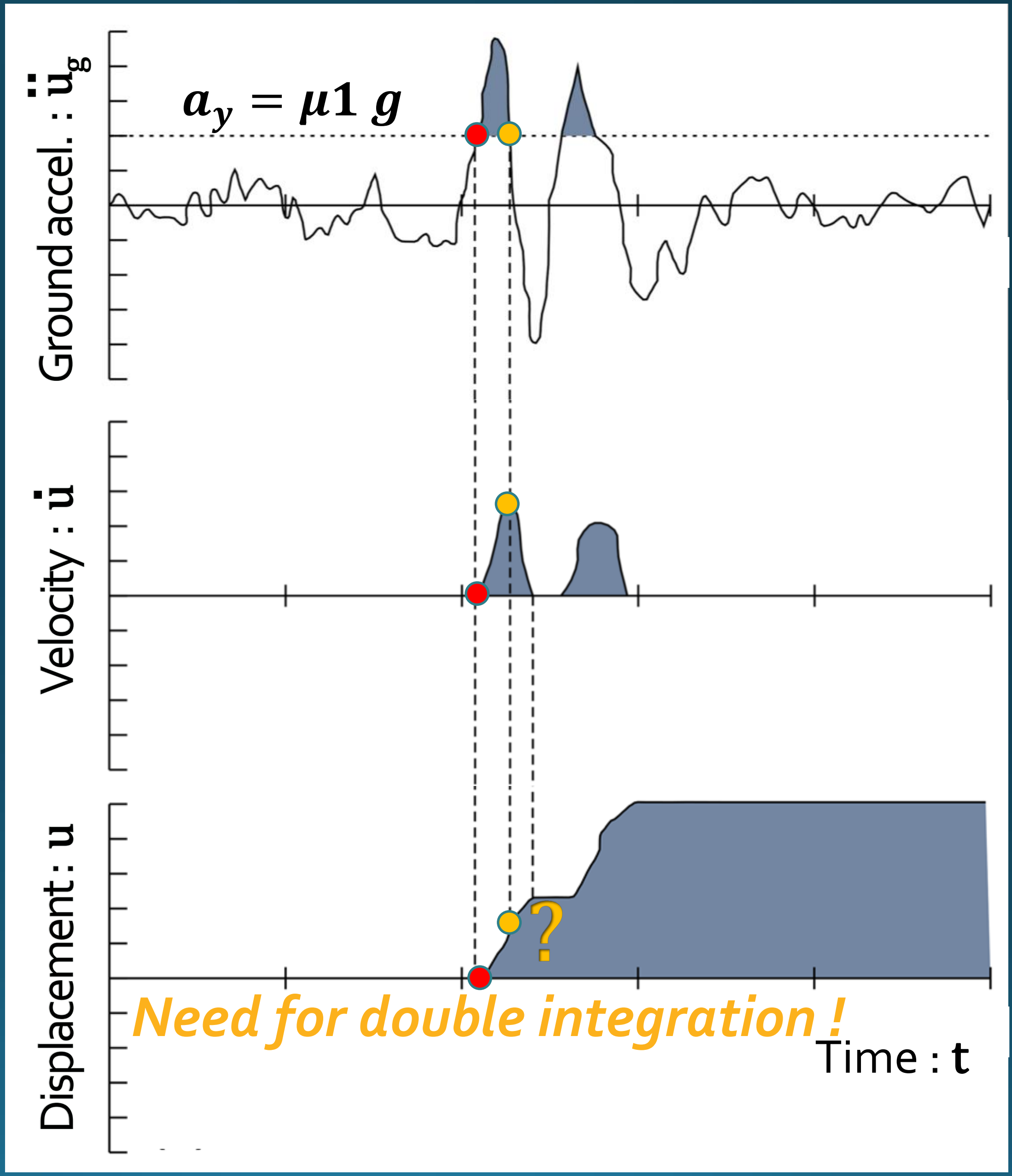
Newmark (1965)

Bilateral sliding



Newmark sliding block (1965)





Newmark's method (1965)

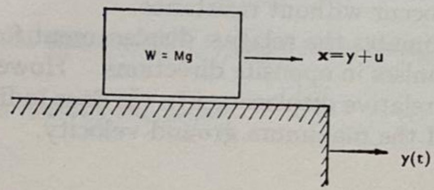


Fig. 16. Rigid block on a moving support

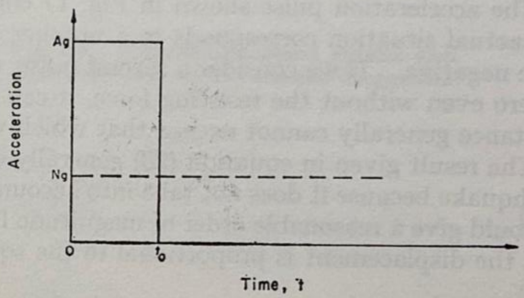


Fig. 17. Rectangular block acceleration pulse

In Fig. 17, the accelerating forces acting on the mass M are shown. The acceleration considered is a single pulse of magnitude Ag, lasting for a time interval t_0. It would be possible to consider a sinusoidal pulse, but this complicates the expressions unnecessarily. The resisting acceleration, Ng, is shown by the dashed line in Fig. 17. The accelerating force lasts only for the short time interval indicated, but the decelerating force lasts until the direction of motion changes.

In Fig. 18, the velocities are shown as a function of time for both the accelerating force and the resisting force. The maximum velocity for the accelerating force has a magnitude V given by the expression

$$V = Agt_0.$$

After the time t_0 is reached, the velocity due to the accelerating force remains constant. The velocity due to the resisting acceleration has the magnitude Ngt. At a time t_m, the two

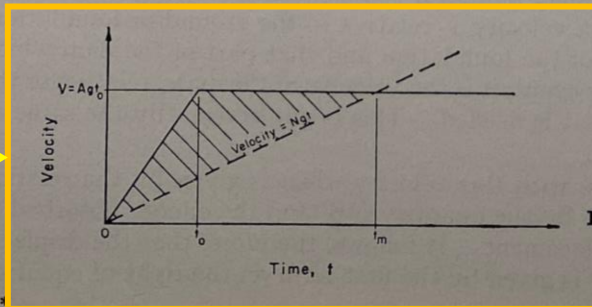


Fig. 18. Velocity response to rectangular block acceleration

Newmark's method (1965)

velocities are equal and the net velocity becomes zero, or the body comes to rest relative to the ground. The formulation for t_m is obtained by equating the velocity V to the quantity Ngt, giving as a result the expression

$$t_m = \frac{V}{Ng} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The maximum displacement of the mass relative to the ground u_m is obtained by computing the shaded triangular area in Fig. 18. The calculation is made as follows:

$$u_m = \tfrac{1}{2}Vt_m - \tfrac{1}{2}Vt_0$$

or

$$u_m = \frac{1}{2}\frac{V^2}{Ng} - \frac{1}{2}\frac{V^2}{Ag}$$

whence

$$u_m = \frac{V^2}{2gN}\left(1 - \frac{N}{A}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The acceleration pulse shown in Fig. 17 corresponds to an infinite ground displacement. The actual situation corresponds to a number of pulses in random order, some positive and some negative. If we consider a second pulse, of a negative magnitude, to bring the velocity to zero even without the resisting force, it can be shown that the net displacement with the resistance generally cannot exceed that which would occur without resistance.

The result given in equation (23) generally overestimates the relative displacement for an earthquake because it does not take into account the pulses in opposite directions. However, it should give a reasonable order of magnitude for the relative displacement. It does indicate that the displacement is proportional to the square of the maximum ground velocity.

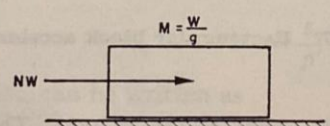
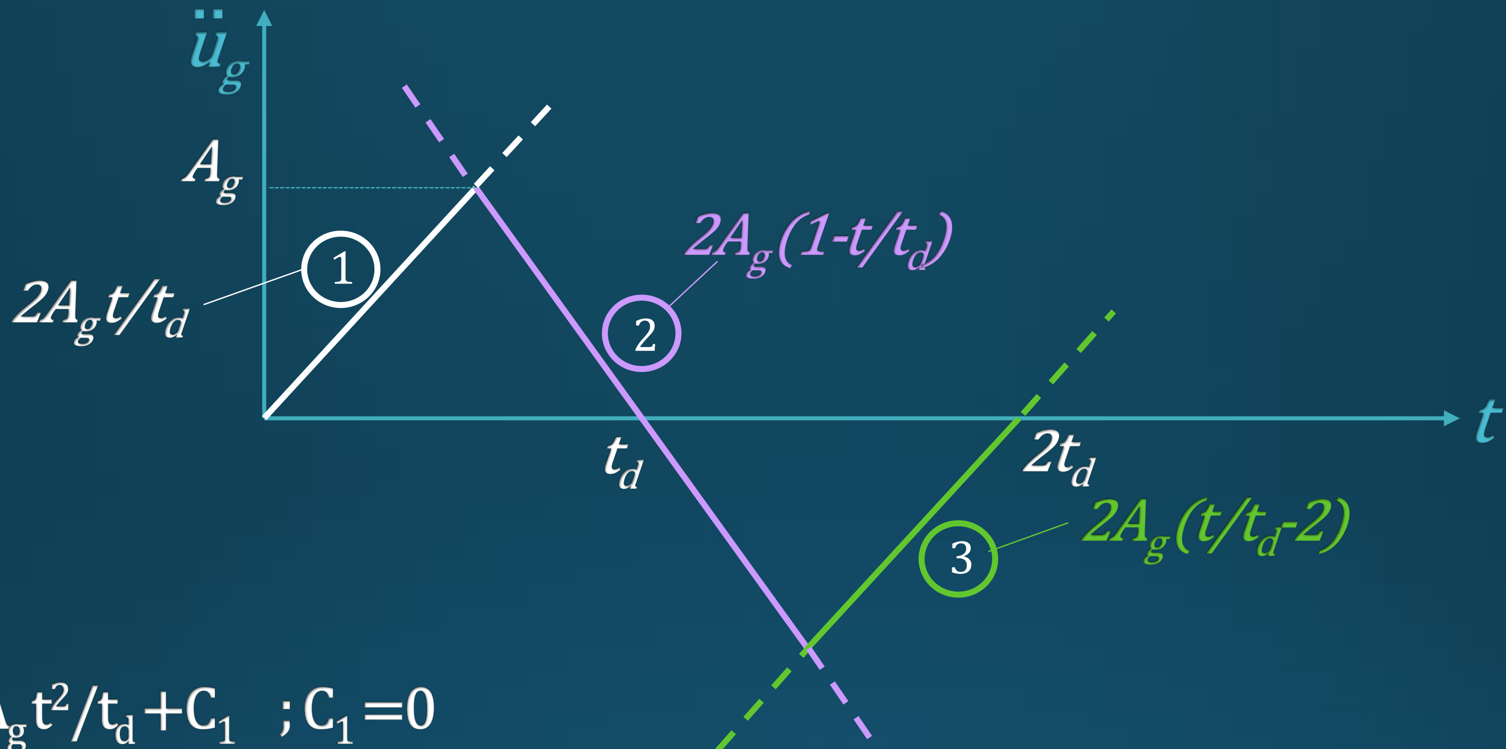


Fig. 19. Mass sliding under constant force

The result derived above is applicable also for a group of pulses when the resistance in either direction of possible motion is the same. For a situation in which the body has a resistance to motion greater in one direction than in another, one must take into account the cumulative effect of the displacements. A simple example where this must be considered would be found if Fig. 16 were rotated clockwise, as in Fig. 19, so that the body has a tendency to slide downhill. In this situation, ground motions in the direction of the downward slope tend to move the mass downhill, but ground motions in the upward direction along the slope leave the mass without relative additional motion except where these are extremely large in magnitude. One may consider that this case is applicable to the dam.

