Response spectrum analysis for peak floor acceleration demands in earthquake excited structures

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Motivation and objective

- ✓ Simplified approaches for estimating peak floor acceleration (PFA) demands
  - Lack usually generality in application
- ✓ Response spectrum methods with appropriate modal combination rules (SRSS, CQC) more accurate
- ✓ SRSS and CQC rules originally developed for relative response quantities (displacement, internal forces)
- ✓ PFA demand absolute response quantity
- ✓ CQC modal combination rule for PFA demands proposed
  - Closed form solution for peak factors and correlation coefficients
  - Based on concepts of normal stationary random vibration theory
  - Related studies: Taghavi and Miranda (2009), Pozzi and Der Kiureghian (2012, 2015)

Modal response history analysis

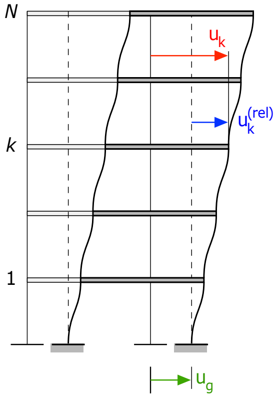
Equations of motion

$$\mathbf{M}\ddot{\mathbf{u}}^{(rel)} + \mathbf{C}\dot{\mathbf{u}}^{(rel)} + \mathbf{K}\mathbf{u}^{(rel)} = -\mathbf{M}\mathbf{e}\ddot{u}_g$$

Modal decomposition of $\mathbf{u}^{(rel)}$

$$\mathbf{u}^{(rel)} = \sum_{i=1}^{N} \phi_i \Gamma_i d_i^{(rel)} \qquad \Gamma_i = \frac{\phi_i^T \mathbf{M}\mathbf{e}}{\phi_i^T \mathbf{M}\phi_i}$$

$$\ddot{d}_i^{(rel)} + 2\zeta_i\omega_i\dot{d}_i^{(rel)} + \omega_i^2 d_i^{(rel)} = -\ddot{u}_g$$



Modal response history analysis

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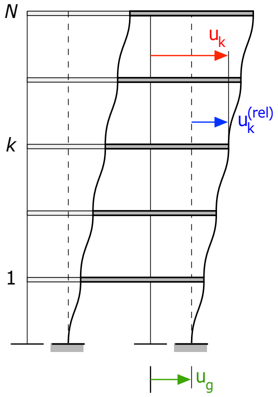


Total accelerations

$$\ddot{\mathbf{u}} = \ddot{\mathbf{u}}^{(rel)} + \mathbf{e}\ddot{u}_g = \sum_{i=1}^{N} \phi_i\Gamma_i \underbrace{\left(\ddot{d}_i^{(rel)} + \ddot{u}_g\right)}_{\ddot{d}_i} = \underbrace{\sum_{i=1}^{N} \phi_i\Gamma_i\ddot{d}_i^{(rel)}}_{\ddot{\mathbf{u}}^{(rel)}} + \underbrace{\sum_{i=1}^{N} \phi_i\Gamma_i\ddot{u}_g}_{\mathbf{e}\ddot{u}_g} = \sum_{i=1}^{N}\phi_i\Gamma_i\ddot{d}_i$$

Modal response history analysis

Total accelerations

$$\ddot{\mathbf{u}} = \sum_{i=1}^{N} \phi_i \Gamma_i \ddot{d}_i = \sum_{i=1}^{N}\phi_i\Gamma_i\ddot{d}_i^{(rel)} + \sum_{i=1}^{N}\phi_i\Gamma_i\ddot{u}_g = \sum_{i=1}^{N}\phi_i\Gamma_i\ddot{d}_i^{(rel)} + \mathbf{e}\ddot{u}_g$$

Separation of modal series into 1,…, n modes and n+1,...,N modal contributions

$$\ddot{\mathbf{u}} = \sum_{i=1}^{n}\phi_i\Gamma_i\ddot{d}_i + \sum_{i=n+1}^{N}\phi_i\Gamma_i\ddot{d}_i^{(rel)} + \sum_{i=n+1}^{N}\phi_i\Gamma_i\ddot{u}_g = \underbrace{\sum_{i=1}^{n}\phi_i\Gamma_i\ddot{d}_i}_{\mathbf{u}^{(n)}} + \underbrace{\sum_{i=n+1}^{N}\phi_i\Gamma_i\ddot{d}_i^{(rel)}}_{\approx\mathbf{0}} + \underbrace{\left(\mathbf{e}-\sum_{i=1}^{n}\phi_i\Gamma_i\right)}_{\mathbf{r}^{(n)}}\ddot{u}_g$$

