Accounting for the non-stationary frequency content of ground motions in Earthquake Engineering: Does it matter? (and can wavelet analysis be useful after all?)

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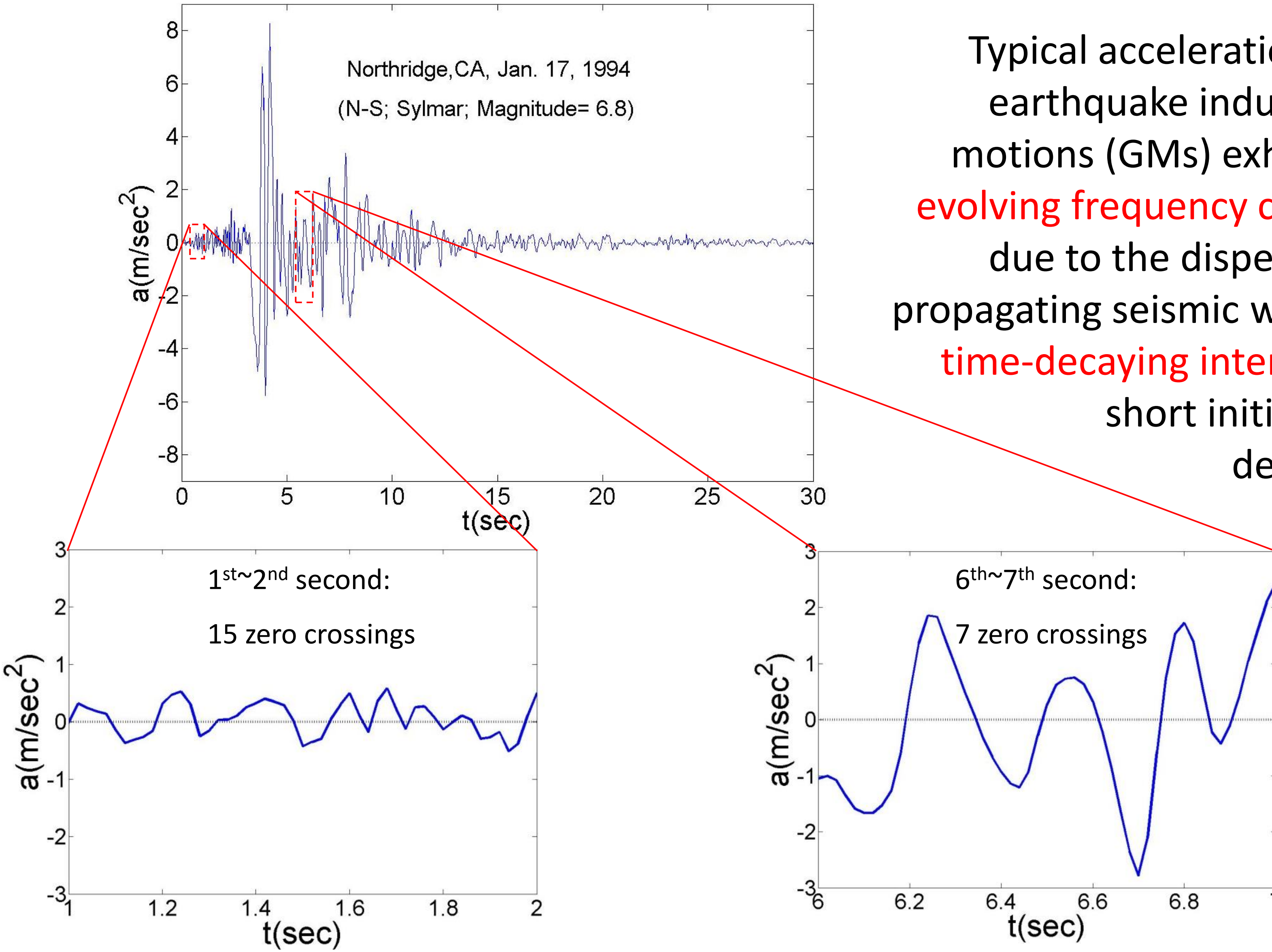
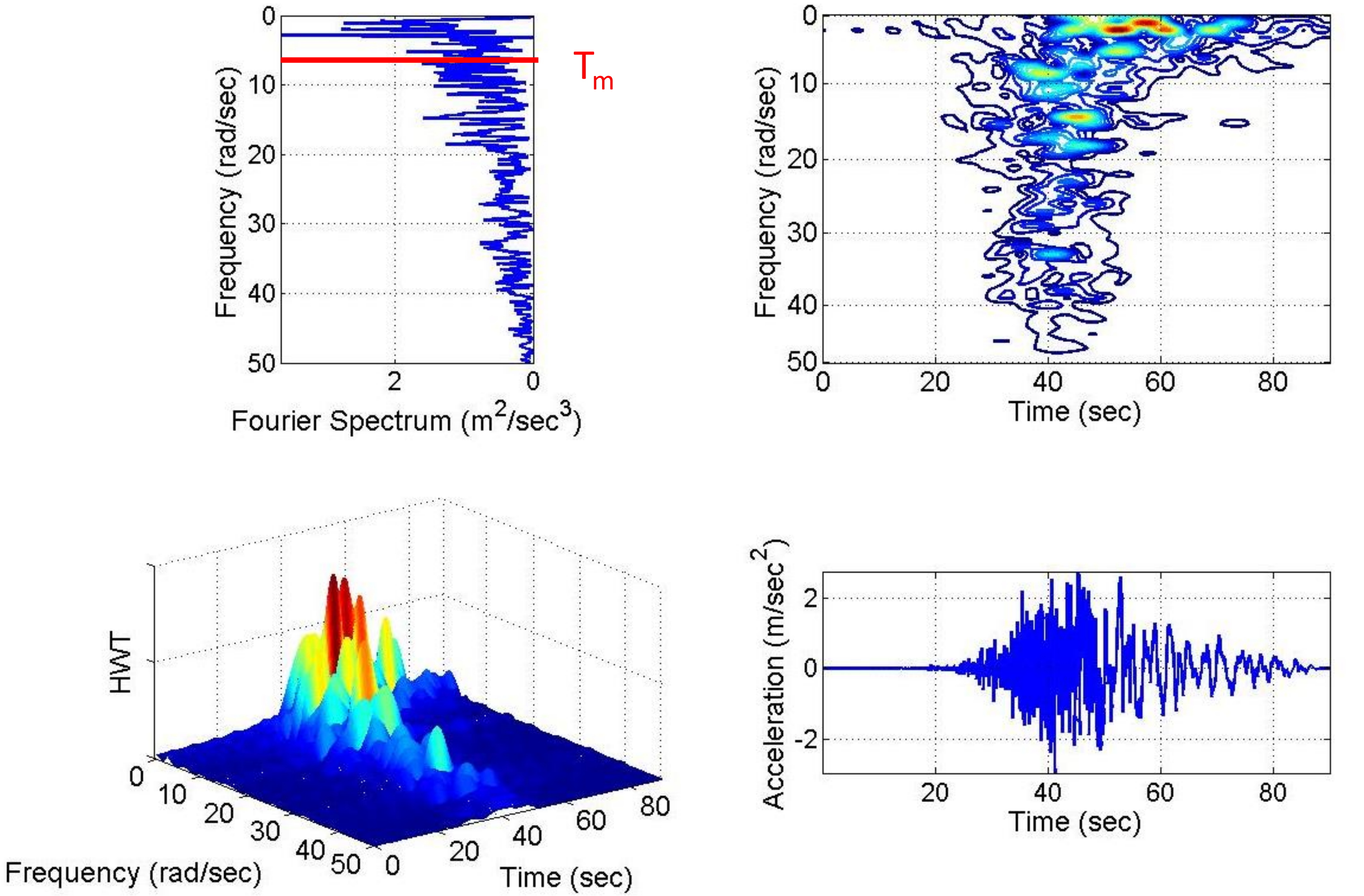
Introduction / Motivation



Typical acceleration traces of earthquake induced ground motions (GMs) exhibit a time-evolving frequency composition due to the dispersion of the propagating seismic waves, and a time-decaying intensity after a short initial period of development.

Introduction / Motivation



Yet, the evolving frequency content of GMs is not taken into account by any of the commonly-used GM properties, widely used to characterize the structural damage potential of GMs.