Calculation of ground displacement



$V_{g1}(t)=A_g t^2/t_d+C_1 \quad ; C_1=0$

$V_{g2}(t)=2A_g t_d(t/t_d-t^2/2t_d^2)+C_2 \quad ; C_2=-A_g t_d/2$

$V_{g3}(t)=2A_g t_d(-2t/t_d+t^2/2t_d^2)+C_3 \quad ; C_3=4A_g t_d$

$D_{g1}(t)=A_g t^3/3t_d+C_1 t+C_4 \quad ; C_4=0$

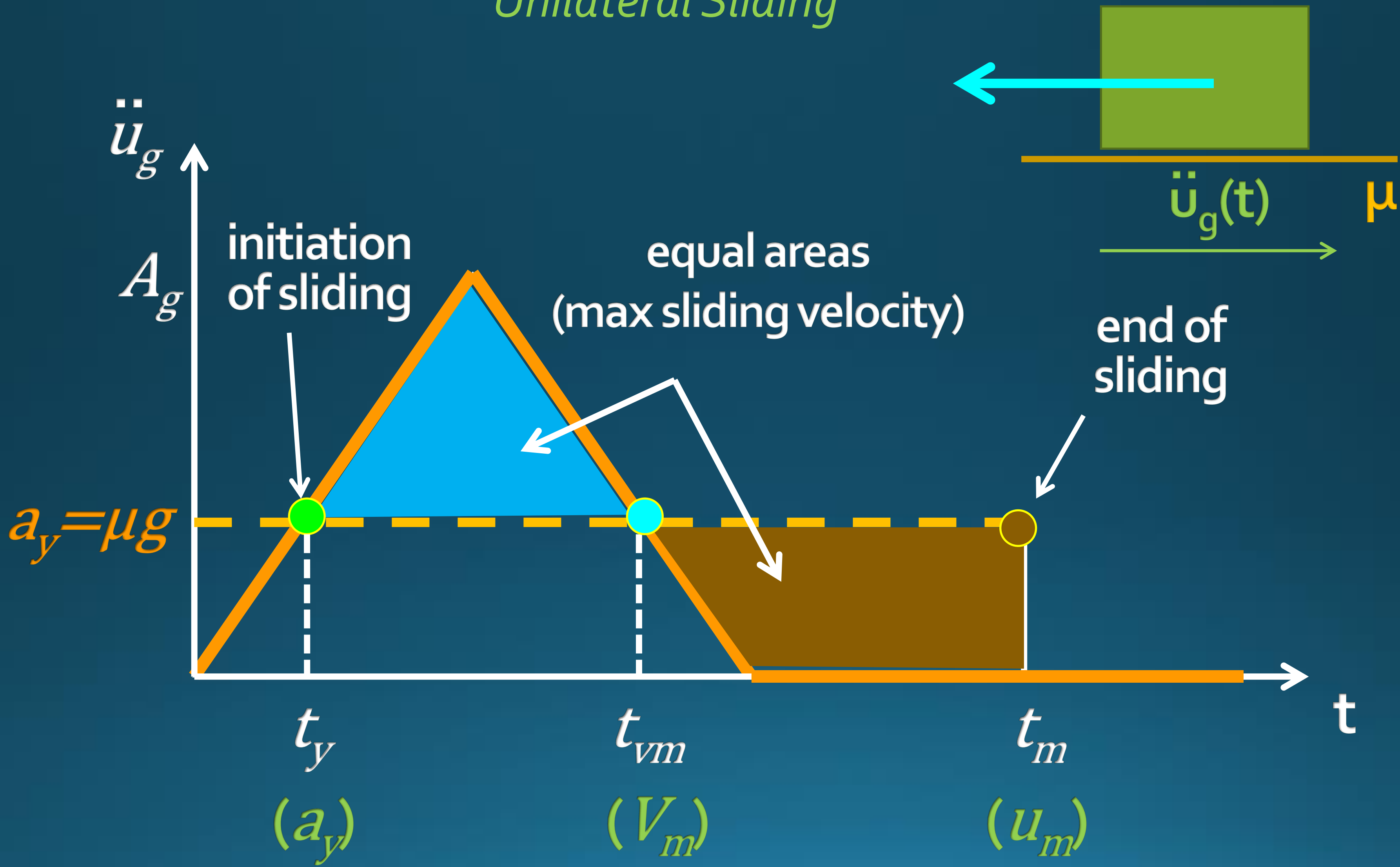
$D_{g2}(t)=A_g(t^2-t^3/3t_d)+C_2 t+C_5 \quad ; C_5=A_g t_d^2/12$

$D_{g3}(t)=A_g(t^3/3t_d-2t^2)+C_3 t+C_6 \quad ; C_6=-13A_g t_d^2/6$

$$D_g = A_g t_d^2/2$$

Graphical solution of Newton's equation

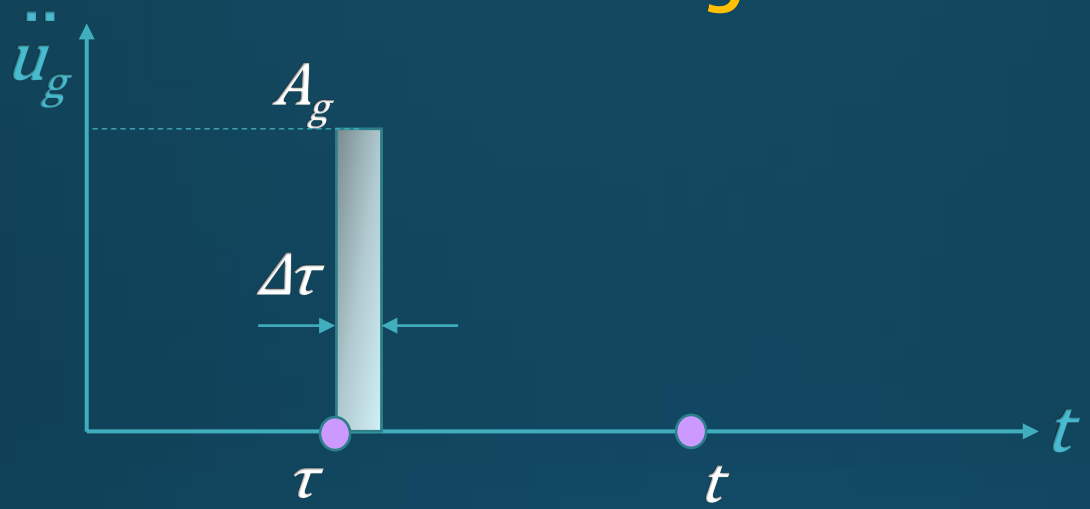
Unilateral Sliding



Newmark (1965)

Solution via single integration

Duhamel's integral



$$u(t) = \frac{1}{\omega_D}\int_0^t \ddot{u}_g(\tau)\, e^{-\xi\omega(t-\tau)}\, sin[\omega_D(t-\tau)]\, d\tau$$

Limit case

$$\left.\begin{matrix}\xi \to 0 \\ \omega_D \to 0\end{matrix}\right\} u(t) = u_g(t)$$

(Ground displacement)

$\ddot{u}_g(\tau) \to A_g$

$\Delta\tau \ll t$

Solution via single integration

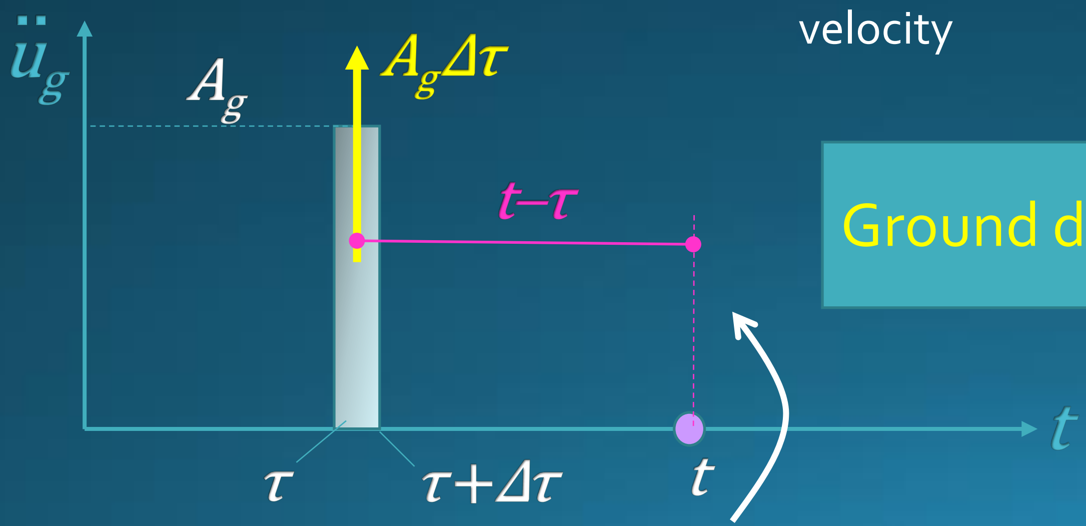
Duhamel's integral

$$u(t) = \int_0^t \ddot{u}_g(\tau)\, e^{-\xi\omega(t-\tau)}\, \frac{\sin[\omega_D(t-\tau)]}{\omega_D}\, d\tau$$

$u(t) \rightarrow u_g(t)$; $\ddot{u}_g(\tau) \rightarrow A_g$; $e^{-\xi\omega(t-\tau)} \rightarrow 1$; $d\tau \rightarrow \Delta\tau$

$$\frac{\sin[\omega_D(t-\tau)]}{\omega_D} = (t-\tau)\,\frac{\sin[\omega_D(t-\tau)]}{\omega_D(t-\tau)}, \quad \frac{\sin[\omega_D(t-\tau)]}{\omega_D(t-\tau)} \rightarrow 1$$

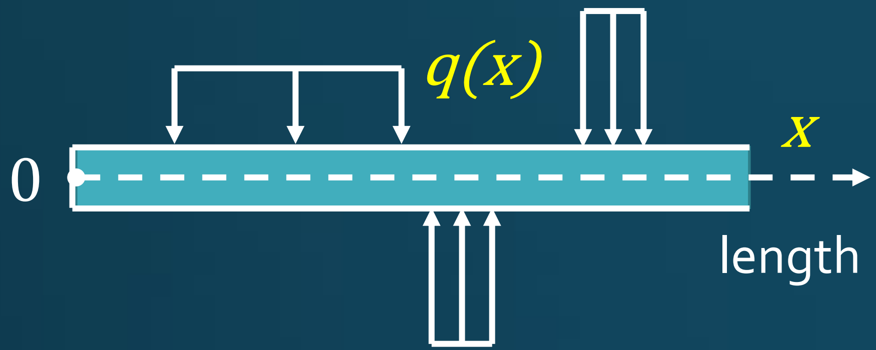
$$u_g(t) = \underbrace{(A_g\,\Delta\tau)}_{\text{pulse velocity}} \times \underbrace{(t-\tau)}_{\text{moment arm}}$$



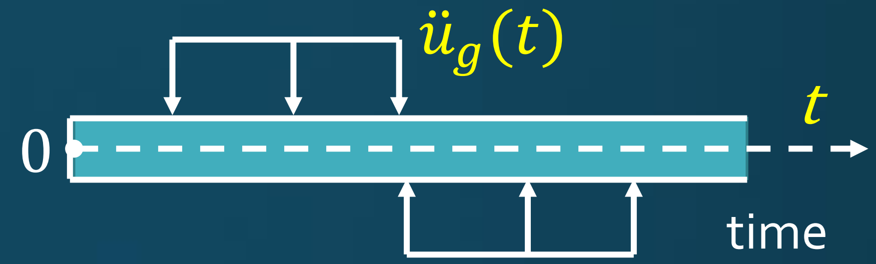
Ground displacement = moment !!!

Beam analog

Loaded beam



Acceleration time history



distributed load $\quad q \longleftrightarrow \ddot{u}_g \quad$ *ground acceleration*