Modal response history analysis

Total accelerations

$$\ddot{\mathbf{u}} = \sum_{i=1}^{N} \phi_i \Gamma_i \ddot{d}_i = \sum_{i=1}^{N}\phi_i\Gamma_i\ddot{d}_i^{(rel)} + \sum_{i=1}^{N}\phi_i\Gamma_i\ddot{u}_g = \sum_{i=1}^{N}\phi_i\Gamma_i\ddot{d}_i^{(rel)} + \mathbf{e}\ddot{u}_g$$

Separation of modal series into 1,…, n modes and n+1,...,N modal contributions

$$\ddot{\mathbf{u}} = \sum_{i=1}^{n}\phi_i\Gamma_i\ddot{d}_i + \sum_{i=n+1}^{N}\phi_i\Gamma_i\ddot{d}_i^{(rel)} + \sum_{i=n+1}^{N}\phi_i\Gamma_i\ddot{u}_g = \underbrace{\sum_{i=1}^{n}\phi_i\Gamma_i\ddot{d}_i}_{\mathbf{u}^{(n)}} + \underbrace{\sum_{i=n+1}^{N}\phi_i\Gamma_i\ddot{d}_i^{(rel)}}_{\approx\mathbf{0}} + \underbrace{\left(\mathbf{e}-\sum_{i=1}^{n}\phi_i\Gamma_i\right)}_{\mathbf{r}^{(n)}}\ddot{u}_g$$

Approximation of total accelerations by the first n modes

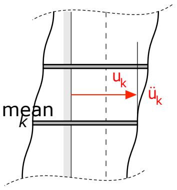
$$\ddot{\mathbf{u}} \approx \underbrace{\sum_{i=1}^{n}\phi_i\Gamma_i\ddot{d}_i}_{\ddot{\mathbf{u}}^{(n)}} + \underbrace{\left(\mathbf{e}-\sum_{i=1}^{n}\phi_i\Gamma_i\right)}_{\mathbf{r}^{(n)}}\ddot{u}_g = \ddot{\mathbf{u}}^{(n)} + \mathbf{r}^{(n)}\ddot{u}_g$$

residual vector -> high frequency contribution of ground acceleration considered

Proposed response spectrum method

Total accelerations in terms of stationary random variables

$$\ddot{\mathbf{U}} \approx \sum_{i=1}^{n} \phi_i \Gamma_i \ddot{D}_i + \mathbf{r}^{(n)} \ddot{U}_g = \ddot{\mathbf{U}}^{(n)} + \mathbf{r}^{(n)} \ddot{U}_g$$



Assumption:

- Set of ground motions represent Gaussian random process with zero mean
- $\ddot{\mathbf{U}}$ is Gaussian with zero mean
- $E\left[\ddot{\mathbf{U}}^2\right] = \text{Var}\left[\ddot{\mathbf{U}}\right]$
- Central peak floor acceleration (PFA) demand $E\left[\max\left|\ddot{U}_k\right|\right] \equiv m_{PFA_k} = \sqrt{\text{Var}\left[\ddot{U}_k\right]}\, p_k$

variance peak factor

quantity to be determined

Variance

$$\text{Var}\left[\ddot{\mathbf{U}}\right] = \text{Var}\left[\ddot{\mathbf{U}}^{(n)}\right] + \left(\mathbf{r}^{(n)}\right)^2 \text{Var}\left[\ddot{U}_g\right] + 2\text{Cov}\left[\ddot{\mathbf{U}}^{(n)}, \mathbf{r}^{(n)} \ddot{U}_g\right]$$

$$\text{Var}\left[\ddot{\mathbf{U}}\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} \text{Cov}\left[\ddot{\mathbf{U}}_i^{(n)}, \ddot{\mathbf{U}}_j^{(n)}\right] + \left(\mathbf{r}^{(n)}\right)^2 \text{Var}\left[\ddot{U}_g\right] + 2\sum_{i=1}^{n} \text{Cov}\left[\ddot{\mathbf{U}}_i^{(n)}, \mathbf{r}^{(n)} \ddot{U}_g\right]$$

Proposed response spectrum method

Modal decomposition of variance

$$\text{Var}\left[\ddot{\mathbf{U}}\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} \text{Cov}\left[\ddot{\mathbf{U}}_i^{(n)}, \ddot{\mathbf{U}}_j^{(n)}\right] + \left(\mathbf{r}^{(n)}\right)^2 \text{Var}\left[\ddot{U}_g\right] + 2\sum_{i=1}^{n} \text{Cov}\left[\ddot{\mathbf{U}}_i^{(n)}, \mathbf{r}^{(n)} \ddot{U}_g\right]$$

$$\text{Var}\left[\ddot{\mathbf{U}}\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} \phi_i \Gamma_i \phi_j \Gamma_j \text{Cov}\left[\ddot{D}_i, \ddot{D}_j\right] + \left(\mathbf{r}^{(n)}\right)^2 \text{Var}\left[\ddot{U}_g\right] + 2\mathbf{r}^{(n)} \sum_{i=1}^{n} \phi_i \Gamma_i \text{Cov}\left[\ddot{D}_i, \ddot{U}_g\right]$$