Introduction / Motivation

GM properties
(some used as intensity measures, IMs, or for record selection to feed in the performance-based earthquake engineering, PBEE, machinery)

- PGA, PGV, PGD
- Arias intensity
- Spectral shape, Sa(T_1), AvgSa,…
- Dominant frequency/period (where the GM Fourier spectrum peaks)
- Mean period T_m (e.g. Rathje et al. 1998)

$$T_m = \frac{\sum_{k=K_1}^{K_2} \left|\hat{X}[k]\right|^2 \frac{2\pi}{\omega_k}}{\sum_{k=K_1}^{K_2} \left|\hat{X}[k]\right|^2}$$

where $$\hat{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-i\omega_k n}$$ with ω_k = [0.25Hz, 20Hz]

(DFT of GM)

Introduction / Motivation

First things first…

We need to come up with a “new” GM property, which:

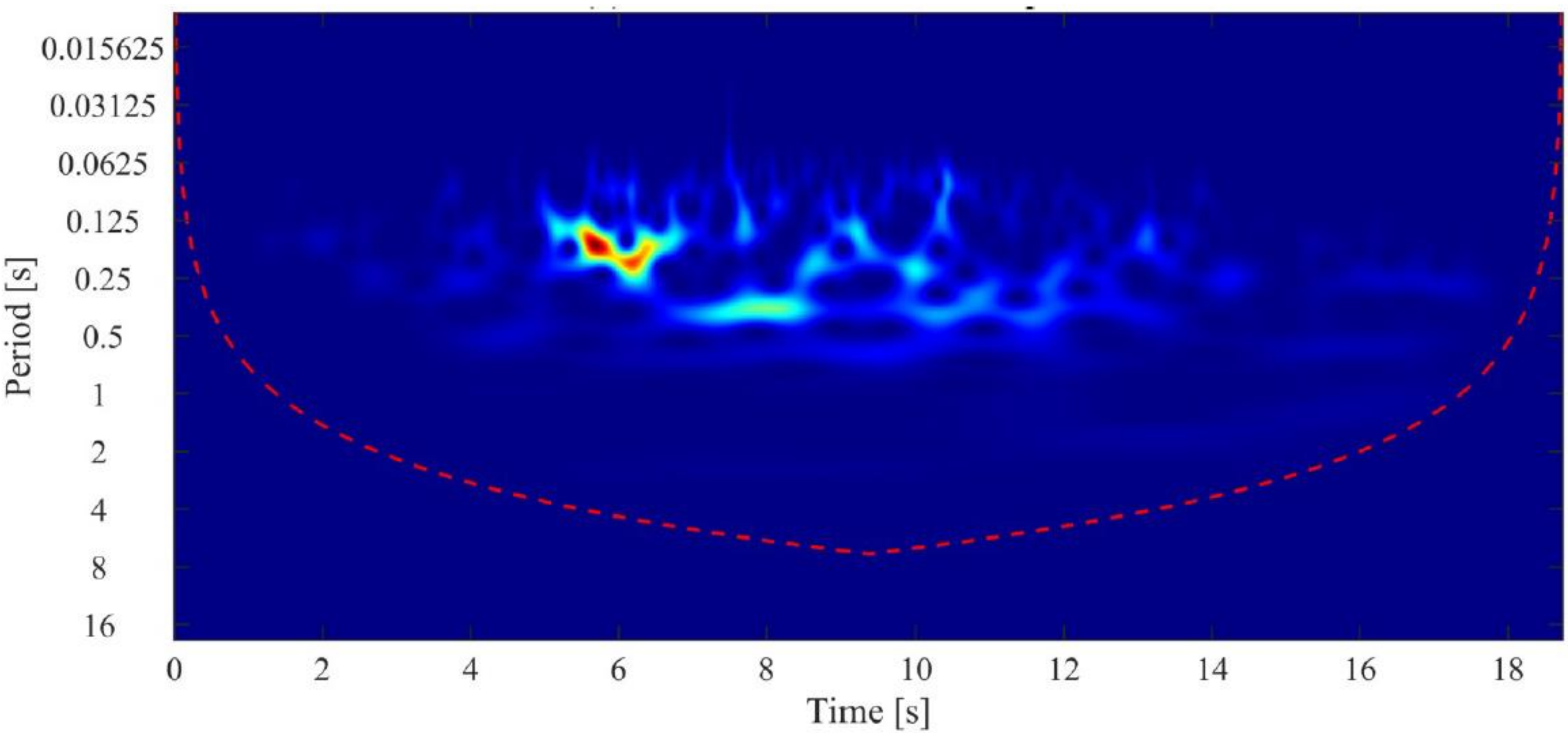
-is a “good” metric of the temporal change of GM frequency content

-is a scalar (number), ideally scaling invariant (to accommodate PBEE)

-is “relatively easy” to compute

The Continuous (Morlet) Wavelet Transform

Imperial Valley, 1979, Plaster City (045) record



Joint time-frequency signal analysis is not only “somewhat complicated”, it also is not “exact science”…

The Continuous (Morlet) Wavelet Transform

- The continuous wavelet transform (CWT) given by the equation

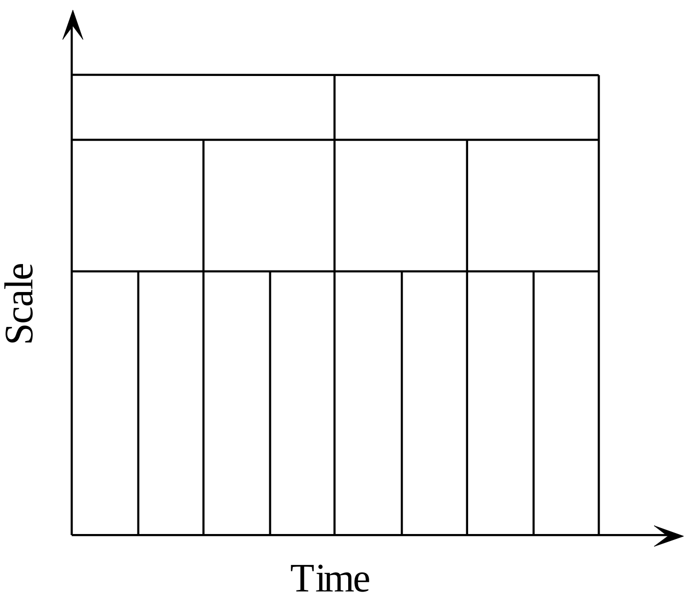
$$W(s,t_o)=\frac{1}{\sqrt{s}}\int_{-\infty}^{\infty}x(t)\psi^*\left(\frac{t-t_o}{s}\right)dt$$

decomposes any finite energy signal *f(t)* onto a basis of functions generated by scaling a single mother wavelet function *ψ(t)* by the scale parameter *α* and by shifting it in time by the parameter *b*.

ψ : analyzing or mother wavelet

$$\psi(s,t_o)=\frac{1}{\sqrt{s}}\psi\left(\frac{t-t_o}{s}\right)$$

- Variable size wavelet "windows" are employed
- Long duration windows capture lower frequencies (large scales)
- Short duration windows are used to capture higher frequencies (small scales)
- Heisenberg's uncertainty principle holds



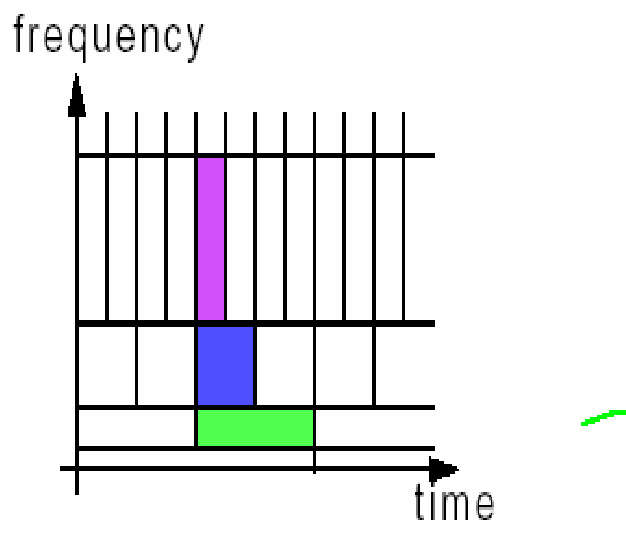
The Continuous (Morlet) Wavelet Transform

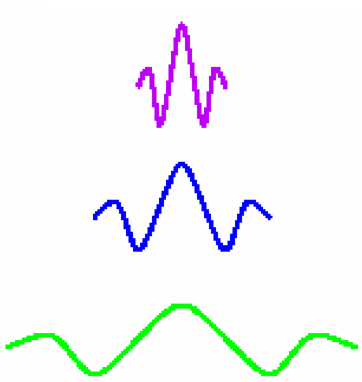
Such an analysis results in a three-dimensional spectrum having the wavelet coefficients plotted versus time and scale (scalogram). A certain wavelet-dependent relationship between scale and frequency should be established to yield a wavelet- based spectrogram.