$\int dx$ $\qquad\qquad$ $\int dt$

shear force $\quad Q \longleftrightarrow \dot{u}_g \quad$ *ground velocity*

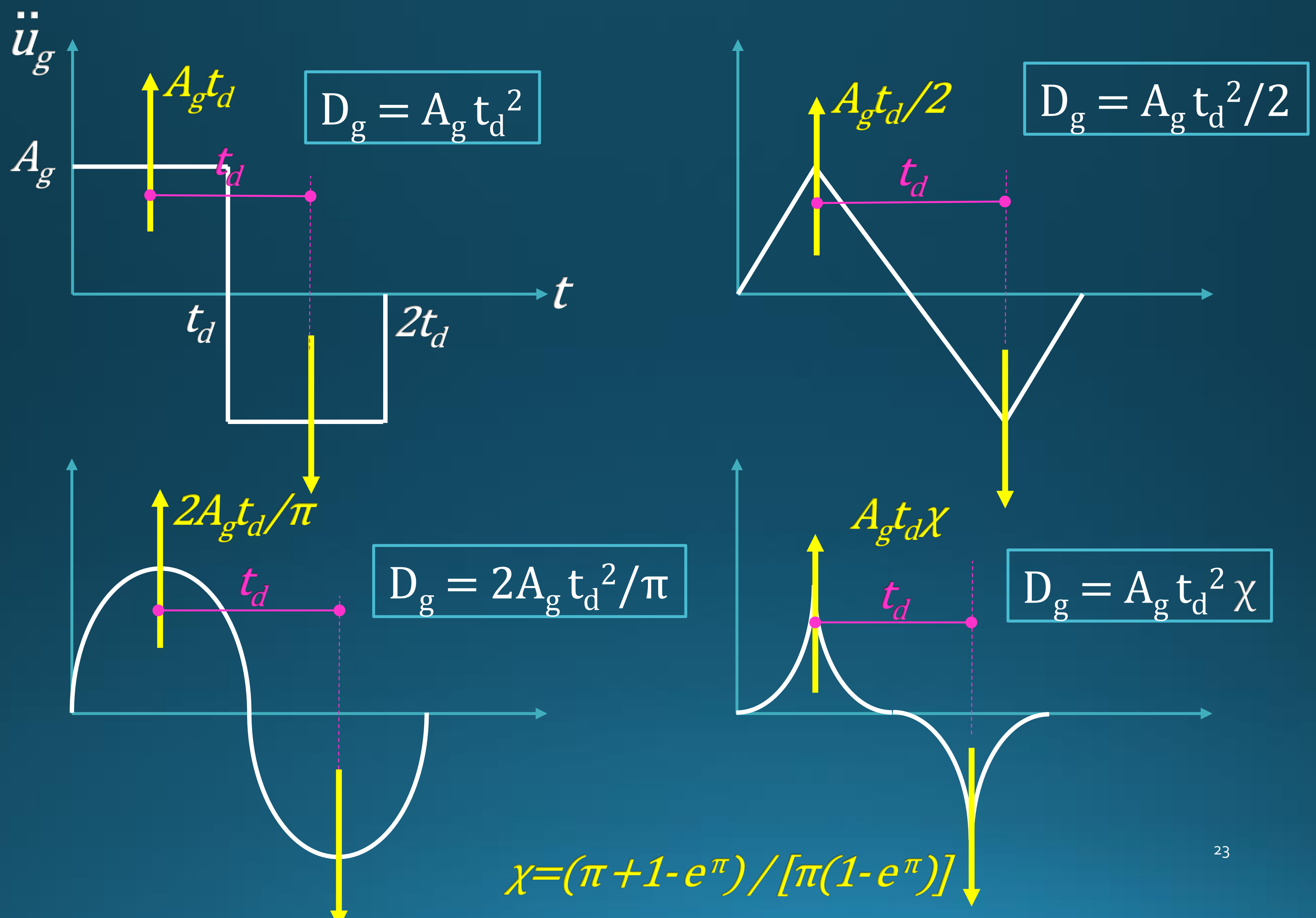
$\int dx$ $\qquad\qquad$ $\int dt$

bending moment $\quad M \longleftrightarrow u_g \quad$ *ground displacement*

Beam analog

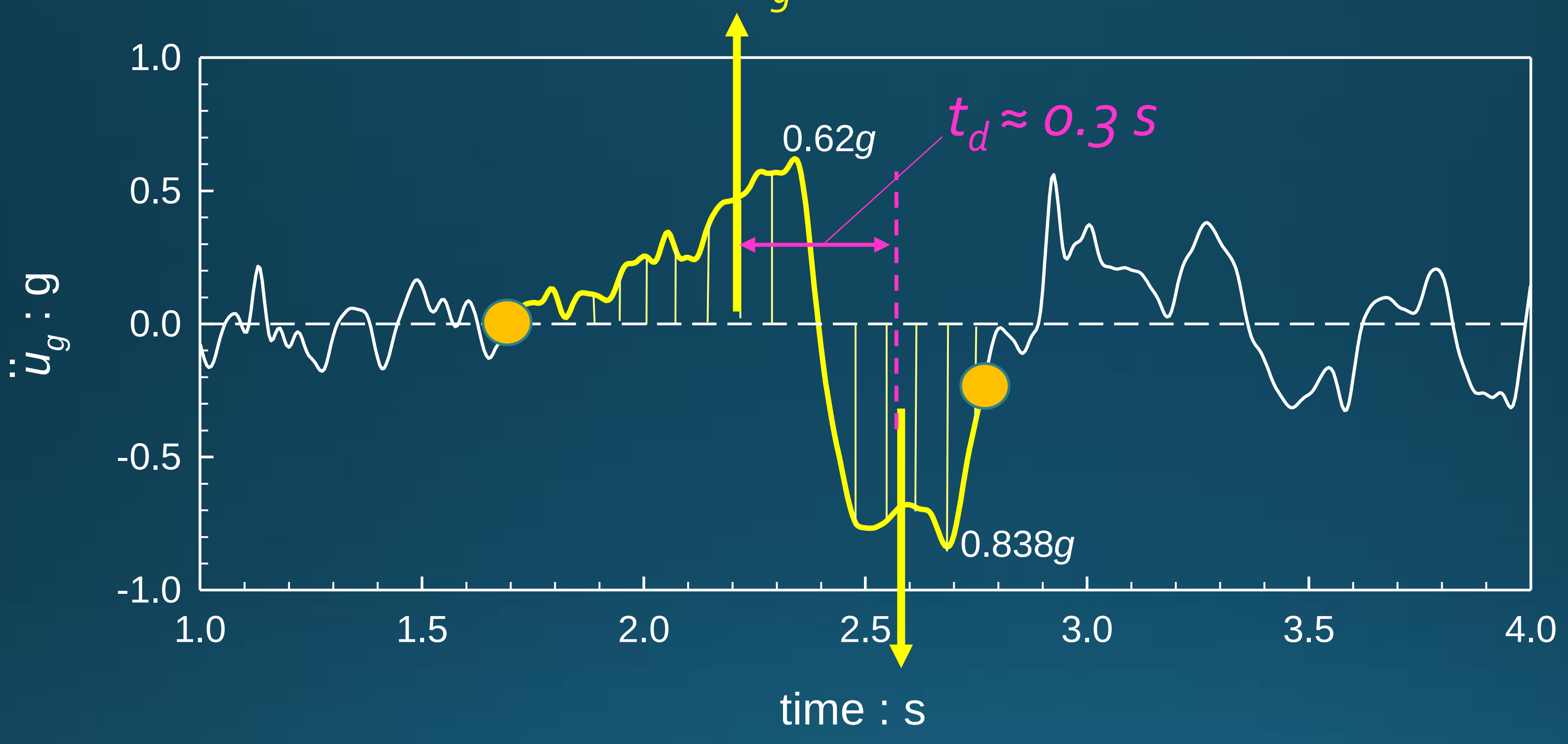
Loaded Beam			Acceleration time history
distributed load	q	$\ddot{u}_g$	*ground acceleration*
$\int dx$			$\int dt$
shear force	Q	$\dot{u}_g$	*ground velocity*
$\int dx$			$\int dt$
bending moment	M	u_g	*ground displacement*
~~$\int dx$~~			
~~*rotation*~~	~~θ~~		
~~$\int dx$~~			
~~*deflection*~~	~~y~~		

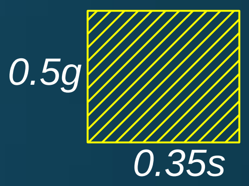
Peak ground displacement for simple pulses



Applications to actual earthquake regordings

$V_g \approx 170$ cm/s



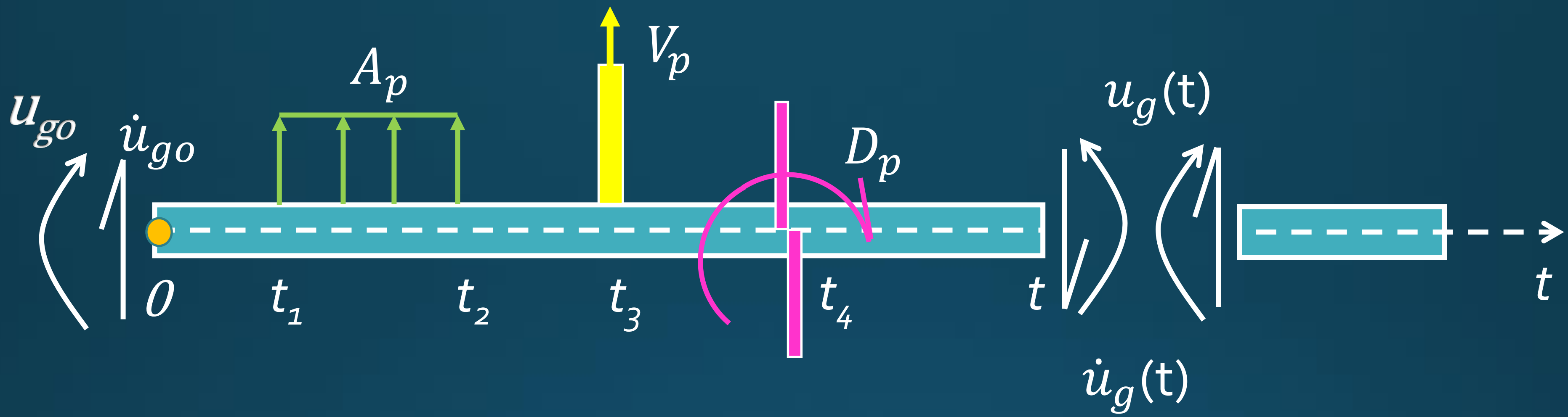


$D_g = V_g\, t_d \approx 170$ cm/s x 0.3 s ≈ 51 cm

Rinaldi 1994

Summary of displacement calculation

based on beam analog



$$\dot{u}_g(t) = \dot{u}_g(0) + A_p\,(t_2 - t_1) + V_p$$

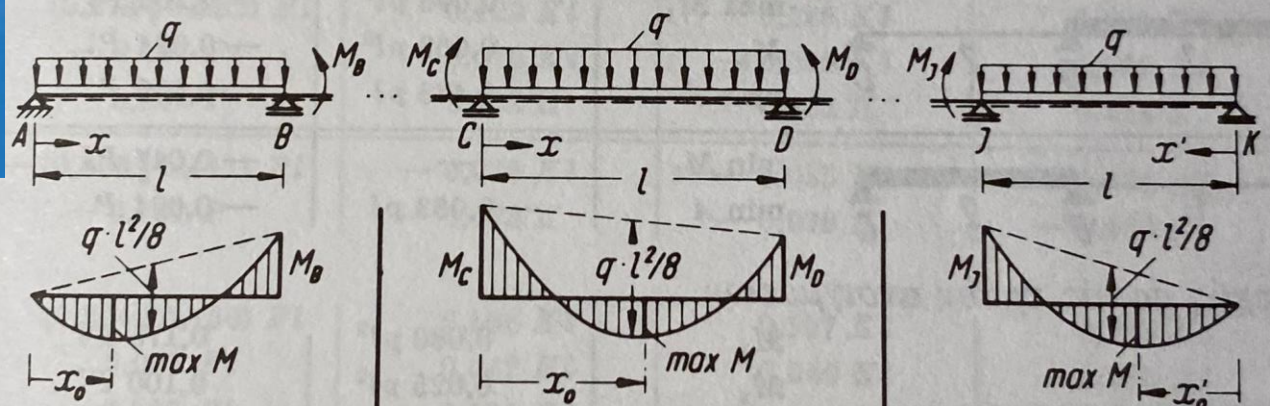
$$u_g(t) = \boxed{u_g(0) + \dot{u}_g(0)\,t} + A_p(t_2 - t_1)\left[t - \frac{(t_2 + t_1)}{2}\right] +$$

$$+\,V_p\,(t - t_3) + D_p$$

influence of
initial conditions

Available solutions in manuals

Πίνακας 10.2. Τέμνουσες δυνάμεις και max M σε συνεχείς δοκούς με φόρτιση κατ᾽ άνοιγμα q = σταθ.

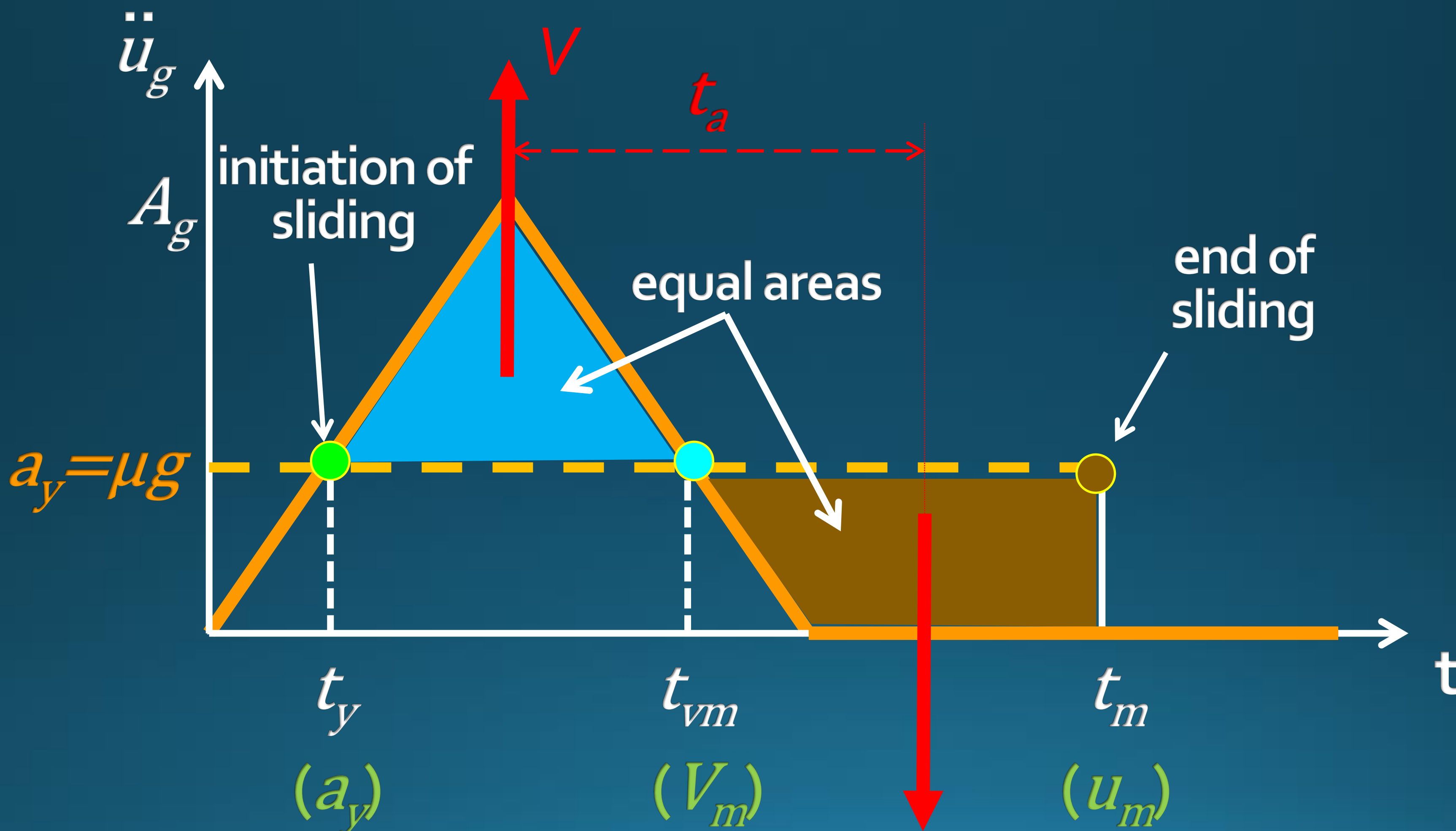


$A = \frac{1}{2} q \cdot l + M_B / l$	$Q_{Cr} = \frac{1}{2} q \cdot l + (M_D - M_C)/l$	$Q_{Jr} = \frac{1}{2} q \cdot l - M_J / l$
$Q_{Bl} = -\frac{1}{2} q \cdot l + M_B / l$	$Q_{Dl} = -\frac{1}{2} q \cdot l + (M_D - M_C)/l$	$K = \frac{1}{2} q \cdot l + M_J / l$
$M = A \cdot x - \frac{1}{2} q \cdot x^2$	$M = M_C + x \cdot Q_{Cr} - \frac{1}{2} q \cdot x^2$	$M = K \cdot x' - \frac{1}{2} q \cdot x'^2$
$\max M = A^2 / 2q$	$\max M = M_C + Q_{Cr}^2 / 2q$	$\max M = K^2 / 2q$
$x_0 = A / q$	$x_0 = Q_{Cr} / q$	$x'_0 = K / q$