- Pearson's cross correlation coefficient $\rho_{i,j}$ and central peak modal coordinate $E\left[\max\left|\ddot{D}_i\right|\right]$

central spectral pseudo-acceleration at i-th period

$$\rho_{i,j} = \frac{\text{Cov}\left[\ddot{D}_i, \ddot{D}_j\right]}{\sqrt{\text{Var}\left[\ddot{D}_i\right]\text{Var}\left[\ddot{D}_j\right]}} \qquad E\left[\max\left|\ddot{D}_i\right|\right] \approx S_{a,i} = \sqrt{\text{Var}\left[\ddot{D}_i\right]}\, p_i$$

$$\text{Cov}\left[\ddot{D}_i, \ddot{D}_j\right] = \rho_{i,j}\sqrt{\text{Var}\left[\ddot{D}_i\right]\text{Var}\left[\ddot{D}_j\right]} = \rho_{i,j} \frac{E\left[\max\left|\ddot{D}_i\right|\right]}{p_i} \frac{E\left[\max\left|\ddot{D}_j\right|\right]}{p_j} \approx \rho_{i,j} \frac{S_{a,i}}{p_i} \frac{S_{a,j}}{p_j}$$

$$\text{Var}\left[\ddot{\mathbf{U}}\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{i,j} \phi_i \Gamma_i \phi_j \Gamma_j \frac{S_{a,i}}{p_i} \frac{S_{a,j}}{p_j} + \left(\mathbf{r}^{(n)}\right)^2 \frac{m_{PGA}^2}{p_g^2} + 2\mathbf{r}^{(n)} \frac{m_{PGA}}{p_g} \sum_{i=1}^{n} \rho_{i,g} \phi_i \Gamma_i \frac{S_{a,i}}{p_i}$$

Proposed response spectrum method

CQC mode superposition rule for central PFA demand of the k-th floor

$$E\left[\max\left|\ddot{U}_k\right|\right] \equiv m_{PFA_k} = \sqrt{Var\left[\ddot{U}_k\right]}\,p_k =$$

$$= \left[\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{p_k}{p_i}\frac{p_k}{p_j}\phi_{i,k}\Gamma_i S_{a,i}\phi_{j,k}\Gamma_j S_{a,j}\rho_{i,j} + \left(\frac{p_k}{p_g}\right)^2 m_{PGA}^2\left(r_k^{(n)}\right)^2 + 2m_{PGA}r_k^{(n)}\frac{p_k}{p_g}\sum_{i=1}^{n}\frac{p_k}{p_i}\phi_{i,k}\Gamma_j S_{a,j}\rho_{i,g}\right]^{1/2}$$

peak factors

correlation coefficients

Proposed response spectrum method

Correlation coefficients

$$\rho_{i,j} = \frac{\overset{\text{zeroth cross-spectral moment between i-th and j-th random modal acceleration}}{\boxed{\lambda_{0,ij}}}}{\sqrt{\lambda_{0,ii}\lambda_{0,jj}}} \qquad \rho_{i,g} = \frac{\overset{\text{zeroth cross-spectral moment between i-th random modal and ground acceleration}}{\boxed{\lambda_{0,ig}}}}{\sqrt{\lambda_{0,ii}\lambda_{0,gg}}}$$

l-th cross-spectral moment

$$\lambda_{l,XY} = \int_0^\infty \nu^l \underbrace{G_{XY}(\nu)}_{\text{cross PSD}} d\nu = \int_0^\infty \nu^l \overbrace{G_g^{(KT)}(\nu)}^{\text{Kanai-Tajimi PSD}} \underbrace{H_X(\nu)H_Y^*(\nu)}_{\text{FRFs}} d\nu \ , \ l = 0,1,2,..$$