- **Uncertainty Principle**

$$\psi(t) \xleftrightarrow{\text{Fourier Pairs}} \hat{\Psi}(\omega)$$

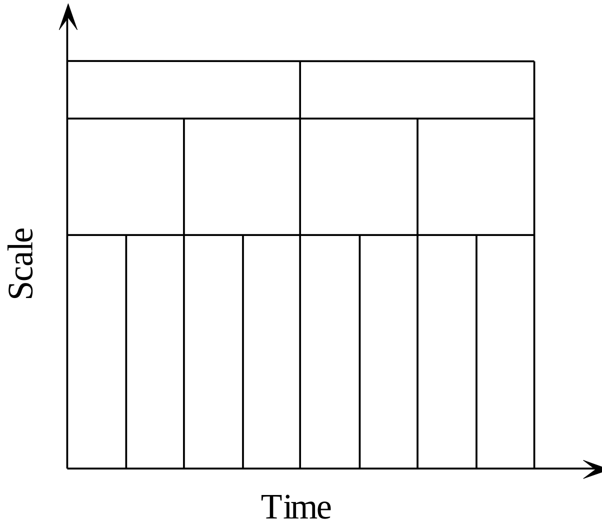
$$\frac{1}{\sqrt{s}}\psi\left(\frac{t-t_o}{s}\right) \xleftrightarrow{\text{Fourier Pairs}} \sqrt{s}\hat{\Psi}(s\omega)\exp(-i\omega s t_o)$$





- **Reciprocal relationship between scale-frequency:**

$$\text{Frequency} = \frac{\text{Constant}}{\text{Scale}}$$



The Continuous (Morlet) Wavelet Transform

So, we need to know what we are aiming for:

Time or Frequency
(resolution/smoothness/bias)???

Uncertainty principle

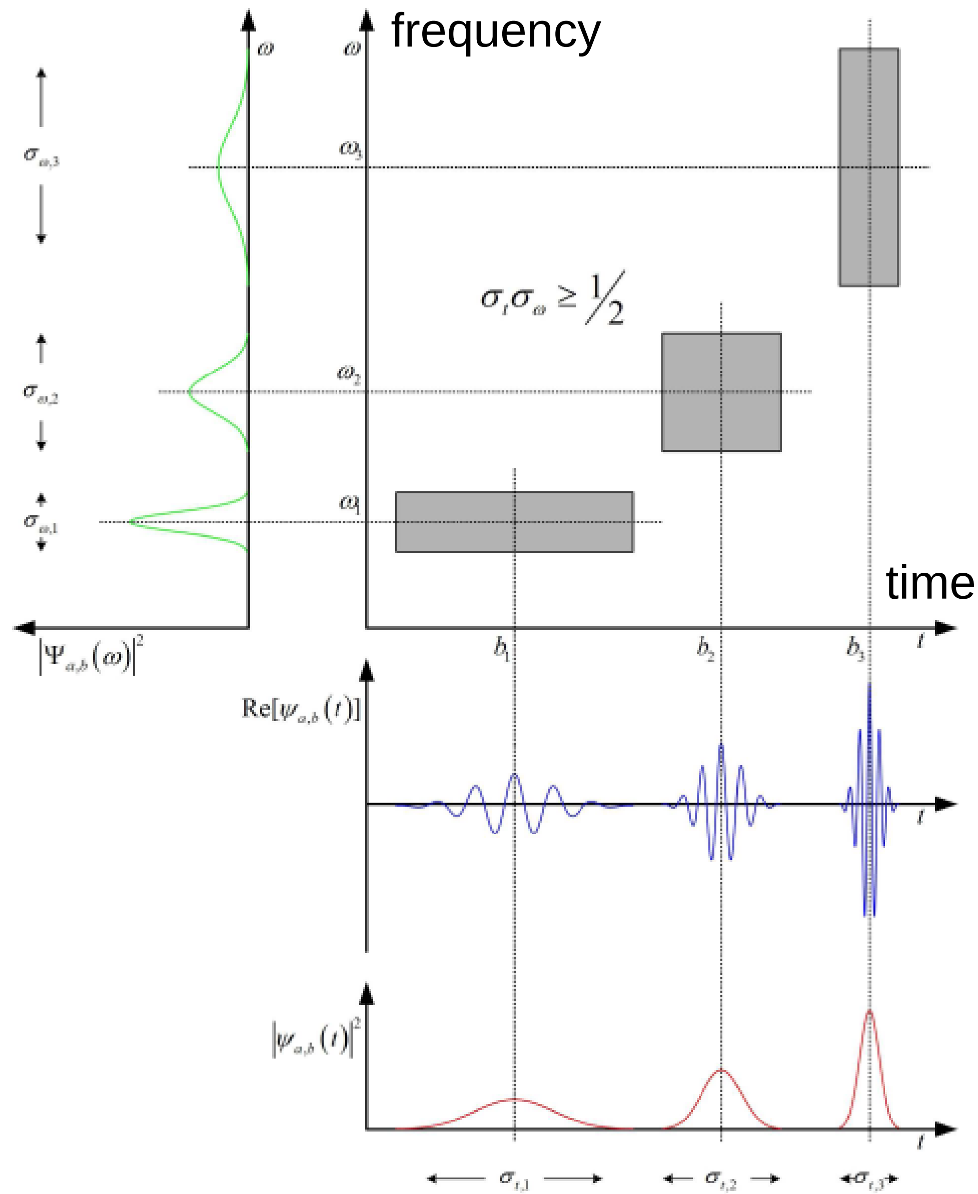
Resolution trade-off

Wavelet shape

Wavelet smoothness

Vanishing moments of wavelets

etc.



The Continuous (Morlet) Wavelet Transform

Analytic (complex) Morlet wavelets

- At scale *α* and time position *b* the modified Morlet wavelet is given by

$$\psi^{M}\left(\frac{t-t_o}{s}\right)=\frac{1}{\sqrt{s\pi\Omega_b}}\exp\left(i\frac{\Omega_c}{a}\left(t-t_o\right)-\frac{\left(t-t_o\right)^2}{s^2\Omega_b}\right)$$

- Its Fourier transform is a shifted Gaussian function, that is:

$$\hat{\Psi}_{t_o}^{M}\left(s\omega\right)=\sqrt{s}\exp\left(-\frac{\Omega_b}{4}\left(s\omega-\Omega_c\right)^2-is\omega t_o\right)$$