M, Q με πρόσημο

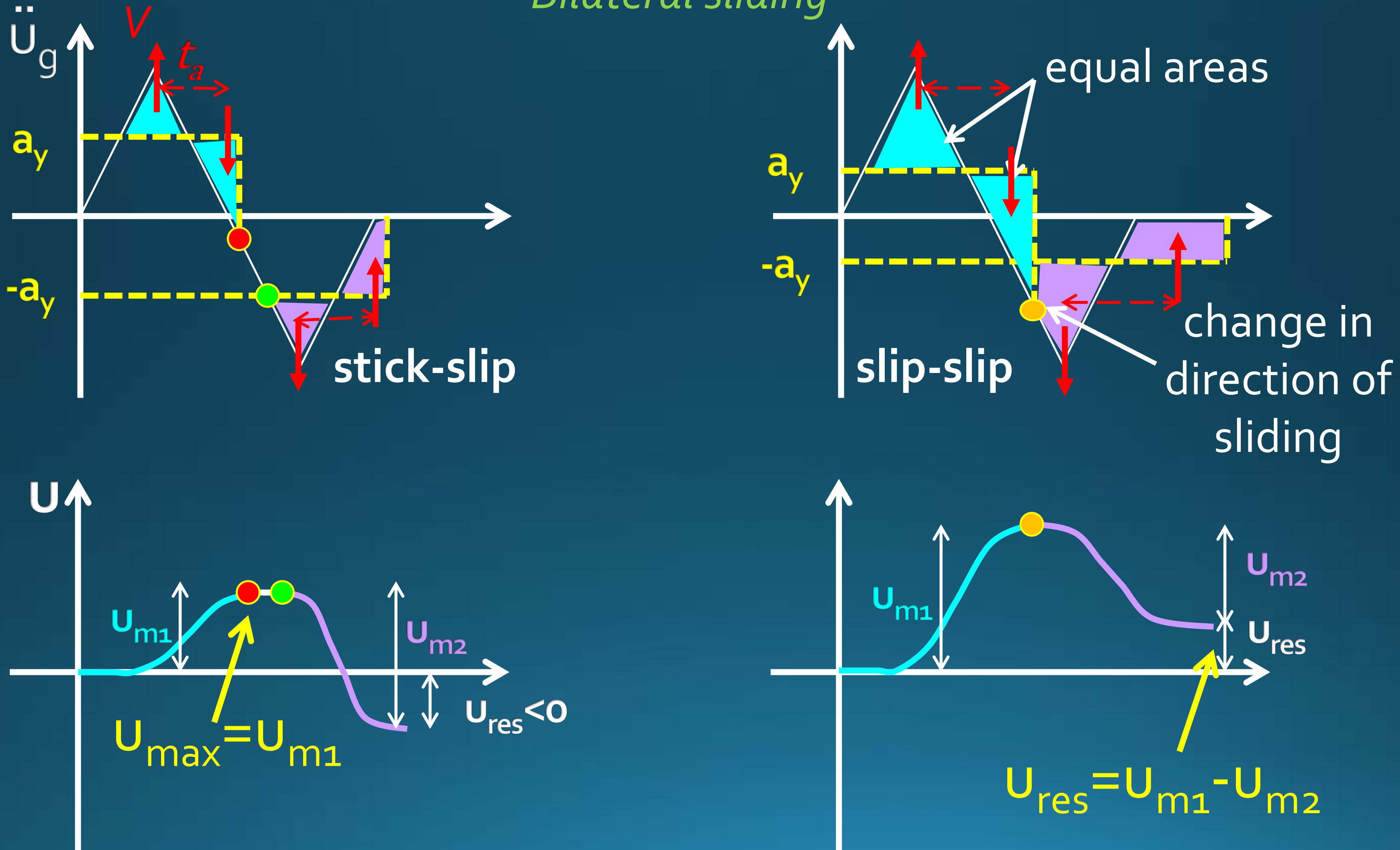
Graphical calculation of sliding using beam analog method

Unilateral Sliding



Graphical calculation of sliding using beam analog method

Bilateral sliding



Dimensionless variables

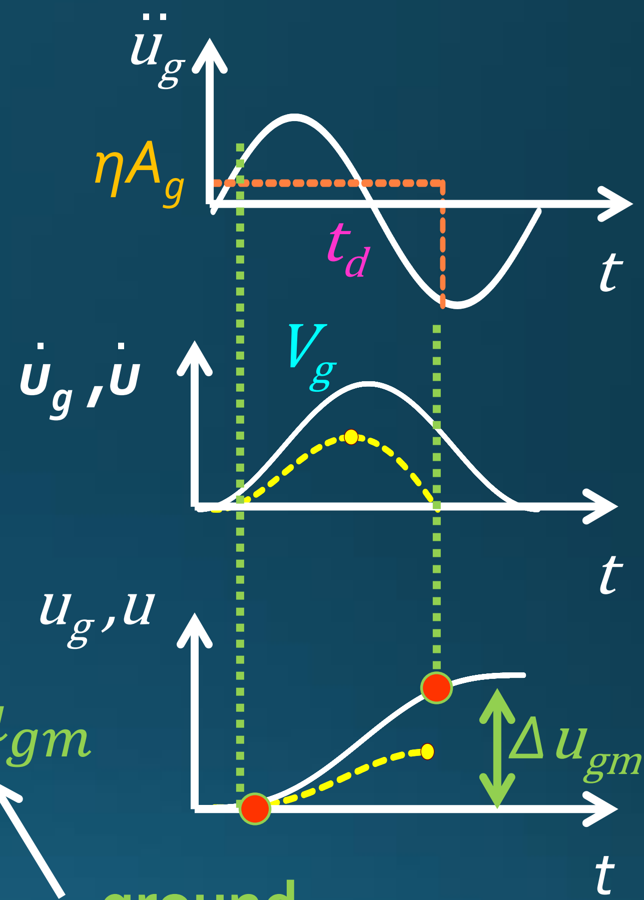
Time $t/t_d \rightarrow \quad \tau, \quad \tau_y, \tau_m$