FRF of i-th modal acceleration

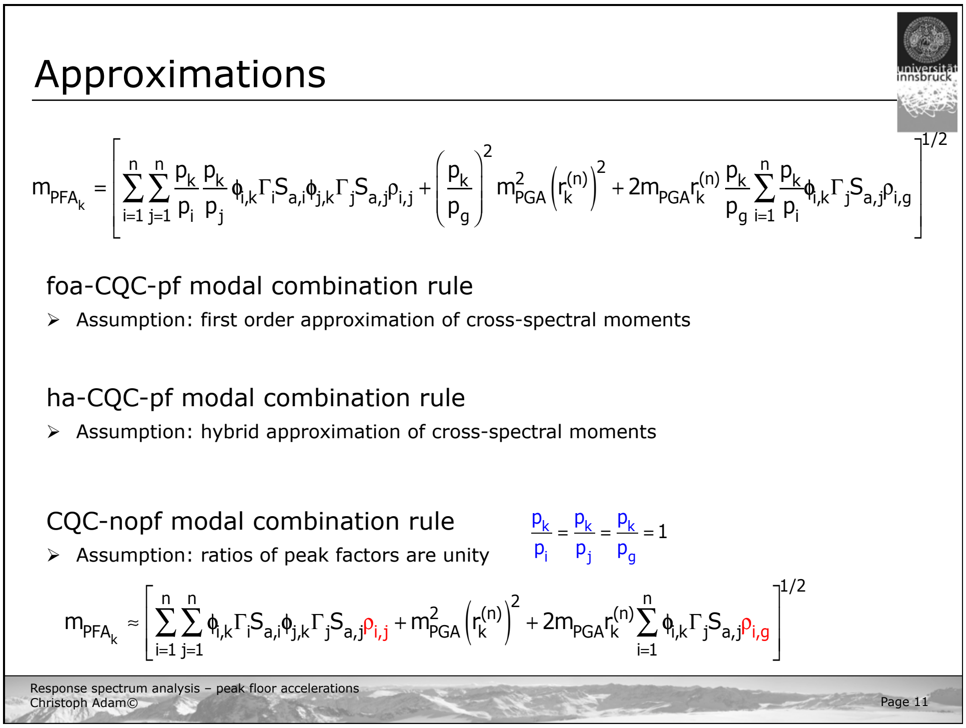
$$H_i(\nu) = \frac{\omega_i^2 + 2i\zeta_i\omega_i\nu}{\omega_i^2 - \nu^2 + 2i\zeta_i\omega_i\nu}$$

FRF of ground acceleration

$$H_g(\nu) = 1$$

Kanai-Tajimi PSD – characterization of ground excitation

$$G_g^{(KT)} = G_0\frac{1 + 4\zeta_g^2(\nu/\nu_g)^2}{\left(1-(\nu/\nu_g)^2\right)^2 + 4\zeta_g^2(\nu/\nu_g)^2}$$



Approximations

$$m_{PFA_k} = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{p_k}{p_i}\frac{p_k}{p_j} \phi_{i,k}\Gamma_i S_{a,i}\phi_{j,k}\Gamma_j S_{a,j}\rho_{i,j} + \left(\frac{p_k}{p_g}\right)^2 m_{PGA}^2 \left(r_k^{(n)}\right)^2 + 2m_{PGA} r_k^{(n)} \frac{p_k}{p_g} \sum_{i=1}^{n} \frac{p_k}{p_i} \phi_{i,k}\Gamma_j S_{a,j}\rho_{i,g} \right]^{1/2}$$

foa-CQC-pf modal combination rule

- Assumption: first order approximation of cross-spectral moments

ha-CQC-pf modal combination rule

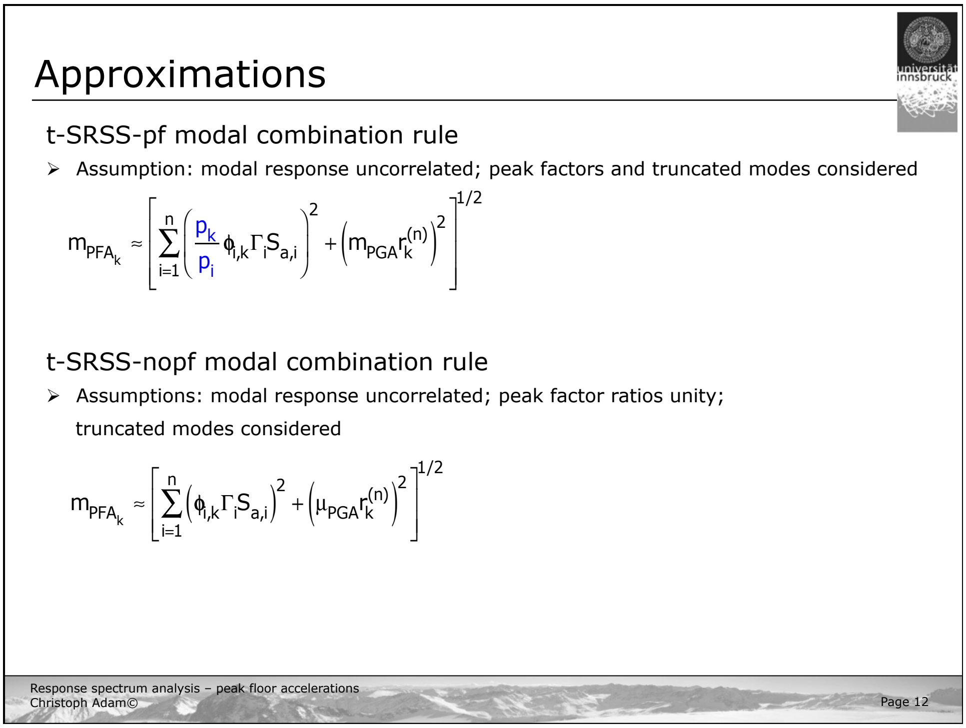
- Assumption: hybrid approximation of cross-spectral moments