- The central (pseudo-) frequency observed at scale *α* is usually computed by

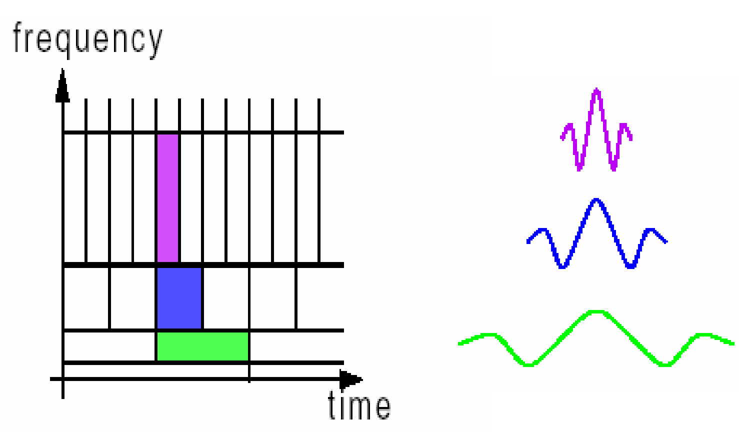
$$\omega_o=\frac{\Omega_c}{a}$$

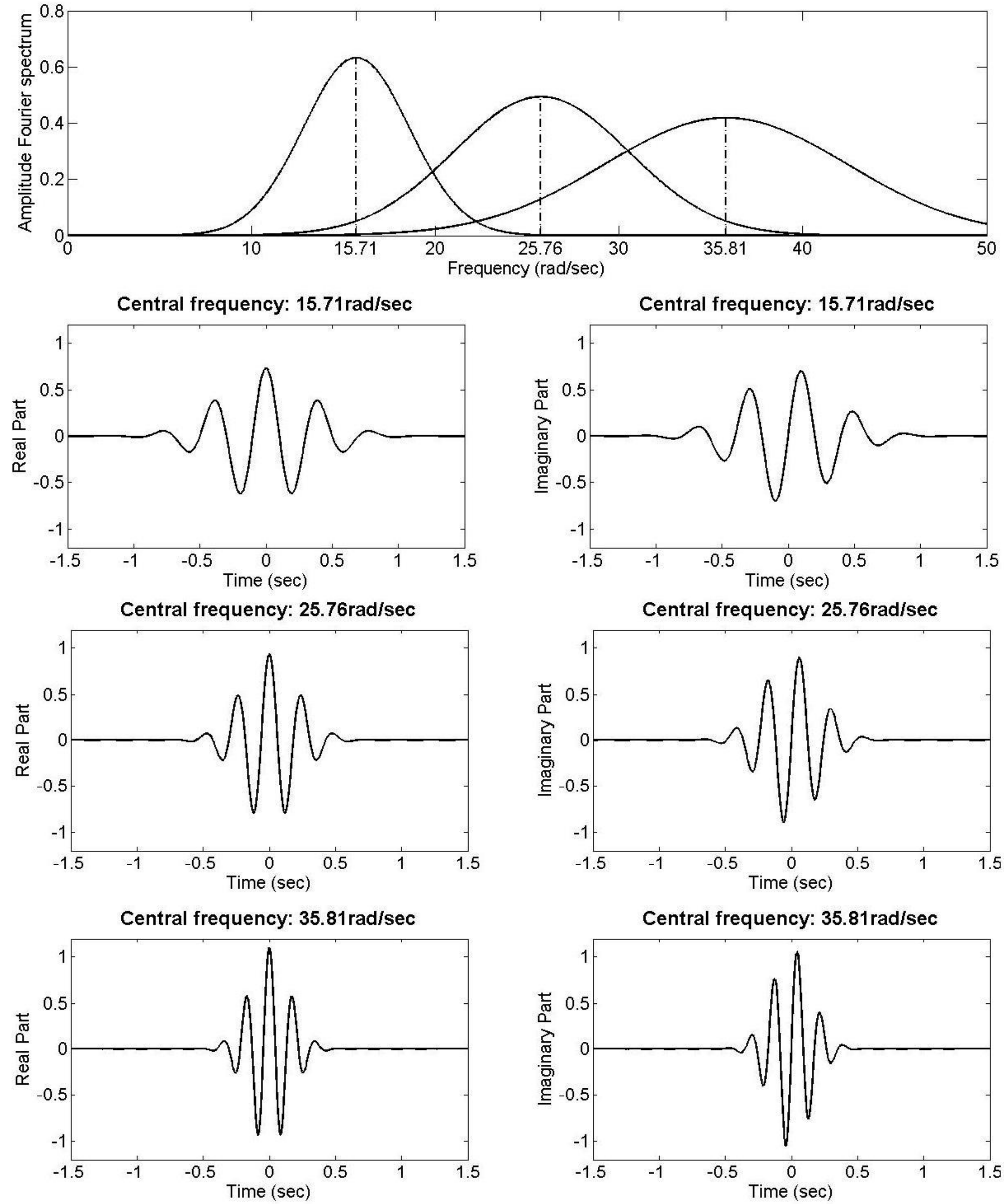
- The constant Ω_b controls the bandwidth of the Gaussian function in the frequency domain

The Continuous (Morlet) Wavelet Transform

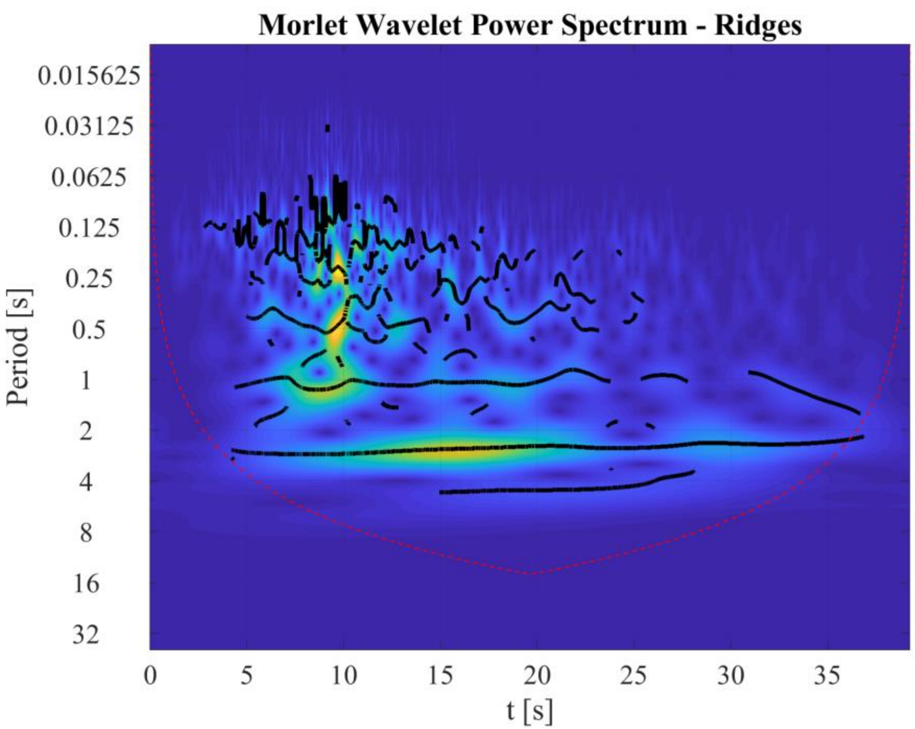
Analytic Morlet wavelets

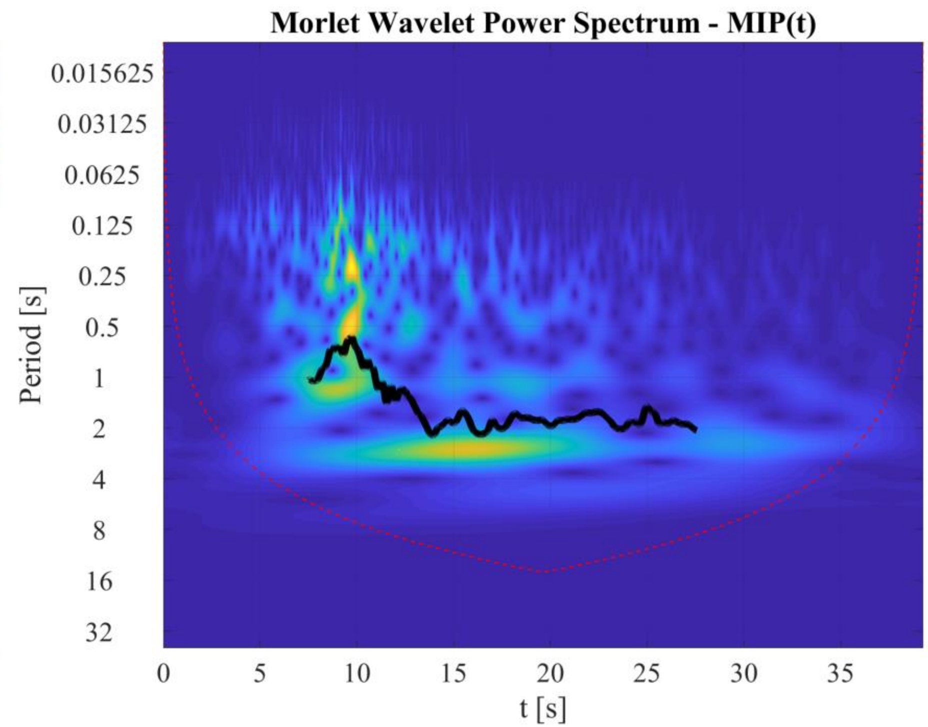
- The scaling operation by $\alpha<1$ moves the central frequency Ω_c/α towards higher frequency levels.
- It also compresses (narrows) the time domain waveforms which leads to reduced resolution in the frequency domain (uncertainty principle).





