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Accounting for non-stationary frequency content in Earthquake Engineering: Can wavelet analysis be useful after all?

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akt II

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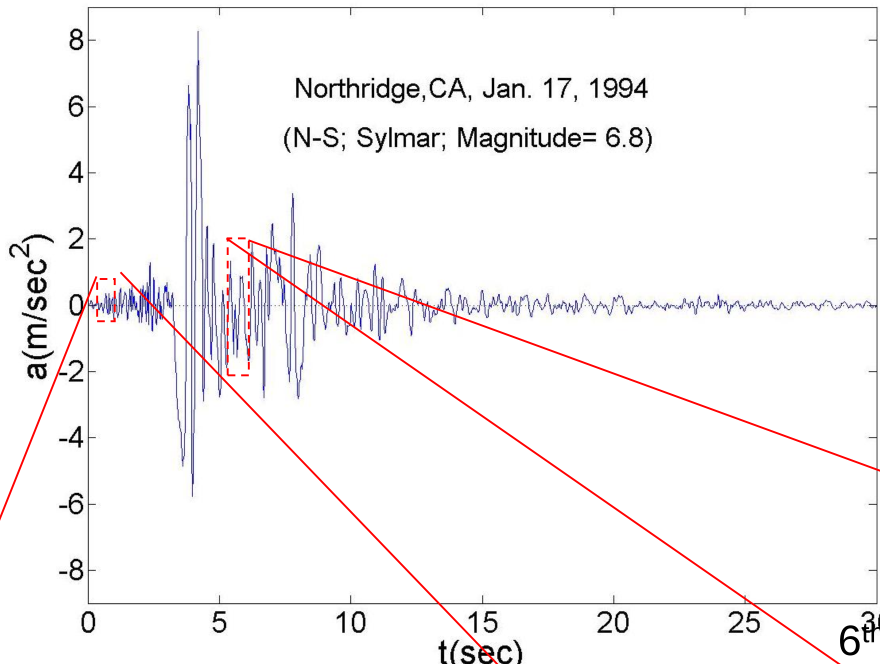
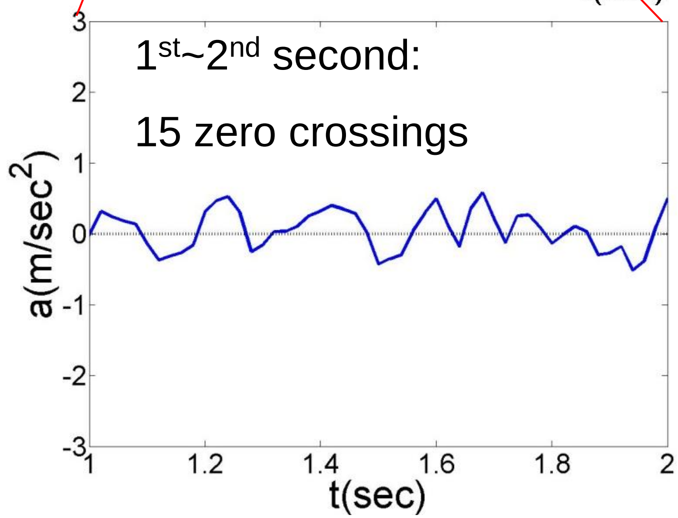
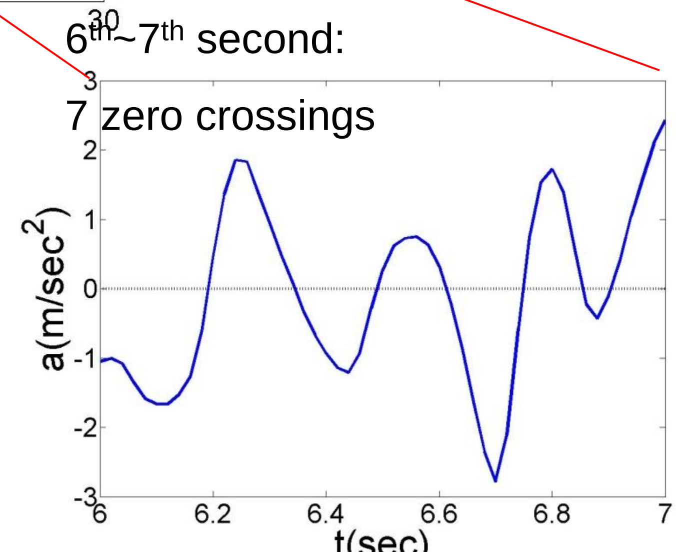
Introduction / Motivation

Typical earthquake accelerograms exhibit a time-evolving frequency composition due to the dispersion of the propagating seismic waves, and a time-decaying intensity after a short initial period of development.







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Introduction / Motivation

Transient signals encountered in earthquake engineering and structural dynamics are inherently **non-stationary:**

Both their **frequency content** and **amplitude** vary with time.

Earthquake induced strong ground motion (accelerograms) GMs:

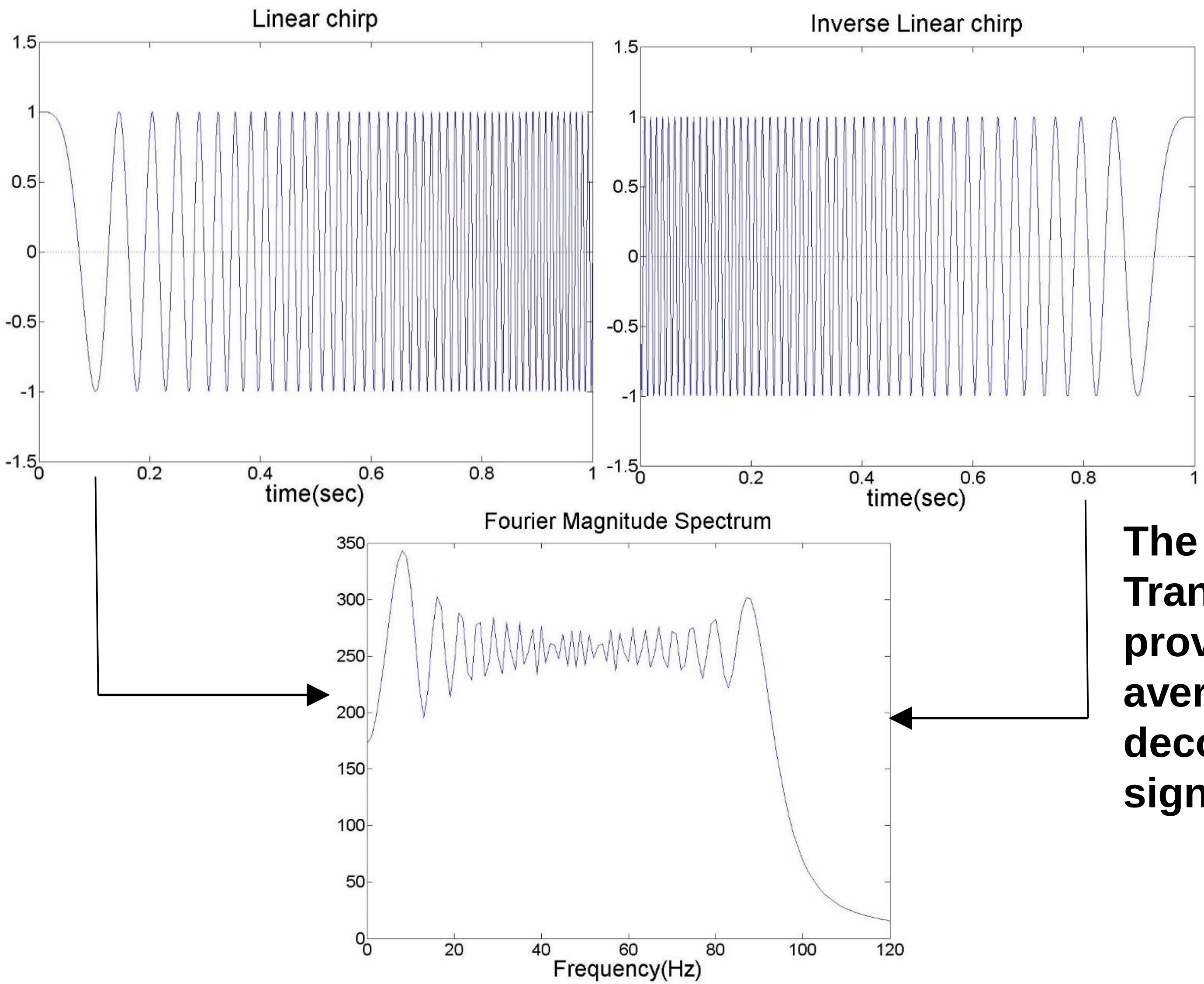
Exhibit a time-evolving frequency composition due to the dispersion of the propagating seismic waves, and a time-decaying intensity after a short initial period of development.

Response time histories of yielding structures under seismic excitation:

Their evolving frequency content carries information about the possible level of (global) structural damage (e.g. degradation of the effective natural frequencies).

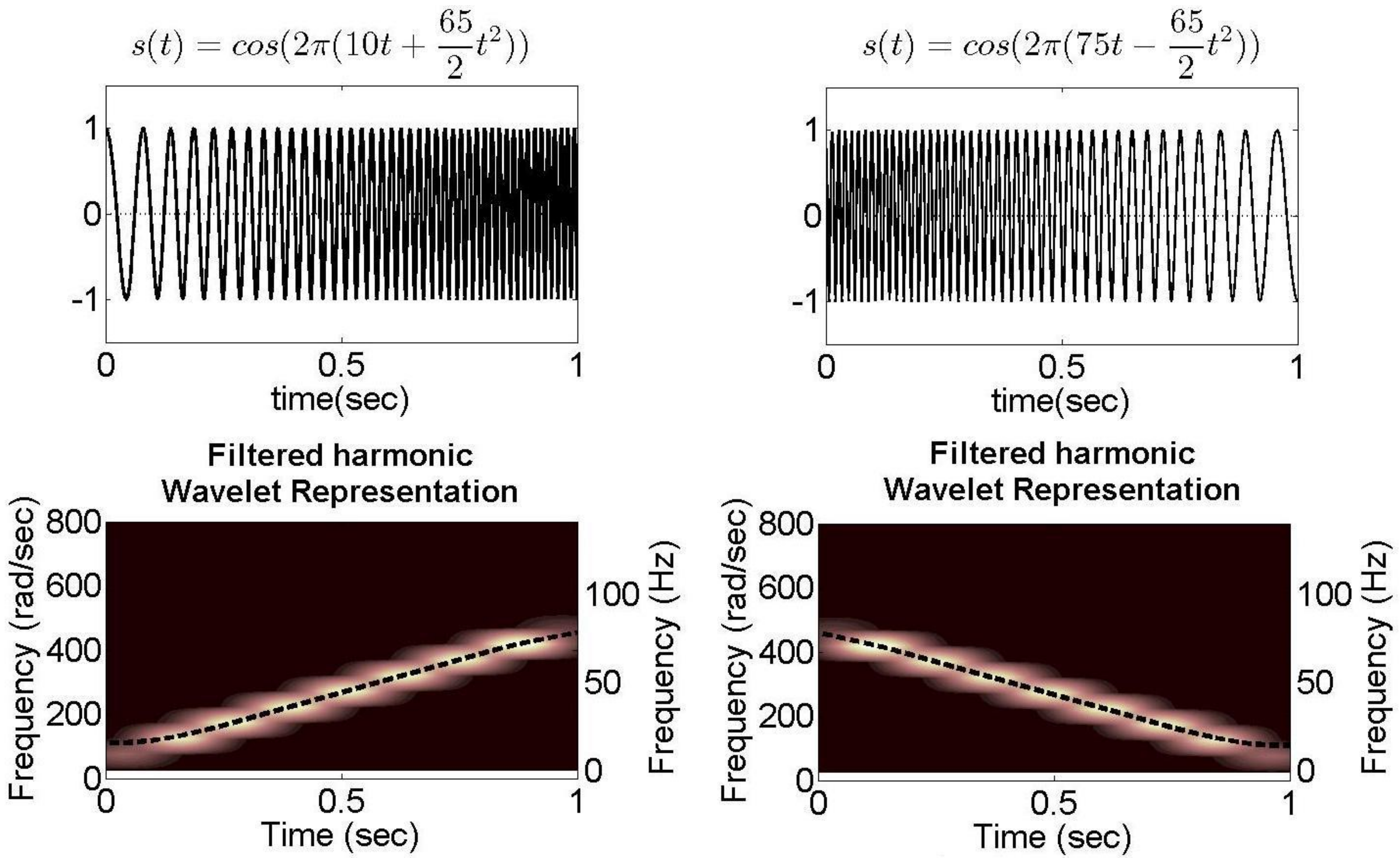
Such signals call for a joint time- frequency analysis; for it is clear that their time- dependent frequency content cannot be adequately represented by the ordinary Fourier analysis.

Introduction / Motivation



The ordinary Fourier Transform (FT) provides only the average spectral decomposition of a signal.

Time-frequency analysis tools provide meaningful non-stationary signal representations



Presentation Outline

- **Introduction / Motivation**
- **The Continuous Wavelet Transform (CWT)**
- **The wavelet-based mean instantaneous period (MIP)**
- **MIP of Recorded Seismic ground motions (GMs)**
- **MIP of Hysteretic Response Signals**
- **The "alpha" *α* angle of the average MIP**
- **The *α* as a GM property for the evolving frequency content**
- **Concluding remarks**

The continuous wavelet transform (CWT)

- The continuous wavelet transform (CWT) given by the equation

$$W(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt$$

decomposes any finite energy signal *f(t)* onto a basis of functions generated by scaling a single mother wavelet function *ψ(t)* by the scale parameter *α* and by shifting it in time by the parameter *b*.

ψ : analyzing or mother wavelet

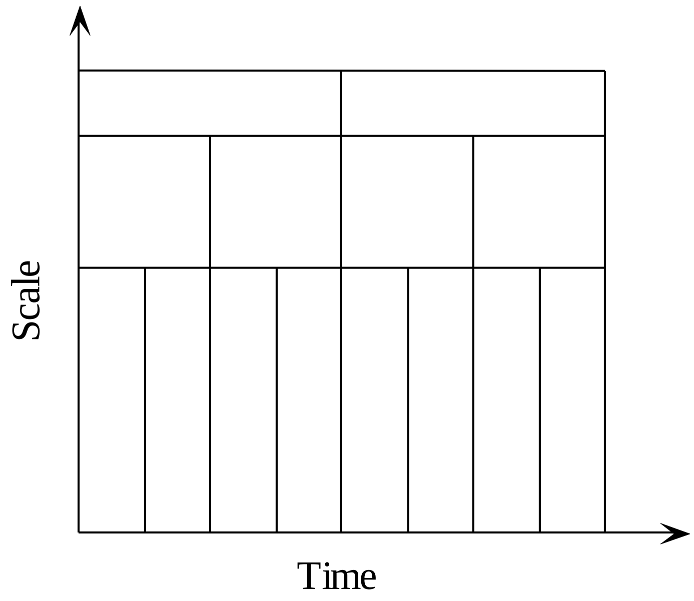
$$\psi(a,b)=\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

Variable size windows are employed

Long duration windows capture lower frequencies (large scales)

Short duration windows are used to capture higher frequencies (small scales)

Heisenberg's uncertainty principle holds



The continuous wavelet transform (CWT)

Such an analysis results in a three-dimensional spectrum having the wavelet coefficients plotted versus time and scale (scalogram). A certain wavelet-dependent relationship between scale and frequency should be established to yield a wavelet- based spectrogram.

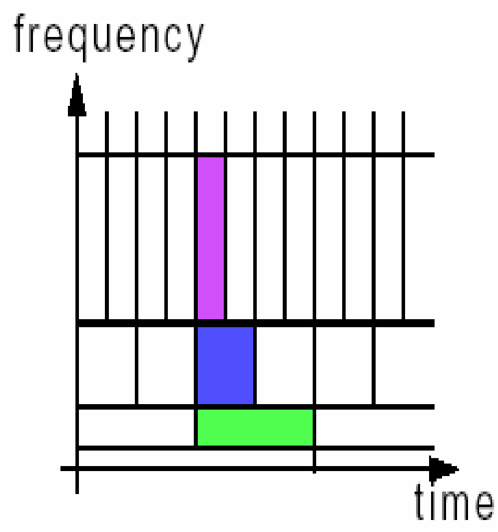
- **Uncertainty Principle**

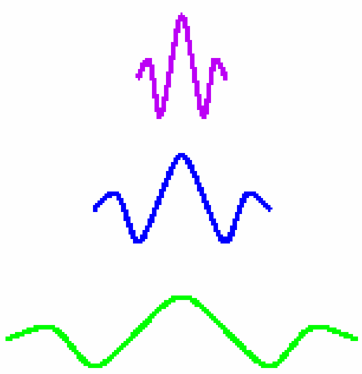
$$\psi(t) \xleftrightarrow{\textit{Fourier Pairs}} \hat{\Psi}(\omega)$$

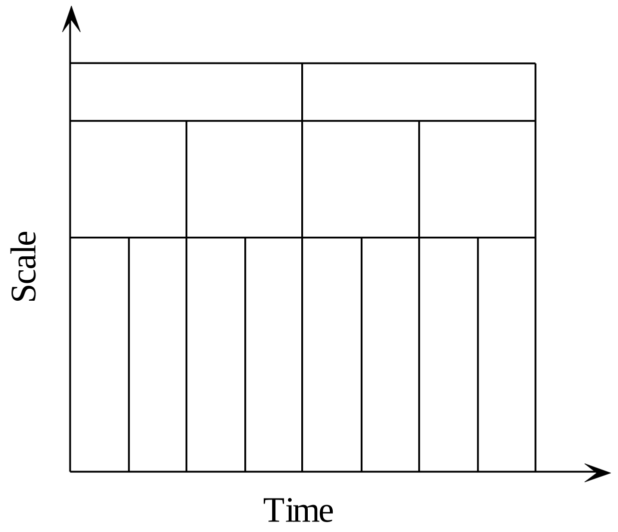
$$\frac{1}{\sqrt{\alpha}}\psi\left(\frac{t-b}{\alpha}\right) \xleftrightarrow{\textit{Fourier Pairs}} \sqrt{\alpha}\hat{\Psi}(\alpha\omega)\exp(-i\omega\alpha b)$$

Reciprocal relationship between scale-frequency:

Frequency= constant/scale

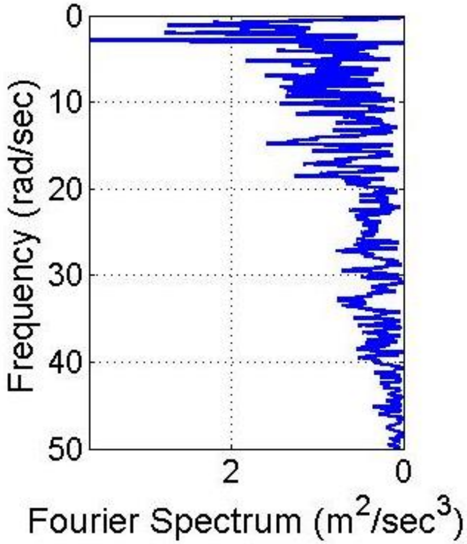


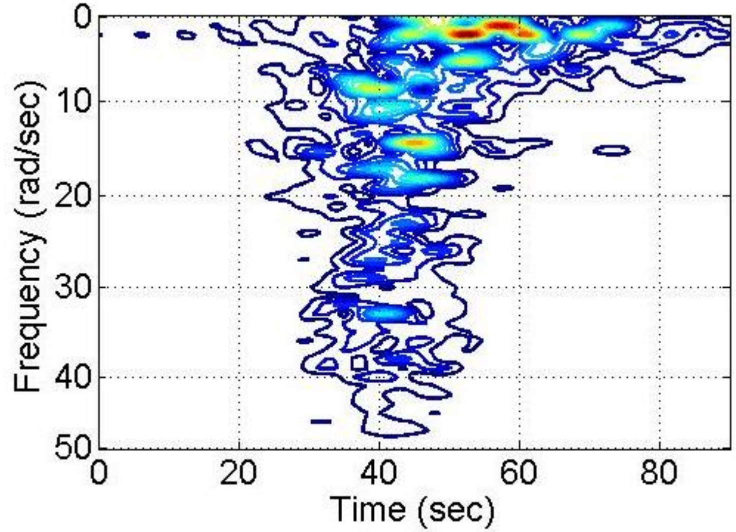




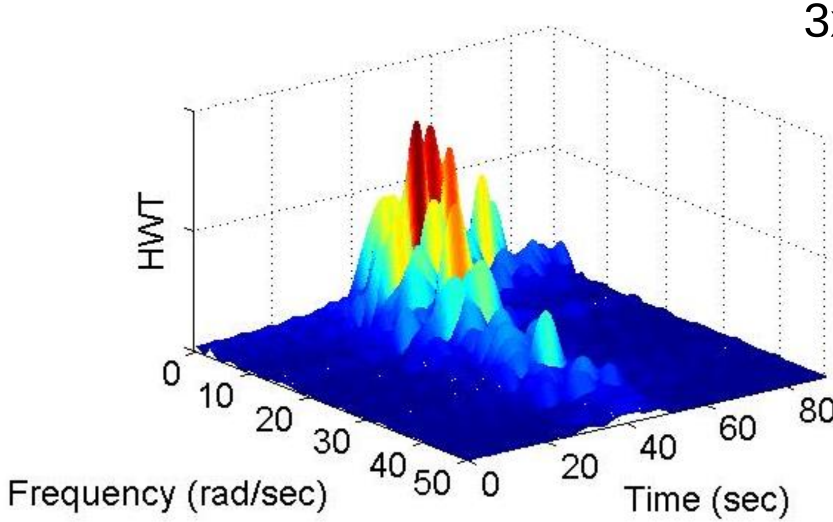
The continuous wavelet transform (CWT)

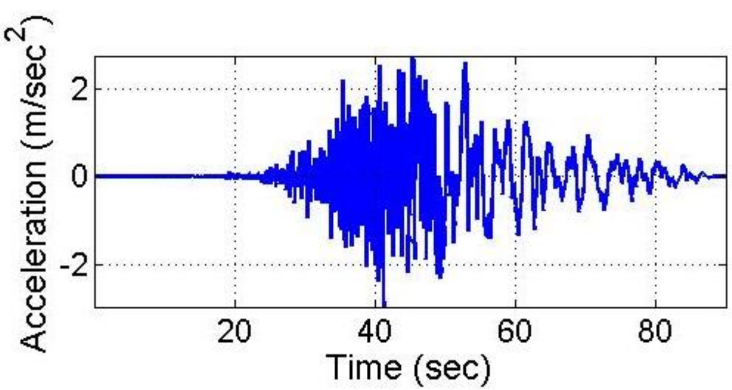
Is CWT useful?





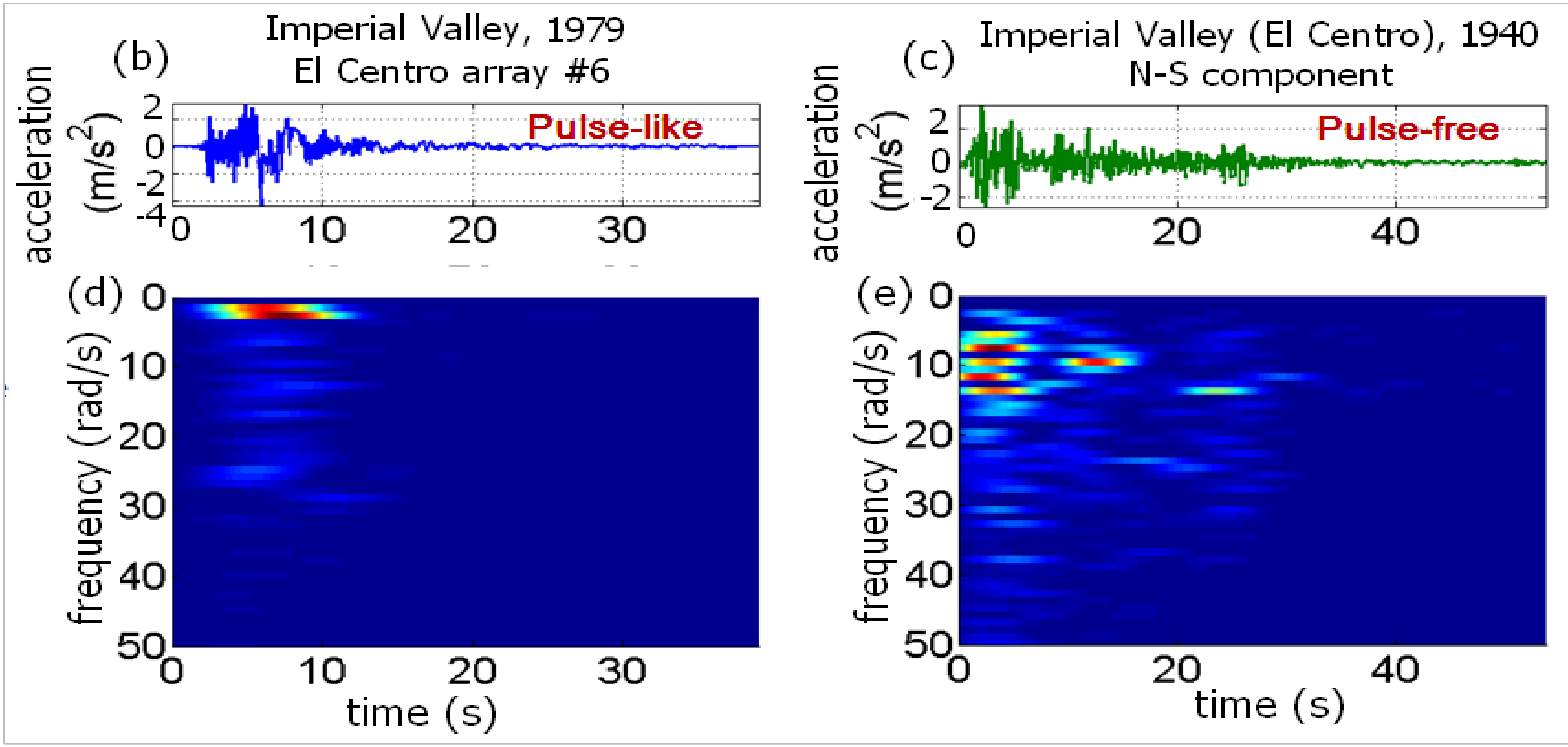
3x 1999 Chi – Chi, Taiwan (station TCU098)





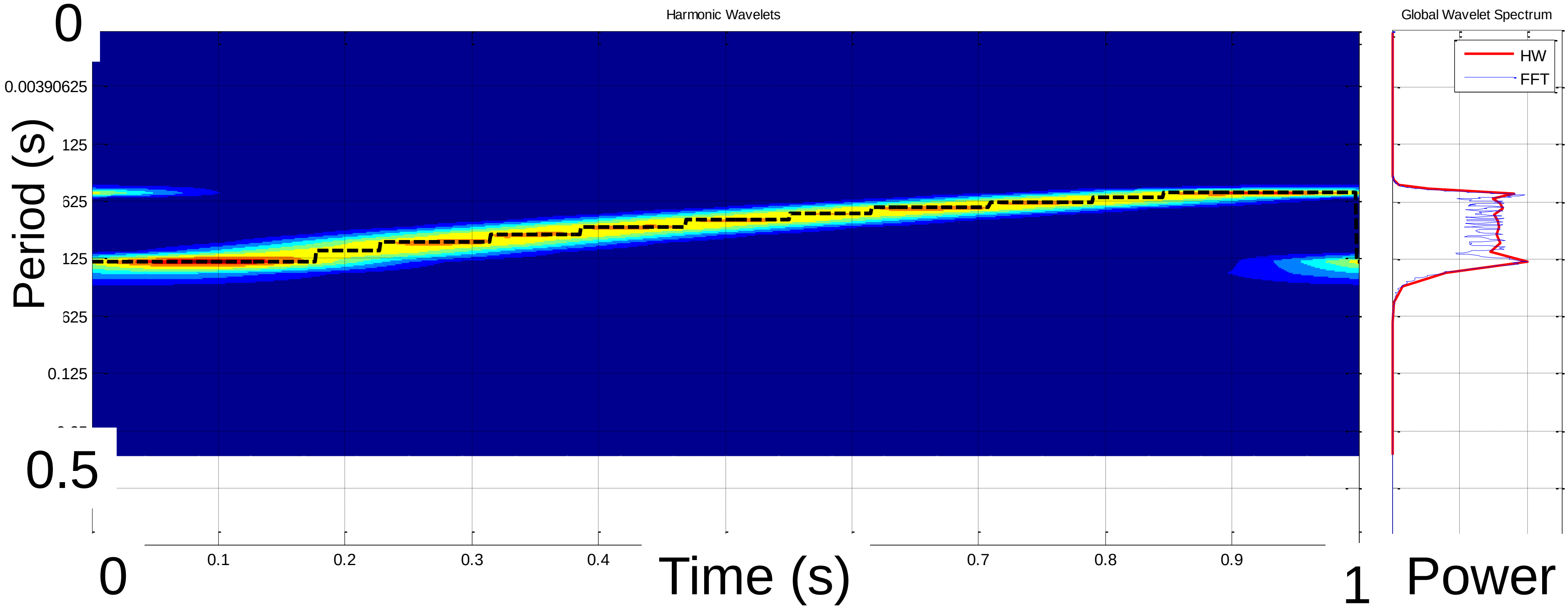
The continuous wavelet transform (CWT)

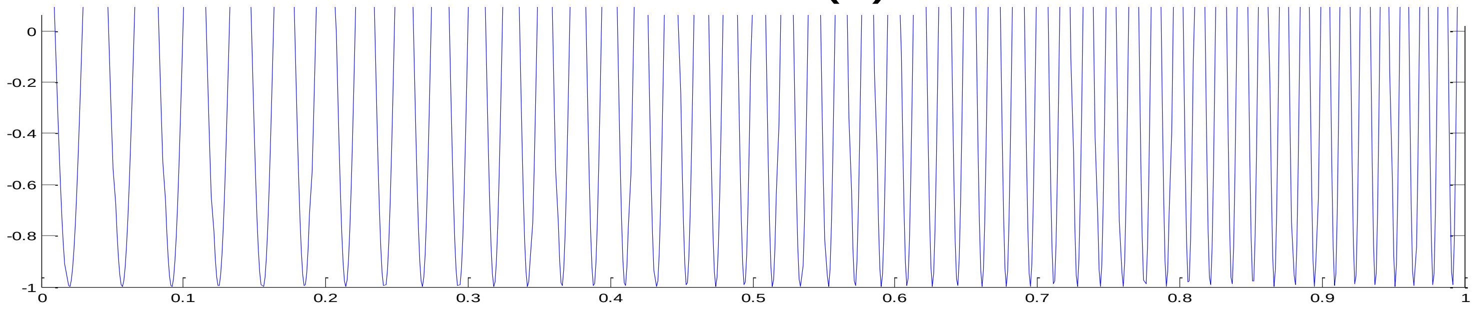
Is CWT useful?



The continuous wavelet transform (CWT)

But we need to know what we are aiming for: Time or Frequency (resolution/ smoothness/bias)???





The continuous wavelet transform (CWT)

But we need to know what we are aiming for:

Time or Frequency
(resolution/smoothness/bias)???

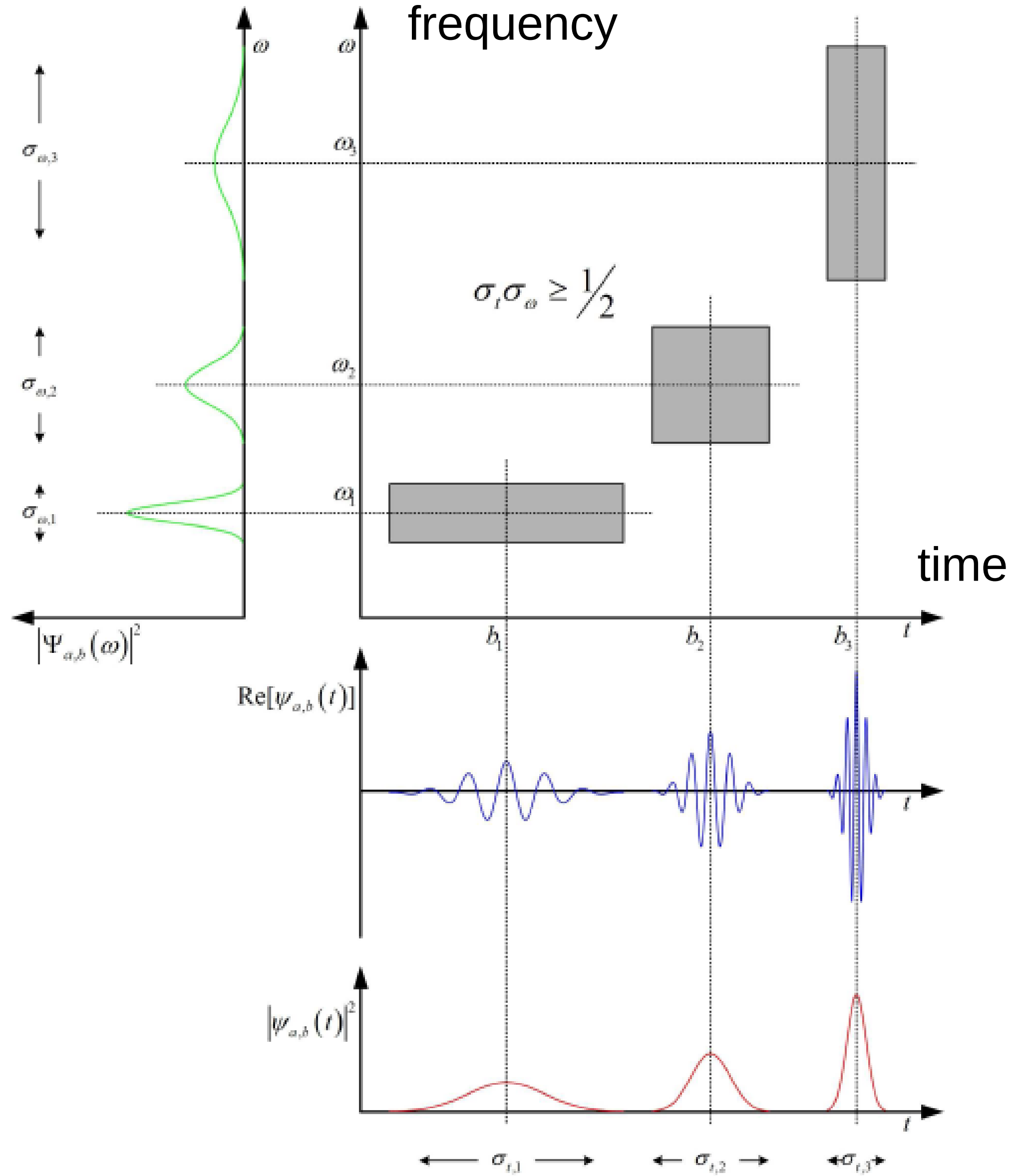
Uncertainty principle

Resolution trade-off

Wavelet shape

Smoothness

etc.



Modified complex Morlet wavelets

- At scale *α* and time position *b* the modified Morlet wavelet is given by

$$\psi^{M}\left(\frac{t-b}{a}\right)=\frac{1}{\sqrt{a\pi\Omega_b}}\exp\left(i\frac{\Omega_c}{a}(t-b)-\frac{(t-b)^2}{a^2\Omega_b}\right)$$

- Its Fourier transform is a shifted Gaussian function, that is:

$$\hat{\Psi}_b^{M}(a\omega)=\sqrt{a}\exp\left(-\frac{\Omega_b}{4}(a\omega-\Omega_c)^2-ia\omega b\right)$$

- The central (pseudo-) frequency observed at scale *α* is usually computed by

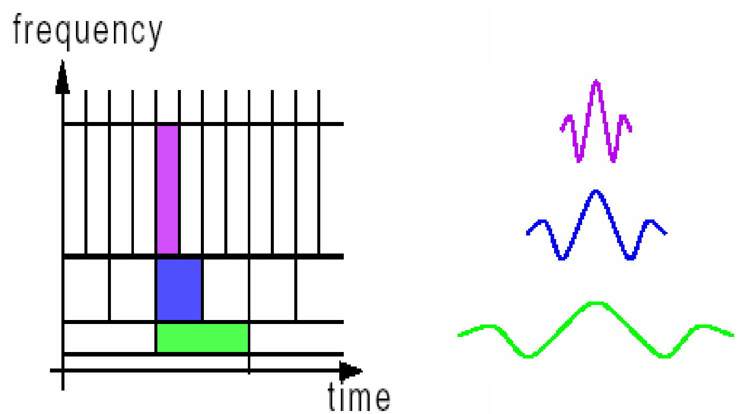
$$\omega_o=\frac{\Omega_c}{a}$$

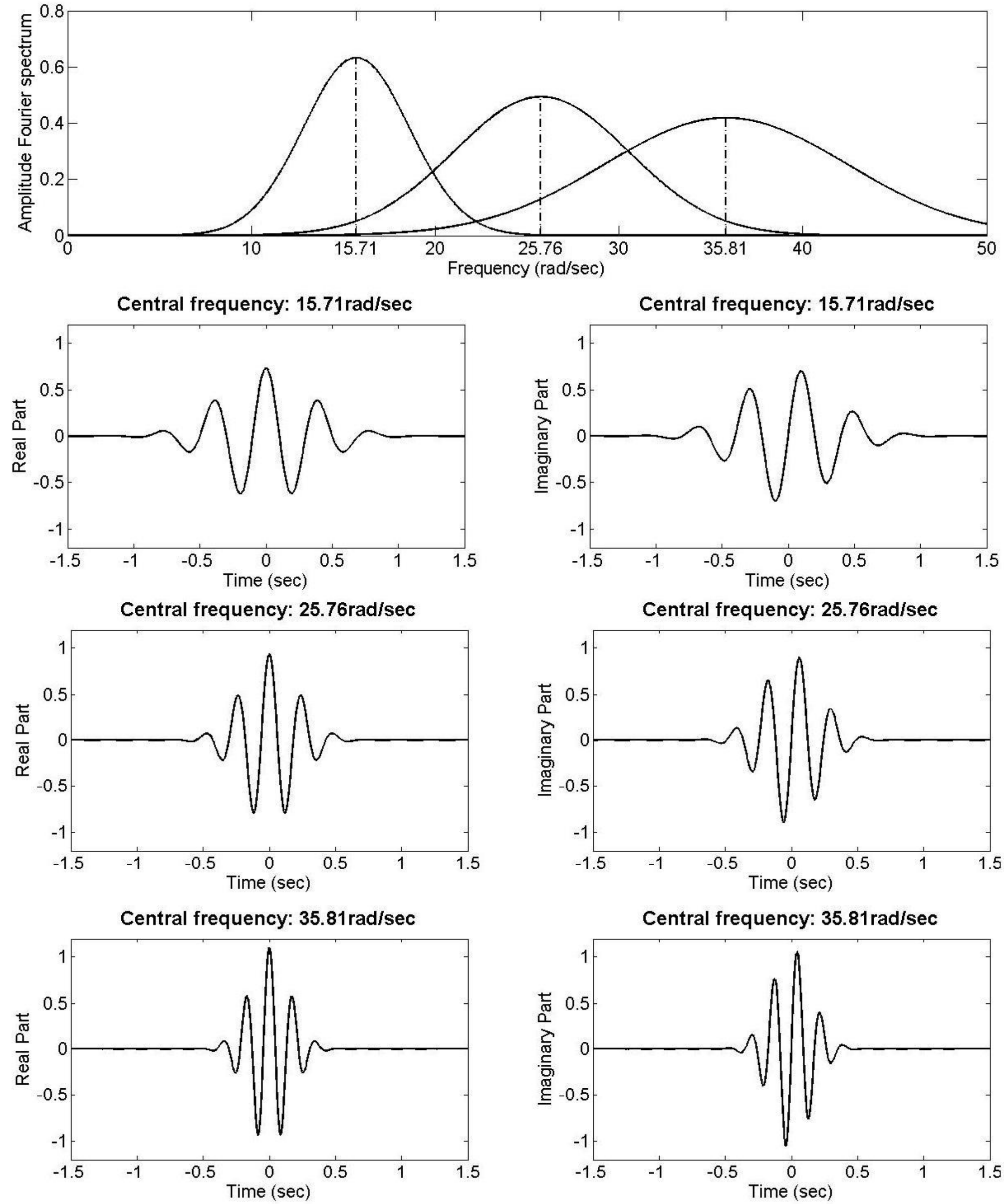
- The constant Ω_b controls the bandwidth of the Gaussian function in the frequency domain

The continuous wavelet transform (CWT)

Modified complex Morlet wavelets

- The scaling operation by $\alpha<1$ moves the central frequency Ω_c/α towards higher frequency levels.
- It also compresses (narrows) the time domain waveforms which leads to reduced resolution in the frequency domain (uncertainty principle).