CQC-nopf modal combination rule

- Assumption: ratios of peak factors are unity

$$\frac{p_k}{p_i} = \frac{p_k}{p_j} = \frac{p_k}{p_g} = 1$$

$$m_{PFA_k} \approx \left[\sum_{i=1}^{n}\sum_{j=1}^{n} \phi_{i,k}\Gamma_i S_{a,i}\phi_{j,k}\Gamma_j S_{a,j}\rho_{i,j} + m_{PGA}^2 \left(r_k^{(n)}\right)^2 + 2m_{PGA} r_k^{(n)} \sum_{i=1}^{n} \phi_{i,k}\Gamma_j S_{a,j}\rho_{i,g} \right]^{1/2}$$



Approximations

t-SRSS-pf modal combination rule

- Assumption: modal response uncorrelated; peak factors and truncated modes considered

$$m_{PFA_k} \approx \left[\sum_{i=1}^{n} \left(\frac{p_k}{p_i} \phi_{i,k}\Gamma_i S_{a,i} \right)^2 + \left(m_{PGA} r_k^{(n)} \right)^2 \right]^{1/2}$$

t-SRSS-nopf modal combination rule

- Assumptions: modal response uncorrelated; peak factor ratios unity; truncated modes considered

$$m_{PFA_k} \approx \left[\sum_{i=1}^{n} \left(\phi_{i,k}\Gamma_i S_{a,i} \right)^2 + \left(\mu_{PGA} r_k^{(n)} \right)^2 \right]^{1/2}$$

Approximations

SRSS-pf modal combination rule

- Assumptions: modal response uncorrelated; truncated modes neglected

$$m_{PFA_k} \approx \left[\sum_{i=1}^{n}\left(\frac{p_k}{p_i}\phi_{i,k}\Gamma_i S_{a,i}\right)^2\right]^{1/2}$$

Classical SRSS modal combination rule (SRSS-nopf)

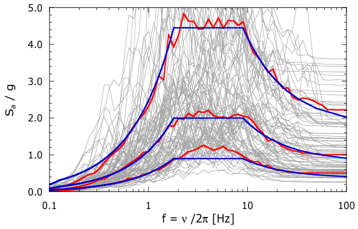
- Assumptions: modal response uncorrelated; peak factor ratios unity; truncated modes neglected

$$m_{PFA_k} \approx \left[\sum_{i=1}^{n}\left(\phi_{i,k}\Gamma_i S_{a,i}\right)^2\right]^{1/2}$$

Ground motion modeling in frequency domain

Reference solution: outcome of RHA

- Seismic hazard representative for Century City (Los Angeles)
- 92 site compatible ground motion records from PEER NGA database
- Median and dispersion of record set match design spectrum and target spectrum $\sigma_t = 0.80$ in period range 0.05s ≤ T ≤ 3.0s

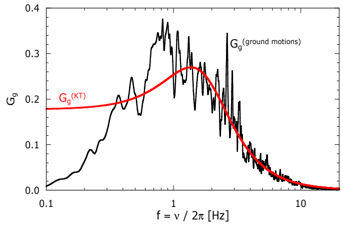


Calibration of Kanai-Tajimi PSD

$G_0 = 0.18 \quad \zeta_g = 0.78$

$\nu_g / (2\pi) = 1.79\text{Hz}$

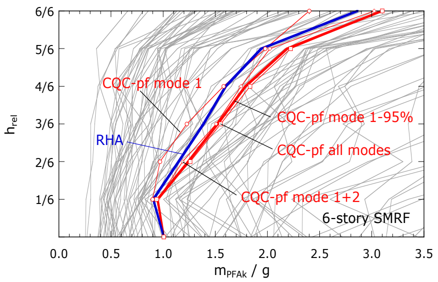
$$G_g^{(KT)} = G_0 \frac{1+4\zeta_g^2(\nu/\nu_g)^2}{\left(1-(\nu/\nu_g)^2\right)^2+4\zeta_g^2(\nu/\nu_g)^2}$$

