Precursors of Iso-Geometric Analysis (pre-IGA)

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OUTLINE

- Progress of CAGD before NURBS (1964 – 2003, ...), and the
- First ideas on CAD/CAE integration (~1972, GM)
- NTUA research 1984-1989: Pre-IGA (C-elements)
  - Using a reduced cardinal B-spline
- Pre-IGA: BEM analysis (1991-1992)
- Bézier patches and relevant macroelements (1992)
- Pre-IGA: Volume Blocks (3D)
- Pre-IGA Collocation Method: 2004-2006-2007
- Applications:
  - Laplace – Poisson, 2D-acoustics, 2D-Elasticity, Plate-bending, 3D problems – Shape optimization

Spline Interpolation (< Schoenberg 1946)

Non-decreasing sequence of breakpoints : $x_0, x_1, \ldots, x_{n-1}, x_n$

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \sum_{j=1}^{n-1} b_j \left\langle x - x_j \right\rangle_+^3$$

$$\left\langle x - x_i \right\rangle_+^3 = \begin{cases} 0, & x \le x_i \\ \left(x - x_i\right)^3, & x > x_i \end{cases}$$

"Modern" consideration of B-splines

Non-decreasing sequence of *knots* : $u_0, u_1, \ldots, u_{m-1}, u_m$

$$\mathbf{C}(u) = \sum_{i=0}^{n_p} N_{i,p}(u)\mathbf{P}_i \qquad 0 \le u \le 1$$

$$m = n_p + p + 1$$

$$N_{i,0} = \begin{cases} 1, & u_i \le u < u_{i+1}, \quad i = 0, \ldots, m-1 \\ 0 & \text{elsewhere} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

de Boor (1972)

Chronological sequence of CAGD

- Coons interpolation (S.A. Coons: 1964)
- Gordon's interpolation (W.J. Gordon: 1971)
- Bézier interpolation (P. Bézier, 1970)
- B-splines (J. Schoenberg 1946, C. de Boor 1972, Cox 1972)
- NURBS (L. Piegl, 1988, between others)→IGA
- T-splines
- Etc……

Some pioneering papers

MIT/LCS/TR-41

SURFACES FOR COMPUTER-AIDED DESIGN OF SPACE FORMS

Steven A. Coons

June 1967

http://publications.csail.mit.edu/lcs/pubs/pdf/MIT-LCS-TR-041.pdf

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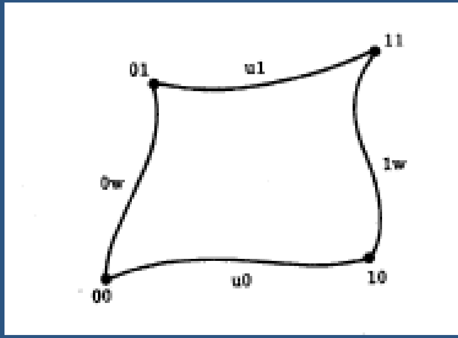

Numer. Math. 21, 109–129 (1973)

Transfinite Element Methods: Blending-Function Interpolation over Arbitrary Curved Element Domains

William J. Gordon

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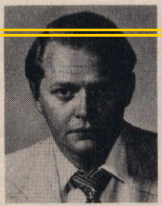
Charles A. Hall

Department of Mathematics, University of Pittsburgh, Pittsburgh, Pennsylvania

Received April 17, 1972

Abstract. In order to better conform to curved boundaries and material interfaces, curved finite elements have been widely applied in recent years by practicing engineering analysts. The most well known of such elements are the "isoparametric elements." As Zienkiewicz points out in [18, p. 132] there has been a certain parallel between the development of "element types" as used in finite element analyses and the independent development of methods for the mathematical description of general free-form surfaces. One of the purposes of this paper is to show that the relationship between these two areas of recent mathematical activity is indeed quite intimate. In order to establish this relationship, we introduce the notion of a "transfinite element" which, in brief, is an invertible mapping $\vec{T}$ from a square parameter domain $\mathscr{S}$ onto a closed, bounded and simply connected region $\mathscr{R}$ in the xy-plane together with a "transfinite" blending-function type interpolant to the dependent variable f defined over $\mathscr{R}$. The "subparametric," "isoparametric" and "superparametric" element types discussed by Zienkiewicz in [18, pp. 137–138] can all be shown to be special cases obtainable by various discretizations of transfinite elements. Actual error bounds are derived for a wide class of semi-discretized transfinite elements (with the nature of the mapping $\vec{T}$: $\mathscr{S} \to \mathscr{R}$ remaining unspecified) as applied to two types of boundary value problems. These bounds for semi-discretized elements are then specialized to obtain bounds for the familiar isoparametric elements. While we consider only two dimensional elements, extensions to higher dimensions is straightforward.

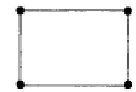
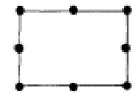
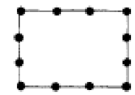
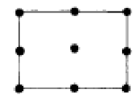
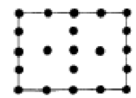
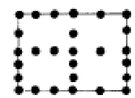
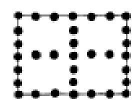


William J. Gordon has been a professor of mathematics and computer science at Drexel University in Philadelphia since 1979. In addition, he is director of Drexel's center for interactive computer graphics. Prior to joining Drexel, he was a visiting scientist at IBM's T. J. Watson Research Center, spent 11 years at the General Motors Research Laboratories, and from 1976 to 1978, served as liaison scientist for the London branch of the US Office of Naval Research. Gordon has taught at the University of Utah, Syracuse University, and the University of Detroit. His current research interests include the mathematical foundations of geometry modeling.

Gordon received his DSc degree from Brown University in 1965. He is a member of ACM, SIAM, and the AMS.

Table 1

$p_1 \ni \|f - \mathscr{P}_s \oplus \mathscr{P}_t[f]\|_\infty = \mathcal{O}(h^{p_1})$ as $h \to 0$
$p_2 \ni \|f - \mathscr{P}_s \mathscr{P}_t[f]\|_\infty = \mathcal{O}(h^{p_2})$ as $h \to 0$
$p_3 \ni \|f - \tilde{f}(m, n, \overline{m}, \overline{n})\|_\infty = \mathcal{O}(h^{p_3})$ as $h \to 0$
N = number of points in stencil
$f(m, n, \overline{m}, \overline{n})$ = Function defined by Eq. (20)

$\tilde{f}(1, 1, 1, 1)$ 	$p_1 = 4$,	$p_2 = 2$,	$p_3 = 2$,	$N = 4$
$\tilde{f}(1, 1, 2, 2)$ 	$p_1 = 4$,	$p_2 = 2$,	$p_3 = 3$,	$N = 8$
$\tilde{f}(1, 1, 3, 3)$ 	$p_1 = 4$,	$p_2 = 2$,	$p_3 = 4$,	$N = 12$
$\tilde{f}(2, 2, 2, 2)$ 	$p_1 = 6$,	$p_2 = 3$,	$p_3 = 3$,	$N = 9$
$\tilde{f}(2, 2, 4, 4)$ 	$p_1 = 6$,	$p_2 = 3$,	$p_3 = 5$,	$N = 21$
$\tilde{f}(2, 2, 5, 5)$ 	$p_1 = 6$,	$p_2 = 3$,	$p_3 = 6$,	$N = 27$
$\tilde{f}(2, 2, 6, 6)$ 	$p_1 = 6$,	$p_2 = 3$,	$p_3 = 6$,	$N = 33$

Numer. Math. 26, 155–178 (1976)

Ritz-Galerkin Approximations in Blending Function Spaces

James C. Cavendish, William J. Gordon, and Charles A. Hall

Received June 24, 1974

Summary. This paper considers the theoretical development of finite dimensional bivariate blending function spaces and the problem of implementing the Ritz-Galerkin method in these approximation spaces. More specifically, the approximation theoretic methods of polynomial blending function interpolation and approximation developed in [2, 11–13] are extended to the general setting of L-splines, and these methods are then contrasted with familiar tensor product techniques in application of the Ritz-Galerkin method for approximately solving elliptic boundary value problems. The key to the application of blending function spaces in the Ritz-Galerkin method is the development of criteria which enable one to judiciously select from a nondenumerably infinite dimensional linear space of functions, certain finite dimensional subspaces which do not degrade the asymptotically high order approximation precision of the entire space. With these criteria for the selection of subspaces, we are able to derive a virtually unlimited number of new Ritz spaces which offer viable alternatives to the conventional tensor product piecewise polynomial spaces often employed. In fact, we shall see that tensor product spaces themselves are subspaces of blending function spaces; but these subspaces do not preserve the high order precision of the infinite dimensional parent space.

Considerable attention is devoted to the analysis of several specific finite dimensional blending function spaces, solution of the discretized problems, choice of bases, ordering of unknowns, and concrete numerical examples. In addition, we extend these notations to boundary value problems defined on planar regions with curved boundaries.

1. Introduction

In this paper we provide a theoretical development of finite dimensional blending function spaces, and we consider the utilization of these spaces in Ritz-Galerkin methods for the approximation of two-dimensional elliptic boundary value problems.

In Section 2 we begin by extending the analytic theory of blended interpolation developed in [2, 11–13] to the more general setting of L-splines. We also consider methods for the discretization of L-spline blended interpolants and present approximation theoretic error bounds for these finite dimensional function subspaces. These results are used to derive asymptotic bounds on the discretization error involved in Ritz-Galerkin approximations to second-order elliptic problems by functions from a finite dimensional subspace of an L-spline blended space. We conclude that by the judicious choice of such subspaces, one may obtain finite dimensional Ritz spaces having the same asymptotically high order of approximation as the original nondenumerably infinite dimensional parent space. Moreover, the resulting finite dimensional blending function spaces are of considerably smaller dimension than conventional piecewise polynomial tensor product spaces of comparable asymptotic accuracy. The results presented in Section 2 serve to

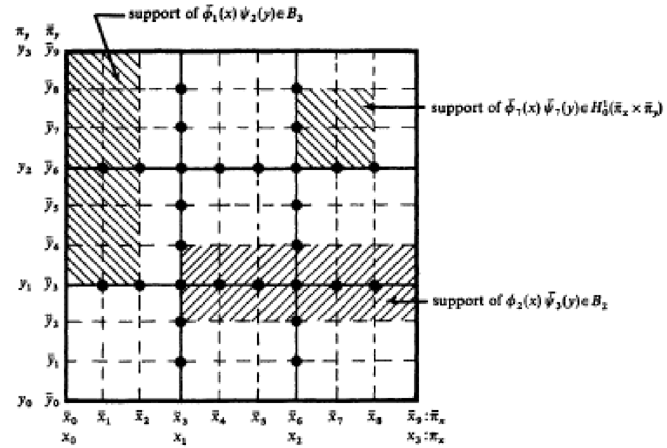


Fig. 1. Regions of support for particular basis functions in B_1, B_3 and $H_0^1(\bar{\pi}_x \times \bar{\pi}_y)$

$\cup\, H_0^1(\bar{\pi}_x \times \pi_y)\rangle$. In Figure 1 we show the regions of support for typical basis elements in B_1 and B_3 and $H_0^1(\bar{\pi}_x \times \bar{\pi}_y)$ for the case $h_x = h_y = 1/3$, $\bar{h}_x = \bar{h}_y = 1/9$.