Velocity $\dot{u}/V_g$

Displacement $u/(u_{gm} - u_{gy}) = u/\Delta u_{gm}$

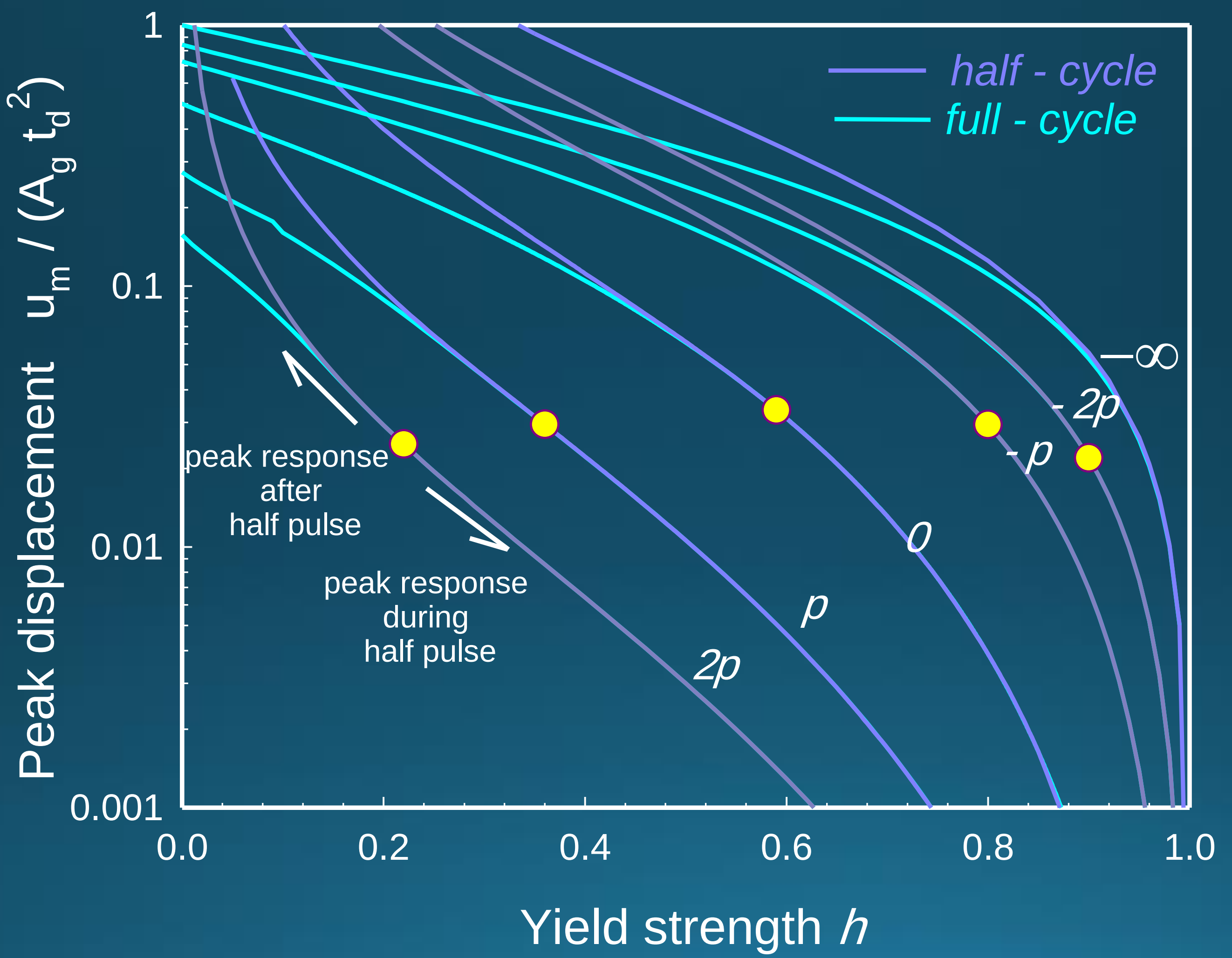
Sliding Resistance

$$\eta = \frac{\alpha_y}{A_g} = \frac{\mu\ g}{A_g}$$

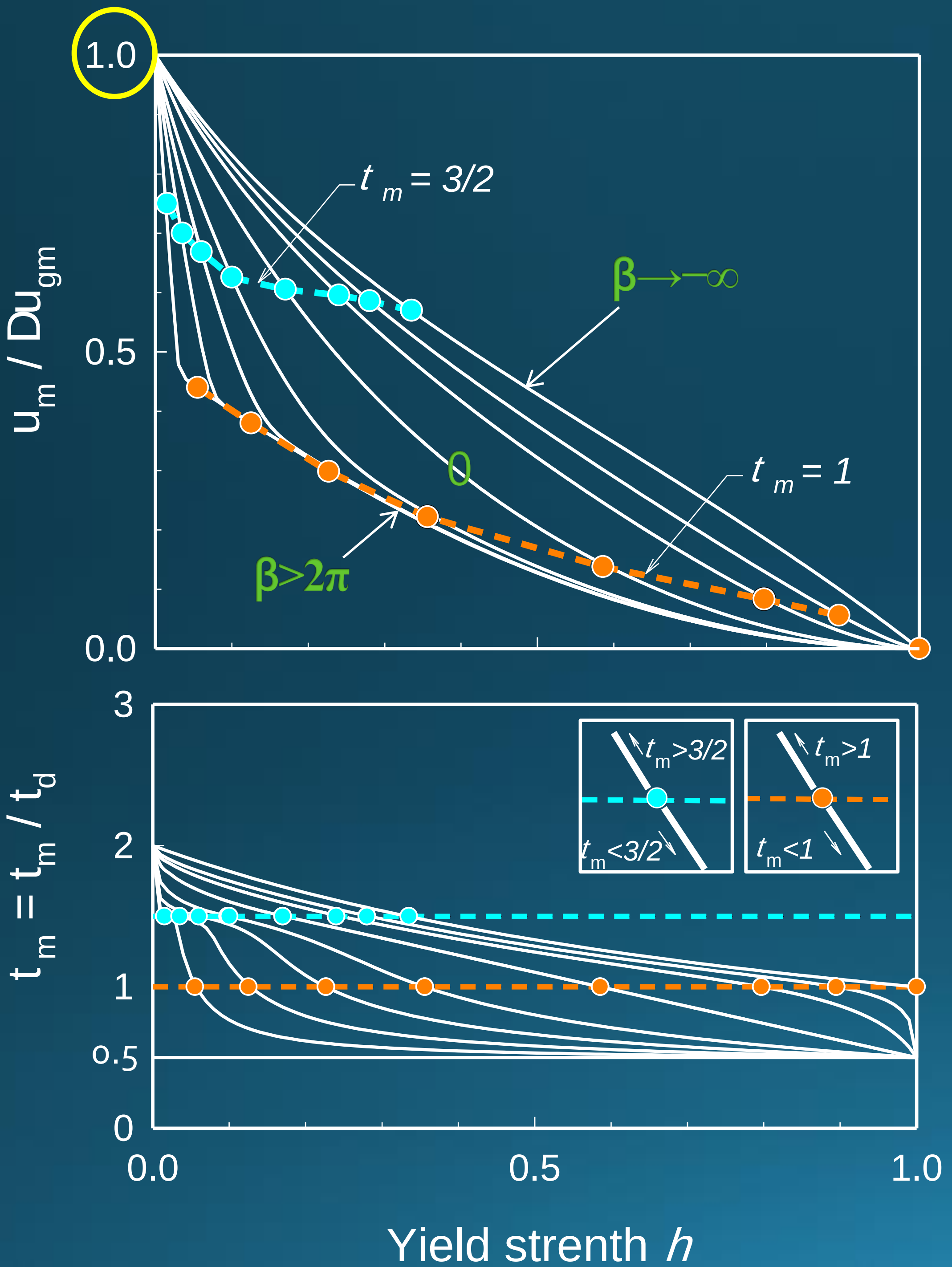


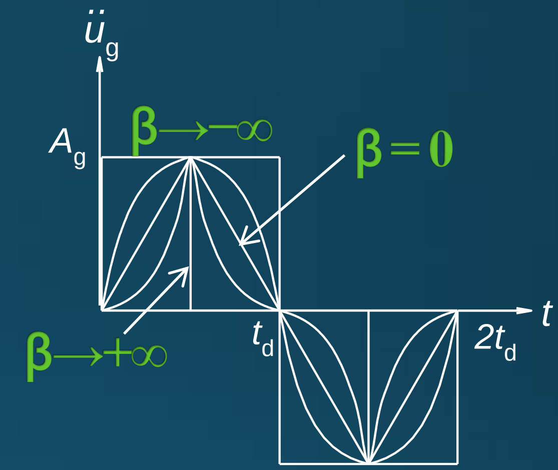
ground displacement step

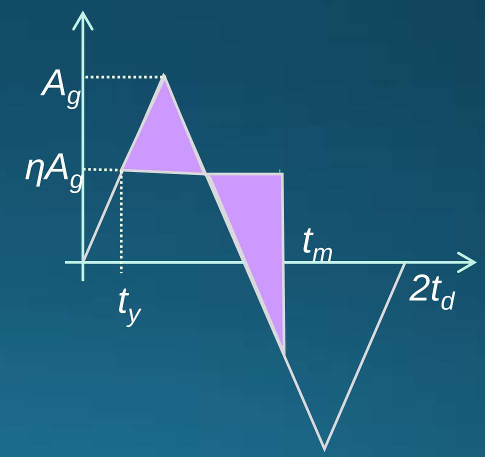
Peak Sliding Displacement



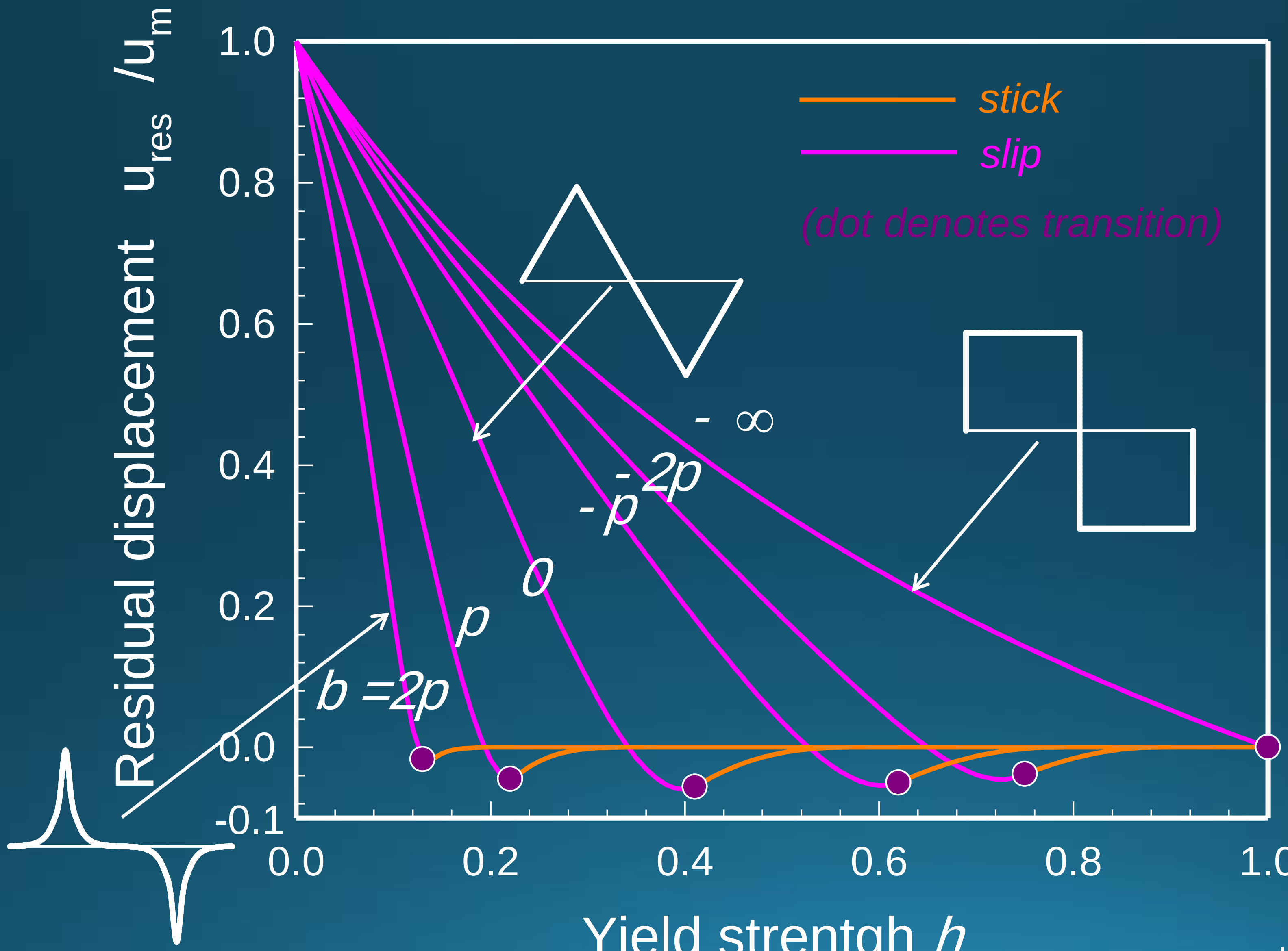
Μέγιστη Ολίσθηση







Παραμένουσα Μετατόπιση



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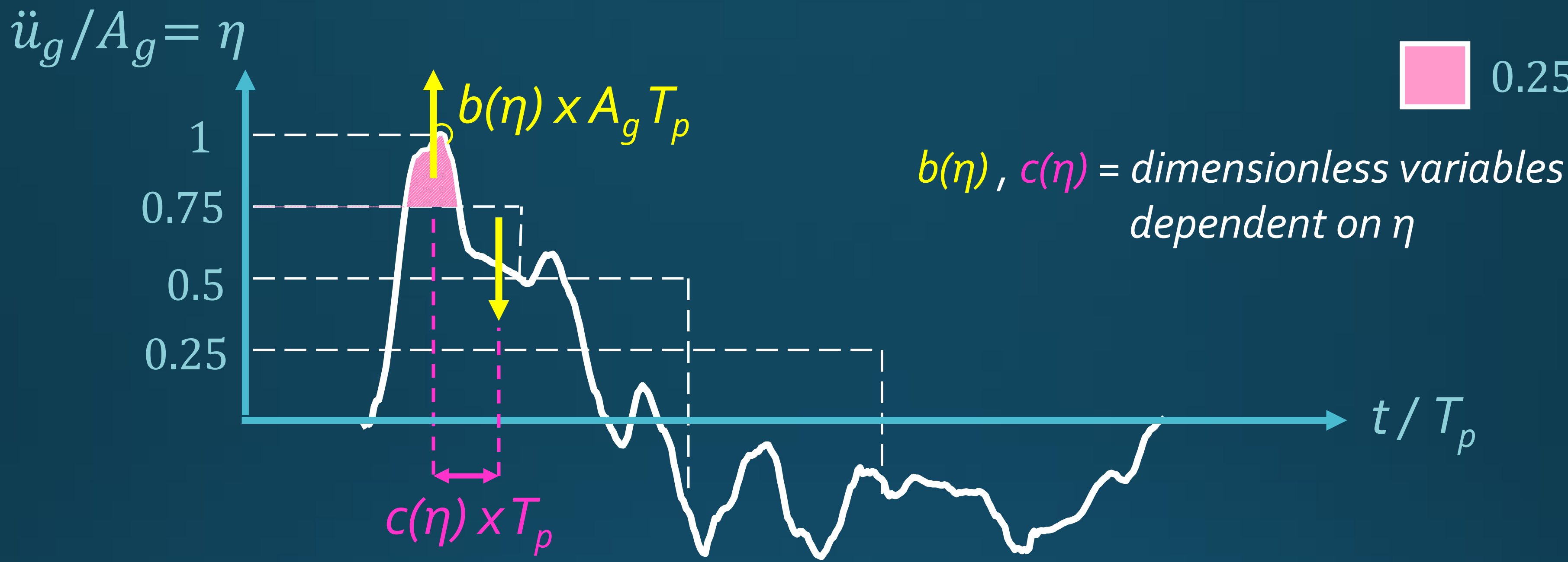
Inverse calculation



$$D = b(\eta)\, A_g T_p \times c(\eta)\, T_p \times \Pi(\chi i)$$

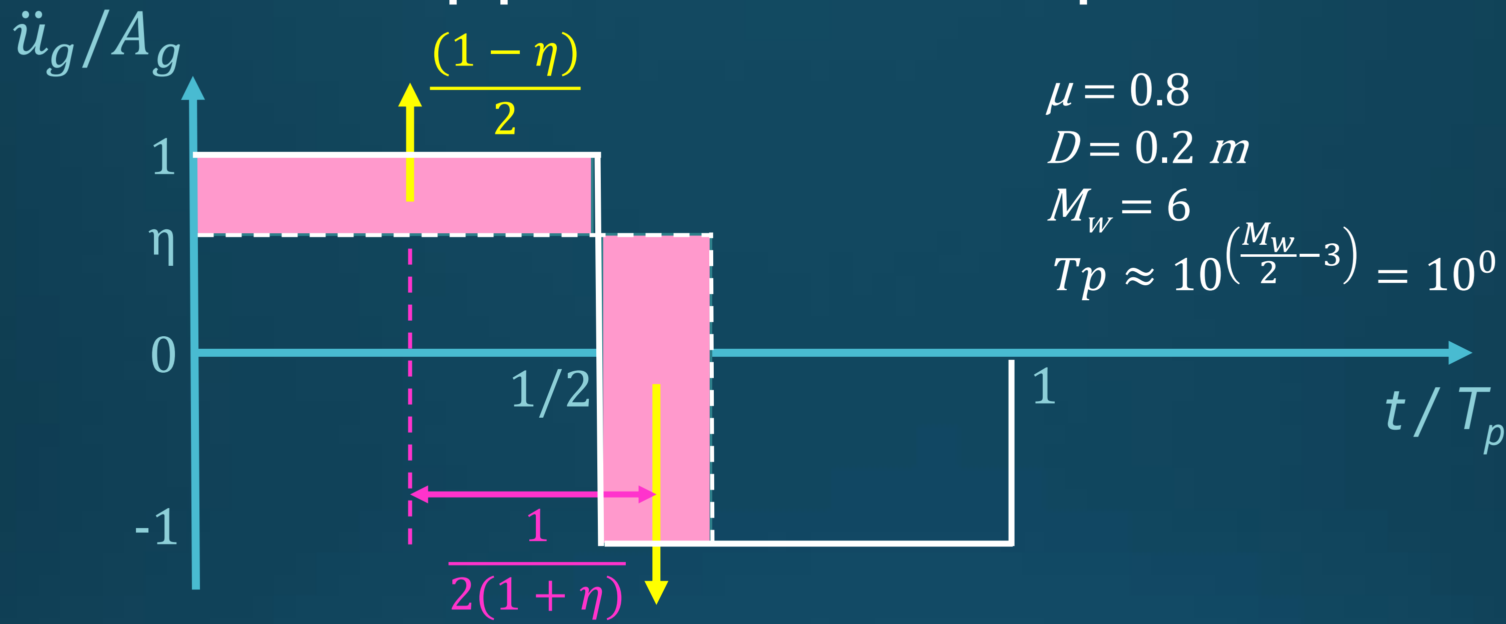
- D: given displacement
- $b(\eta)\, A_g T_p$: peak sliding velocity
- $c(\eta)\, T_p$: effective duration
- $\Pi(\chi i)$: aggravation factors