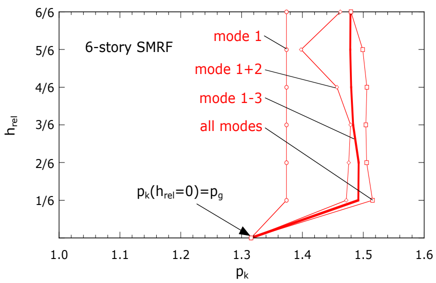


Application

Planar SMRF structures

- 6-story ($\omega_1 = 7.33\,\mathrm{rad/s}$), 12-story ($\omega_1 = 4.22\,\mathrm{rad/s}$), and 24-story ($\omega_1 = 2.42\,\mathrm{rad/s}$) structures
- Linear mode shape
- Seismic active mass concentrated to BC connections. Roof only half mass.
- 5% Rayleigh damping





Application

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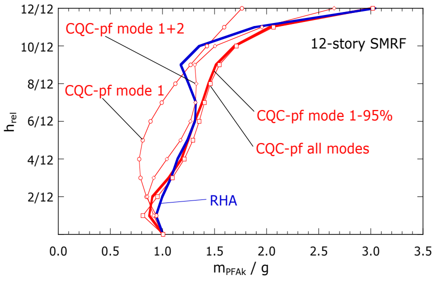
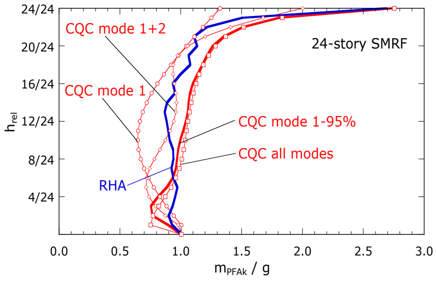
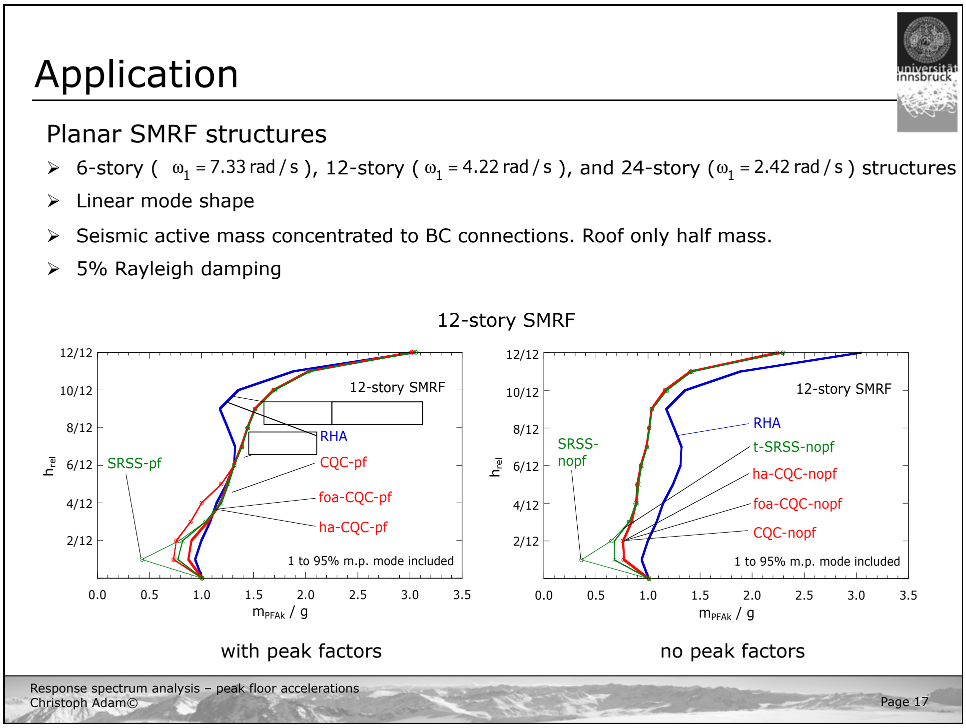