Determination of a Ritz-Galerkin approximation to (29a–c) in a finite dimensional function space S_M is equivalent to solving a linear system of equations

$$K\,\bar{x} = \bar{b} \tag{39}$$

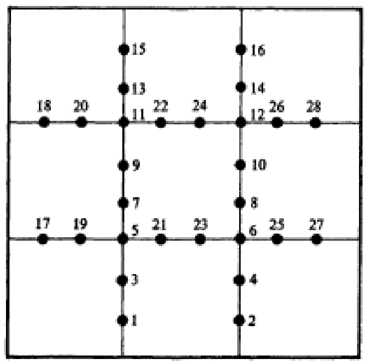


Fig. 2. Ordering unknowns for the stencil in Figure 1

Year 1976

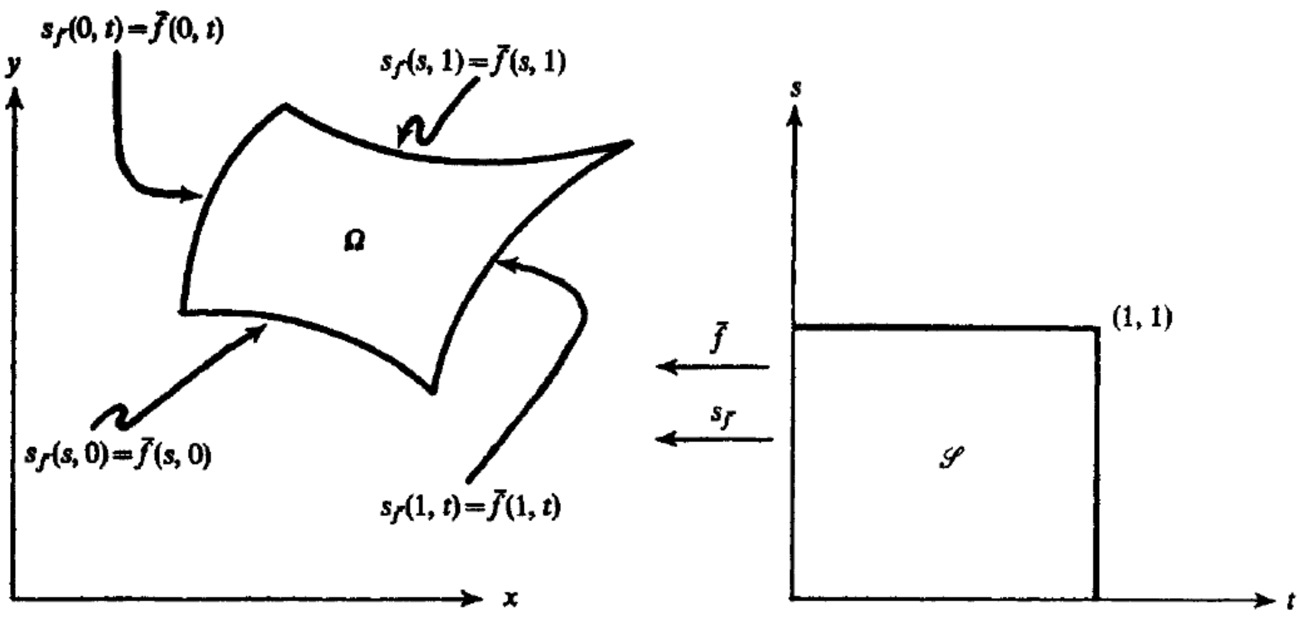


Fig. 5. A domain transformation $s_f: \mathscr{S} \to \Omega$ such that $s_f: \partial\mathscr{S} \to \partial\Omega$

Example 4. Consider the problem

$$-\nabla^2 u = f, \quad (x,y) \in \Omega \tag{64}$$

$$u = 0 \quad (x,y) \in \partial\Omega \tag{65}$$

where Ω is the region in Figure 7.

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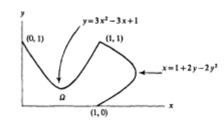


Fig. 7. Problem domain Ω for (64), (65)

Table 12. Problem (64), (65)—Linearly blended-linearly decomposed Ritz approximations, $\hat{u}$

$\bar{h}$	Dimension of Ritz space	$\Vert u-\hat{u}\Vert_{L_\infty(\Omega)}$	α
1/9	28	6.16×10^{-1}	—
1/16	81	2.78×10^{-1}	1.40
1/25	176	1.41×10^{-1}	1.51
1/36	325	7.64×10^{-2}	1.68
1/49	540	4.27×10^{-2}	1.90

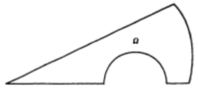
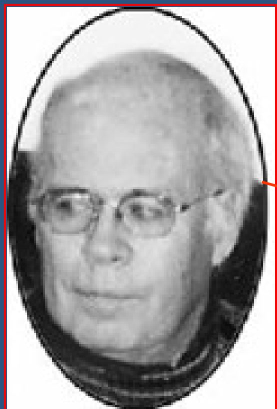


Fig. 8.



INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, VOL. 11, 1405–1421 (1977)

SUBSTRUCTURED MACRO ELEMENTS BASED ON LOCALLY BLENDED INTERPOLATION

JAMES C. CAVENDISH AND WILLIAM J. GORDON

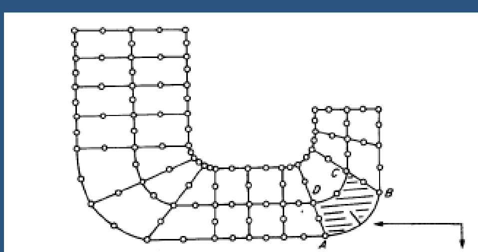
Mathematics Department, General Motors Corporation, General Motors Technical Center, Warren, Michigan, U.S.A.

CHARLES A. HALL

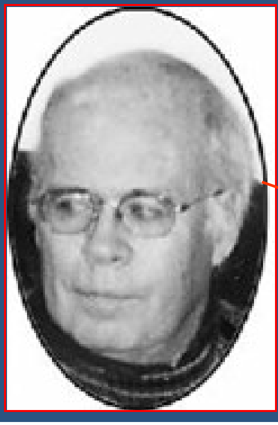
Department of Mathematics and Statistics, University of Pittsburgh, Pittsburgh, Pennsylvania, U.S.A.

SUMMARY

In this paper we describe a new class of locally refined macro finite elements which are especially amenable to the use of substructuring techniques for the efficient solution of the resulting idealization. The tools and guidelines illustrated by the examples of modelling crack tips, point load singularities and singularities at re-entrant corners should enable an analyst to construct other such blended macro elements specifically tailored to his particular class of problems. The use of such substructured macro elements in finite element calculations permits substantial reduction in the manual effort of data preparation and the computational cost of numerical solution.



GMSOLID



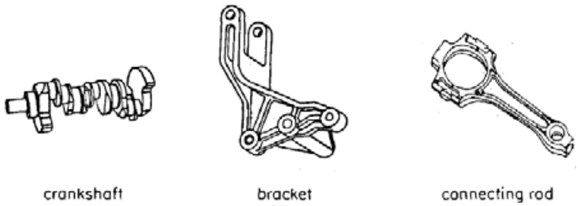


Figure 2. Solid parts designed on GMSOLID

GMSOLID

INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, VOL. 20, 241–253 (1984)

A NEW CLASS OF TRANSITIONAL BLENDED FINITE ELEMENTS FOR THE ANALYSIS OF SOLID STRUCTURES†

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Department of Mathematics and Statistics, University of Pittsburgh, Pittsburgh, Pennsylvania, U.S.A.

SUMMARY

Recently developed computer aided design systems for the design and modification of complex physical solids using interactive computer graphics offer the exciting possibility of an integrated design/analysis system. Called geometric modellers, these systems build complex solids from primitive solids (cubes, cylinders, spheres, etc.) and macro solids (combinations of primitives). To provide an effective finite element analysis capability for these systems, methods must be devised to ease the burden of discretizing the solid geometry into a user controlled finite element mesh. In this paper we describe a new class of transitional blended finite elements which make substantially simpler the task of finite element mesh generation and local mesh refinement. Computational experience indicates that numerical accuracy is not compromised by use of these flexible elements.

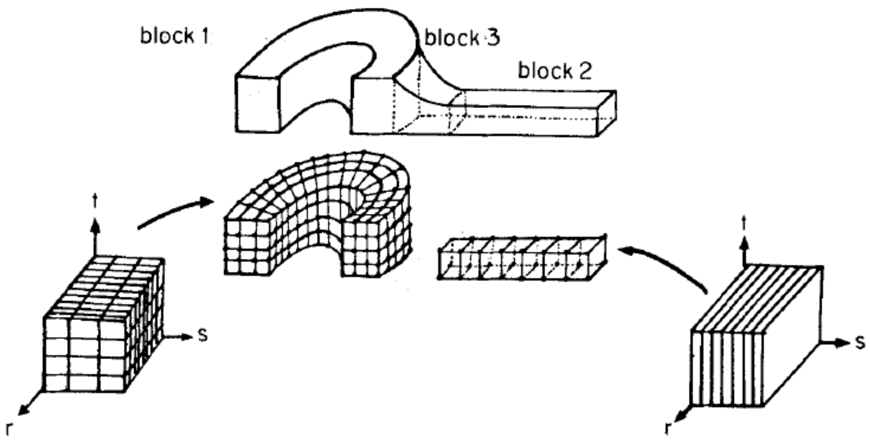

Figure 4. Mesh generation via isoparametric or blended mappings

GMSOLID
(1984)

J. C. CAVENDISH AND C. A. HALL

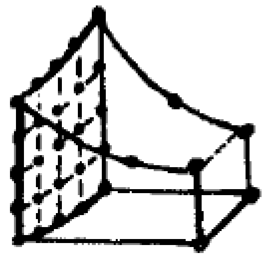

Figure 5. A blended transition element

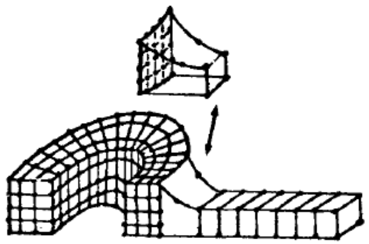

Figure 6. An assembly of compatible finite elements

INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, VOL. 20, 241–253 (1984)

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INTRODUCTION

Computer aided design (CAD) systems have proved to be extremely useful for the automation of two-dimensional design and drafting procedures. Notable success has also been achieved by three-dimensional CAD systems for the design of curves and sculptured surfaces in the automobile, aerospace and shipbuilding industries. Although such so-called wire-frame CAD systems based on sculptured curve/surface technology are useful for the design of smooth exterior surfaces (for example, automobile sheet metal panels), they are awkward and difficult to use for the design of solid functional components such as automobile pistons, connecting rods, crankshafts, housings or other parts that are typically cast, moulded or machined. Several CAD systems have recently been built for the design of such solid objects. Among the most interesting approaches have been systems which combine (via unions, differences and intersections) many copies of a few basic primitive solids (blocks, cylinders, cones and spheres) in the design of complex parts. Figure 1 illustrates a simple example of these set operations applied to a block and a cylinder.