The continuous wavelet transform (CWT)

Generalized harmonic wavelets

- A *generalized harmonic wavelet of (m,n) scale and k position in time* is constructed as a box-like function *in the frequency domain* (Newland, 1994), that is:

$$\hat{\Psi}_{(m,n),k}(\omega)=\begin{cases}\dfrac{T_o}{2\pi(n-m)}\exp\left(\dfrac{-i\omega kT_o}{(n-m)}\right), & \dfrac{m2\pi}{T_o}\le\omega<\dfrac{n2\pi}{T_o}\\ 0, & otherwise\end{cases}$$

; where T_o is the effective duration of the signal to be analyzed.

- ***In the time domain*** it is a complex- valued function given by

$$\psi_{(m,n),k}(t)=\frac{\sin\left\{\pi\left(\dfrac{t}{T_o}-\dfrac{k}{n-m}\right)(n-m)\right\}}{\pi\left(\dfrac{t}{T_o}-\dfrac{k}{n-m}\right)(n-m)}\exp\left(i\pi\left(\frac{t}{T_o}-\frac{k}{n-m}\right)(m+n)\right)$$

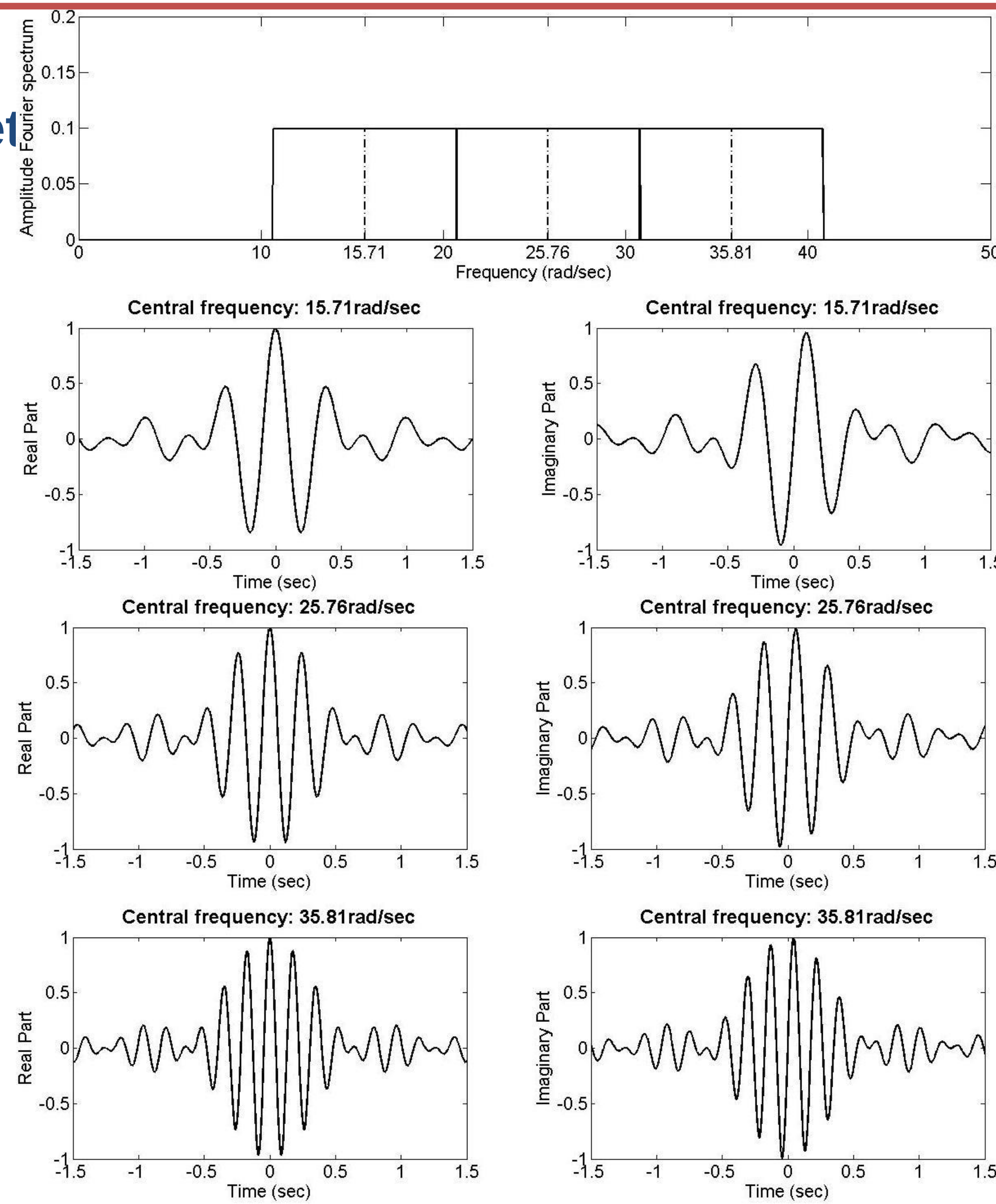
- Central frequency at scale *(m,n): (m+n)π/*T_o

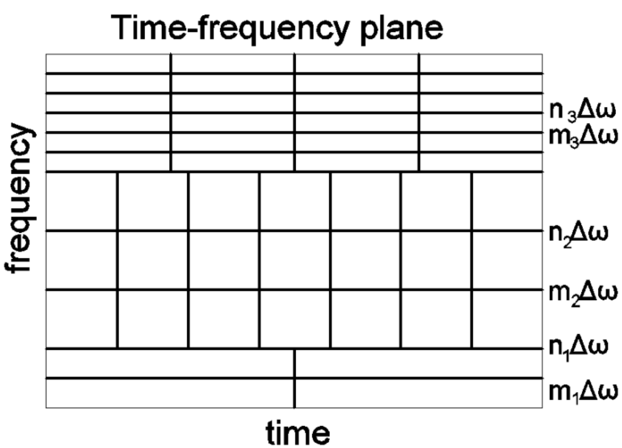
- Bandwidth in the frequency domain at scale *(m,n): (n-m)2π/*T_o

The continuous wavelet transform (CWT)

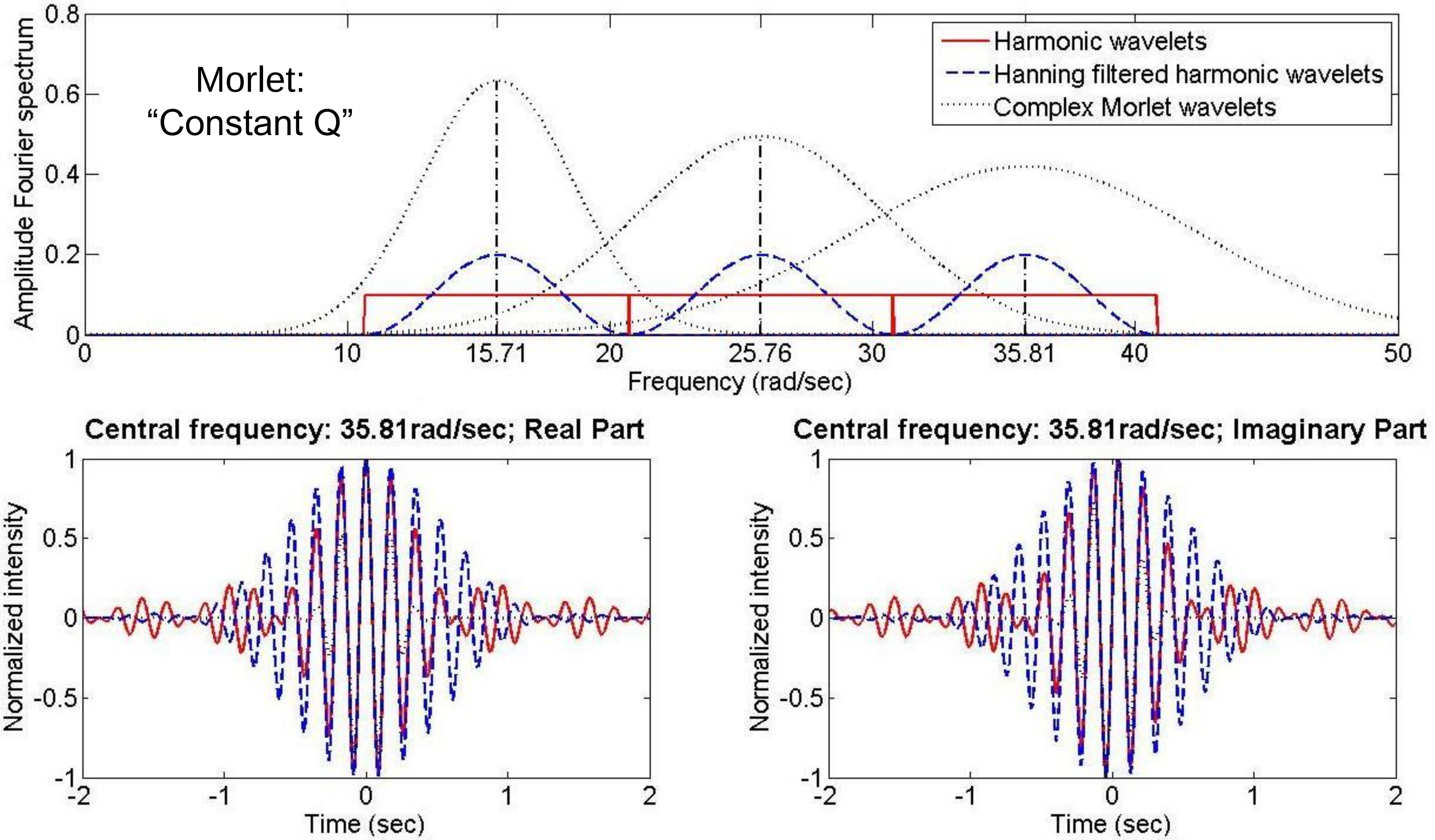
Generalized harmonic wavelets

- Harmonic wavelets of different scales can have arbitrarily chosen bandwidths throughout the frequency domain. This is because the scales are defined by two parameters (*m*,*n*), as opposed to one (*α*) in the case of common wavelets used in the context of the CWT.





The continuous wavelet transform (CWT)



The wavelet-based mean instantaneous period (MIP)

Can we make CWT more useful?
-> GM non-stationary frequency content characterization?

Mean Period (e.g. Rathje et al.1998) is defined starting from DFT as:

$$\hat{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-i\omega_k n} \quad \Rightarrow \quad T_m = \frac{\sum_{k=K_1}^{K_2} \left|\hat{X}[k]\right|^2 \frac{2\pi}{\omega_k}}{\sum_{k=K_1}^{K_2} \left|\hat{X}[k]\right|^2}$$

Evolution in time of the Mean period

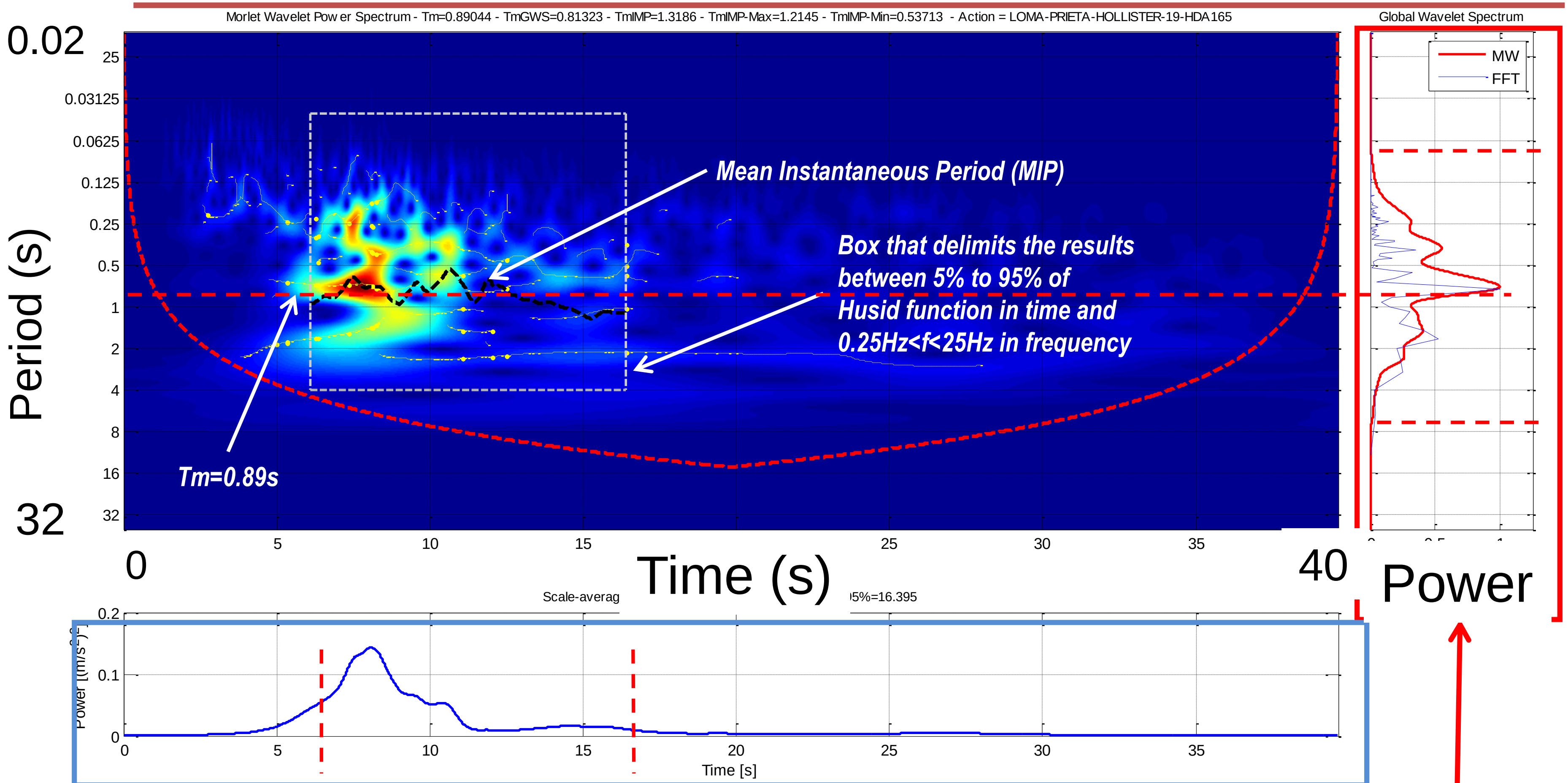
A wavelet based ***time-varying instantaneous period (MIP)*** *can be defined as (Margnelli/Giaralis 2015):*

$$W(s,n) = \sum_{n'=0}^{N-1} x[n'] \psi^* \left(\frac{(n'-n)\Delta t}{s} \right) \quad \Rightarrow \quad \mathrm{MIP}[n] = \mathrm{MIP}(n\Delta t) = \frac{\sum_{s=S_1}^{S_2} \left|W(s,n)\right|^2 T_{eff}(s)}{\sum_{s=S_1}^{S_2} \left|W(s,n)\right|^2}$$

Frequency range: [0.25 25]Hz

$$for \quad floor\left(\frac{t_{05}}{\Delta t}\right) \le n \le ceil\left(\frac{t_{95}}{\Delta t}\right)$$

The wavelet-based mean instantaneous period (MIP)



$$\overline{W}_n^{\,2} = \frac{\delta j\,\delta t}{C_\delta} \sum_{j=j_1}^{j_2} \frac{\left|W_n\left(s_j\right)\right|^2}{s_j}$$

Averaging in scale
(scale-averaged wavelet power)

$$\overline{W}^{\,2}(s) = \frac{1}{N} \sum_{n=0}^{N-1} \left|W_n(s)\right|^2$$

Averaging in time
(global wavelet spectrum)