12-story SMRF



24-story SMRF



Application

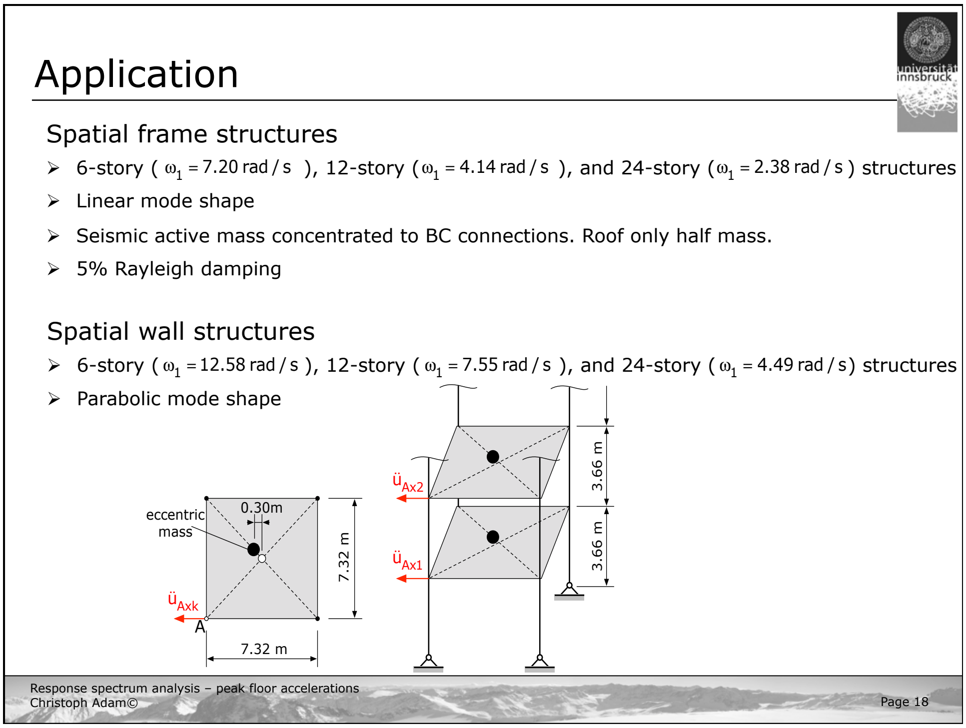
Planar SMRF structures

- 6-story ($\omega_1 = 7.33\,\mathrm{rad/s}$), 12-story ($\omega_1 = 4.22\,\mathrm{rad/s}$), and 24-story ($\omega_1 = 2.42\,\mathrm{rad/s}$) structures
- Linear mode shape
- Seismic active mass concentrated to BC connections. Roof only half mass.
- 5% Rayleigh damping

12-story SMRF

with peak factors

no peak factors



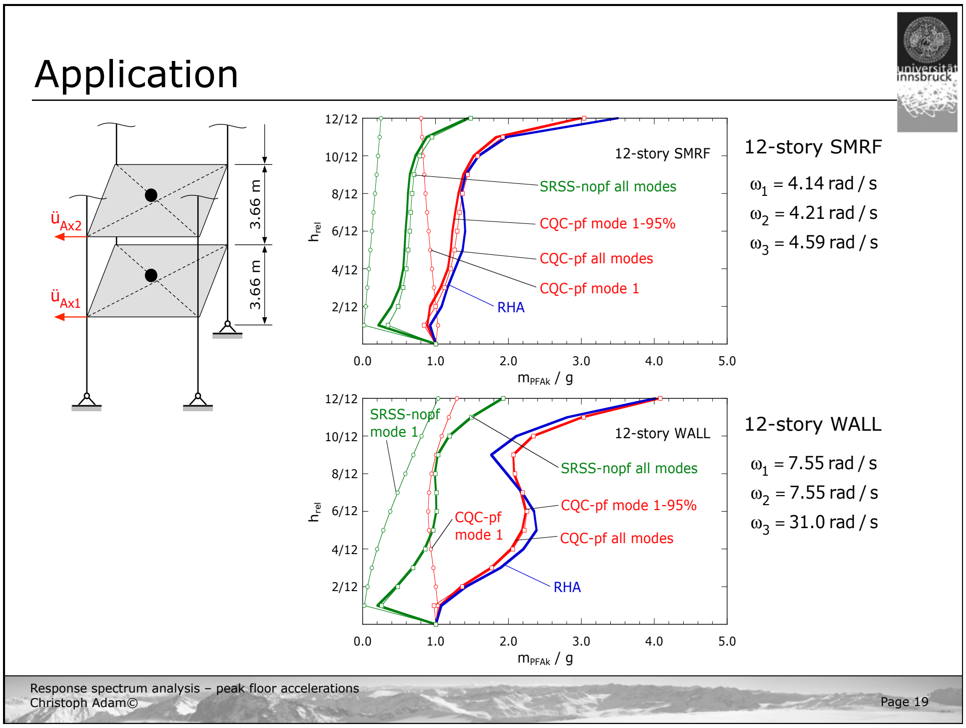
Application

Spatial frame structures

- 6-story ($\omega_1 = 7.20\,\mathrm{rad/s}$), 12-story ($\omega_1 = 4.14\,\mathrm{rad/s}$), and 24-story ($\omega_1 = 2.38\,\mathrm{rad/s}$) structures
- Linear mode shape
- Seismic active mass concentrated to BC connections. Roof only half mass.
- 5% Rayleigh damping

Spatial wall structures

- 6-story ($\omega_1 = 12.58\,\mathrm{rad/s}$), 12-story ($\omega_1 = 7.55\,\mathrm{rad/s}$), and 24-story ($\omega_1 = 4.49\,\mathrm{rad/s}$) structures
- Parabolic mode shape