A very relevant question is whether or not a part designer can (with reasonable experience) successfully manipulate solid primitives (instead of lines and curves) in a two-dimensional graphics setting to generate computer representations of functional solid parts. Figure 2 contains examples of such functional parts designed on the solid geometric modelling system, GMSOLID, developed at the General Motors Research Laboratories.[1] GMSOLID is a

† Presented at the SIAM 30th Anniversary Meeting, Stanford University, 19–23 July 1982.

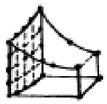

Figure 5. A blended transition element

of the difficulty of the problem. Suppose, however, the user had in his finite element library a solid element with the node/edge configuration displayed in Figure 5. This transitional finite element would be compatible with the decompositions of block 1 and block 2 (that is, the entire finite element assembly would yield a continuous displacement field) and could be used to model block 3, as illustrated in Figure 6.

Although such transitional blended elements currently do not exist in commercial finite element codes (cf. the later section, 'Some computational experiences'), the formal interpolation formulae needed for their construction are well understood and have been documented and analysed in, for example, Reference 3. In addition, convergence results can be developed for these solid element types as simple extensions of the analyses established in Reference 2 for planar blended elements. Expanding a finite element library to include blended transitional solid elements provides basic building blocks which considerably reduce the task of structure decomposition and mesh refinement. One possible approach which exploits the solid geometric modeller during the design phase would be to define finite element decompositions of the modeller's building blocks (the primitives or macro solids). Transitional blended finite elements could then be used to join together the building blocks as the part was being designed (see Figure 7).

In the next section we give a brief account of the construction of transitional blended finite elements. Then, in the third section, we discuss the computational experience we have had with the use of blended elements for modelling two problems: (1) modelling stress distributions around a circular hole in a loaded plate, and (2) modelling stress intensity factors associated with cracks in three-dimensional structures. Our numerical results indicate that using blended finite elements with a small total number of degrees-of-freedom compares very favourably with standard finite element idealizations using more elements with far more total degrees-of-freedom. We conclude the paper with a summary and some comments in the final section.

CONSTRUCTION OF TRANSITIONAL BLENDED ELEMENTS

A blended finite element is any element whose definition is based upon blended interpolation methods.[3] For ease of exposition we consider first a specific two-dimensional blended transi-

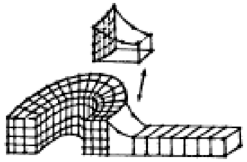

Figure 6. An assembly of compatible finite elements

NUMERICAL GRID GENERATION
Joe F. Thompson, editor

TRANSFINITE MAPPINGS AND THEIR APPLICATION TO GRID GENERATION

WILLIAM J. GORDON AND LINDA C. THIEL
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Philadelphia, PA 19104 USA

SUMMARY

The two essential ingredients of any boundary value problem are the field equations which describe the physics of the problem and a set of relations which specify the geometry of the problem domain. Mesh generators or grid generators are preprocessors which decompose the problem domain into a large number of interconnected finite elements or curvilinear finite difference stencils. A number of such techniques have been developed over the past decade to alleviate the frustration and reduce the time involved in the tedious manual subdividing of a complex-shaped region or 3-D structure into finite elements. Our purpose here is to describe how the techniques of bivariate and trivariate "blending function" interpolation, which were originally developed for and applied to geometric problems of computer-aided design of sculptured surfaces and 3-D solids, can be adapted and applied to the geometric problems of grid generation. In contrast to other techniques which require the numerical solution of complex partial differential equations (and, hence, a great deal of computing), the transfinite methods proposed herein are computationally inexpensive.

1. INTRODUCTION

Over the past decade, a number of schemes have been developed for automating the generation of finite element and curvilinear finite difference grids. Among these, the *transfinite mapping* technique of Gordon and Hall[5] has been shown to have a number of advantages (cf. [6],[7]). Some of these are:

1. Exact modeling of boundaries
2. Minimal input effort
3. Automatic node connectivity definition
4. Well-suited to interactive graphics implementation
5. Good correlation between boundary nodes and interior mesh
6. Computationally efficient
7. Easy extension to three dimensions.

We use the term "*transfinite*" to describe this class of techniques since, unlike classical methods of higher dimensional interpolation which match the *primitive function* $\vec{F}$ at a finite number of points, the transfinite methods match

Gordon's last work

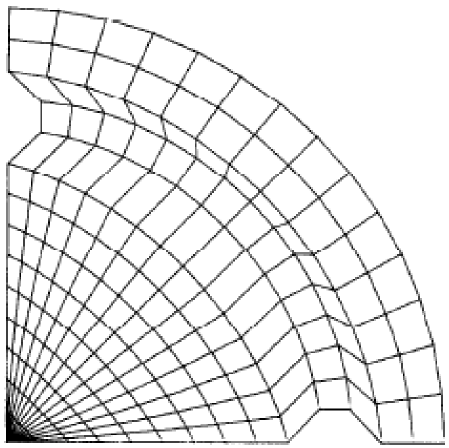

Figure 8

This mapping was generated via (18) by mapping the point (11/14,1/2) in the s,t-plane onto the point (.326,.334) in the x,y-plane. The boundary segments are:

$$\vec{F}(0,t) = \begin{bmatrix} -3 \\ -3 \end{bmatrix}, \quad \vec{F}(1,t) = \begin{bmatrix} -3+6\cos(\pi t/2) \\ -3+6\sin(\pi t/2) \end{bmatrix}$$

$$\vec{F}(s,0) = \begin{bmatrix} 6s-3 \\ \begin{cases} -3+\sqrt{.25-(6s-4.5)^2}, & .67 \le s \le .83 \\ -3, & \text{otherwise} \end{cases} \end{bmatrix}$$

$$\vec{F}(s,1) = \begin{bmatrix} \begin{cases} -3+\sqrt{.25-(6s-4.5)^2}, & .67 \le s \le .83 \\ -3, & \text{otherwise} \end{cases} \\ 6s-3 \end{bmatrix}$$

INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, VOL. 12, 1841–1851 (1978)

Cavendish's ~last work

BLENDED INFINITE ELEMENTS FOR PARABOLIC BOUNDARY VALUE PROBLEMS

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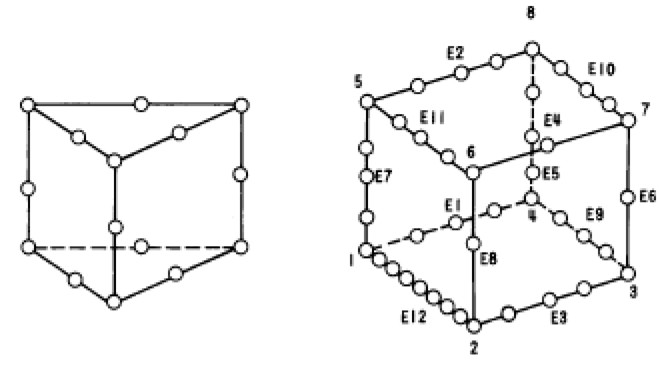

(a) 45 d.o.f. WEDGE (b) VARIABLE d.o.f. BLENDED BRICK

Figure 4. Micro elements used to design the macro element

International Journal of Fracture, Vol. 15, No. 3, June 1979 231

Hall's ~last work

A macro element approach to computing stress intensity factors for three dimensional structures

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(Received November 25, 1977; in revised form July 27, 1978)

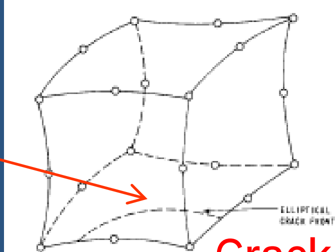

Figure 1. Macro element

Crack

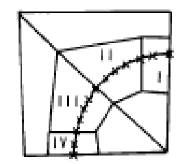

Figure 6a. Groups I to IV of the channel

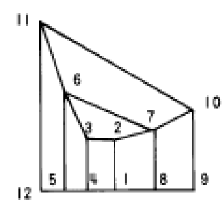

Figure 6c. Orbit designation

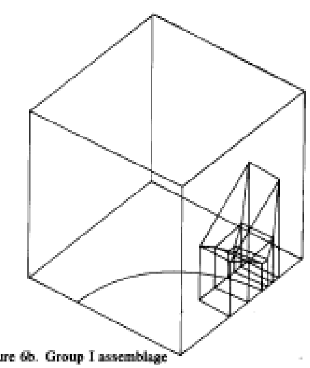

Figure 6b. Group I assemblage

Figure 6. Micro element assemblage

NTUA-attempts since 1984

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Finite Elements in Analysis and Design 5 (1989) 97–109   97
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ON THE SOLUTION OF LAPLACE AND WAVE PROPAGATION PROBLEMS USING "C-ELEMENTS"

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Received June 1988
Revised November 1988

Abstract. This paper tackles the solution of Laplace and wave propagation problems using large B-splines-finite-elements based on Coons's interpolation method ("C-elements"). C-elements are characterized by degrees of freedom appearing only at the element boundaries and can be used in the solution of both static and dynamic problems. They are described in detail and compared to boundary and finite elements. Finally, numerical results are given that sustain theoretical statements.

Introduction

The development of "large" elements with the purpose of reducing mesh generation work load, the total number of degrees of freedom, as well as the computational effort in both the static and dynamic regimes, has kept researchers busy for a long time.

In the context of finite element methods (FEM), isoparametric elements were first introduced in [13] and [31]. It was Irons [18] who generalized the idea of arbitrarily noded elements. Isoparametric elements were analyzed from a strictly mathematical point of view by Zlamal [33] and then also by Ciarlet and Raviart [6]. Blending function methods based on the ideas put forward in [8] have been used to produce some interesting element families [16]. These methods have been extended and generalized in [1–3, 15]. El-Zafrany and Cookson in [12] also use Coons's ideas.

On the other hand, "large" elements are introduced in a most natural way by the boundary element method (BEM) [4] and by schemes [19–21] based on Trefftz's method [30]. These require knowledge of the fundamental solution of the problem under consideration.

The present work proposes the use of "large" B-splines finite-elements ("C-elements") based on Coon's interpolation theory, with degrees of freedom appearing only at the element boundaries. C-elements can be applied to both static and dynamic problems.

In the sequel, fundamental features of C-elements are presented. At a second stage their performance is compared to the finite element, as well as to the boundary element method.

C-elements methodology

Let us pose the following problems: First, the two-dimensional Laplace equation

$$u_{xx}+u_{yy}=\nabla^2 u=0 \tag{1a}$$

and, second, the wave propagation equation

$$(1/c^2)u_{tt}-\nabla^2 u=0. \tag{1b}$$

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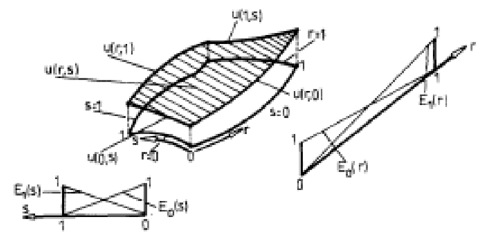


Fig. 1. Boundary curves $u(0, s)$, $u(1, s)$, $u(r, 0)$, $u(r, 1)$ and "blending functions" E_0 and E_1 of the C-element.