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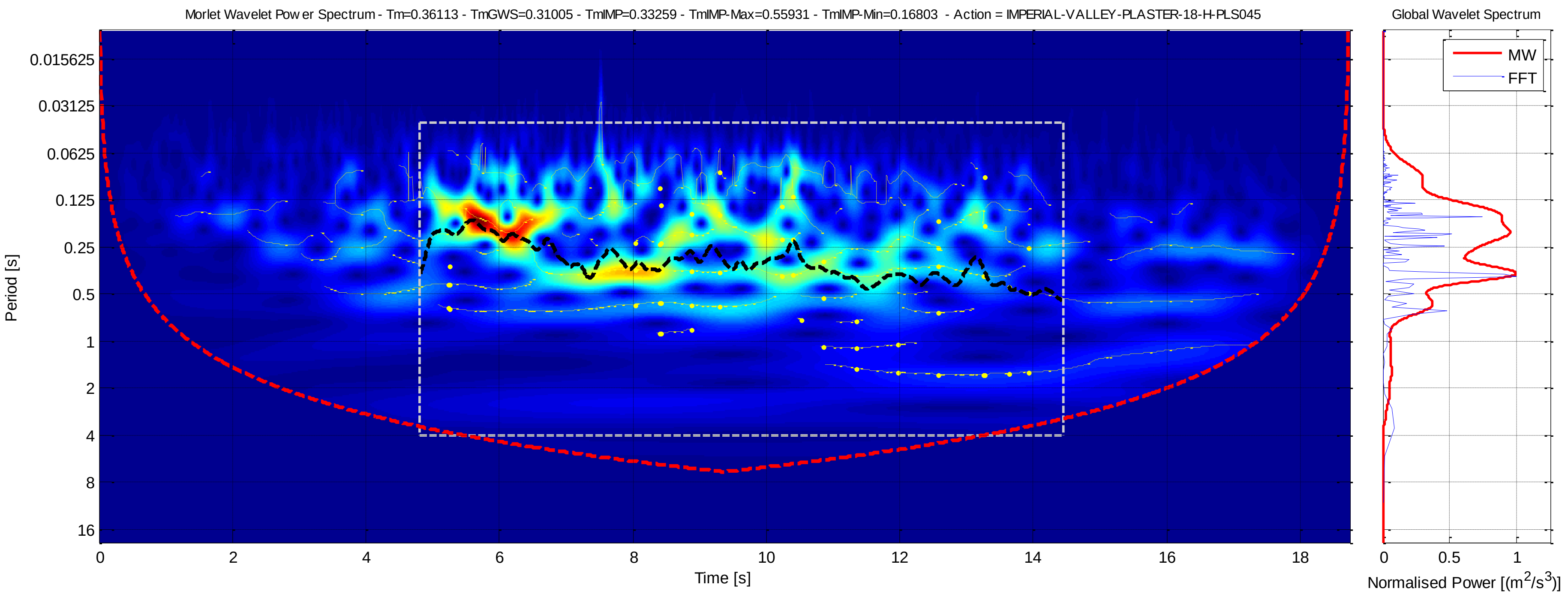
The wavelet-based mean instantaneous period (MIP)

MIP is a generalization of T_m: Temporal averaging of MIP "should" yield T_m



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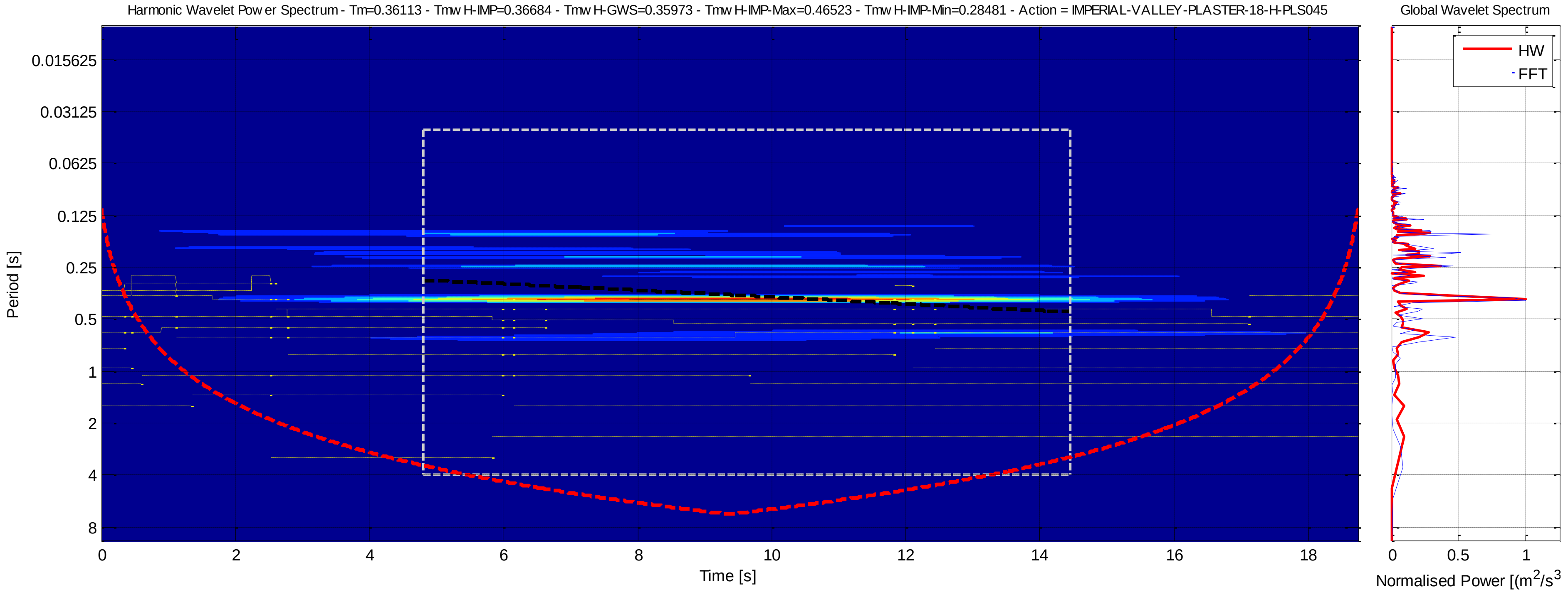

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The wavelet-based mean instantaneous period (MIP)

MIP is a generalizatio
“s

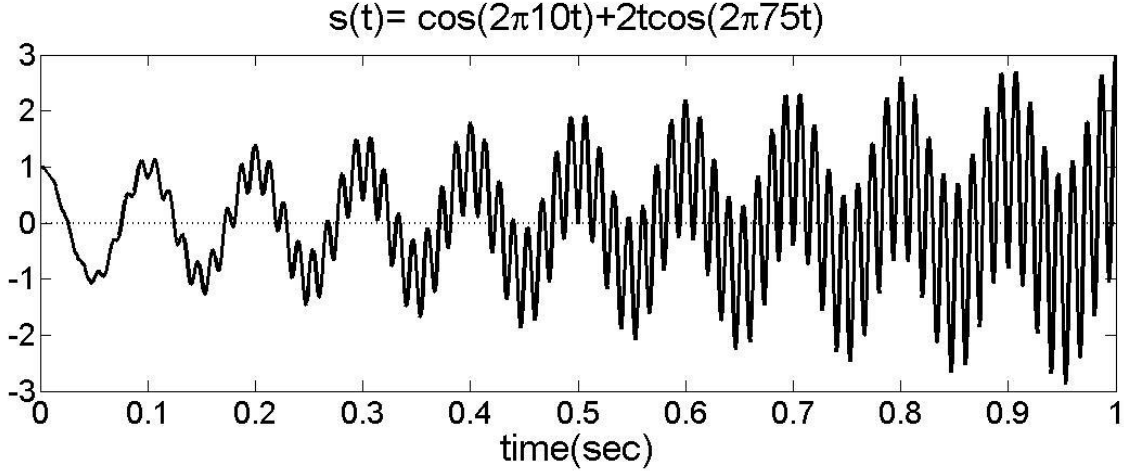
Table 1. Properties of the 20 ground motions considered by Vamvatsikos and Cornell (2004) and averaged over time mean instantaneous periods

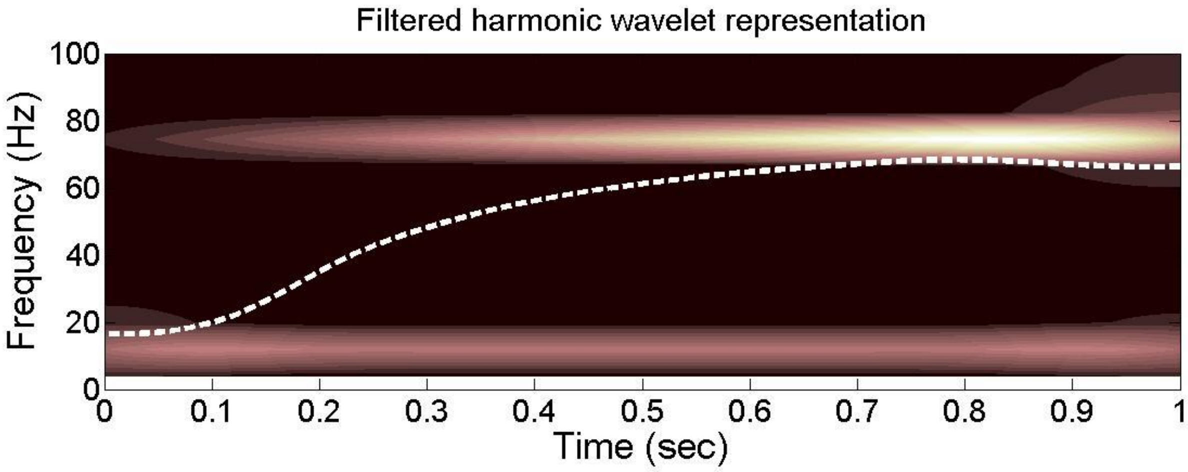
No.	Event (Year)	Station (Component)	M[1]	R[2] (km)	PGA (g)	Tm[3] (s)	MIP Morlet[4] (s)	MIP harmonic[5] (s)
1	Loma Prieta (1989)	Agnews State Hospital (090)	6.9	28.2	0.159	0.957	1.079	0.978
2	Imperial Valley (1979)	Plaster City (135)	6.5	31.7	0.057	0.378	0.369	0.406
3	Loma Prieta (1989)	Hollister Diff. Array (255)	6.9	25.8	0.279	0.798	1.212	0.942
4	Loma Prieta (1989)	Anderson Dam Downstrm (270)	6.9	21.4	0.244	0.467	0.495	0.476
5	Loma Prieta (1989)	Coyote Lake Dam Downstrm (285)	6.9	22.3	0.179	0.538	0.601	0.540
6	Imperial Valley (1979)	Cucapah (085)	6.5	23.6	0.309	0.558	0.706	0.602
7	Loma Prieta (1989)	Sunnyvale Colton Ave (270)	6.9	28.8	0.207	1.502	1.430	1.532
8	Imperial Valley (1979)	El Centro Array #13 (140)	6.5	21.9	0.117	0.585	0.725	0.644
9	Imperial Valley (1979)	Westmoreland Fire Station (090)	6.5	15.1	0.074	0.849	1.308	1.162
10	Loma Prieta (1989)	Hollister South & Pine (000)	6.9	28.8	0.371	0.935	1.439	0.998
11	Loma Prieta (1989)	Sunnyvale Colton Ave (360)	6.9	28.8	0.209	1.380	1.397	1.465
12	Superstition Hills (1987)	Wildlife Liquefaction Array (090)	6.7	24.4	0.180	0.854	1.015	1.024
13	Imperial Valley, 1979	Chihuahua (282)	6.5	28.7	0.254	0.701	0.699	0.709
14	Imperial Valley, 1979	El Centro Array #13 (230)	6.5	21.9	0.139	0.470	0.785	0.575
15	Imperial Valley, 1979	Westmoreland Fire Station (180)	6.5	15.1	0.110	0.985	1.059	1.058
16	Loma Prieta (1989)	WAHO (000)	6.9	16.9	0.370	0.275	0.232	0.269
17	Superstition Hills (1987)	Wildlife Liquefaction Array (360)	6.7	24.4	0.200	1.137	1.251	1.235
18	Imperial Valley (1979)	Plaster City (045)	6.5	31.7	0.042	0.361	0.333	0.367
19	Loma Prieta (1989)	Hollister Diff. Array (165)	6.9	25.8	0.269	0.890	1.098	0.988
20	Loma Prieta (1989)	WAHO (090)	6.9	16.9	0.638	0.271	0.257	0.278

[1] Moment Magnitude; [2] Closest distance to fault rupture; [3] Fourier-based mean period defined by Eq. (2) (Rathje et al. 1998); [4] Temporal averaged MIP derived using Morlet wavelets; [5] Temporal averaged MIP derived using harmonic wavelets.

The wavelet-based mean instantaneous period (MIP)

MIP may not correspond to any actual frequency component in multi-chromatic signals… it only coincides with the wavelet ridge for mono-chromatic signals

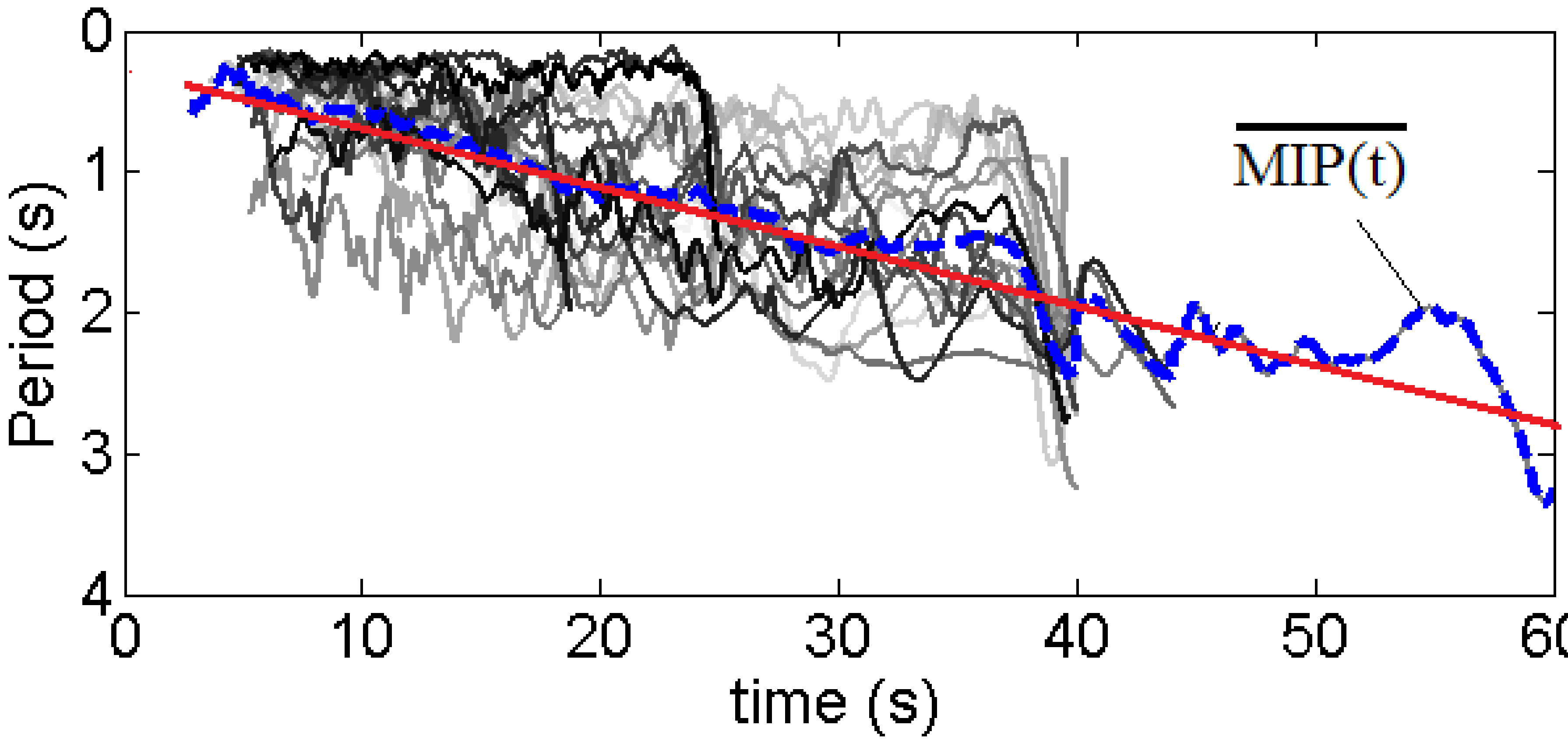




The wavelet-based mean instantaneous period (MIP)

How useful MIP is?

It does capture what we expect to see
AND it is only a time-history rather than a matrix (CWT)

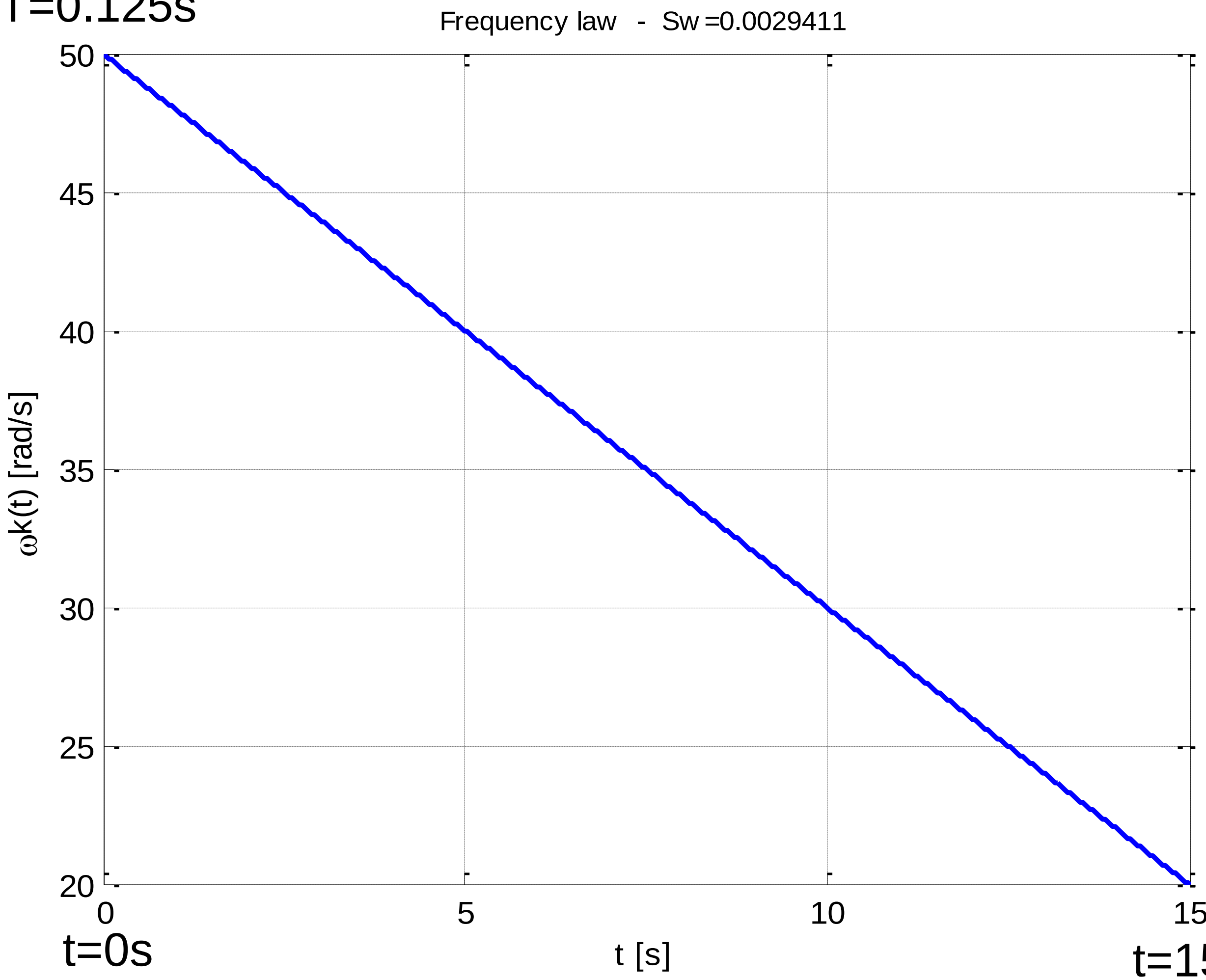


Wavelet analysis of hysteretic response signals

How useful CWT is for hysteretic structural response?