The mean instantaneous period (MIP) of GMs





A wavelet based **time-varying instantaneous period (MIP)** can be defined as (Margnelli/Giaralis 2015):

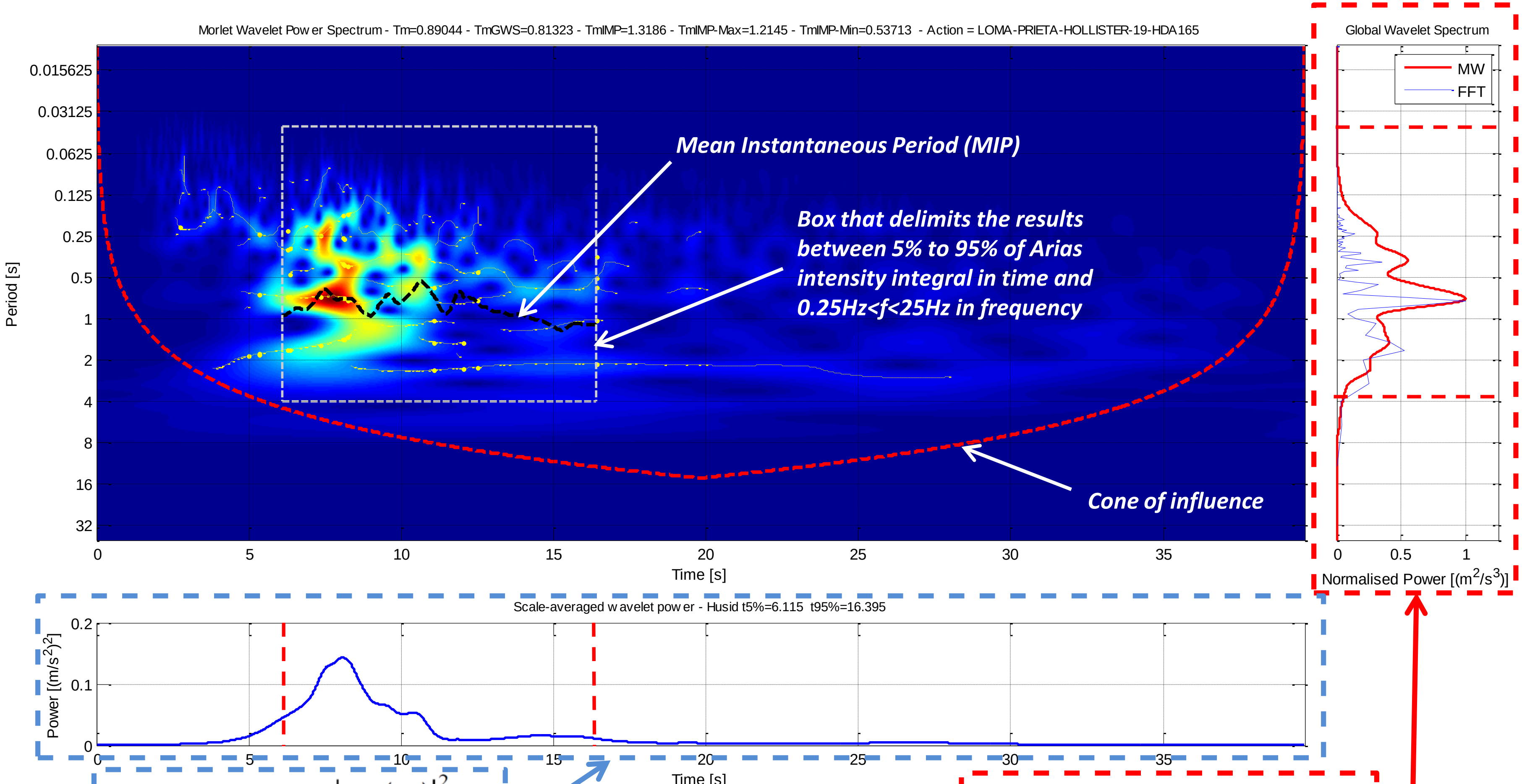
$$W(s,n) = \sum_{n'=0}^{N-1} x[n'] \psi^{*}\left(\frac{(n'-n)\Delta t}{s}\right)$$

$$\text{MIP}[n] = \text{MIP}(n\Delta t) = \frac{\sum_{s=S_1}^{S_2} \left|W(s,n)\right|^2 T_{eff}(s)}{\sum^{S_2} \left|W(s,n)\right|^2}$$

$$for \quad floor\left(\frac{t_{05}}{\Delta t}\right) \leq n \leq ceil\left(\frac{t_{95}}{\Delta t}\right)$$

Frequency range: [0.25 25]Hz

The mean instantaneous period (MIP) of GMs



$$\overline{W}_n^{\,2} = \frac{\delta j\,\delta t}{C_\delta}\sum_{j=j_1}^{j_2}\frac{\left|W_n\left(s_j\right)\right|^2}{s_j}$$

Averaging in scale
(scale-averaged wavelet power)

$$\overline{W}^{\,2}(s) = \frac{1}{N}\sum_{n=0}^{N-1}\left|W_n(s)\right|^2$$

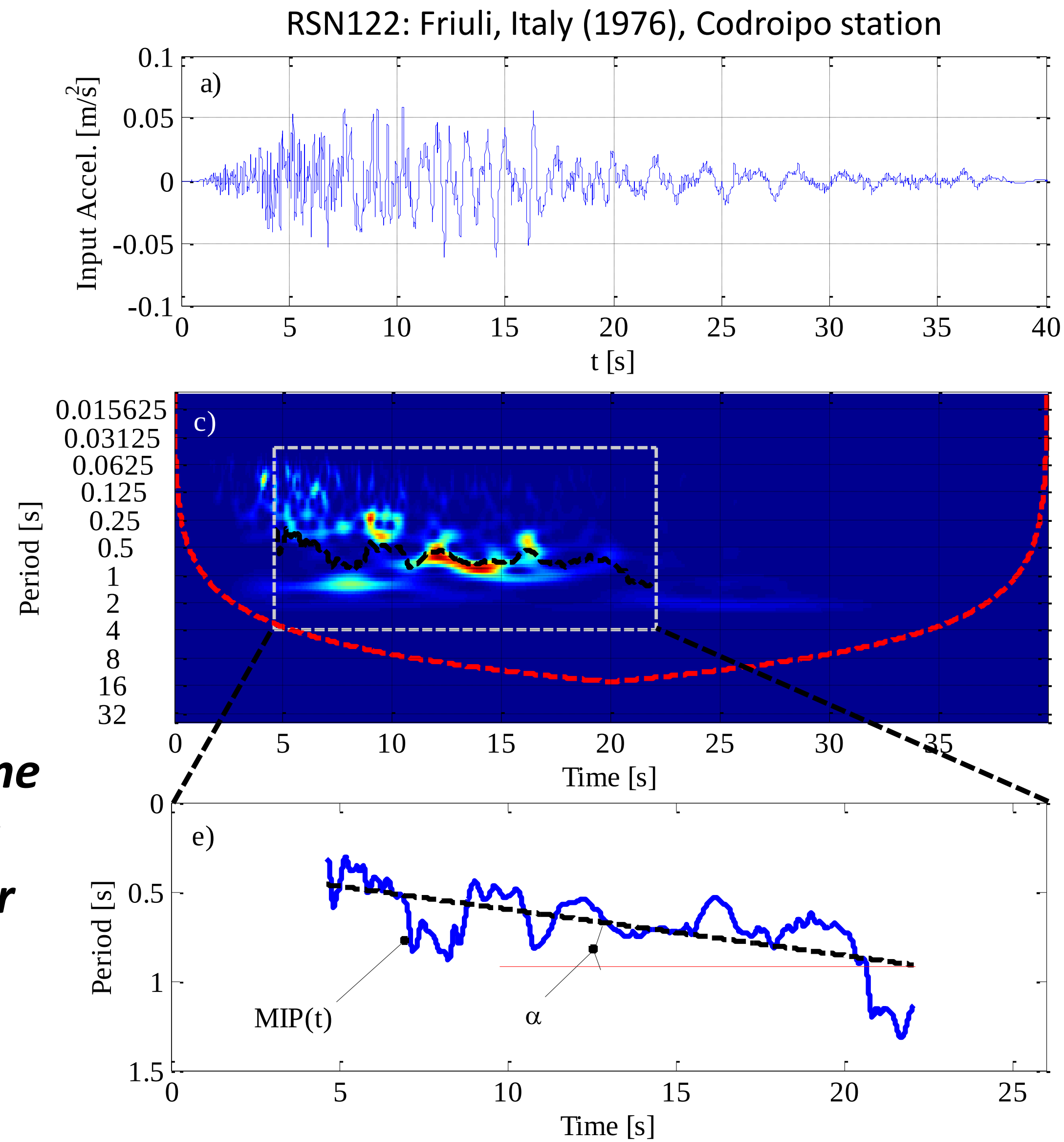
Averaging in time
(global wavelet spectrum)