For C^0-continuity discrete problems the blending functions are linear and equal to:

$$E_0(n)=1-n, \qquad E_1(n)=n. \tag{11}$$

The next step will be to determine a suitable discretization scheme along the C-element boundaries. Having prescribed q (different) degrees of freedom on each boundary, $u(0, s_i)$, $u(1, s_i)$, $u(r_i, 0)$ and $u(r_i, 1)$, $i=1, 2, \dots, q$, appropriate interpolating formulae for the functions $u(0, s)$, $u(1, s)$, $u(r, 0)$ and $u(r, 1)$ are sought. Considering that q may be allowed to be a large number, a Lagrangian interpolation polynomial would tend to produce undesirable oscillations between two arbitrary abscissae n_i and n_{i+1}, as it may possess as many as $(q-1)$ maxima and minima over its entire interval of variation. For this reason, the use of splines is envisaged:

Given q degrees of freedom on a C-element-boundary at $n_1, n_2, \dots, n_q$, a spline function $B(n)$ of degree m is a function having the two following properties [10,14]:

(1) In each interval (n_i, n_{i+1}), $i=1, 2, \dots, q-1$, $B(n)$ is given by a polynomial of degree m or less.

(2) $B(n)$ and its derivatives of order $1, 2, \dots, m-1$ are continuous everywhere.

A commonly used spline function is the truncated power function $\langle n-n_i\rangle^m$, for any variable $n-n_i$ and for any positive integer m. This function is defined by:

$$\langle n-n_i\rangle^m=(n-n_i)^m, \quad \text{for } n-n_i>0; \qquad \langle n-n_i\rangle^m=0, \quad \text{for } n-n_i<0. \tag{12}$$

It is easily seen [17] that the function $B(n)$ has a unique representation of the form:

$$B(n)=b_0+b_1n+b_2n^2+\dots+b_{m-1}n^{m-1}+\sum_{i=1}^{q-1}a_i\langle n-n_i\rangle^m = P(n)+\sum_{i=1}^{q-1}a_i\langle n-n_i\rangle^m, \tag{13}$$

B-splines
<Schoenberg

with $P(n)$ denoting a polynomial of degree $(m-1)$ and a_i properly chosen constants. The most common case is that of splines of order $m=4$ (degree 3), that is of the cubic B-splines. If now $B_j(n)$, where n is either r or s, denote cardinal splines of degree m, i.e.

$$B_j(n_i)=\delta_{ji}, \tag{14}$$

NTUA-attempts since summer 1984

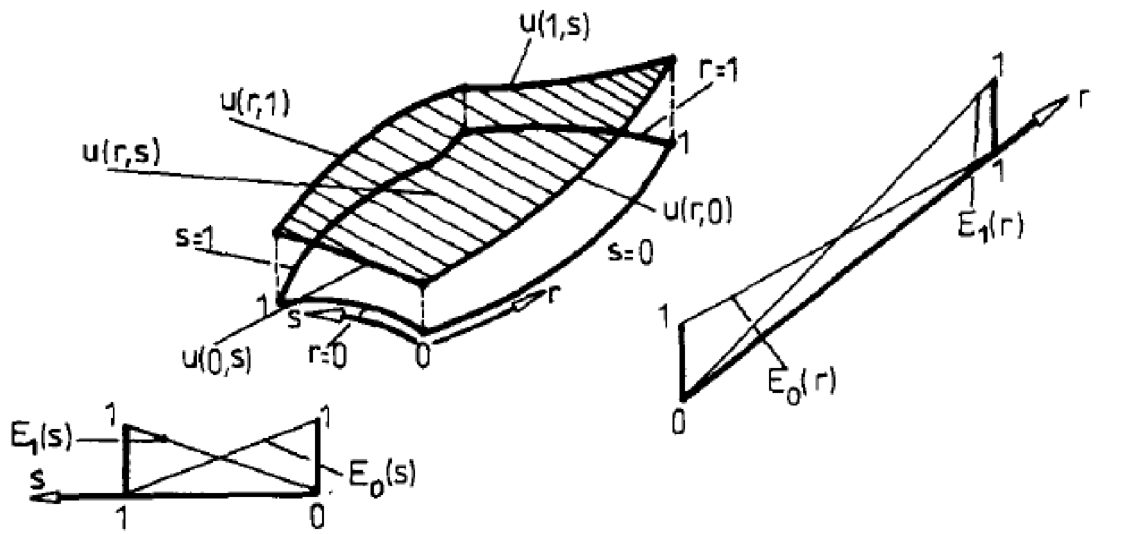


Fig. 1. Boundary curves $u(0, s)$, $u(1, s)$, $u(r, 0)$, $u(r, 1)$ and “blending functions” E_0 and E_1 of the C-element.

$$u(r, s) = P_r[u] + P_s[u] - P_r P_s[u],$$

The same interpolation for both the **Geometry** and the **Analysis**

NTUA-attempts since 1984

then the functions $u(0,s)$, $u(1,s)$, $u(r,0)$ and $u(r,1)$ could be written in the following form:

$$u(0,s)=\sum_{j=1}^{q}B_j(s)u(0,s_j),\qquad u(1,s)=\sum_{j=1}^{q}B_j(s)u(1,s_j);$$
$$u(r,0)=\sum_{j=1}^{q}B_j(r)u(r_j,0),\qquad u(r,1)=\sum_{j=1}^{q}B_j(r)u(r_j,1). \tag{15}$$

In addition, the following equation holds:

$$u(r,s)=E_0(r)u(0,s)+E_1(r)u(1,s)+E_0(s)u(r,0)+E_1(s)u(r,1)+\sum_{i=0}^{1}\sum_{j=0}^{1}E_i(r)E_j(s)u(r_i,s_j)=\sum_{k=1}^{K}N_k(r,s)u_k. \tag{16}$$

Natural cardinal cubic B-splines

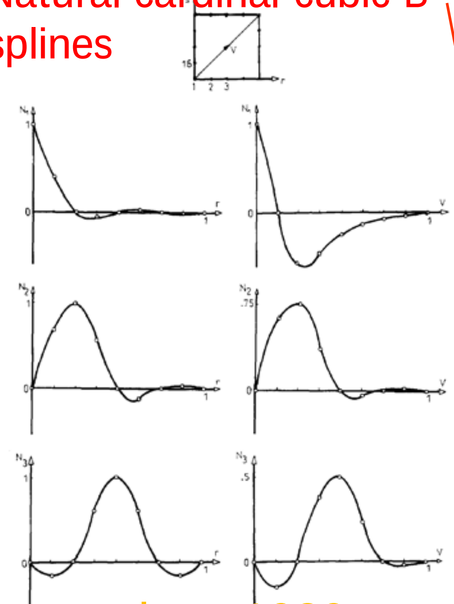
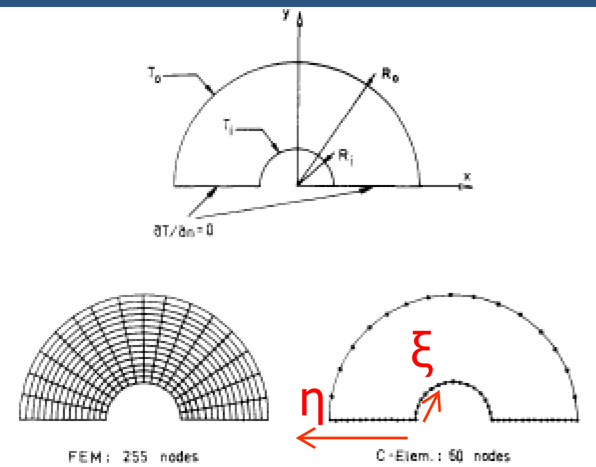


Fig. 2. Shape functions of a 15-node C-element.

Kanarachos, 1989



Radius R(i)	Exact Solution	ERRORS IN %	
		FEM 255 DOF	C-Elem. 60 DOF
1.0000	1000.	0.00	0.00
3.2143	663.1003	3.23	2.86
5.4285	511.8654	3.67	1.60
7.6428	413.1776	3.82	1.68
9.8571	339.7661	3.89	1.62
12.0714	281.2951	3.93	1.63
14.2857	232.6997	3.95	1.62
16.5000	191.1211	3.97	1.62
18.7143	154.7664	3.98	1.62
20.9285	122.5196	3.99	1.62
23.1428	93.5009	4.00	1.62
25.3571	67.1359	4.00	1.63
27.5714	42.9795	4.00	1.62
29.7857	20.6908	4.01	1.73
32.0000	0.	0.00	0.00

Using FEAP

Fig. 4. Example 1: Temperature distribution in a long hollow cylinder with $R_o=4$, $R_i=1$ and radial heat flow. Comparison of the 60-DOF C-element with the FEM (255-DOF, 4-nodes bilinear elements).

[14] FAUX, I.D. and M.J. PRATT, *Computational Geometry for Design and Manufacture*, Ellis Horwood, Chichester, England, 1979.
[15] GORDON, W.J., “Blending functions methods of bivariate multivariate interpolation and approximation”, *SIAM (Soc. Ind. Appl. Math.) J. Numer. Anal.* **8**, pp. 158–177, 1971.
[16] GORDON, W.J. and C.A. HALL, “Transfinite element methods blending function interpolation over arbitrary curved element domains”, *Numer. Math.* **21**, pp. 109–129, 1973.