Moving average ≠ period elongation

ω= 50 rad/s or
T=0.125s



ω= 15 rad/s or
T=0.42s

t=0s

t=15s

Wavelet analysis of hysteretic response signals

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ω= 50 rad/s or
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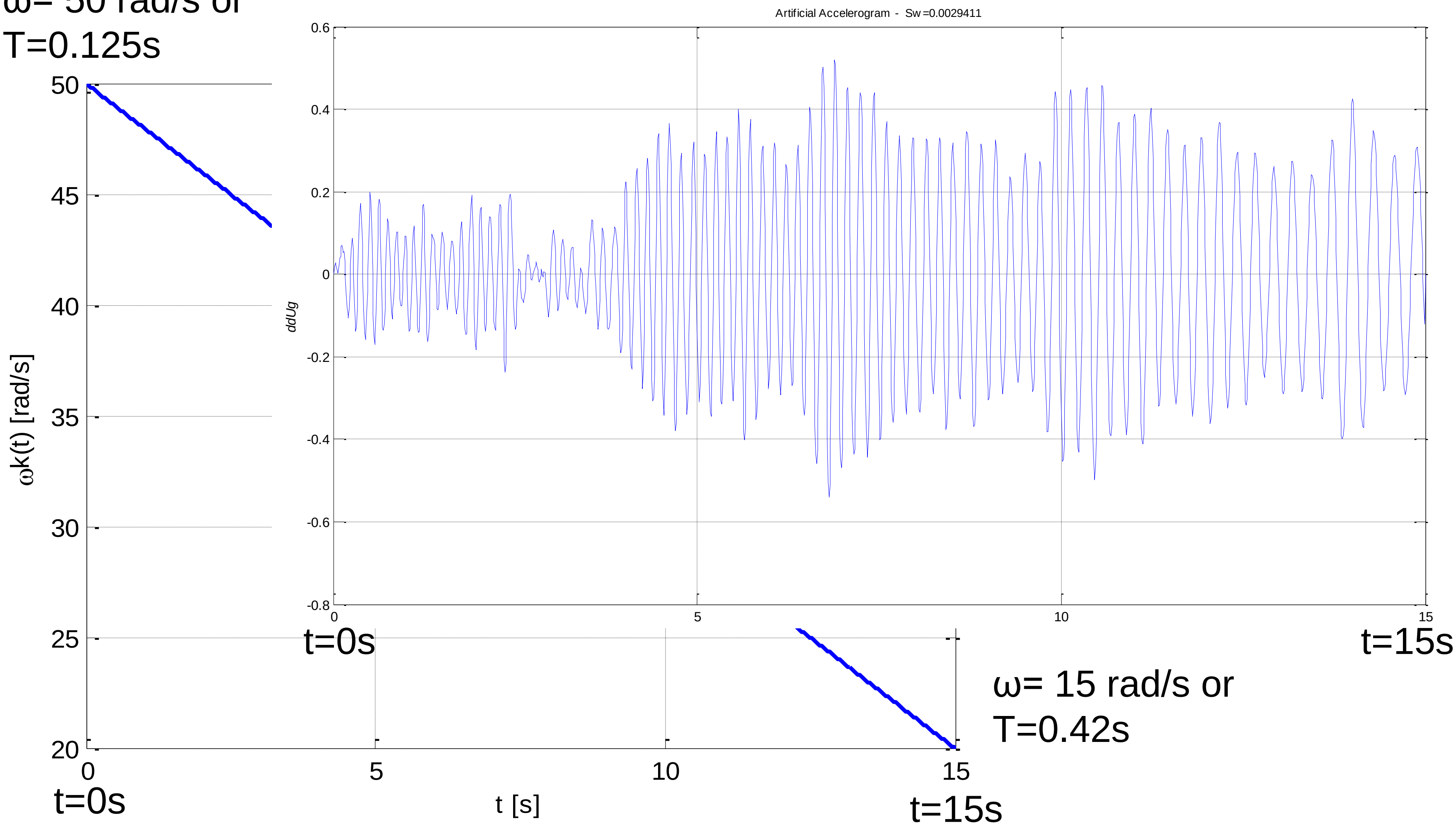

ω= 15 rad/s or
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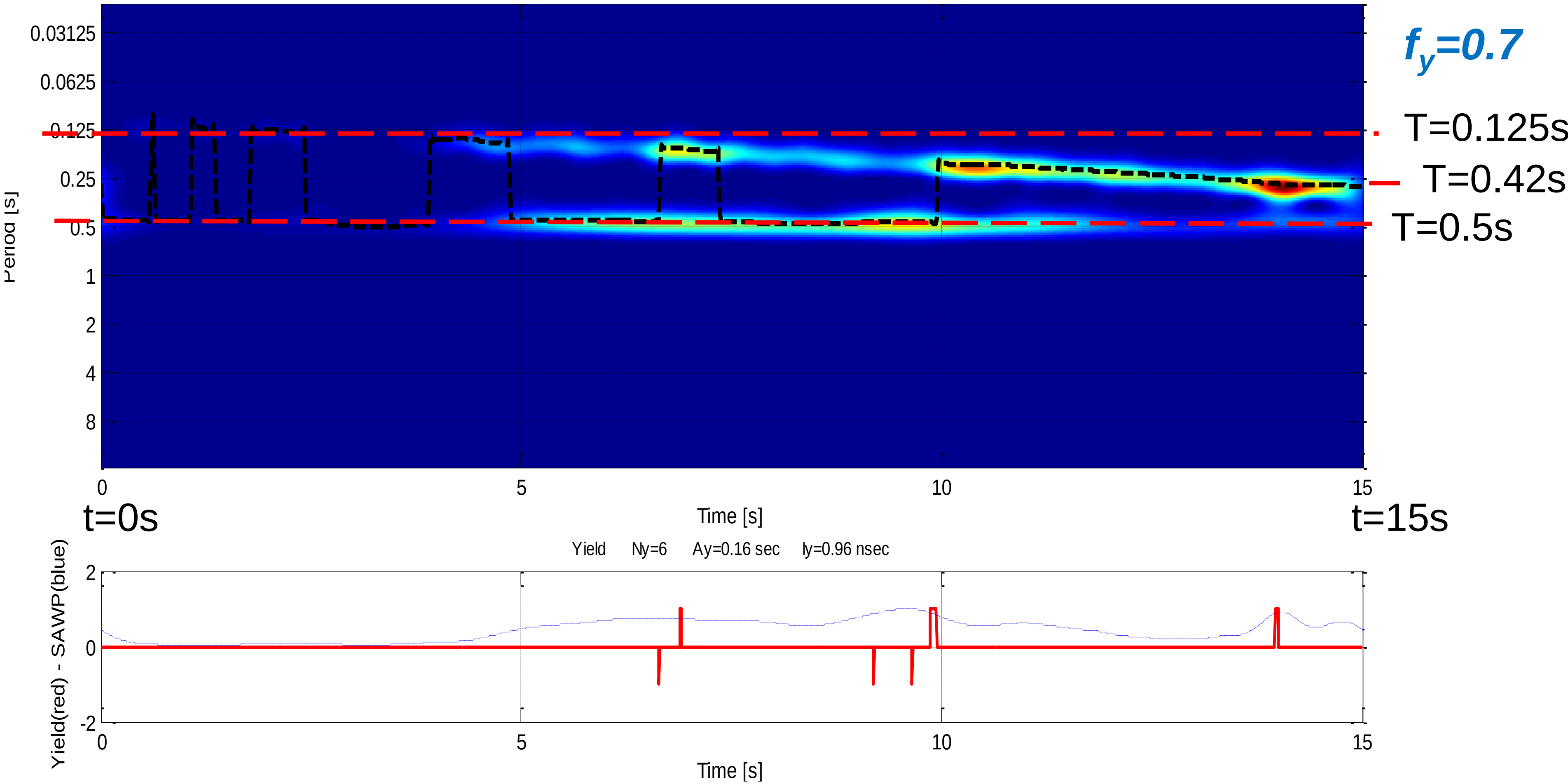
Wavelet analysis of hysteretic response signals

How useful CWT is for studying the hysteretic structural response?

Moving average ≠ period elongation

Elasto-plastic SDOF oscillator with fundamental pre-yielding period: T=0.5s



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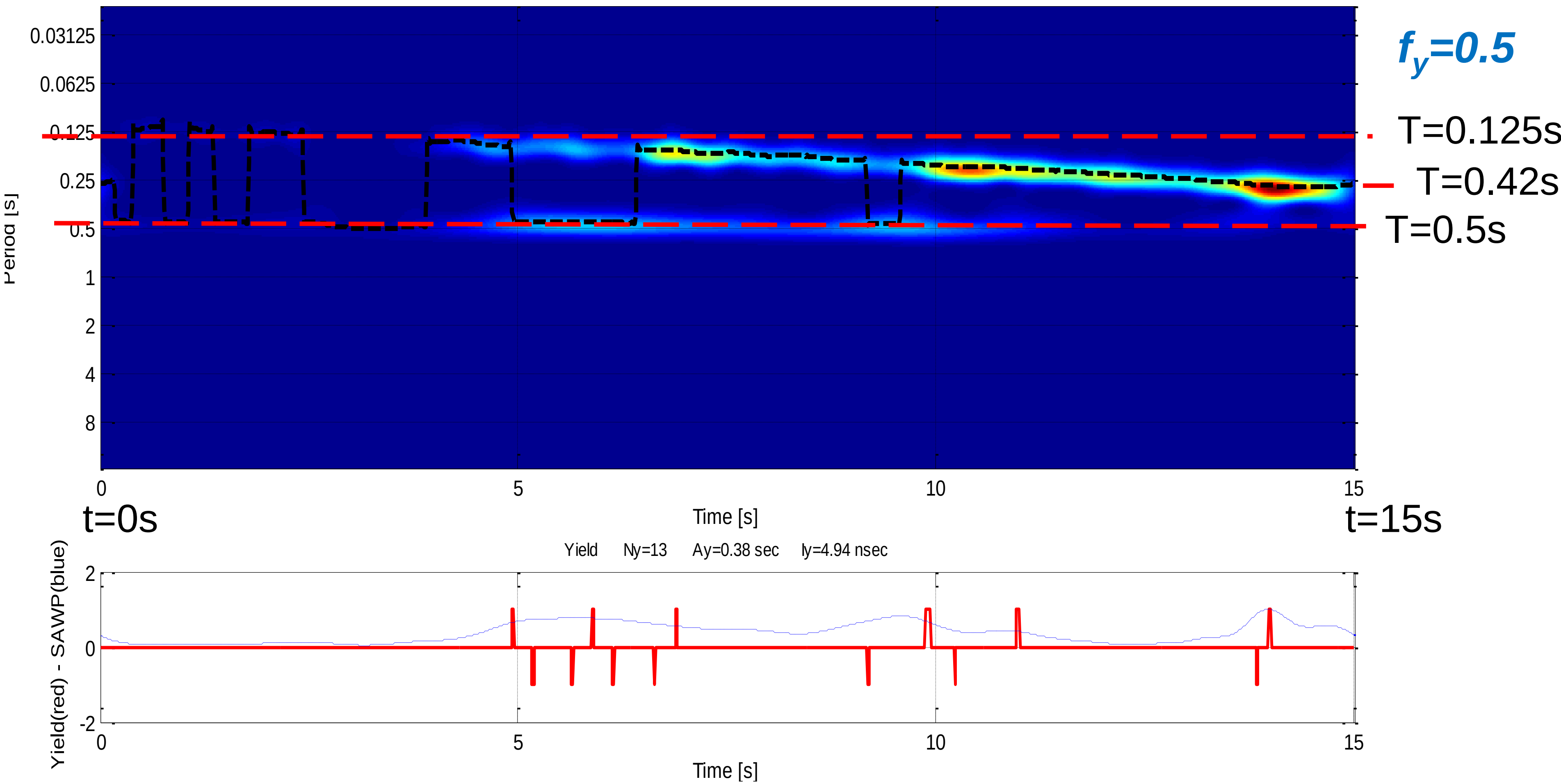

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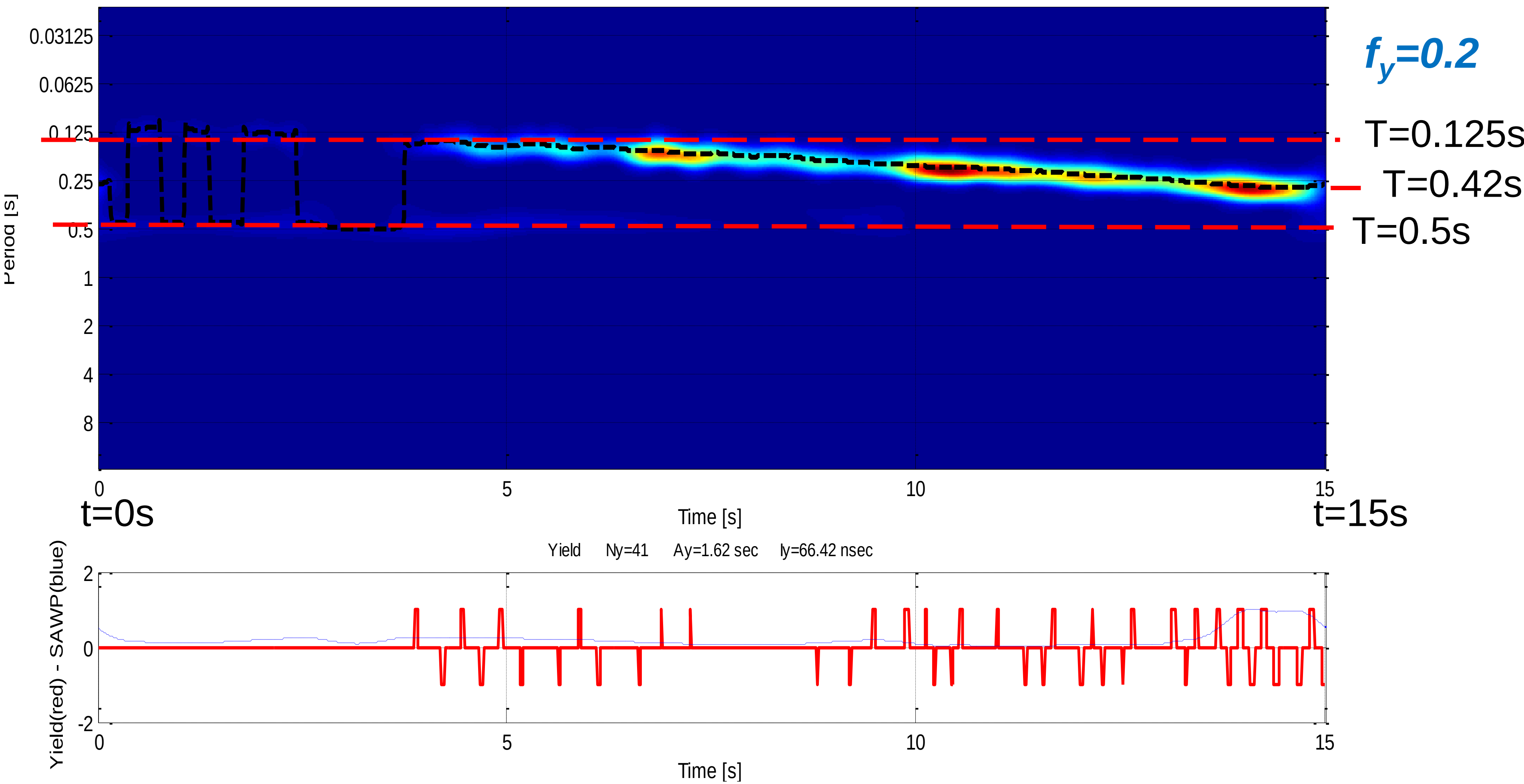
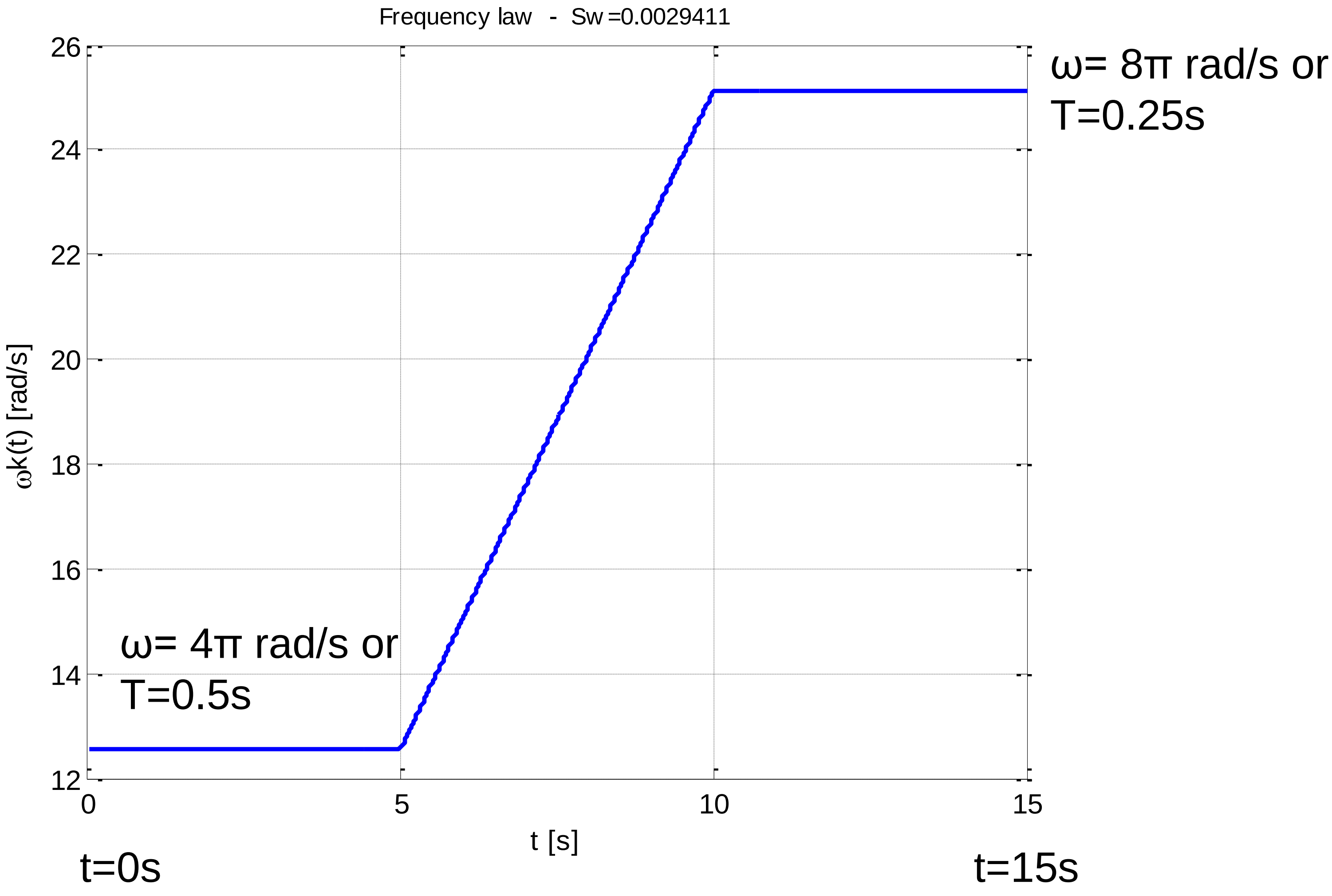


How useful CWT is for hysteretic structural response?