Application

12-story SMRF

$\omega_1 = 4.14$ rad / s

$\omega_2 = 4.21$ rad / s

$\omega_3 = 4.59$ rad / s

12-story WALL

$\omega_1 = 7.55$ rad / s

$\omega_2 = 7.55$ rad / s

$\omega_3 = 31.0$ rad / s

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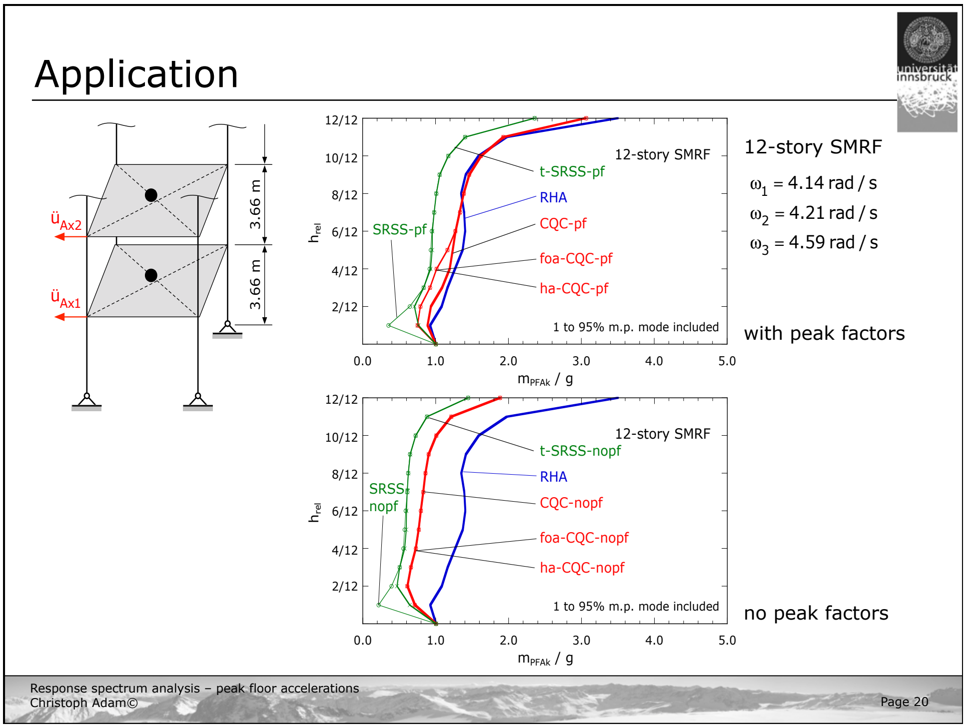

Application

12-story SMRF

$\omega_1 = 4.14$ rad / s

$\omega_2 = 4.21$ rad / s

$\omega_3 = 4.59$ rad / s

with peak factors

no peak factors

Response spectrum analysis – peak floor accelerations
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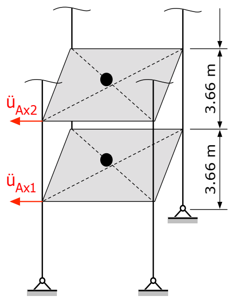
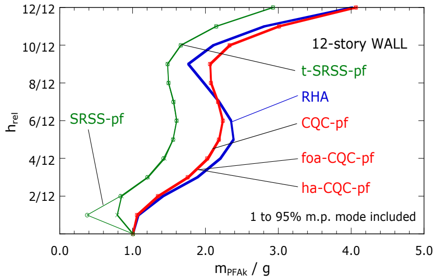
Application





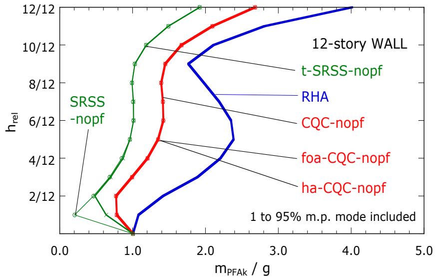
12-story WALL

$\omega_1 = 7.55$ rad / s

$\omega_2 = 7.55$ rad / s

$\omega_3 = 31.0$ rad / s

with peak factors



no peak factors

Summary and conclusions

- ✓ Robust CQC modal combination rule for central PFA demands proposed
- ✓ Analytical expressions and approximations for
  - Correlation coefficients
  - Peak factors
- ✓ Reliable assessment of the central PFA demand
- ✓ Peak factors
  - Significant for midrise to highrise structures
- ✓ Correlation coefficients
  - Insignificant for planar structures
  - Significant for spatial structures with closely spaced modes
- ✓ Truncated modes
  - Significant for lower stories

Thank you very much
for your attention!