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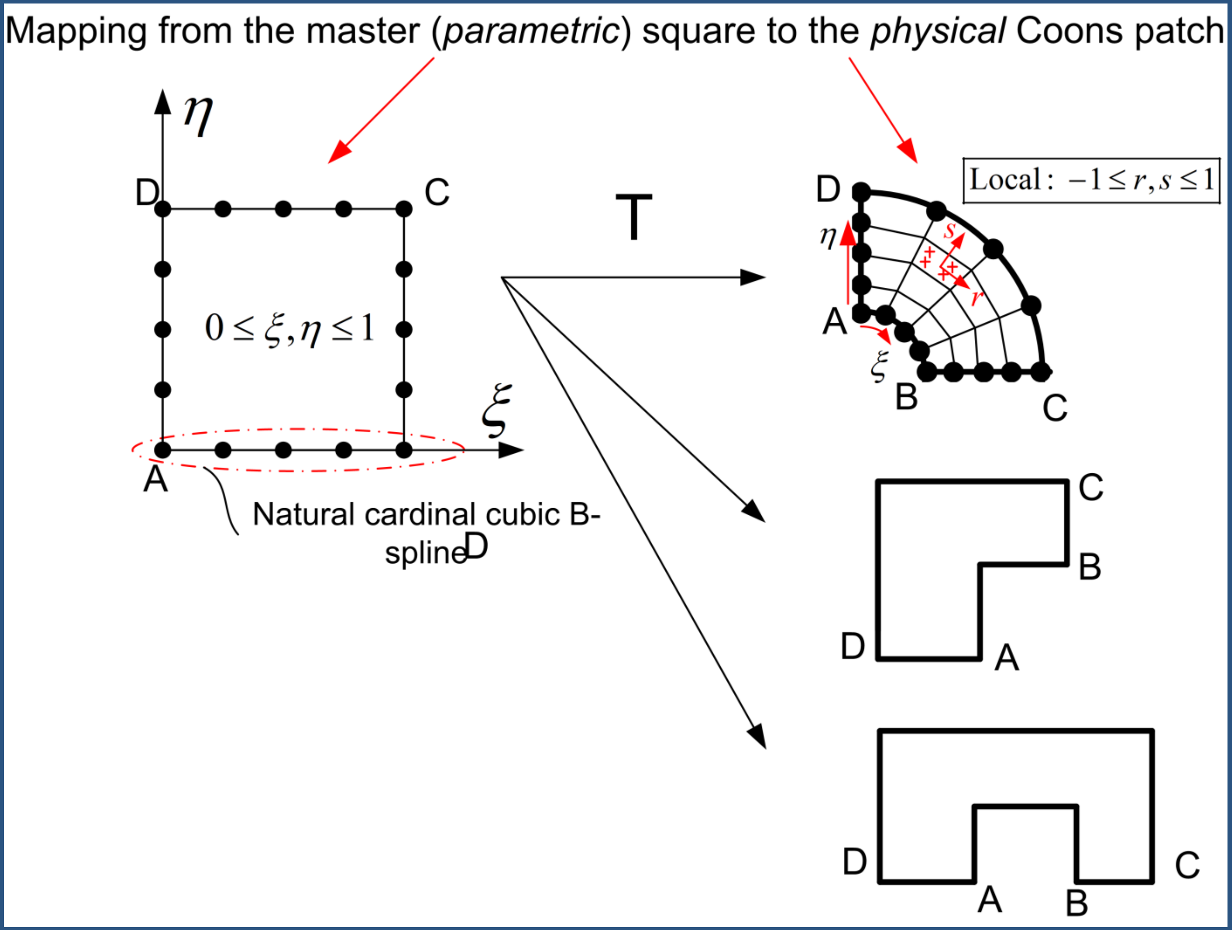
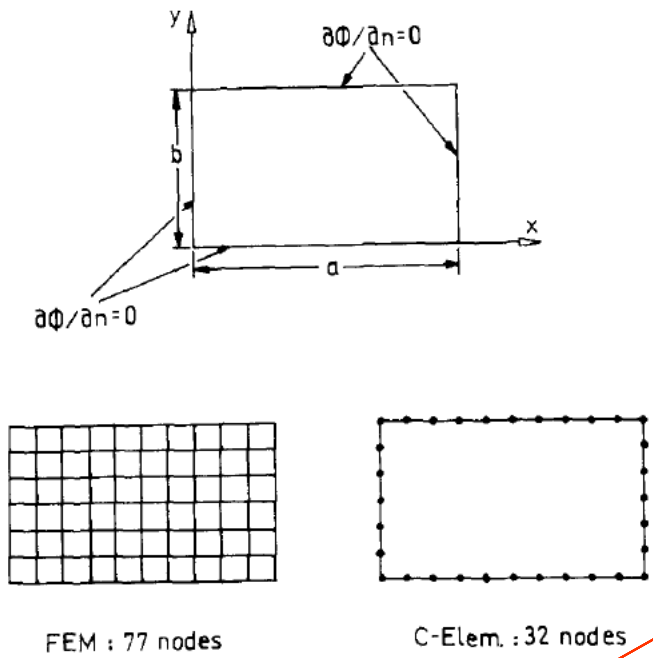


Numerical integration on internal cells (Gaussian quadrature)

NTUA-attempts since 1984



Exact Eigenvalues	ERRORS IN %		
	FEM 77 DOF	BEM 120 DOF	C-Elem. 32 DOF
0.	0.00	0.00	0.00
1.579	0.83	1.26	0.00
6.316	3.34	6.09	0.15
8.157	2.30	12.46	0.19
9.736	2.06	6.86	0.44
14.212	4.64	10.17	0.29
14.473	5.66	18.58	12.23
22.369	5.67	12.31	7.96
25.266	13.71	30.91	0.37

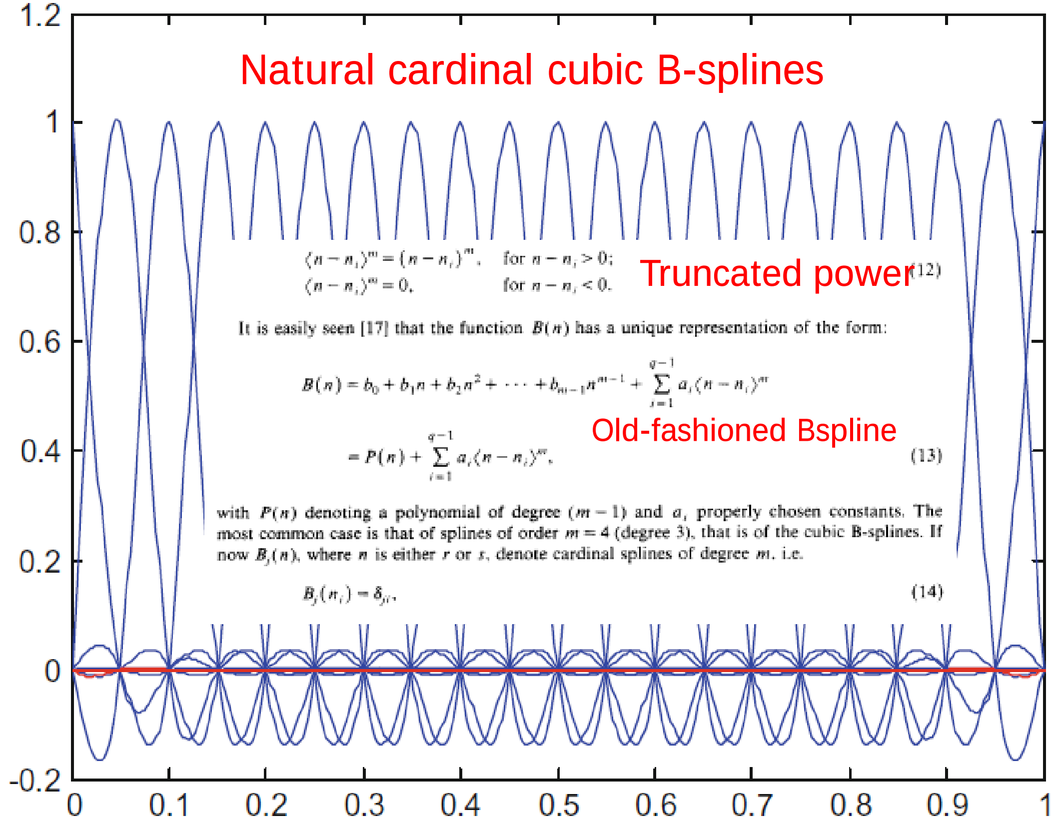
C-element: much better than DR/BEM (Nardini-Brebbia)

Provatidis, 1987

Kanarachos et al, 1989

Fig. 6. Example 3: Eigenvalues of an acoustic cavity with $c=1$, $a=2.5$, $b=1.1$ and Neumann boundary conditions. Comparison of the 32-DOF C-element with the FEM (77-DOF, 4-nodes bilinear elements) and the BEM method (120-DOF).

NTUA-attempts since 1984



$$\langle n - n_i \rangle^m = (n - n_i)^m, \quad \text{for } n - n_i > 0;$$
$$\langle n - n_i \rangle^m = 0, \quad \text{for } n - n_i < 0. \tag{12}$$

It is easily seen [17] that the function $B(n)$ has a unique representation of the form:

$$B(n) = b_0 + b_1 n + b_2 n^2 + \cdots + b_{m-1} n^{m-1} + \sum_{i=1}^{q-1} a_i \langle n - n_i \rangle^m$$
$$= P(n) + \sum_{i=1}^{q-1} a_i \langle n - n_i \rangle^m, \tag{13}$$

with $P(n)$ denoting a polynomial of degree $(m-1)$ and a_i properly chosen constants. The most common case is that of splines of order $m = 4$ (degree 3), that is of the cubic B-splines. If now $B_j(n)$, where n is either r or s, denote cardinal splines of degree m, i.e.

$$B_j(n_i) = \delta_{ji}, \tag{14}$$

Fig. 3.10 Shape functions for nele = 20 subdivisions (natural cubic B-splines)

Implemented in FEAP

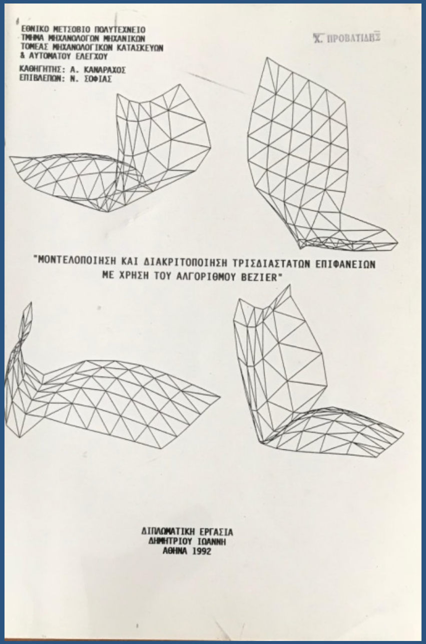
Details in: Provatidis, 2019, ~p. 109

Activities 1990-1998

1990: (~1988) Greece purchased 40 pieces of Mirage-2000 aircrafts → OFFSETS.

- MATRADATAVISION-Palaiseau (Paris), EUCLID_IS: 6-weeks training in Bézier/B-splines/NURBS.
- We proposed the implementation of *C-elements* in EUCLID_IS but Matra-DataVision rejected the idea.
- Essen: VDI-Tagung (neighbors of Babuska-Szabo: PROBE, Hinton-Owen, etc.)
- 1992-1993: A PhD in Bézier-FEA (J. Dimitriou)
- Seven PhD students abandoned (Tutankhamun's Curse)

Activities ~1992-1993



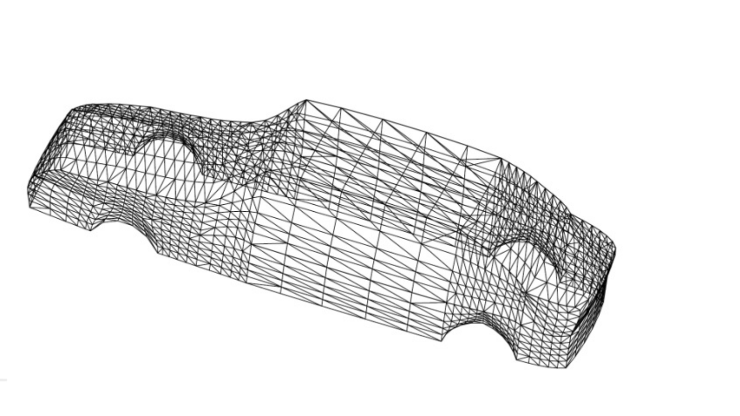
ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ
ΤΜΗΜΑ ΜΗΧΑΝΟΛΟΓΩΝ ΜΗΧΑΝΙΚΩΝ
ΤΟΜΕΑΣ ΜΗΧΑΝΟΛΟΓΙΚΩΝ ΚΑΤΑΣΚΕΥΩΝ
& ΑΥΤΟΜΑΤΟΥ ΕΛΕΓΧΟΥ