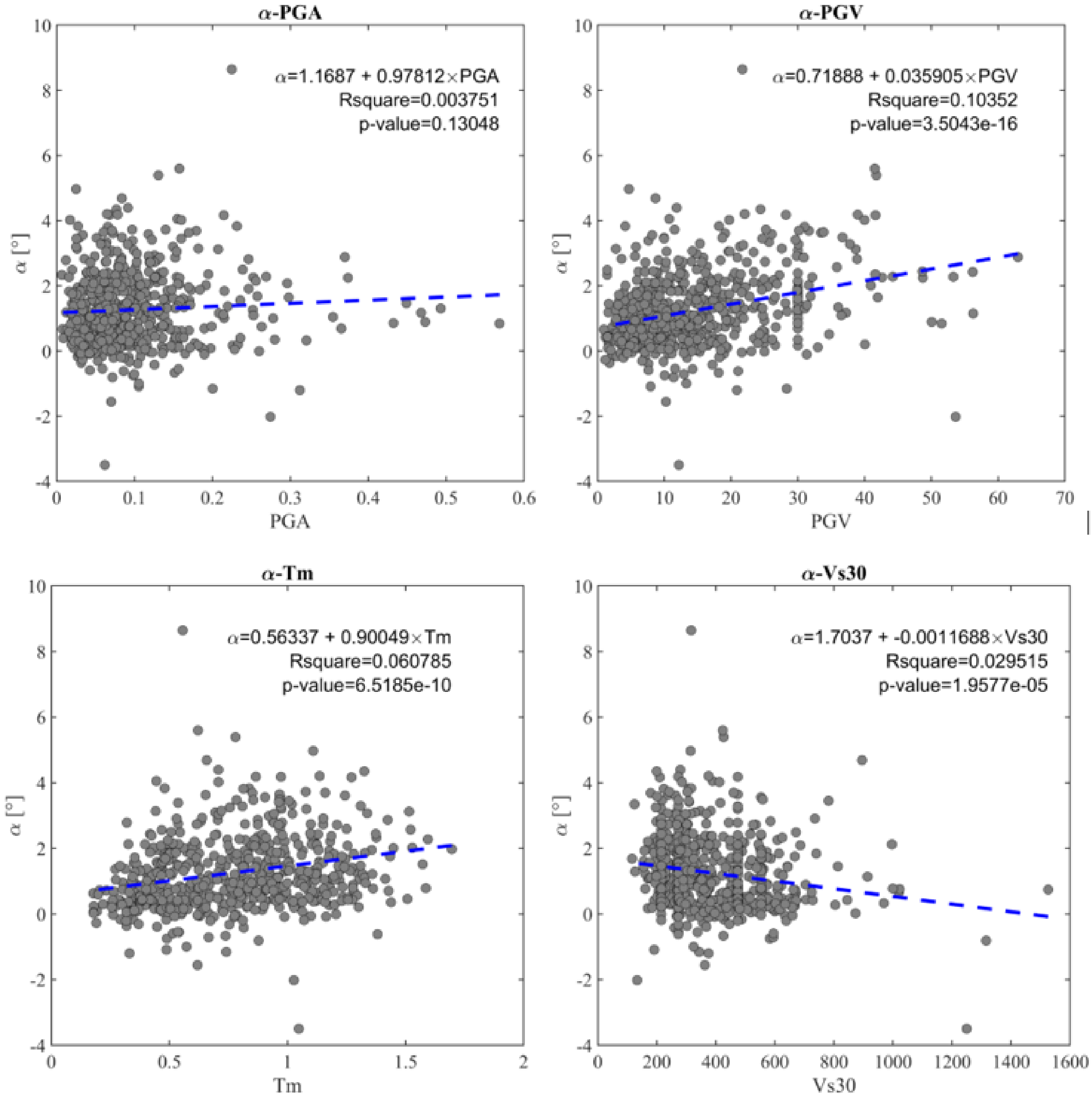
The average slope "alpha" ***α*** of the MIP

The angle "alpha" (α) is a scalar that serves well the purpose of quantifying the **evolution of the mean frequency content in time.** (Margnelli/Giaralis 2017)

It is the **average** slope of the MIP in time measured in degrees.

Higher α== faster variation in time of the mean frequency content from high to low frequencies (or short to long periods)





α-PGA
α=1.1687 + 0.97812×PGA
Rsquare=0.003751
p-value=0.13048
α [°]
PGA
α-PGV
α=0.71888 + 0.035905×PGV
Rsquare=0.10352
p-value=3.5043e-16
α [°]
PGV
α-Tm
α=0.56337 + 0.90049×Tm
Rsquare=0.060785
p-value=6.5185e-10
α [°]
Tm
α-Vs30
α=1.7037 + -0.0011688×Vs30
Rsquare=0.029515
p-value=1.9577e-05
α [°]
Vs30