Moving resonance does not always occur



Wavelet analysis of hysteretic response signals

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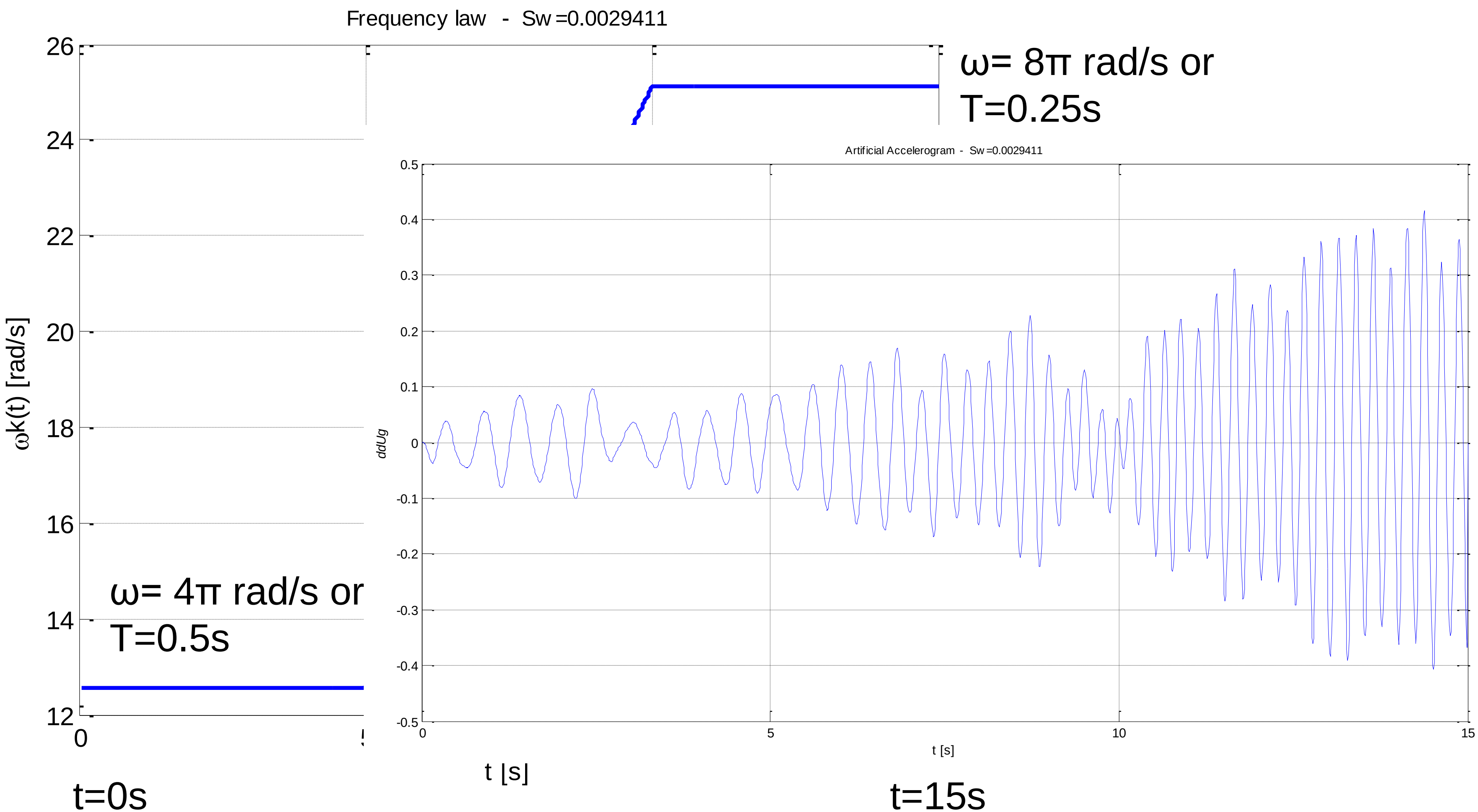

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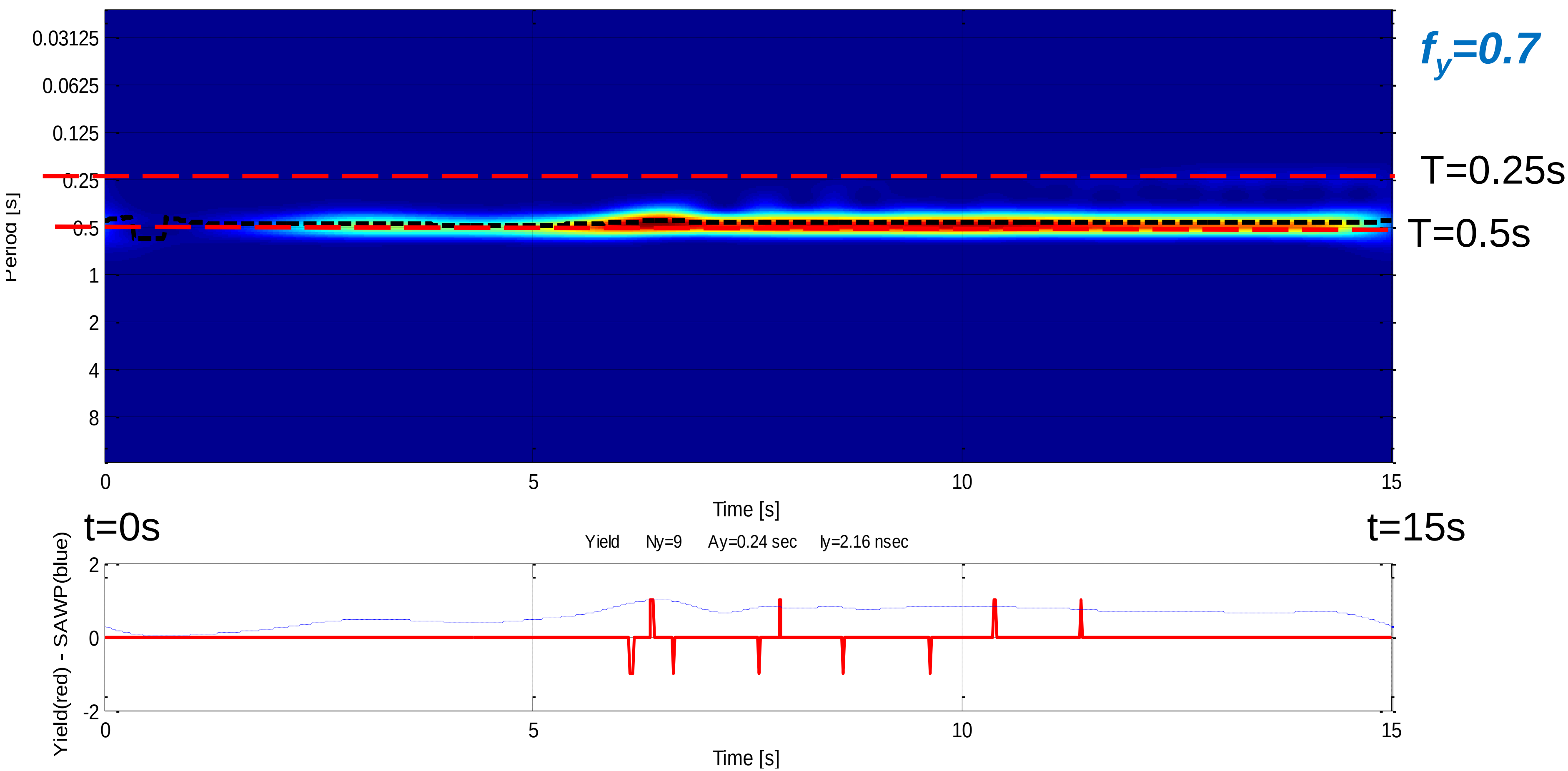

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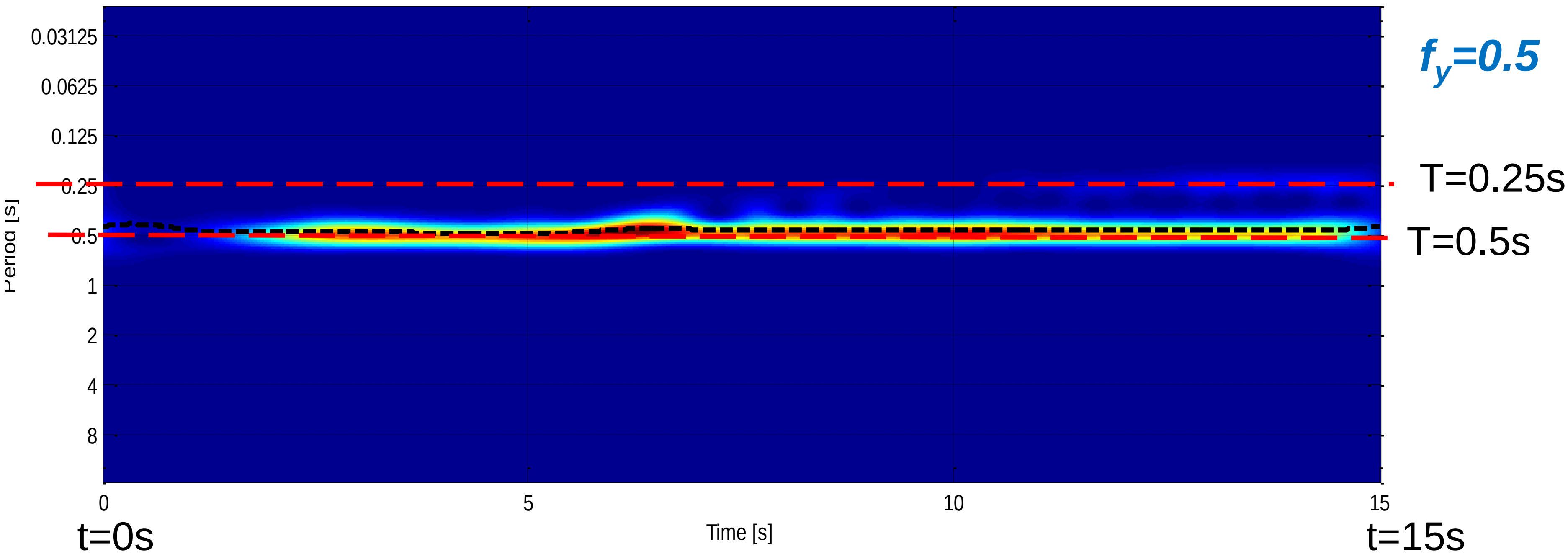

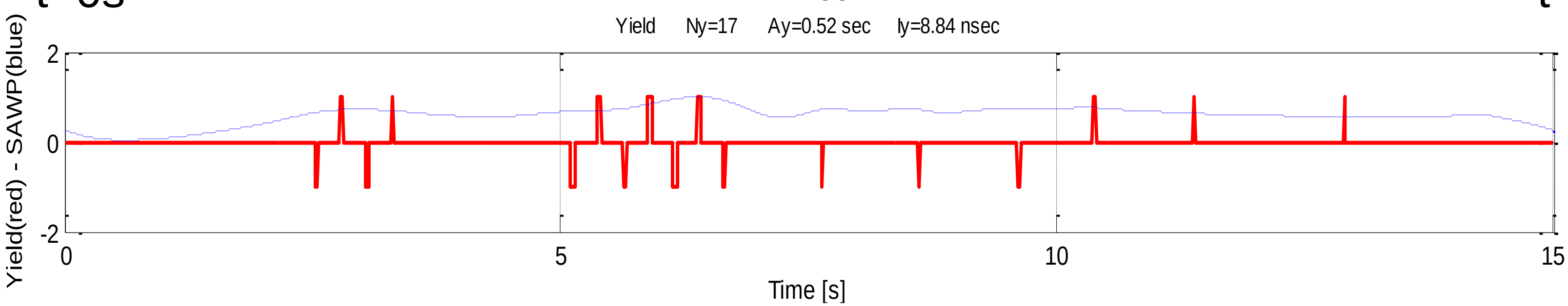

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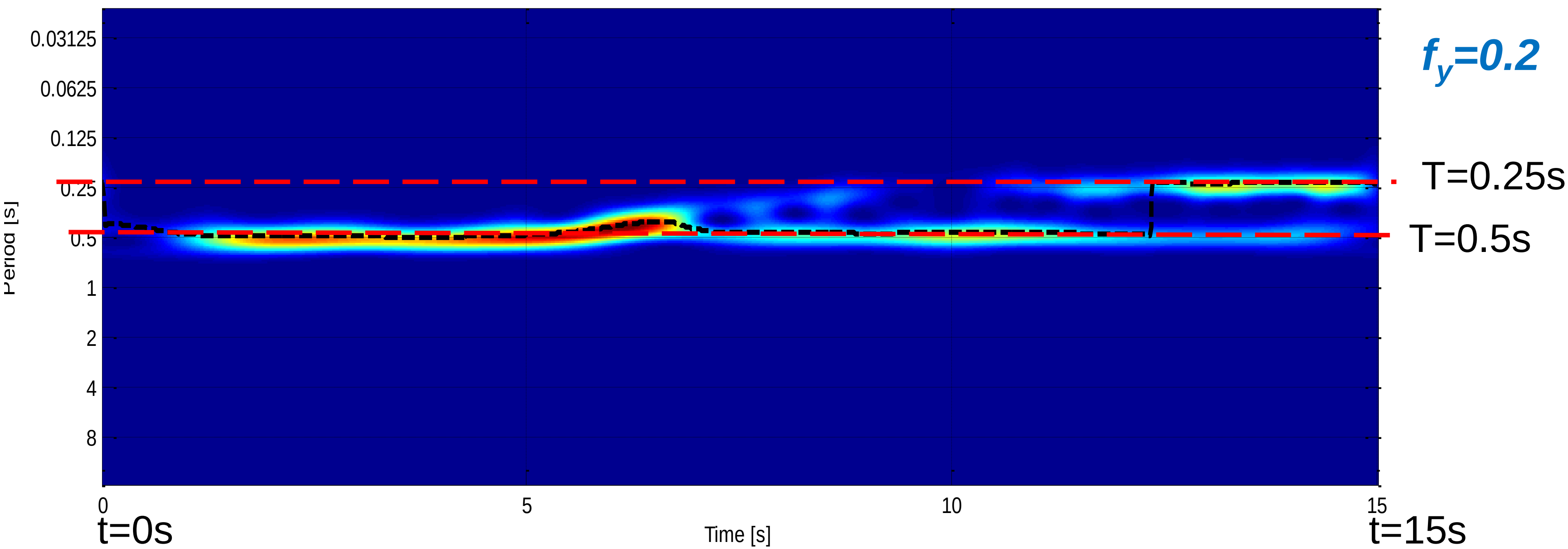
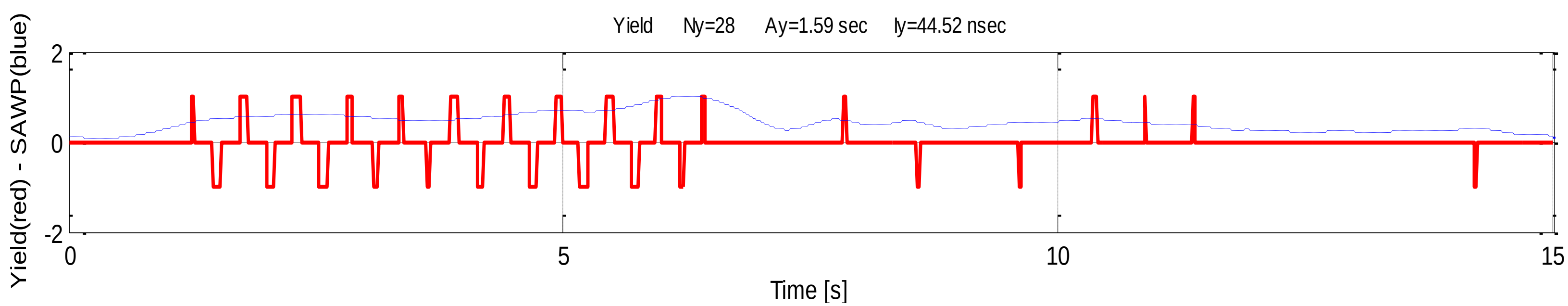
Moving resonance does not always occur

Elasto-plastic SDOF oscillator with fundamental pre-yielding period: T=0.5s





Wavelet analysis of hysteretic response signals

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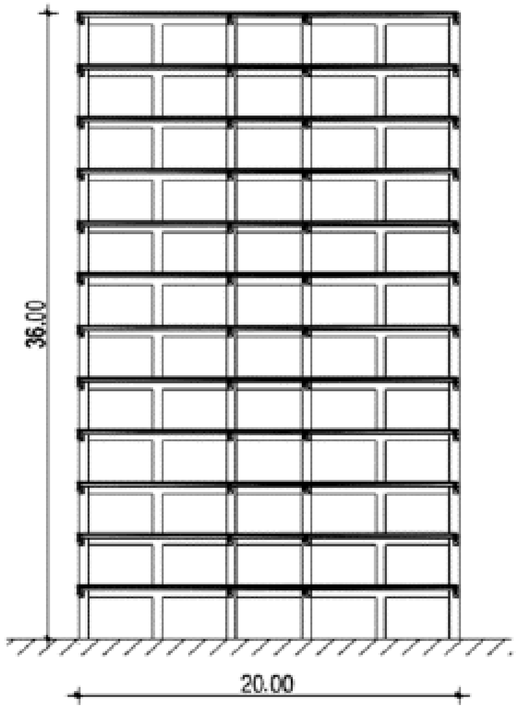
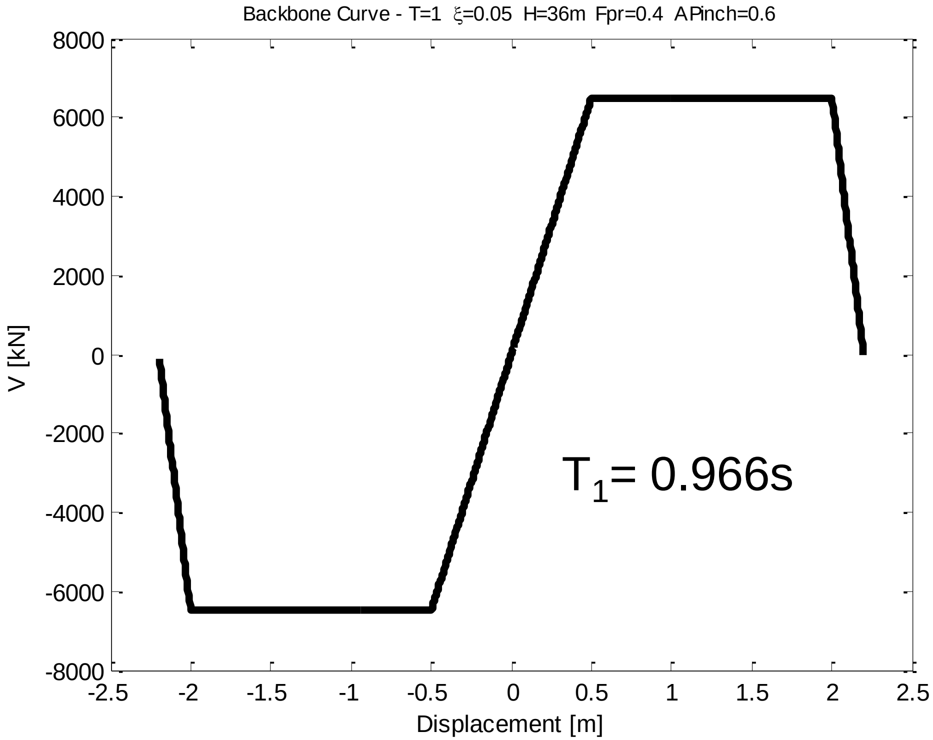