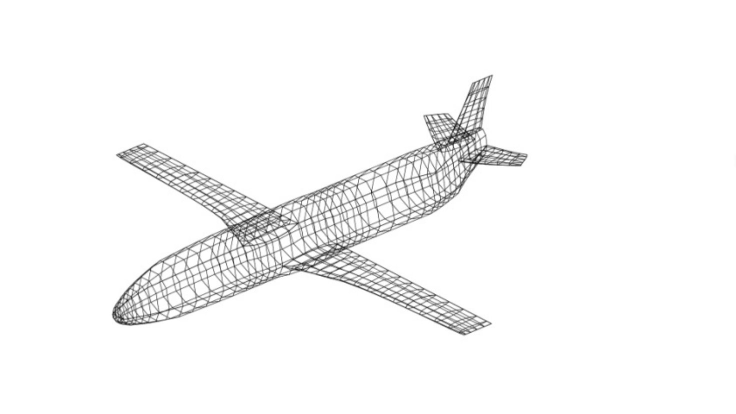
ΚΑΘΗΓΗΤΗΣ: Α. ΚΑΝΑΡΑΧΟΣ
ΕΠΙΒΛΕΠΩΝ: Ν. ΣΟΦΙΑΣ

Χ. ΠΡΟΒΑΤΙΔΗΣ

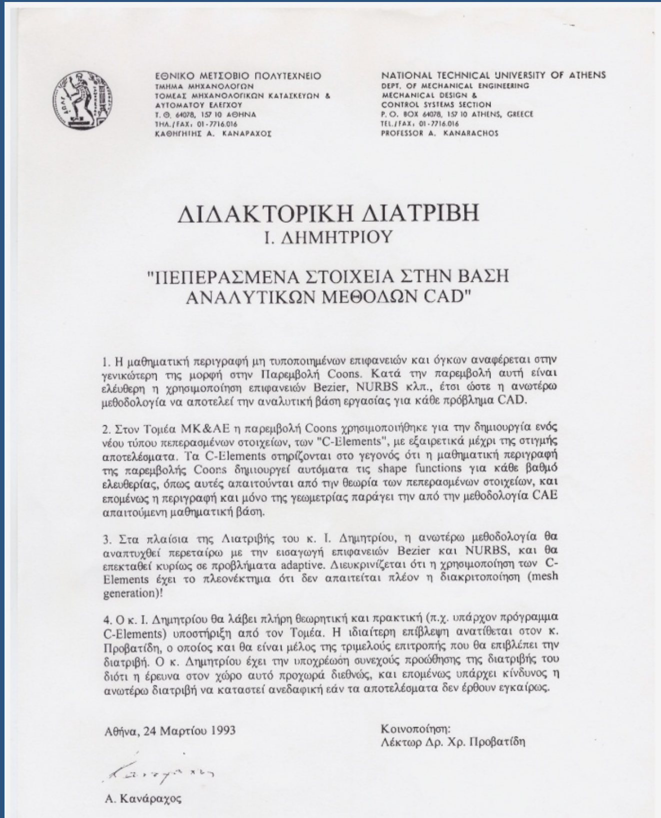
"ΜΟΝΤΕΛΟΠΟΙΗΣΗ ΚΑΙ ΔΙΑΚΡΙΤΟΠΟΙΗΣΗ ΤΡΙΣΔΙΑΣΤΑΤΩΝ ΕΠΙΦΑΝΕΙΩΝ ΜΕ ΧΡΗΣΗ ΤΟΥ ΑΛΓΟΡΙΘΜΟΥ BEZIER"

ΔΙΠΛΩΜΑΤΙΚΗ ΕΡΓΑΣΙΑ
ΔΗΜΗΤΡΙΟΥ ΙΩΑΝΝΗ
ΑΘΗΝΑ 1992





Mini-UNISURF
(>10,000 commands
in Turbo C v.2.0,
Borland)



ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ
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ΤΗΛ./FAX: 01-7716.016
ΚΑΘΗΓΗΤΗΣ Α. ΚΑΝΑΡΑΧΟΣ

NATIONAL TECHNICAL UNIVERSITY OF ATHENS
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PROFESSOR A. KANARACHOS

ΔΙΔΑΚΤΟΡΙΚΗ ΔΙΑΤΡΙΒΗ
Ι. ΔΗΜΗΤΡΙΟΥ

"ΠΕΠΕΡΑΣΜΕΝΑ ΣΤΟΙΧΕΙΑ ΣΤΗΝ ΒΑΣΗ ΑΝΑΛΥΤΙΚΩΝ ΜΕΘΟΔΩΝ CAD"

1. Η μαθηματική περιγραφή μη τυποποιημένων επιφανειών και όγκων αναφέρεται στην γενικώτερη της μορφή στην Παρεμβολή Coons. Κατά την παρεμβολή αυτή είναι ελέυθερη η χρησιμοποίηση επιφανειών Bezier, NURBS κλπ., έτσι ώστε η ανωτέρω μεθοδολογία να αποτελεί την αναλυτική βάση εργασίας για κάθε πρόβλημα CAD.

2. Στον Τομέα ΜΚ&ΑΕ η παρεμβολή Coons χρησιμοποιήθηκε για την δημιουργία ενός νέου τύπου πεπερασμένων στοιχείων, των "C-Elements", με εξαιρετικά μέχρι της στιγμής αποτελέσματα. Τα C-Elements στηρίζονται στο γεγονός ότι η μαθηματική περιγραφή της παρεμβολής Coons δημιουργεί αυτόματα τις shape functions για κάθε βαθμό ελευθερίας, όπως αυτές απαιτούνται από την θεωρία των πεπερασμένων στοιχείων, και επομένως η περιγραφή και μόνο της γεωμετρίας παράγει την από την μεθοδολογία CAE απαιτούμενη μαθηματική βάση.

3. Στα πλαίσια της Διατριβής του κ. Ι. Δημητρίου, η ανωτέρω μεθοδολογία θα αναπτυχθεί περεταίρω με την εισαγωγή επιφανειών Bezier και NURBS, και θα επεκταθεί κυρίως σε προβλήματα adaptive. Διευκρινίζεται ότι η χρησιμοποίηση των C-Elements έχει το πλεονέκτημα ότι δεν απαιτείται πλέον η διακριτοποίηση (mesh generation)!

4. Ο κ. Ι. Δημητρίου θα λάβει πλήρη θεωρητική και πρακτική (π.χ. υπάρχον πρόγραμμα C-Elements) υποστήριξη από τον Τομέα. Η ιδιαίτερη επίβλεψη ανατίθεται στον κ. Προβατίδη, ο οποίος και θα είναι μέλος της τριμελούς επιτροπής που θα επιβλέπει την διατριβή. Ο κ. Δημητρίου έχει την υποχρέωση συνεχούς προώθησης της διατριβής του διότι η έρευνα στον χώρο αυτό προχωρά διεθνώς, και επομένως υπάρχει κίνδυνος η ανωτέρω διατριβή να καταστεί ανεδαφική εάν τα αποτελέσματα δεν έρθουν εγκαίρως.

Αθήνα, 24 Μαρτίου 1993

Κοινοποίηση:
Λέκτωρ Δρ. Χρ. Προβατίδη

Α. Κανάραχος

PhD Thesis: Finite Elements based on Analytical CAD-methods

ECCM'99, Munich, 28 Aug-3 Sept. 1999.

ECCM '99

European Conference on
Computational Mechanics

August 31 – September 3
München, Germany

A NEW APPROACH OF THE FEM ANALYSIS OF TWO-DIMENSIONAL ELASTIC STRUCTURES USING GLOBAL (COONS'S) INTERPOLATION FUNCTIONS

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Key words: Finite elements, Coons interpolation, Macro-elements, Elasticity

Abstract. *This paper introduces the use of global two-dimensional finite elements (macro elements) based on the Coons's interpolation theory, with degrees of freedom appearing only at the boundaries of the domain. The method can be also combined with B-splines interpolation and it is generally applicable to any arbitrary shaped two-dimensional domain. If it is necessary, the domain can be partitioned in a small number of regularly shaped macro-elements, where displacement (C^0) continuity is satisfied per se. It is shown that the proposed macro-elements are successfully applicable to both elasto-static and elasto-dynamic problems. The proposed methodology is general and can be also applied to other engineering problems such as potential problems, acoustics, plate bending, and so on.*

http://users.ntua.gr/cprovat/yliko/763.pdf

(Two PhD Theses discontinued)

On the use of Coons' interpolation in CAD/CAE applications

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National Technical University of Athens
9 Heroon Polytechniou Avenue, Zografos Campus, GR-157 73 Athens
GREECE
prowaton@central.ntua.gr

Abstract: - This paper presents the several possibilities of using the Coons' interpolation in both CAD and CAE applications. It is shown that not only curvilinear surfaces may be interpolated (as it is well known from the literature), but also finite element meshes may be developed in any arbitrarily-shaped domain. Moreover, it is shown how it is possible to built-up isoparametric macro-finite-elements with degrees of freedom appearing at the boundaries only, using global shape functions that are based on the same interpolation. The theory is sustained by one typical two-dimensional application of a U-notched elastic structural member.

Key-Words: - CAD/CAE, Coons' interpolation, Finite Element Method (FEM), Mesh generation, Macro-elements, Elastic Structures, Notched members, Stress concentration.

1 Introduction

The technique of bivariate «blending» function interpolation of S.A. Coons [1], was developed and applied to geometrical problems of computer-aided design and numerically controlled machining of free-form surfaces. Since then, the method has been used also in many engineering applications that require the description of three-dimensional surfaces in a form suitable for numerical analysis, such as in the finite element method and in mesh generation problems [17-19].

In two-dimensional problems, the domain is a patch, sometimes defined by its four surrounding sides, which may be meshed using any coordinate-mapping technique such as blending processes [8-10] and others [6,27].

With respect to the polynomial degree of the finite elements, after many years of using small-size isoparametric ones [6], the development of "large" elements with the purpose of reducing mesh generation work load, the total number of degrees of freedom, as well as the computational effort in both static and dynamic regimes, has kept researchers busy for a long time. Historically, it was Irons [7] who generalized the idea of arbitrarily noded elements, but also blending function methods based on the ideas put forward in [8] have been used to produce some interesting element families [9,10,11]. On the other hand, "large" elements were introduced by schemes [12-15] based on Trefftz's method [4]. These, as well as the Boundary Element Method (BEM) [16], require knowledge of the fundamental solution of the problem under consideration.

Coons's interpolation method has been generalised in a unique formula that describes C^0-, C^1-, C^2- etc. continuity of the first-, second- and third-derivative, respectively [17]. In the context of the FEM, Coons's interpolation is practically used for mesh generation in structured four-sided curvilinear patches [18-20].

El-Zafrany and Cookson [11] use Coons's idea for two-dimensional problems in conjunction with Lagrange and Hermite interpolation functions, allowing a small number of degrees of freedom per element. Also, Zhaobei and Zhiqiang [24] apply Coons's surface method to fit boundary conditions in some families of finite element of plates and shells. The use of large B-splines finite-elements based on Coon's interpolation theory, with degrees of freedom appearing only at the element boundaries has appeared in two-dimensional potential [25] and elasticity problems [26].