Rearranging terms:

$$A_g = \frac{D}{b(\eta)\, c(\eta)\, T_p^2}$$

Check if: $A_g = \mu g / \eta$ and iterate …

Application example



$\mu = 0.8$
$D = 0.2\ m$
$M_w = 6$
$Tp \approx 10^{\left(\frac{M_w}{2}-3\right)} = 10^0 = 1s$

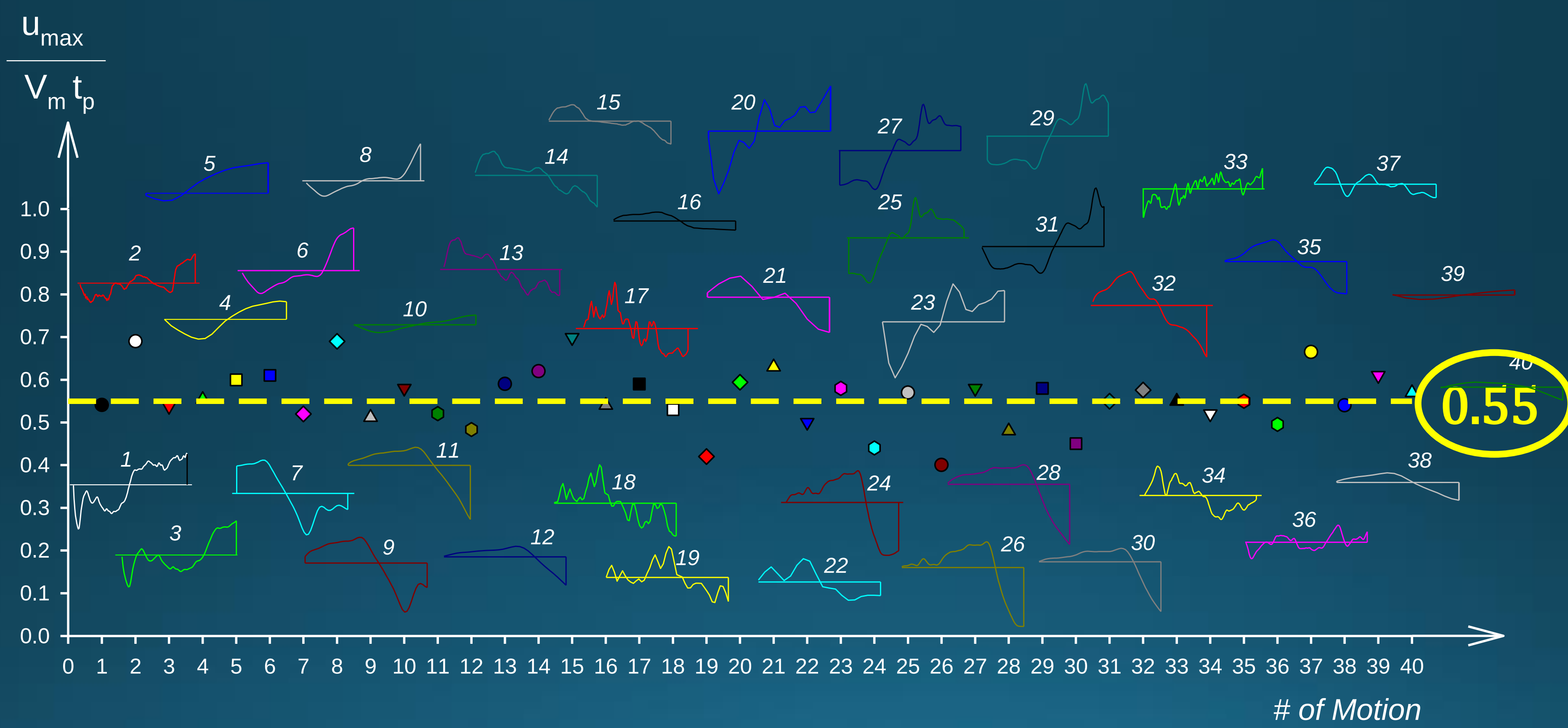
$$D = \underbrace{A_g(1-\eta)\frac{1}{2}Tp}_{\text{peak sliding velocity}} \times \underbrace{\frac{1}{2(1+\eta)}Tp}_{\text{effective duration}}$$

$$\left.\begin{aligned} A_g &= \frac{4D}{Tp^2}\frac{1+\eta}{1-\eta} \\ A_g &= \mu g/\eta \end{aligned}\right\} \text{using 2-3 iterations}$$

$\mu \approx 0.83$

$A_g \approx 8.1\ m/s^2$

Effective durations of sliding



Conclusions

- New graphical solution for the forward and inverse calculation of peak sliding displacement on horizontal and inclined plane (& peak ground displacement itself)
- Solution is based on the similarity ("analog") between a loaded beam and an acceleration time history, which provides insight into the physics of the problem
- The inversion can be generalized to incorporate the difference between peak and residual sliding, multi-axial excitation and the compliance of the sliding body

Publications

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- Newmark, N. M. (1965). “Effects of earthquakes on dams and embankments.” Geotechnique, 15(2), 139–160.
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- **Voyagaki, E**., Mylonakis, G., and Psycharis, I. N. “Plastic Input Motion: a Transformation for the Response of Yielding Oscillators”, *Journal of Engineering Mechanics ASCE* 2012, **138**(7): 749–760, https://doi.org/10.1061/(ASCE)EM.1943-7889.0000373
- **Voyagaki, E.**, Mylonakis, G., and Psycharis, I. N. “A Shift Approach for the Dynamic Response of Rigid-Plastic Systems”, *Earthquake Engineering & Structural Dynamics* 2011, **40**(8): 847-866, https://doi.org/10.1002/eqe.1063
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Thank you !

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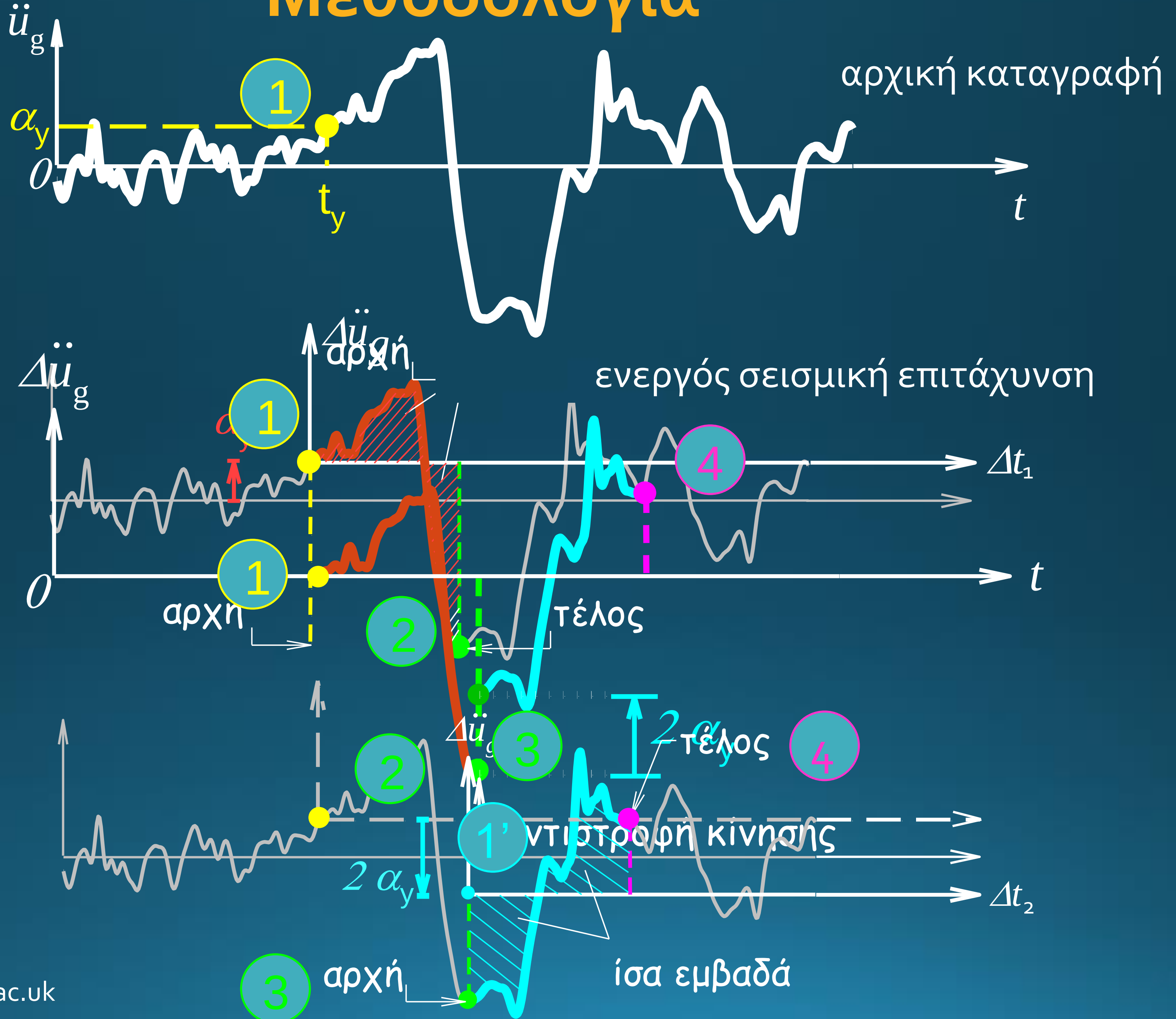
- Elia Voyagaki
- Christos Giarlelis
- Dimitris Karamitros
- Ioannis Psycharis

Ground motion database

40 near-fault ground motions

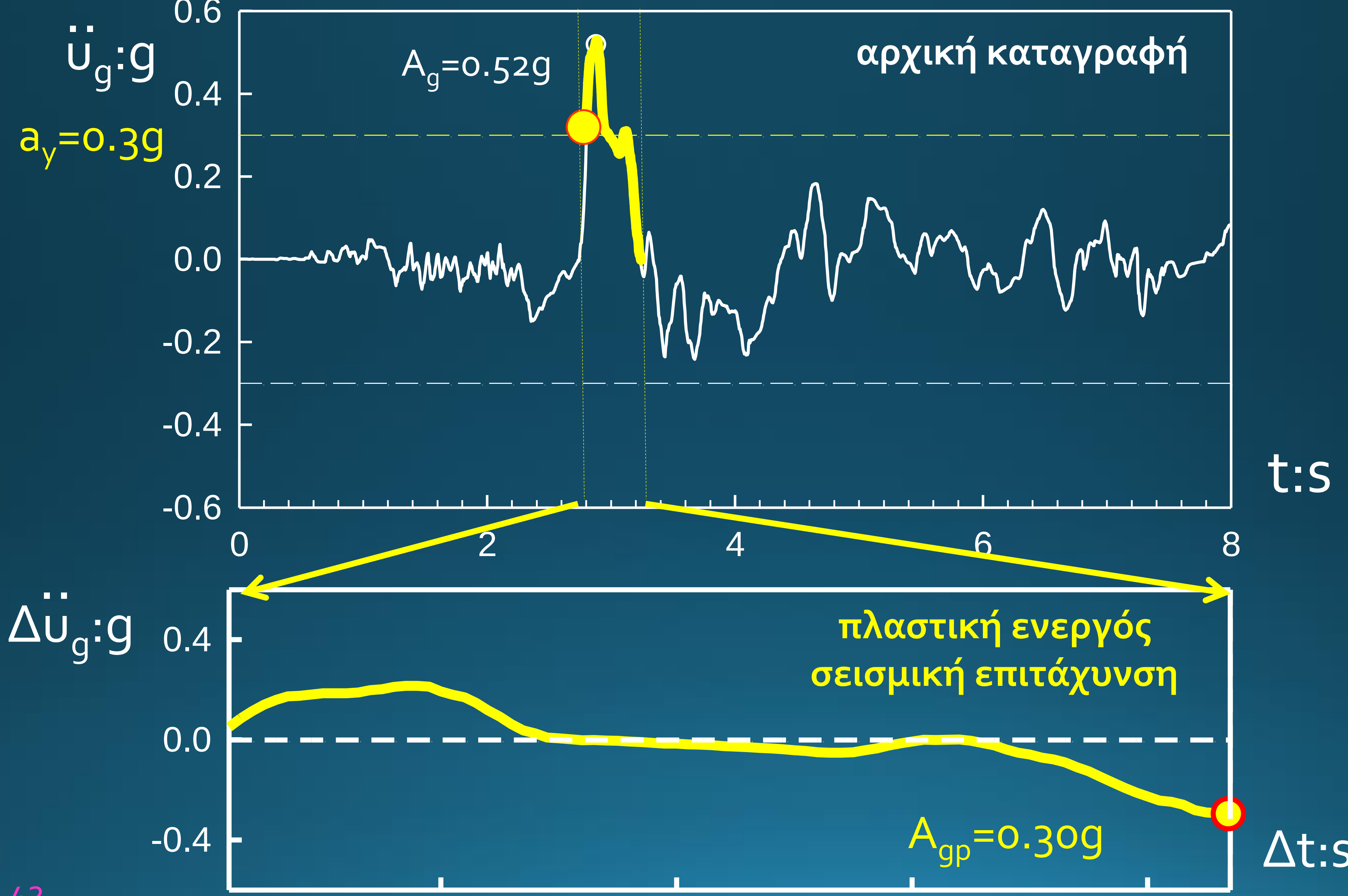
A/A	Σεισμός	Ημερομηνία	Μηχ/σμός	M_w	Σταθμός
1	Parkfield	6/27/1966	SS	6.20	C02
2	San Fernando	2/9/1971	RV	6.55	PCD
3	Gazli	5/17/1976	RV	6.80	KAR
4	Bucharest	5/4/1977	RV	7.27	BRI
5	Tabas	9/16/1978	RV	7.11	TAB
6	Coyote Lake	6/8/1979	SS	5.63	GA6
7	Imperial Valley	10/15/1979	SS	6.50	E04
8					E05
9					E06
10					E07
11					EMO
12	Morgan Hill	4/24/1984	SS	6.15	HAL
13	Palm Springs	6/8/1986	OB	6.09	NPS
14					DSP
15	Whittier Narrows	10/10/1987	RV	5.93	DOW
16					NWK
17	Superstition Hill	11/24/1987	SS	6.40	PTS
18					ELC
19	Loma Prieta	10/17/1989	OB	6.90	LGP
20					STG
21	Sierra Madre	6/28/1991	RV	5.56	COG
22	Erzican	5/13/1992	SS	6.63	ERZ
23	Landers	6/28/1992	SS	7.20	LUC
24	Northridge	1/17/1994	RV	6.70	JFA
25					RRS
26					SCG
27					SCH
28					NWS
29	Aigion	6/15/1995	NM	6.33	AEG
30					AEG
31	Izmit	8/17/1999	SS	7.40	ARC
32					SKR
33					YPT
34					GBZ
35					GBZ
36	Chi-Chi	9/20/1999	OB	7.60	TCU052
37					TCU068
38					TCU075
39					TCU076
40					TCU129