Codified name: 12RFDCH/12RFDCL
Number of stories 12
Frame
Regular in elevation
Story height: 3.0m
Ductility Class: High/Low
Natural period: 0.715s/0.752s

SDOF Oscillator	Deformation (m)		Base Shear (kN)		Stiffness (kN/m)	
	Yield	Collapse	Yield	Collapse	Pre-yield	Post-yield
12RFDCH	0.356	1.801	6505.62	6615.62	18259.49	76.17
12RFDCL	0.357	1.881	5501.93	5612.26	15422.24	72.42
8SWDCH	0.282	1.202	8333.64	8653.99	29564.88	348.10
8SWDCL	0.248	0.822	6489.67	6900.92	26156.20	716.52
8IFDCH	0.287	1.276	4578.75	4712.67	15978.69	135.32

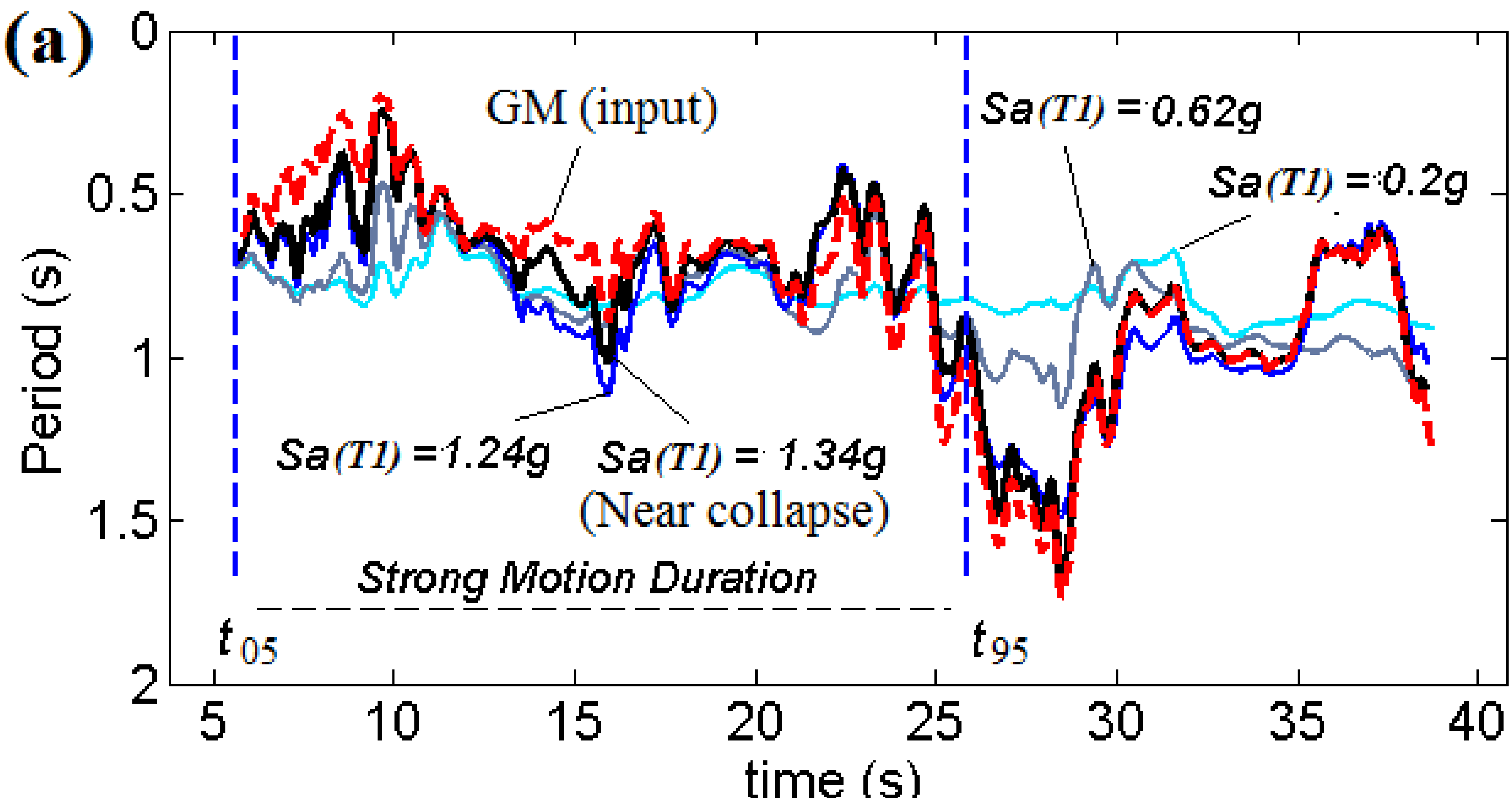


Katsanos/Sextos/Elnashai (2014)

Ibarra/Medina/Krawinkler (2005) model with strength+stiffness degradation

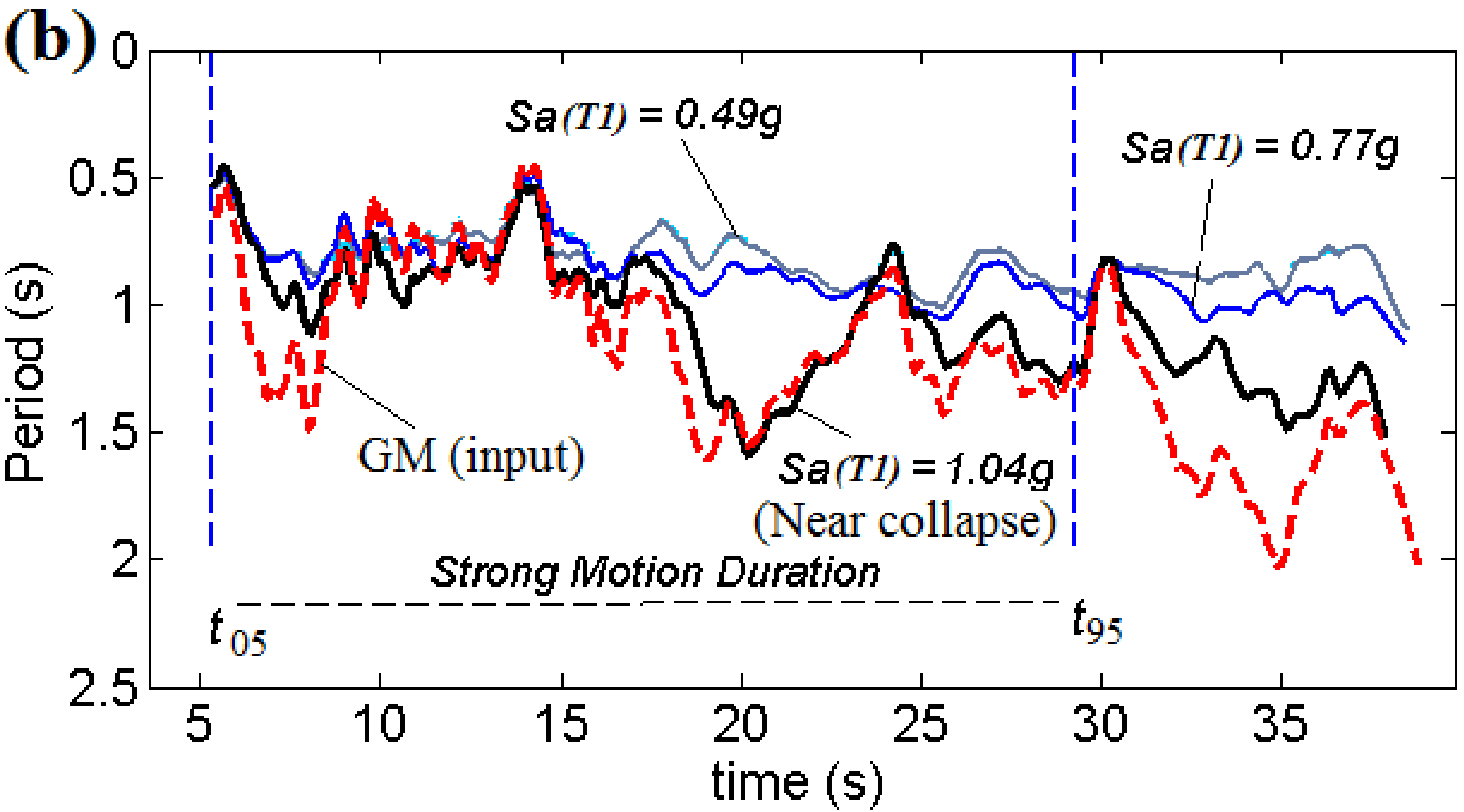
MIP of hysteretic response signals

IDA (1 GM is considered)
MIPs of input and of output for various IMs



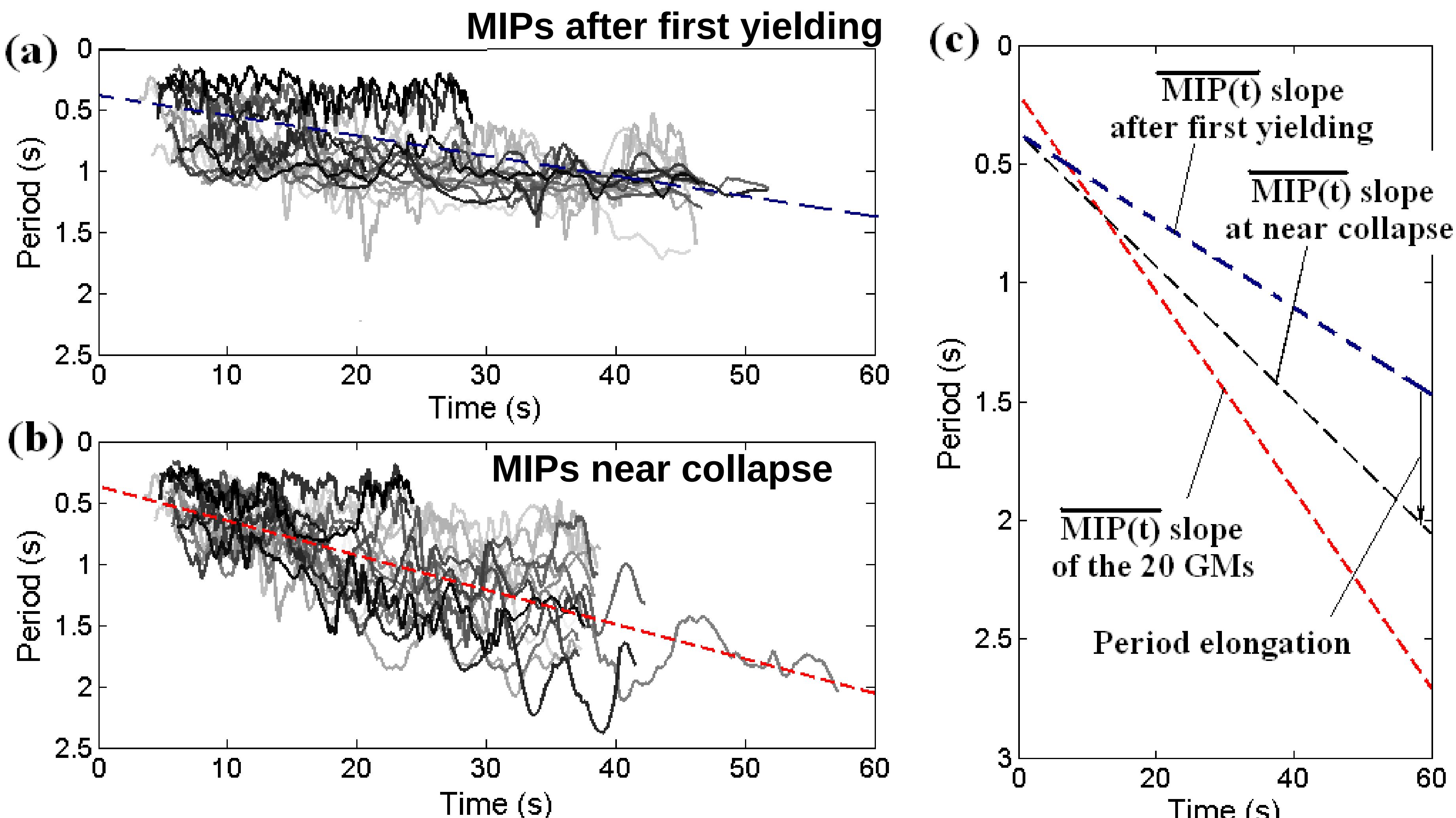
MIP of hysteretic response signals

IDA (1 GM is considered)
MIPs of input and of output for various IMs



MIP of hysteretic response signals

IDA (20 GMs is considered)

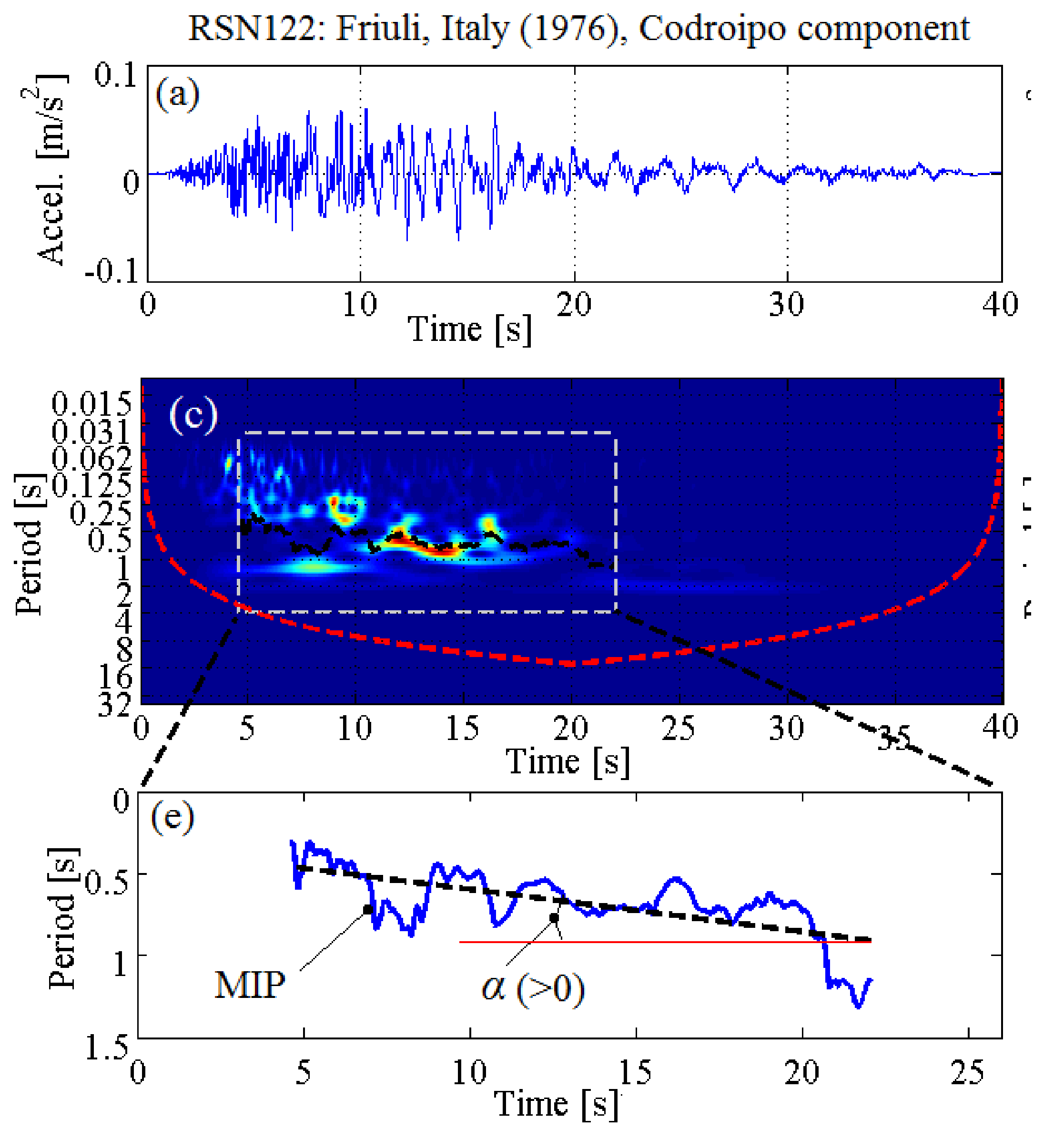


The angle "alpha" α of the average MIP

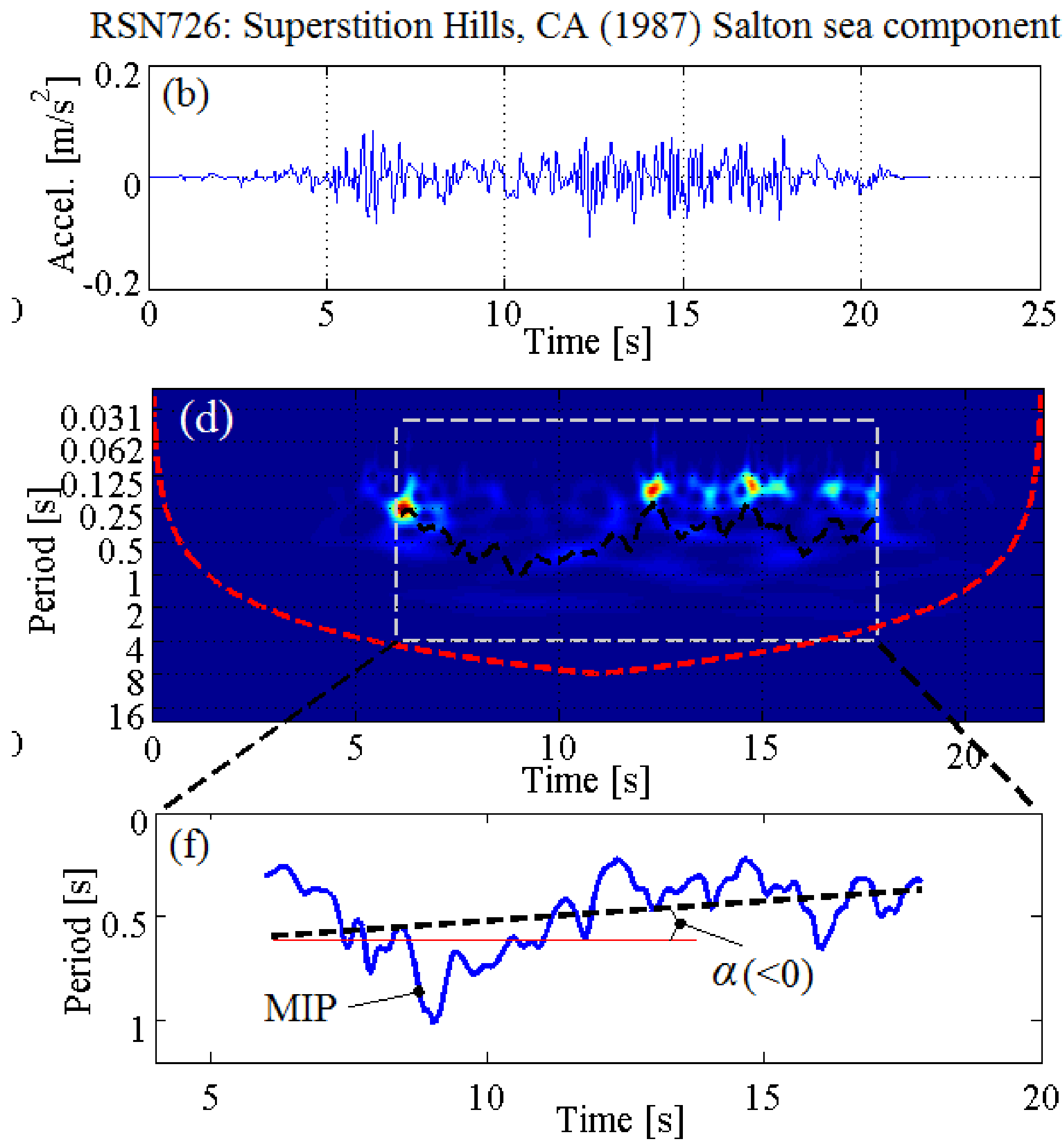
MIP is useful… but still it is a time-history, while all GM properties and intensity measures (IMs) are scalars…

We would ideally like to have a wavelet-based scalar quantity to capture the evolving frequency content of GMs

Angle "alpha" α is a scalar!!!