In this paper, the theory of Coons' is briefly presented for two-dimensional interpolation problems with C^0-continuity in curvilinear coordinates. Then, it is explained how it is possible to use this formula in order to generate a structured finite -element mesh. A smoothening procedure that may significantly improve the quality of the mesh is discussed, too. In the sequence, the general theory of creating "large" finite elements (macro-elements) is presented and global cardinal (1-0-type) shape functions are illustrated. Finally, the theoretical aspects are applied to a structural member with two U-notches of rectangular section.

http://www.wseas.us/e-library/conferences/athens2000/Papers2000/547.pdf

Session Chair: Robert L. Taylor

(developer of FEAP)

- ~100 ATTENDEES at the Session:
  - Olek C. Zienkiewicz
  - A. Samuelson
  - T. J. R. Hughes
  - ...

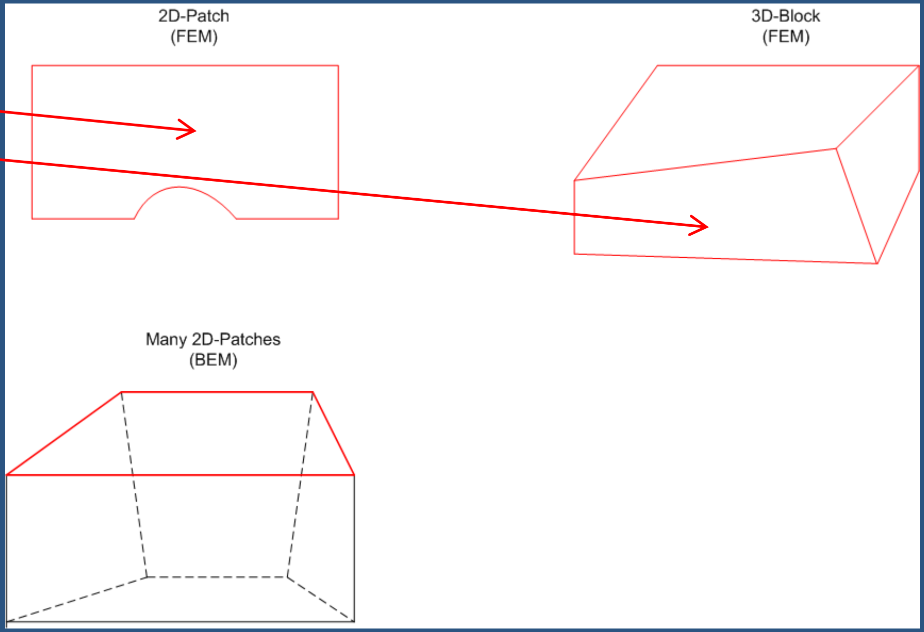
The Session was Video-recorded!

C-element

- Minimizes errors in *data transfer* from CAD (geometric) model to FEA modeling (Casale 1989)
- Needs less “*labor costs*” for meshing
- Accuracy in high-frequency analysis
- Uses as *fewer DOF* as possible (it is a *p*-version)
- Convenience in ***shape changes*** during the design cycle (control points, whatever it means)
- Accurate representation of *geometry* (as much as possible)

Two OPTIONS: Patches & Blocks

- The problem domain
  - 2D patches
  - 3D blocks
- Global approximation in
  - Quadrilateral patches
  - Volume blocks
  - Triangular patches
- Applications
  - Laplace – Poisson
  - 2D-acoustics,
  - 2D-Elasticity
  - Plate-bending
  - 3D problems



CAD/CAE Integration using B-splines (1/5)

- Casale, Bobrow & Underwood (1989, 1992): Trimmed-patch boundary elements: bridging the gap between solid modeling and engineering analysis. *Computer-Aided Design* 1992;24:193-199
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- Kagan, P., Fischer, A., and Bar-Yoseph, P. Z. (1998). New B-spline finite element approach for geometrical design and mechanical analysis. *International Journal of Numerical Methods in Engineering*, 41:435–458.

CAD/CAE Integration (2/5)

- Kagan P, Fischer A. Integrated mechanical based CAE systems using B-spline based finite elements. *Comput Aid Des* 2000;32(8-9), 539-552.
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CAD/CAE Integration (3/5)

- Clark BW, Anderson DC. The penalty boundary method. *Finite Elements in Analysis and Design* 2003; 39: 387-401.
- Clark BW, Anderson DC. The penalty boundary method for combining meshes and solid models in finite element analysis. *Engineering Computations* 2003; 20(4): 344-365.
- Natekar D., Zhang X, Subbarayan G. Constructive solid analysis: a hierarchical, geometry-based meshless analysis procedure for integrated design and analysis. *Computer-Aided Design*, 36(5), 2004, pp. 473-486.
- Etc.

CAD/CAE Integration (4/5)

- The solution $U(x, y)$ of a PDE in a 2D domain constitutes (represents) a curved surface.
- A curved surface can be approximated using CAD formulas.
- CAD formulas for a surface (Farin, 1990):
  - Coons (1964)
  - Gordon (1971)
  - Bézier (1970)
  - B-splines (Schoenberg: 1946, De Boor: 1972)
  - NURBS (Piegl & Tiller ≅ 1985, among others)
- Using any NEW CAD formula, a NEW macro-element can be created

CAD/CAE Integration (5/5)

- In some CAD interpolation formulas (Coons, Gordon-transfinite) the functional set of global basis functions is *hidden*.
- In the rest formulas (Bézier, B-splines, NURBS) it is more *apparent*.

Around 1990, the Journal COMPUTERS & STRUCTURES published a lot about B-splines FEM.

Extension to Axisymmetric Analysis

PERGAMON

Computers and Structures 79 (2001) 1769–1779

Computers & Structures

www.elsevier.com/locate/compstruc

Performance of a macro-FEM approach using global interpolation (Coons') functions in axisymmetric potential problems

Ch. Provatidis *, A. Kanarachos

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Received 5 May 2000; accepted 28 June 2001

Abstract

This paper extends a previously presented global functional set and investigates its performance in axisymmetric potential problems. The main idea is to build-up isoparametric finite elements based on the interpolation formula developed by S.A. Coons for arbitrary-shaped CAD patches. This formula allows the global interpolation of the potential within the whole domain and leads to "large" elements, called "macro-elements". The degrees of freedom appear only at the element boundaries and can be used in the solution of both static and dynamic problems. Numerical results sustain the proposed method, which is successfully compared with conventional finite elements, boundary elements and exact analytical solutions.

Keywords: Finite elements; Macro-elements; Axisymmetric; Potential problems; Global interpolation; Coons' interpolation; Computer aided design

(2001)

ELSEVIER

Finite Elements in Analysis and Design 39 (2003) 535–558

FINITE ELEMENTS IN ANALYSIS AND DESIGN

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Analysis of axisymmetric structures using Coons' interpolation

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Received 1 November 2000; accepted 16 July 2001

Abstract

This paper investigates the performance of global shape functions in axisymmetric elastic structures. The main idea behind the proposed theory is the use of the interpolation formula developed by Coons for CAD purposes in automotive industry, in which it is shown that it is capable of interpolating the displacements within a large patch on the axial cross-section of a structure. The degrees of freedom appear only at the patch boundaries and can be used in the solution of displacement and stress analysis problems. Two different schemes for the interpolation of the boundary data, using B-splines and/or linear global shape functions, are investigated. Numerical results are presented for three typical test cases where the proposed method is successfully compared with conventional finite elements and closed analytical solutions.

Keywords: Large elements; Macro-elements; Global interpolation; CAD/CAE systems

(2003)

Forschung im Ingenieurwesen 67 (2002) 19–26 © Springer-Verlag 2002
DOI 10.1007/s10010-002-0076-1

Coons-patch macroelements in potential Robin problems

C. G. Provatidis

Abstract This paper investigates the performance of large isoparametric finite elements based on the Coons' patch interpolation formula in the analysis of two-dimensional and axisymmetric potential problems with mixed (Robin) boundary conditions. This formula allows the global interpolation of the potential, e.g. temperature, within the whole domain and leads to the so-called "macroelements", where the degrees of freedom appear only at the element boundaries. Numerical results including a steady-state axisymmetric thermal problem and simple test problems in two dimensions from literature, sustain the proposed method, which is successfully compared with conventional finite elements, boundary elements and exact analytical solutions.

Coons-patch Makroelemente in Robin Potentialproblemen

Zusammenfassung In diesem Beitrag wird die Anwendung von grossen isoparametrischen Finiten Elementen, mit Hilfe der *Coons*' schen Formel, für die Lösung von zwei dimensionalen sowie rotationssymmetrischen Potenzialproblemen mit gemischten (Robin) Randbedingungen untersucht. Diese Formel erlaubt die globale Interpolation des Potenzials, wie z.B. der Temperatur, im gesanten Gebiet und ermöglicht die Bildung von "Makroelementen", in denen die Freiheitsgrade nur am Rande des Elements berücksichtigt werden. Numerische Ergebnisse aus einem stetigen rotationssymmetrischen thermischen Problem sowie aus einfachen zweidimensionalen Testproblemen sind aus der Literatur genommen und mit den Ergebnissen aus der Anwendung der vorgeschlagenen Methode verglichen. Damit wird gezeigt, dass die vorgeschlagene Methode im Vergleich mit der konventionellen Finite Elemente Methode, der Randelementverfahren und mit genauen analytischem Lösungen erfolgreich ist.

List of symbols

E	blending functions
$\mathbf{f}$	force vector
h	heat transfer coefficient
k	thermal conductivity factor
$\mathbf{K}^c$	conductivity matrix
$\mathbf{K}^k$	convection matrix
L_m	length of the m-th boundary segment
N_e	number of nodes on the Coons' patch macroelement
q	heat flux
r	radius
u	temperature/potential
u_a	temperature of an ambient medium
$\mathbf{x}$	co-ordinate vector
ξ	normalized co-ordinate (abscissa)
ζ	normalized co-ordinate (ordinate)
Γ	boundary of the structure
Ω	area of the structure

Subscripts

a	ambient temperature related quantities
j	degree of freedom
l	left
m	m-th boundary segment, along *Robin*-type boundary Γ_3
r	right
A, B, C, D	corner points of the four-sided Coons' patch
1, 2, 3	*Dirichlet*, *Neumann* and *Robin* boundary conditions

Received: 25. October 2001

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1 Introduction

Current computer aided engineering (CAE) methods such as the finite element method (FEM), boundary element method (BEM) and finite difference/volume methods (FDM/FVM) are based on the local approximation of the unknown variable within each element or volume. However, the origin of the computational methods is rather the use of global approximation [1–3] than local approximation [4]. After many years of using small-size isoparametric finite elements [5], the development of "large" elements with the purpose of reducing mesh generation effort and relevant total number of degrees of freedom, as well as the computational effort, has kept researchers busy for a long time. The idea of arbitrarily noded elements is due to Irons [6], but also blending function methods based on the ideas put forward by Coons [7] have been used to produce some interesting element families [8–10].

On the other hand, "large" elements were introduced by Jirousek and associates [11–14] based on Trefftz's method [3]. These as well as the BEM (e.g. [15]), require knowledge of the fundamental solution of the problem under consideration, which satisfies the partial differential equation.