Μεθοδολογία



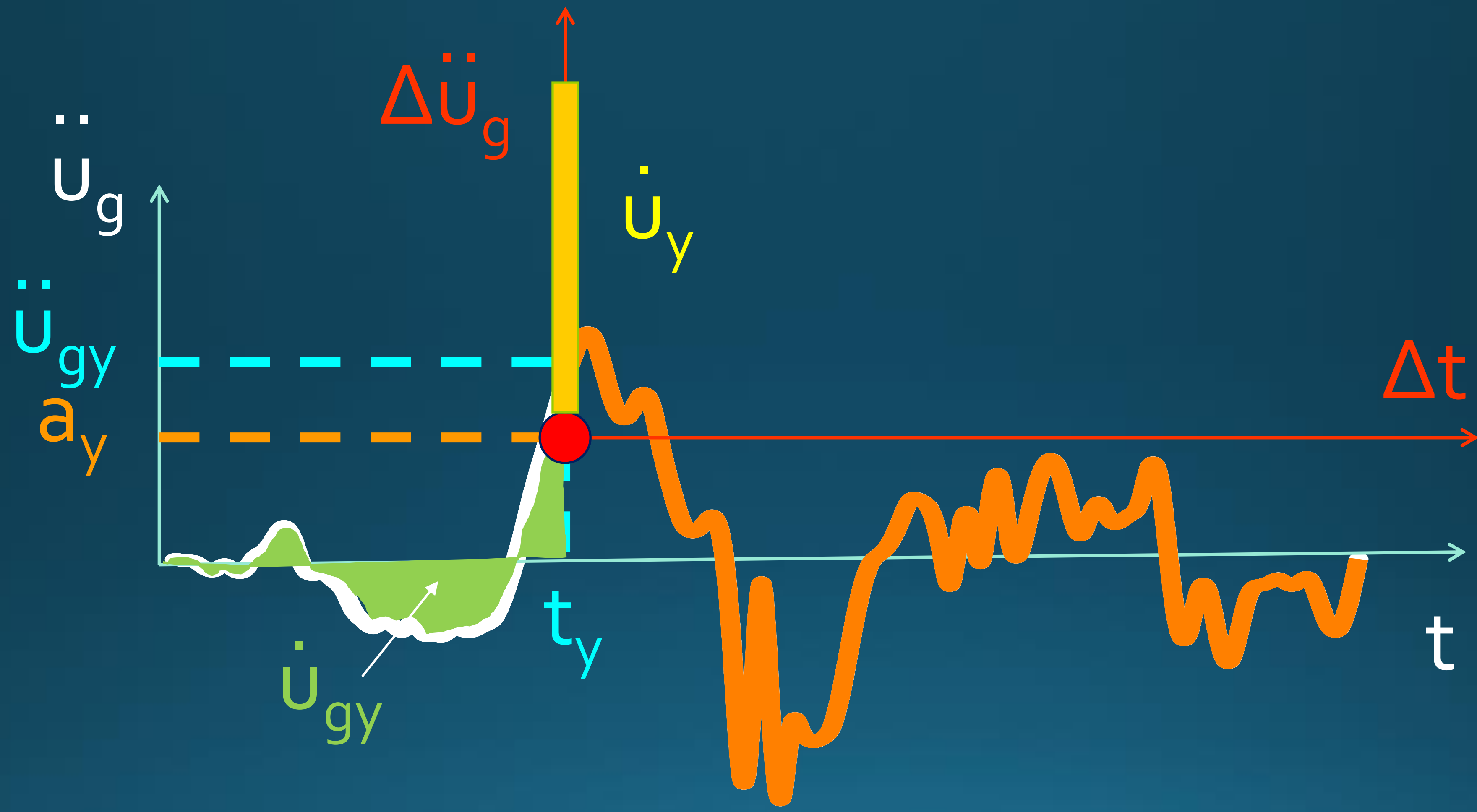
e.voyagaki@bristol.ac.uk

Εφαρμογή : Erzican 1992



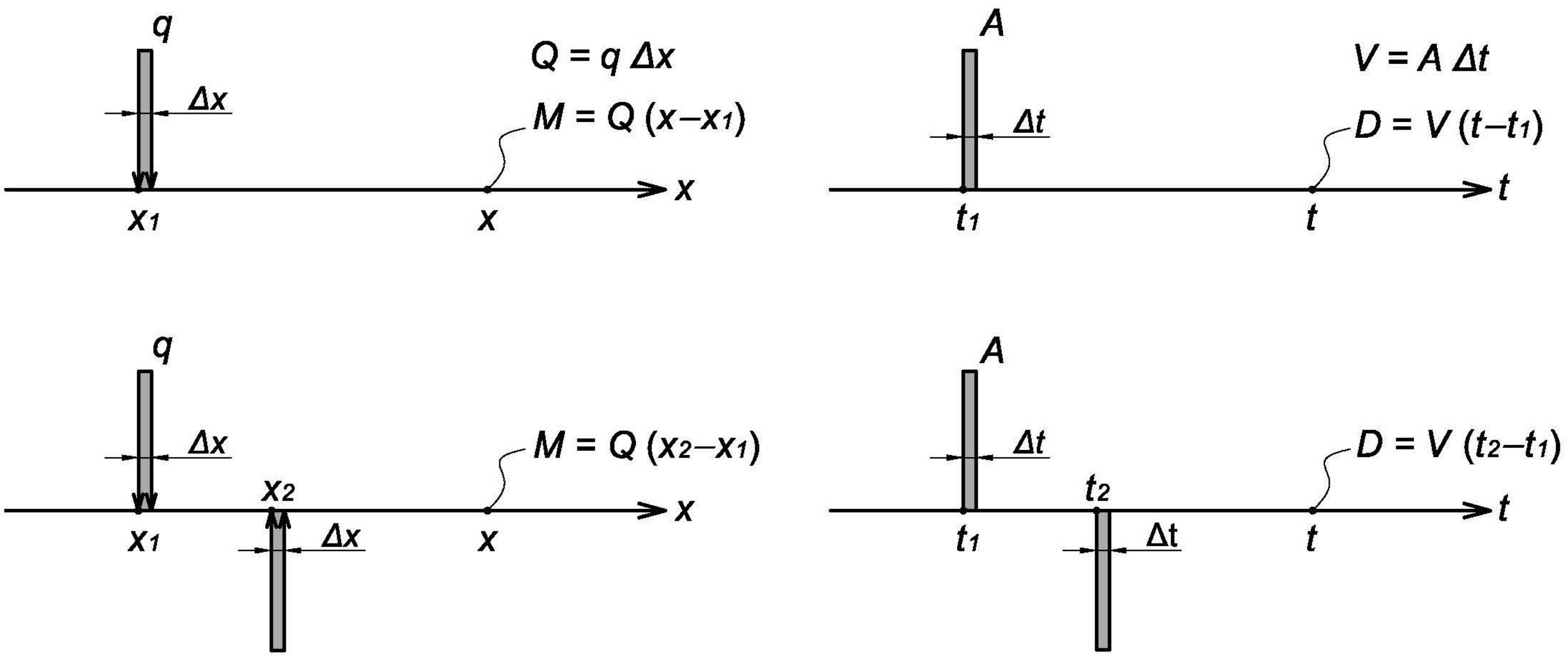
Τεχνική Μετάθεσης Αξόνων

Ελαστικό – πλήρως πλαστικό σύστημα

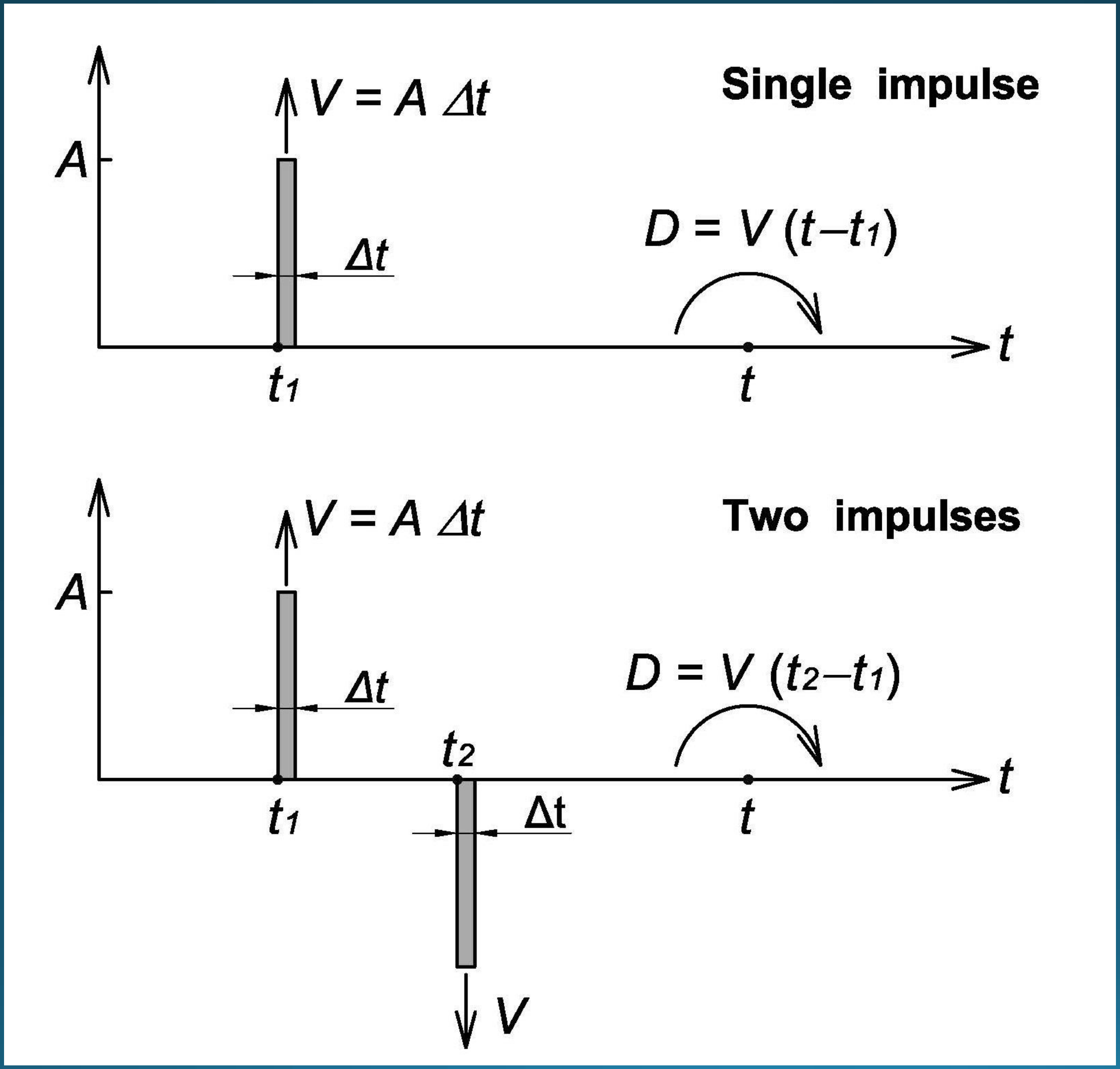


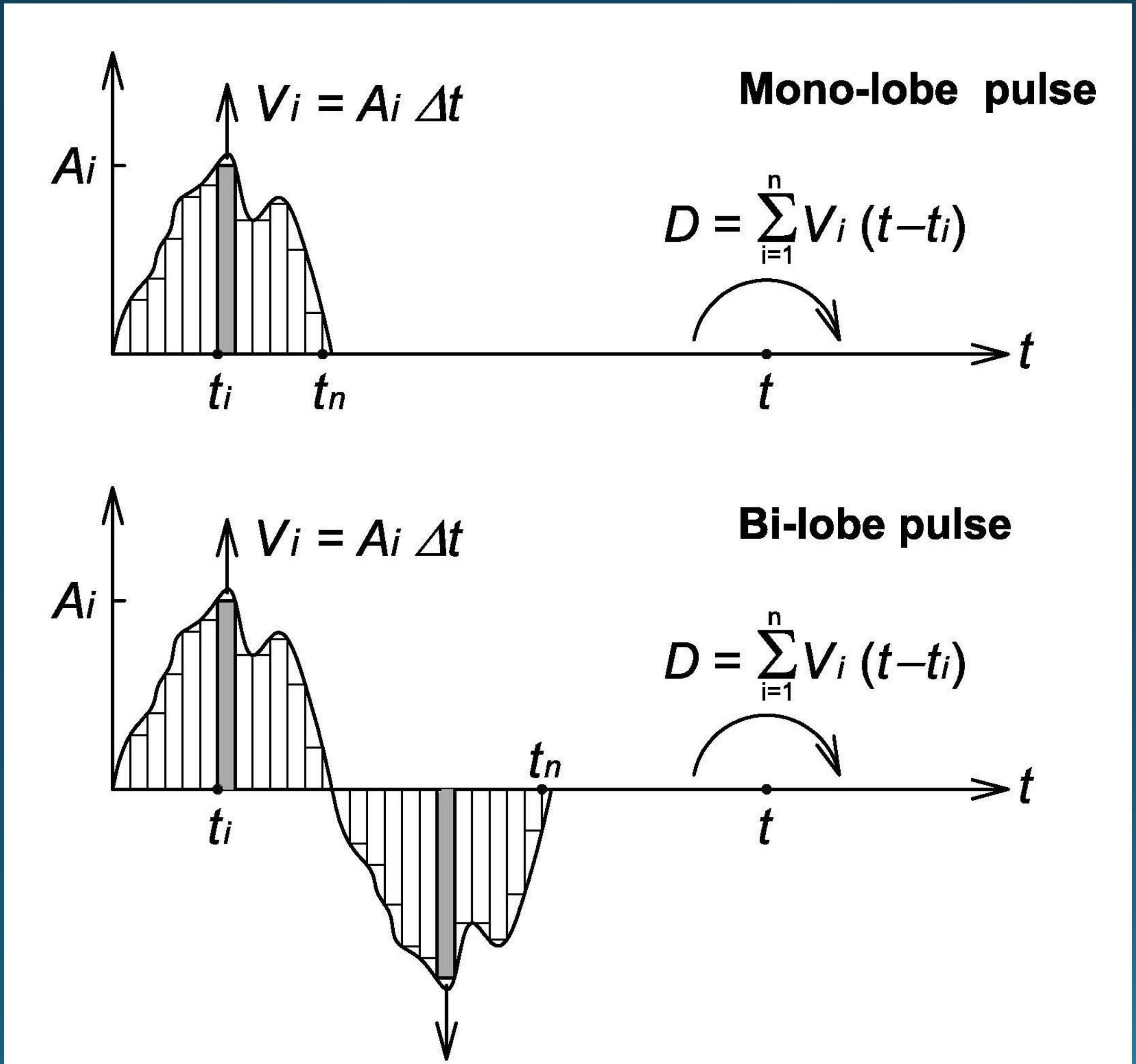
$$\ddot{u} + a_y\,\mathrm{sgn}(\dot{u}) = +\ddot{u}_g + \delta(t - t_y)\dot{u}_y + \dot{\delta}(t - t_y)u_y$$

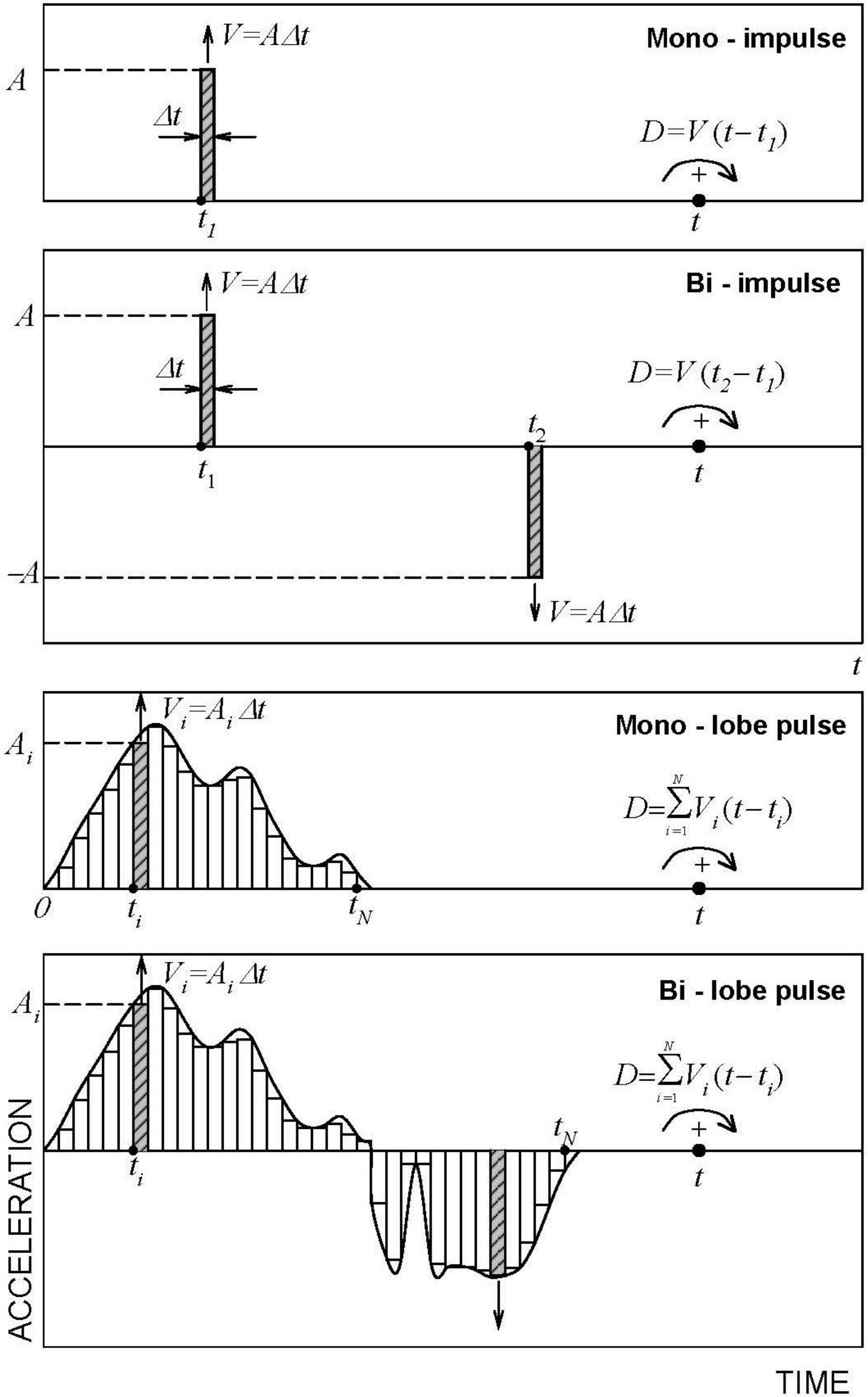
Beam Analog Approach
graphical representation

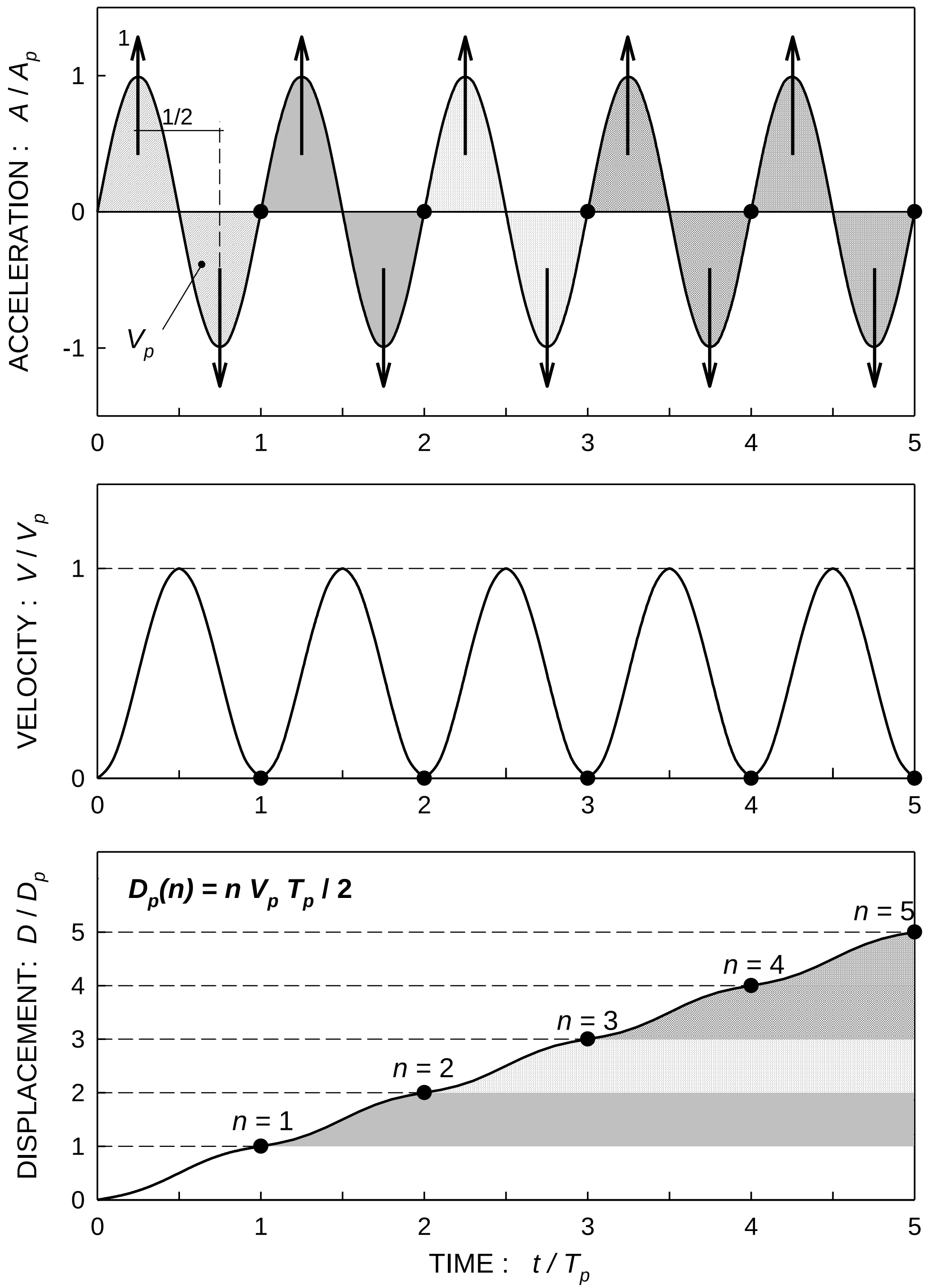


Displacements due to elementary pulses




Mono-lobe pulse
$V_i = A_i \Delta t$
A_i
$D = \sum_{i=1}^{n} V_i (t-t_i)$
t_i
t_n
t
t
Bi-lobe pulse
$V_i = A_i \Delta t$
A_i
$D = \sum_{i=1}^{n} V_i (t-t_i)$
t_n
t_i
t
t


Mono - impulse
V=AΔt
A
Δt
D=V(t−t1)
t1
t
Bi - impulse
V=AΔt
A
Δt
D=V(t2−t1)
t2
t1
t
−A
V=AΔt
t
Mono - lobe pulse
Vi=Ai Δt
Ai
D=Σ Vi(t−ti)
0
ti
tN
t
Bi - lobe pulse
Vi=Ai Δt
Ai
D=Σ Vi(t−ti)
tN
ti
t
ACCELERATION
TIME


ACCELERATION : A / A_p
1
1/2
V_p
0
-1
VELOCITY : V / V_p
DISPLACEMENT: D / D_p
D_p(n) = n V_p T_p / 2
n = 1
n = 2
n = 3
n = 4
n = 5
0
1
2
3
4
5
TIME : t / T_p