The angle “alpha” *α* of the average MIP

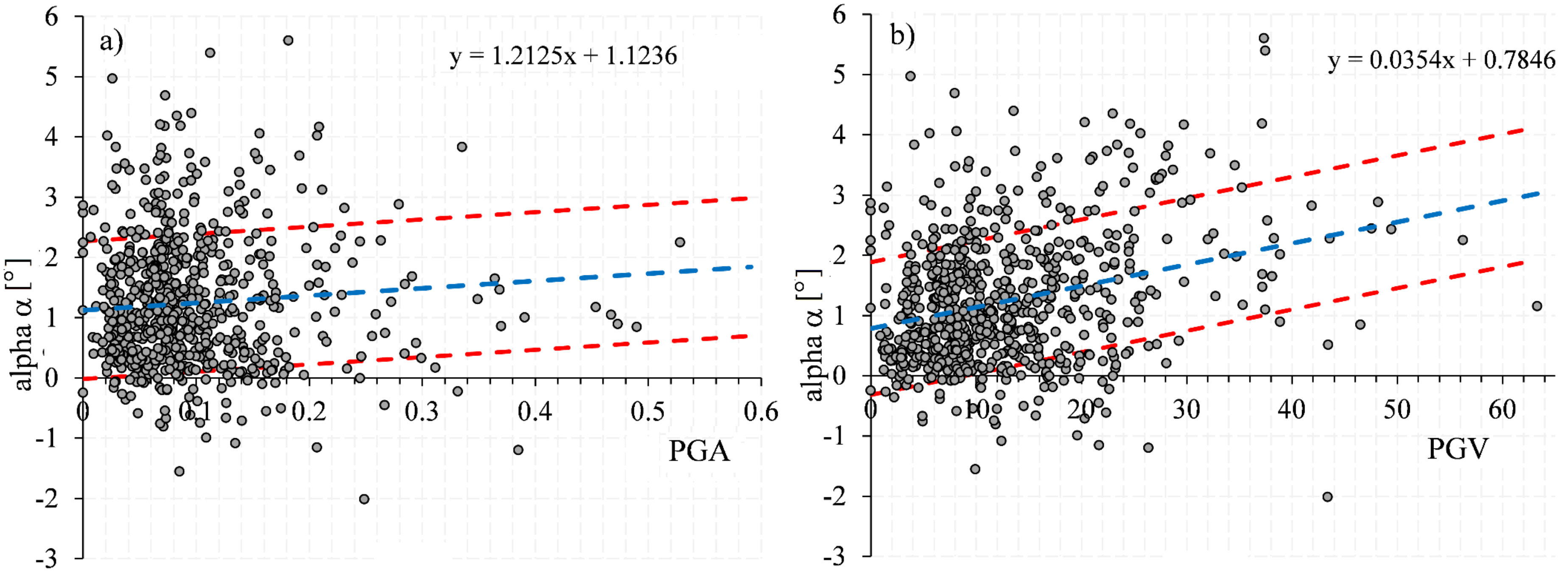


Angle “alpha” *α* may not be always positive…

Relation of *α* with other GM properties: trends and statistics

684 far-field GMs

-No pulses; 6.5<M<8.0; 20km<R_{rup}<120km



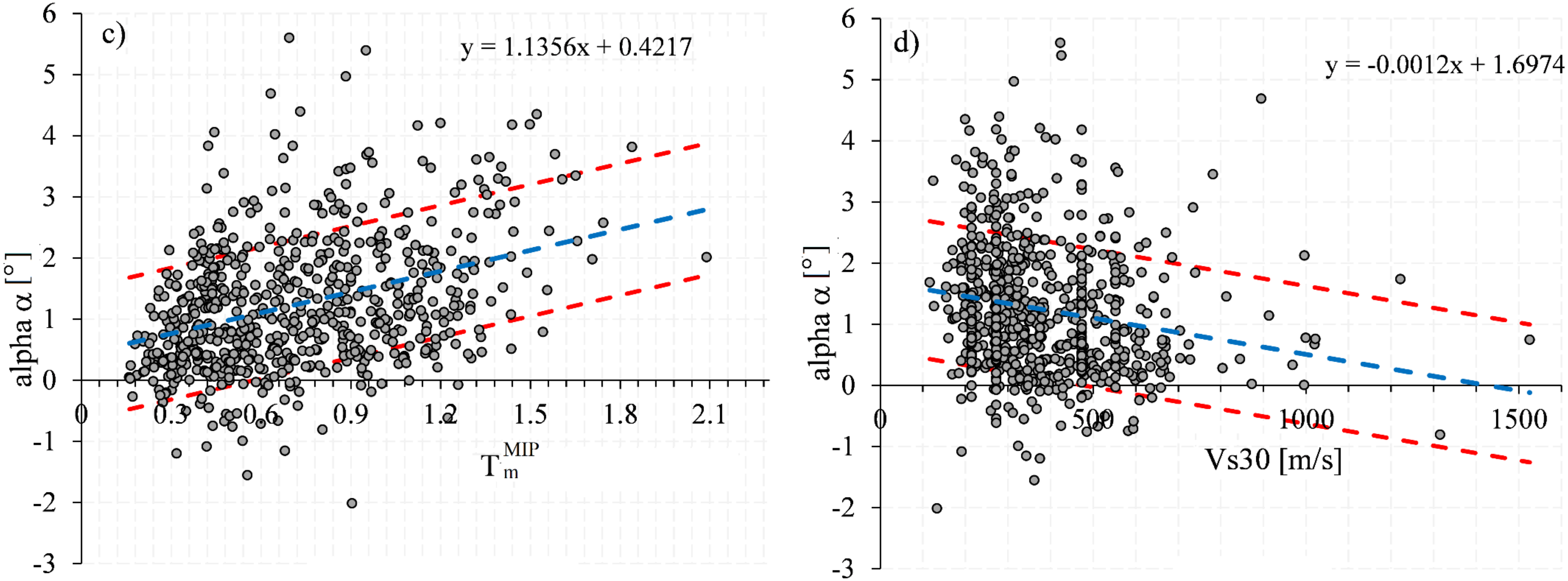
Average value of *α* increases with PGV (but not so much with PGA)

(High PGV values == rich frequency content (presumably towards the end of the GM) == mean frequency content shifts faster from high to low frequencies…)

Relation of *α* with other GM properties: trends and statistics

684 far-field GMs

-No pulses; 6.5<M<8.0; 20km<R_{rup}<120km



Average value of *α* increases with T_m and decreases with $V_{s,30}$

(Rich frequency content (presumably towards the end of the GM) == mean frequency content shifts faster from high to low frequencies…)

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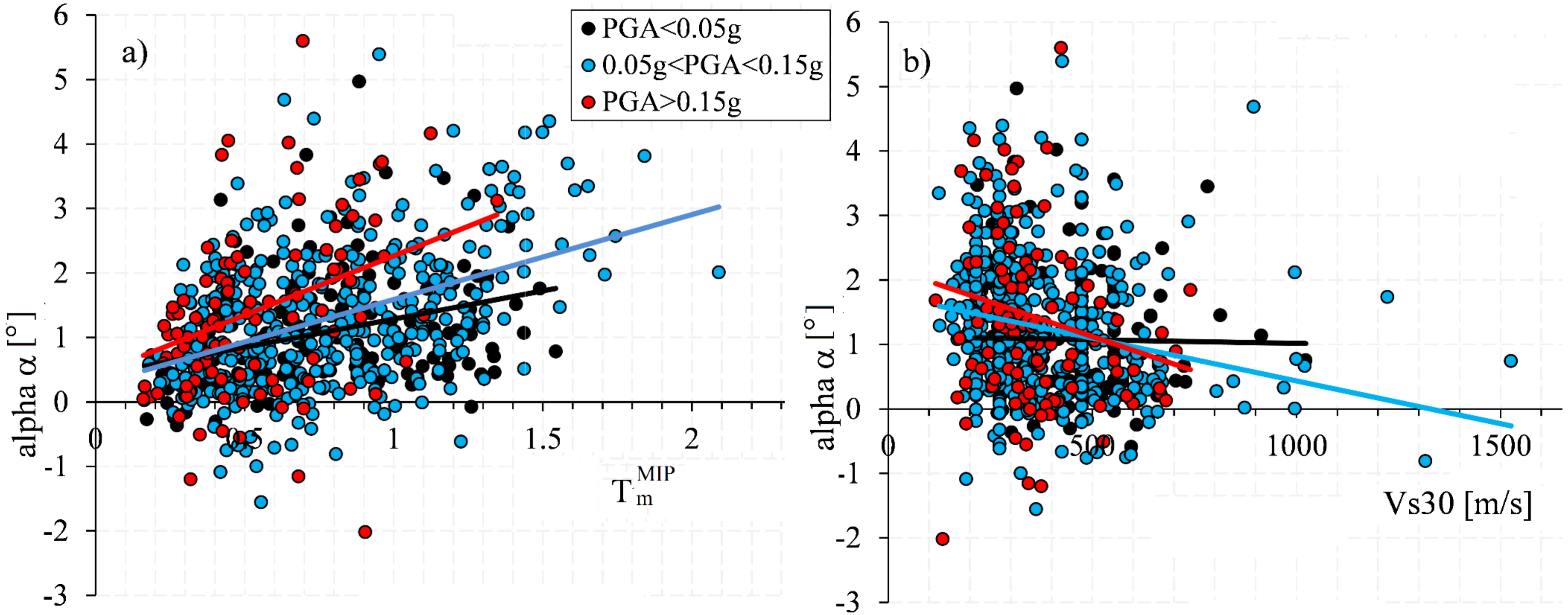
Relation of *α* with other GM properties: trends and statistics

684 far-field GMs

-No pulses; 6.5<M<8.0; 20km<R_{rup}<120km



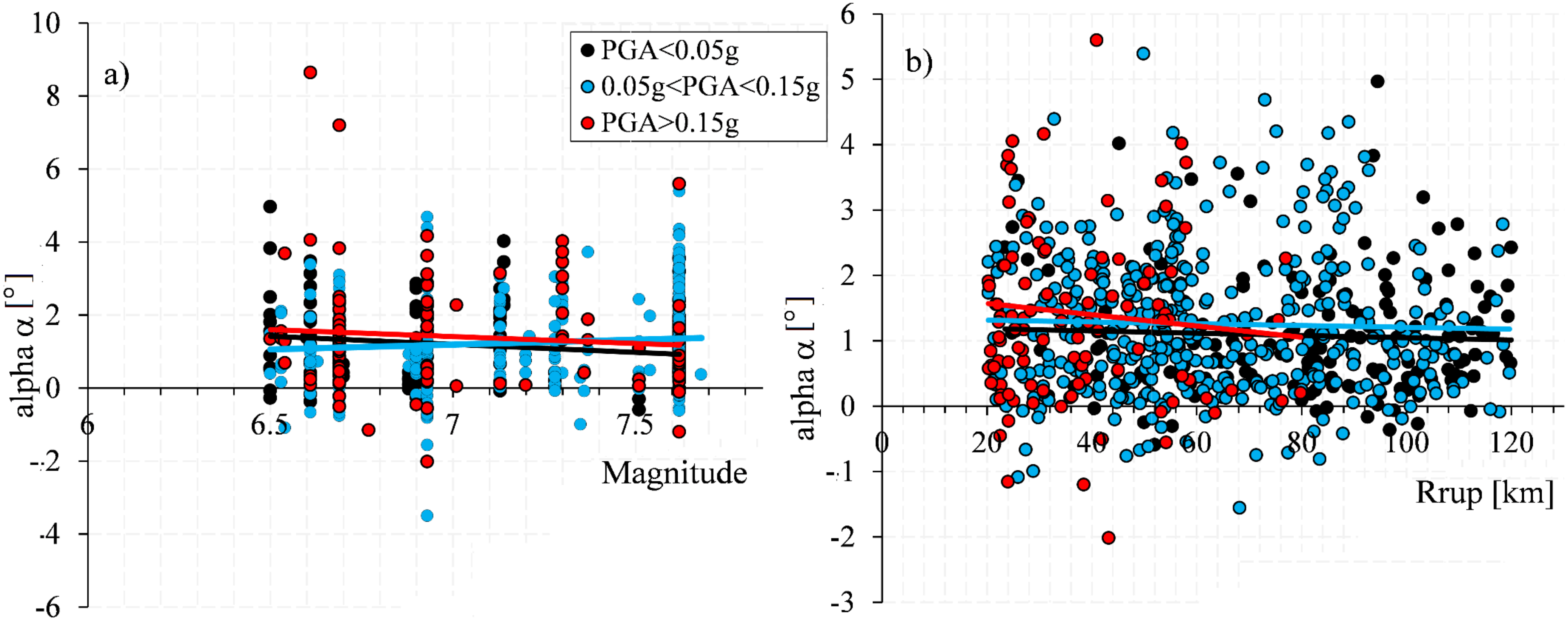
Higher intensity in terms of PGA has a profound impact on the average *α* trends with T_m and $V_{s,30}$

(for example, higher intensity == soft soils yield == Richer frequency content presumably towards the end of the GM) == mean frequency content shifts faster from high to low frequencies…)

Relation of *α* with other GM properties: trends and statistics

684 far-field GMs

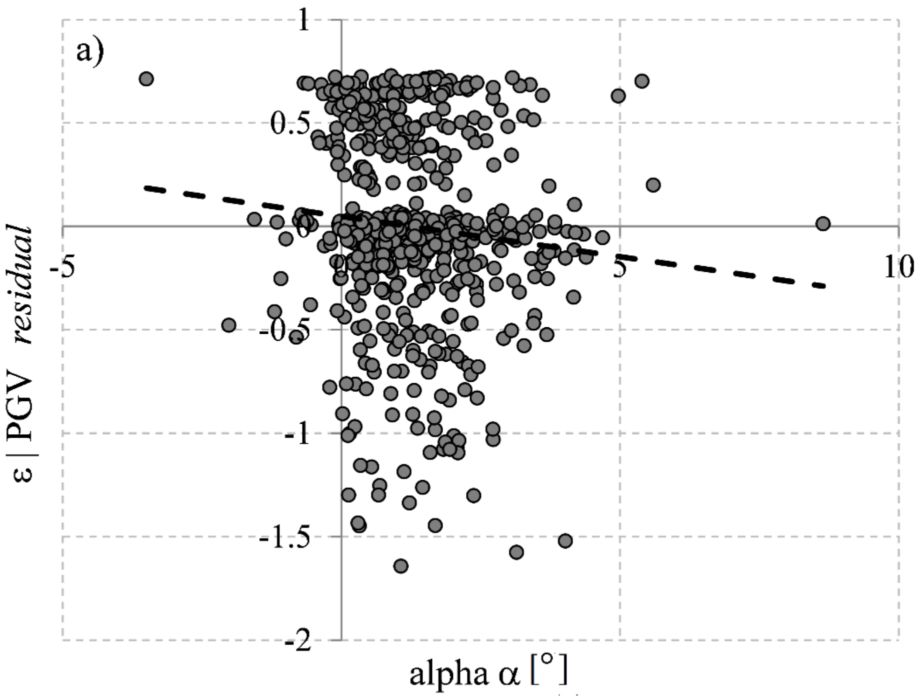
-No pulses; $6.5<M<8.0$; $20km<R_{rup}<120km$



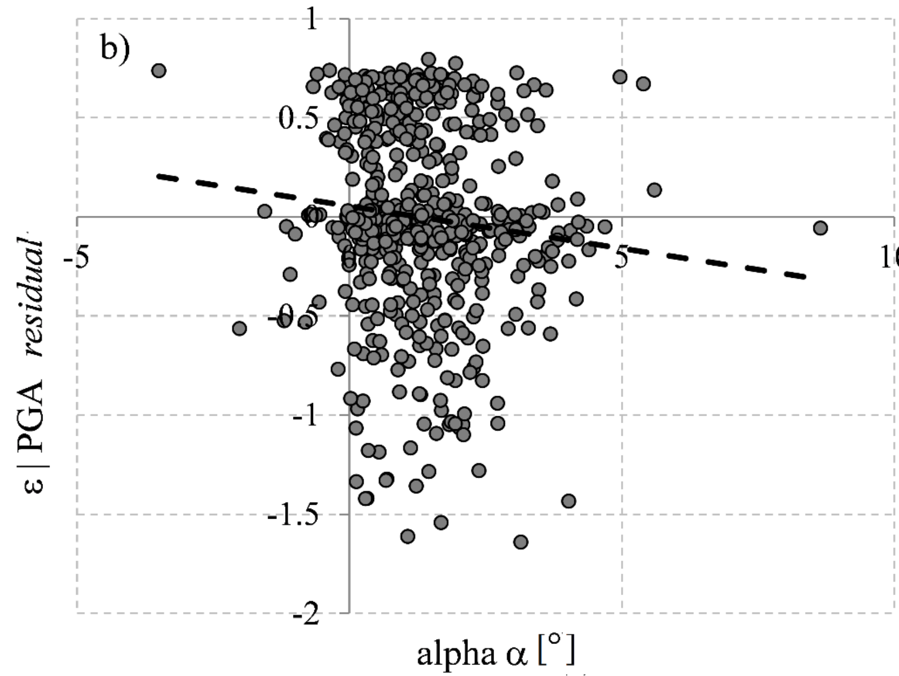
Relation of *α* with other GM properties: trends and statistics

IDA for the previous SDOF system and for the previous 684 far-field GMs using PGA and PGV as IMs

Residual analysis for sufficiency of PGA and PGV with respect to *α*



p-value: 0.0096



p-value: 0.0186

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Major concluding remarks

- The CWT is useful and meaningful in studying GMs… but care must be exercised in appreciating its limitations (e.g., uncertainty principle) and the fact that there is not a single “best” wavelet family to use.

- The MIP seems to be a useful “reduction” of the CWT in studying the evolution of the **mean** frequency content of GMs and in capturing the non-linear behaviour of yielding structures (e.g., moving resonance, period elongation…).

- The “alpha” α angle of the MIP appears to be a meaningful scalar to quantify the evolution of the frequency content of GMs and could be used for record selection in PBEE especially to study flexible structures near collapse.

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Acknowledgments

THANK YOU FOR YOUR ATTENTION

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