Spectrally equivalent ground motion ensembles of different α values

Selection of 611 recorded GM pairs (1222 GMs) from PEER database

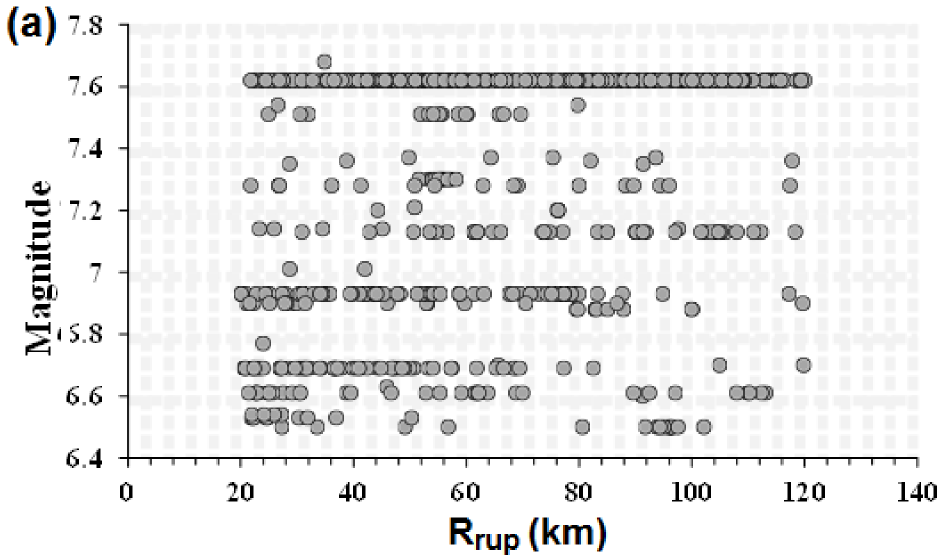
-30 different seismic events

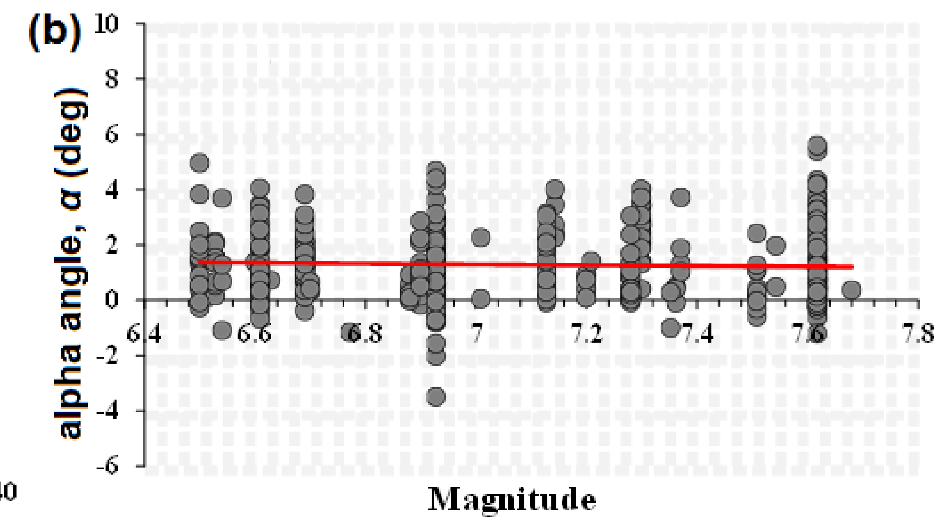
-6.5<M<8.0;

-20km<R_{rup}<120km;

-No pulses

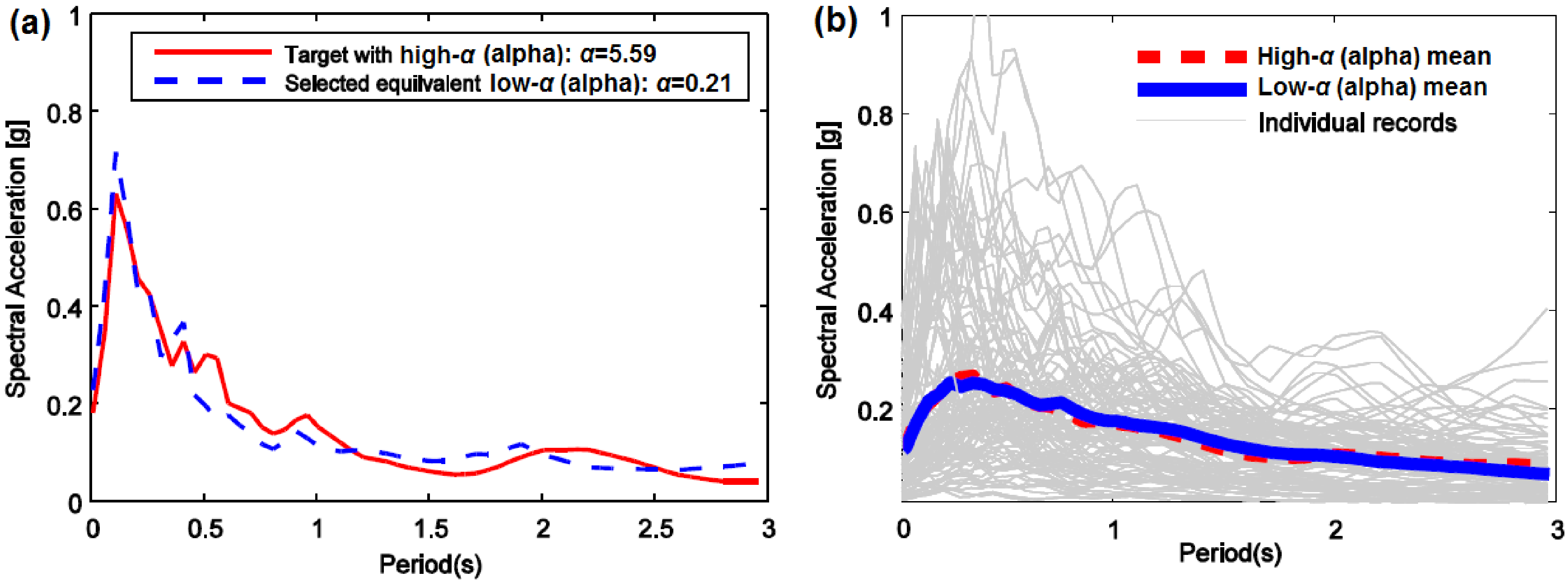
Magnitude-distance, and angle α- magnitude scatter plots for all components





103 GMs with α<0 discarded (less than 10% of GMs)

Spectrally equivalent ground motion ensembles of different α values



A high-α GM set is constructed by taking the 50 GMs with the highest a values from the GM database

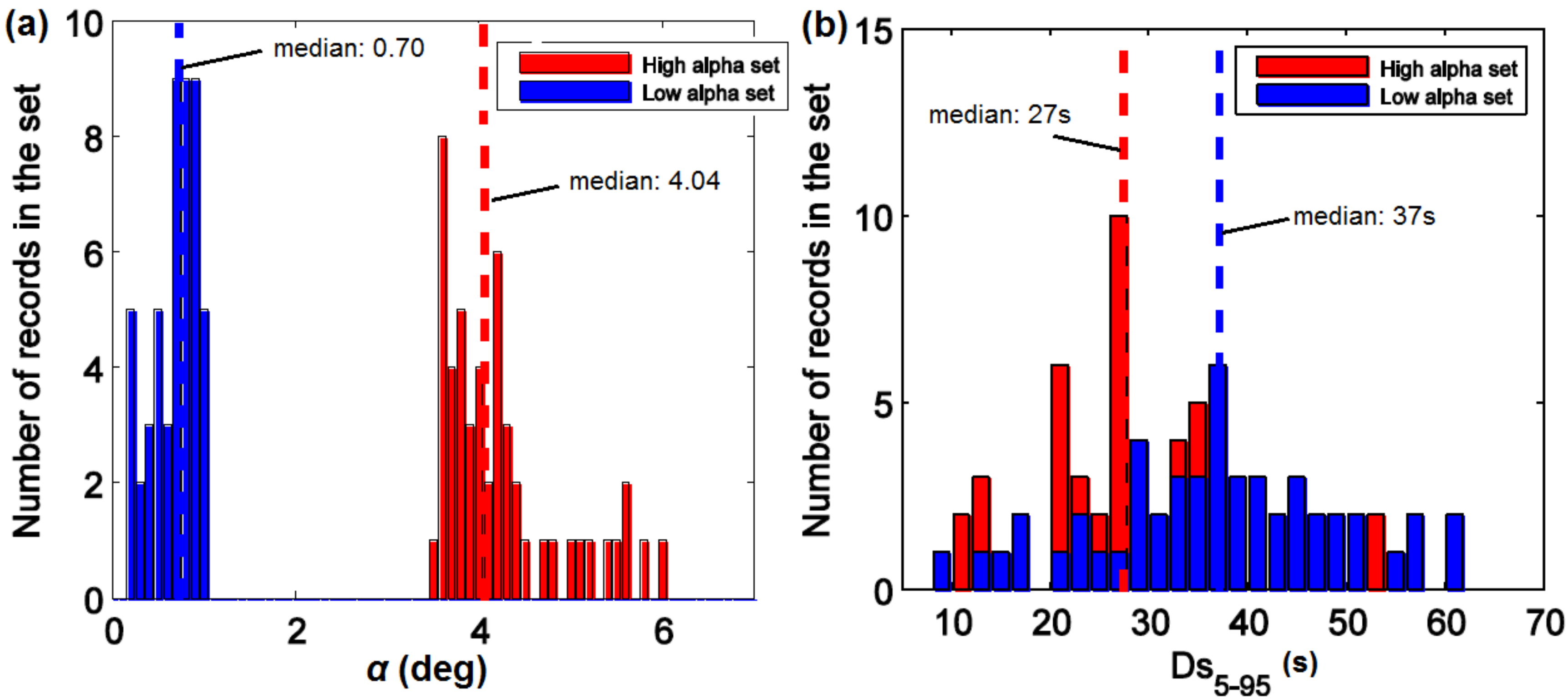
A low-α GM set is constructed by choosing 50 GMs out of half the GMs of the original database with the lowest positive α value possessing equivalent spectral shapes with the GMs of the high-a set using a **greedy matching-pursuit algorithm with scaling** (Chandramohan/Baker/Deirlein 2015)

Matching period range: 0.1s to 3s; max scaling factor allowed= 5

Spectrally equivalent ground motion ensembles of different α values

α histogram of the two sets

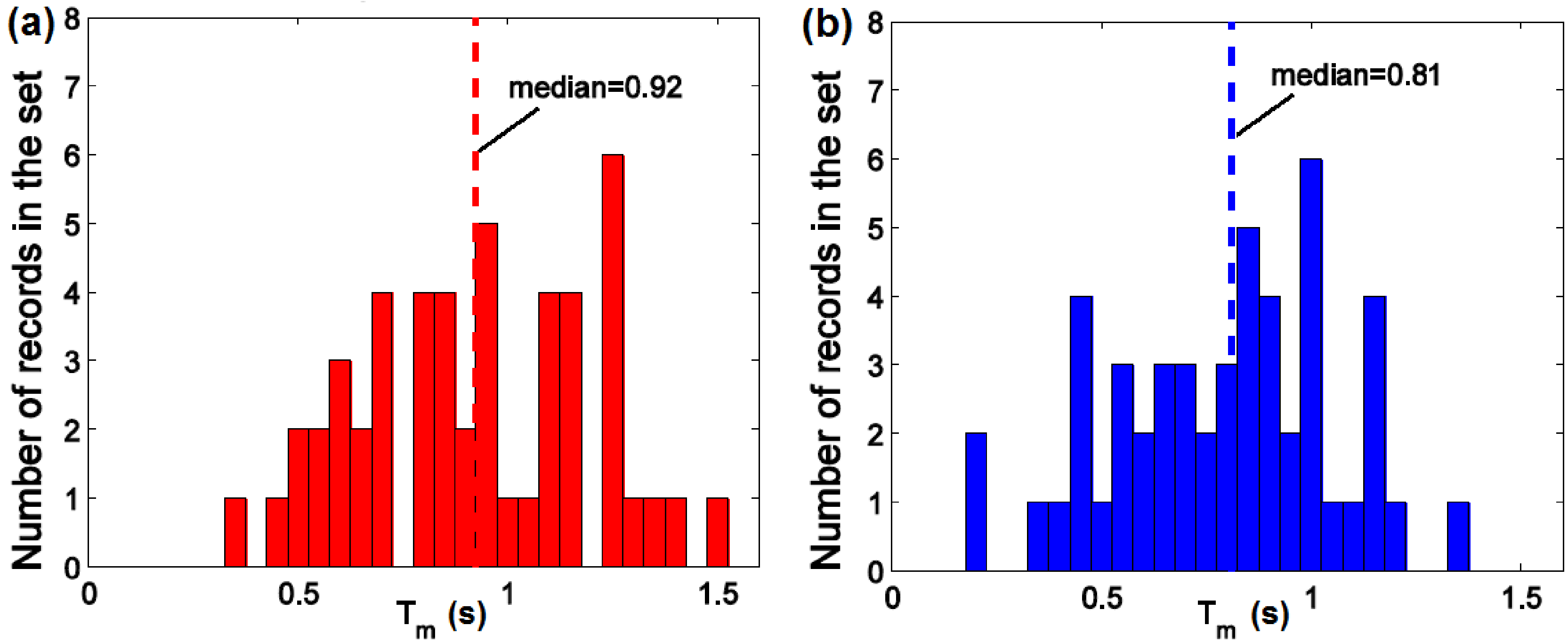
Effective duration histogram of the two sets



The two sets have significantly different α values in terms of median and spread, while non-significant differences in terms of duration which is known to affect peak response of yielding structures (Chandramohan/Baker/Deirlein 2015)

Spectrally equivalent ground motion ensembles of different α values

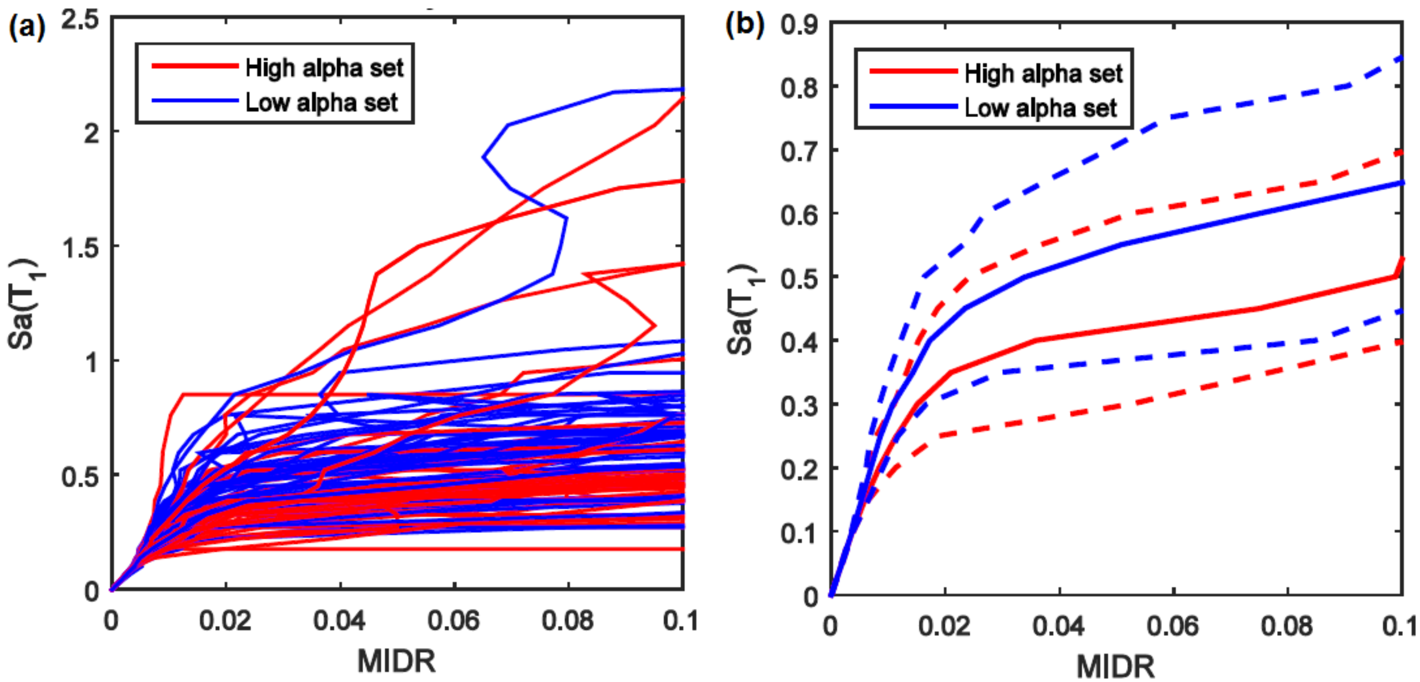
Average frequency content T_m histograms of the two sets



The two GM sets have significantly different rate of change of frequency content in time (i.e., different α values) *but* their average in time stationary frequency content T_m statistics are very similar (T_m is known to affect peak inelastic response of yielding structures: Katsanos/Sextos/Elnashai 2014)

Numerical evaluation of the influence of α to GM damage potential using Incremental dynamic analysis

IDA curves of benchmark 7-storey reinforced concrete MRF designed for PGA=0.60g for the two sets (OpenSees inelastic modelling Kazantzi/Vamvatsikos 2015)



- High-*α* GM set imposes significantly higher drift demands to the structure across a wide range of post-yield limit states
- *20% higher scaling is required to the median Sa(T_1) of the low-α GM set to induce the same MIDR as the high-α GM set for MIDR= 0.02*
- *Up to 25% higher scaling is required to the median Sa(T_1) of the low-α GM set to induce the same MIDR as the high-α GM set for MIDRs≥0.04*

Numerical evaluation of the Influence of α to GM damage potential using Incremental dynamic analysis

Difference in the median seismic structural collapse demand persists even after re-scaling using more efficient IMs, e.g:

$$\mathrm{AvgSA} = \left(\prod_{j=1}^{J} \mathrm{Sa}\left(\mathrm{T}_j\right) \right)^{1/J}$$

Set of *J* natural periods, T_j, equally spaced in the range T_2 to $1.5T_1$ by an increment of 0.1s
(Kohrangi/Bazzuro/Vamvatsikos/Spillatura 2017)