(2004a)

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Coons-patch macroelements in two-dimensional eigenvalue and scalar wave propagation problems

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Received 1 November 2002; accepted 10 October 2003

Abstract

Recently, a global functional set has been proposed for the construction of large finite elements, here called "Coons-patch macroelements", with degrees of freedom at the boundaries only. The background of the method is "Coons interpolation formula"; a mathematical formula capable of describing CAD surfaces initially applied to automobile applications. So far, these macroelements were found to be accurate in potential and plane elasticity problems. Based on these encouraging results, this paper continues the investigation on the universality which Coons interpolation formula offers in the development of a class of macroelements including members of the well-known serendipity family. In this context, it is shown in a systematic way that it is not necessary to use Lagrange polynomials to interpolate the potential along each side of the macroelement but one may choose piecewise-linear and/or cubic B-splines interpolants. Moreover, the paper investigates the performance of Coons-patch macroelements in eigenvalue and scalar wave propagation problems by implementing standard time-integration schemes.

Keywords: Finite element; Macroelement; Global approximation; Coons interpolation; Scalar wave propagation; Computer-aided-design (CAD)

1. Introduction

Historically, before the application of the conventional finite element method [1,2,43], which may be probably characterized as a local approximation method, early numerical methods used to treat the entire problem domain as a *global* approximation procedure [3,4]. In the sequence, apart from the use of small-size isoparametric finite elements, the development of "large" elements aiming at reducing the mesh generation workload, the total number of degrees of freedom, as well as the computational effort, has kept researchers busy for a long time [5–7]. According to Zienkiewicz [2, pp. 155–159], besides the well-known bilinear and quadratic elements, cubic and quartic elements were originally derived by inspection. Progression to yet higher members, which constitute the 'Serendipity' family, is difficult and requires some ingenuity. A systematic way of generating the 'serendipity' shape functions was first introduced by Zienkiewicz et al. [8] but a simpler formulation is that of Taylor [9].

Moreover, Taig [10] appears to be the first who mentioned the concept of using shape functions for establishing curvilinear co-ordinates in the context of finite element analysis. In his first application basic linear quadrilateral relations were established, while Irons [11] generalized the idea of arbitrarily noded elements. It is interesting that in Ref. [2, pp. 181–182] there is a special remark that "...quite independently the exercises of devising various practical methods of generating curved surfaces for purposes of engineering design led to the establishment of similar definitions by Coons [12], and indeed today the subjects of surface definitions and analysis are drawing closer together due to this

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(2004b)

2003

C:\PaperTexts\Provatidis_Engng_with_Computers\ PC-NTUA 11-09-2003

CAD-FEA integration using Coons interpolation

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Abstract

This paper reviews recent advances and presents new findings towards an attempt to integrate FEA with solid modeling. The objective is to handle any sufficiently smooth mechanical component by control lines of its boundary only, for both purposes: geometry description and structural/multi-physics analysis. First, it is shown that bivariate Coons interpolation is capable of developing two-dimensional large finite macroelements without any internal nodes, as well as patches of large three-dimensional boundary elements. Second, it is reminded that trivariate Coons interpolation is capable of generating three-dimensional finite element meshes within boxlike regions, for which a new smoothening procedure is here proposed for the first time. Finally, it is shown that trivariate Coons interpolation is also capable of developing large three-dimensional finite macroelements with the nodal points over the boundary only and in many cases along the twelve edges of the solid region (considered as a curvilinear paralleloidal), which can properly adapted to each mechanical component. Aspects of scientific visualization and differences from NURBS representation are also discussed.

1. Introduction

Integration between different communities seems to be a strategic aim nowadays. As an example, geometric modeling (CAD) and computer-aided analysis (CAE) are usually individually powerful, but they do not always work well together. In addition to that, integration between geometric design and scientific visualization or between CAE and visualization is not a trivial procedure. Within the last years, some solutions have been proposed by using trivariate NURBS as a *unifying* representation. Also, the

http://users.ntua.gr/cprovat/yliko/Provatidis_CAD_CAE_Integration.pdf

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From: John Woodwark [jrw@johnwoodwark.com]
Sent: Τρίτη, 9 Σεπτεμβρίου 2003 2:45 μμ
To: C. G. Provatidis
Subject: Your CAD submission

Prof Provatidis,

Thank you for your manuscript "On the integration between CAD and CAE in engineering design". I have looked through the paper, and I have to say that I don't really think it's suitable for CAD journal.

Firstly, I find the title undescriptive. The "integration of CAD and CAE" might involve machining, DfA, STEP, customization... and 1001 other things: certainly not just FE. A more accurate title might be something like "A finite element based on Coons interpolation", and I think it then starts to become obvious why this is not really a CAD journal paper: certainly we carry *some* stuff about FE and meshing but, as you know, there is a large specialist literature. It's not an area in which we are looking to get more deeply involved -- and, I would say, especially not in new types of element.

Secondly, there seem to be some aspects of a review paper in this manuscript (not least its length), including a summary of your own work. I think your letter and the ms. are admirably straightforward about this, but it's not very clear that there remains enough new material to merit a journal publication. You do itemize the new contributions on p. 5, but I have to say that I consider these to be too specific to support the superstructure you have erected around them.

Thirdly, while the results in Tables 1 and 2 look impressive, I'm not entirely convinced -- even as a non-expert on FE -- by the comparisons. For instance I would certainly have liked to see computation times in the tables. And the components you are meshing certainly don't look particularly challenging. When you are up against a very well established technology like 'conventional FE', I think you have to present a very thorough argument. But in any case I fear that CAD journal isn't the right place to do it.

I'm sorry to disappoint you.

--

Regards

John Woodwark
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Editor, CAD journal (www.elsevier.nl/locate/cad)

Engineering with Computers
An International Journal for Simulation-Based Engineering

Professor Mark S. Shephard
Scientific Computation Research Center
Rensselaer Polytechnic Institute
Troy, New York 12180-3590, USA

X.П.

October 15, 2004

Prof. Christopher Provatidis
National Technical University of Athens
Mechanical Engineering Dept
9, Heroon Polytechniou Str., Zografos Campus
GR-157 73 Athens
Greece

RE: Paper EWC03-019

Title: "CAD-FEA integration using Coons interpolation"

Author(s): Christopher G. Provatidis

Dear Prof. Provatidis:

Based on the review, it has been recommended that your paper be rejected for publication in Engineering with Computers. A copy of the review is enclosed.

Thank you for your interest in Engineering with Computers.

Sincerely,

Mark Shephard

Mark S. Shephard
Editor

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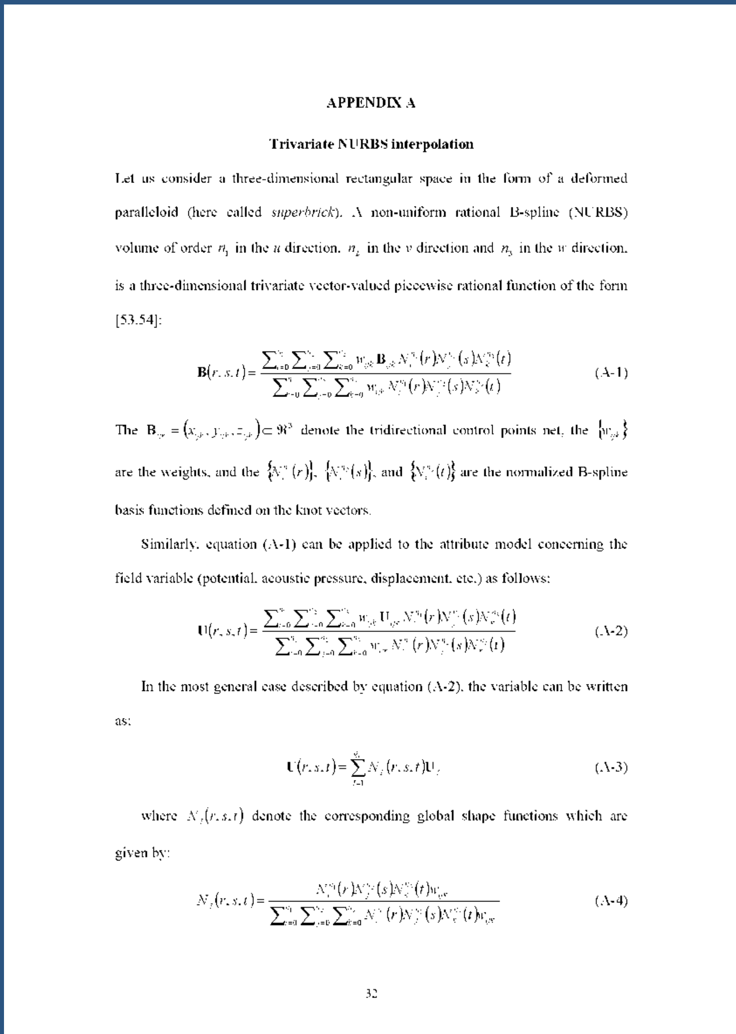
(Unlucky)

(Unlucky)



APPENDIX A

Trivariate NURBS interpolation

Let us consider a three-dimensional rectangular space in the form of a deformed paralleloid (here called *superbrick*). A non-uniform rational B-spline (NURBS) volume of order n_1 in the u direction, n_2 in the v direction and n_3 in the w direction, is a three-dimensional trivariate vector-valued piecewise rational function of the form [53,54]:

$$\mathbf{B}(r,s,t)=\frac{\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}\sum_{k=0}^{n_3} w_{ijk}\,\mathbf{B}_{ijk}\,N_i^{p_1}(r)N_j^{p_2}(s)N_k^{p_3}(t)}{\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}\sum_{k=0}^{n_3} w_{ijk}\,N_i^{p_1}(r)N_j^{p_2}(s)N_k^{p_3}(t)} \qquad \text{(A-1)}$$

The $\mathbf{B}_{ijk}=(x_{ijk},y_{ijk},z_{ijk})\subset\Re^3$ denote the tridirectional control points net, the $\{w_{ijk}\}$ are the weights, and the $\{N_i^{p_1}(r)\}$, $\{N_j^{p_2}(s)\}$, and $\{N_k^{p_3}(t)\}$ are the normalized B-spline basis functions defined on the knot vectors.

Similarly, equation (A-1) can be applied to the attribute model concerning the field variable (potential, acoustic pressure, displacement, etc.) as follows:

$$\mathbf{U}(r,s,t)=\frac{\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}\sum_{k=0}^{n_3} w_{ijk}\,\mathbf{U}_{ijk}\,N_i^{p_1}(r)N_j^{p_2}(s)N_k^{p_3}(t)}{\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}\sum_{k=0}^{n_3} w_{ijk}\,N_i^{p_1}(r)N_j^{p_2}(s)N_k^{p_3}(t)} \qquad \text{(A-2)}$$

In the most general case described by equation (A-2), the variable can be written as:

$$\mathbf{U}(r,s,t)=\sum_{J=1}^{n}N_J(r,s,t)\mathbf{U}_J \qquad \text{(A-3)}$$

where $N_J(r,s,t)$ denote the corresponding global shape functions which are given by:

$$N_J(r,s,t)=\frac{N_i^{p_1}(r)N_j^{p_2}(s)N_k^{p_3}(t)w_{ijk}}{\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}\sum_{k=0}^{n_3} N_i^{p_1}(r)N_j^{p_2}(s)N_k^{p_3}(t)w_{ijk}} \qquad \text{(A-4)}$$

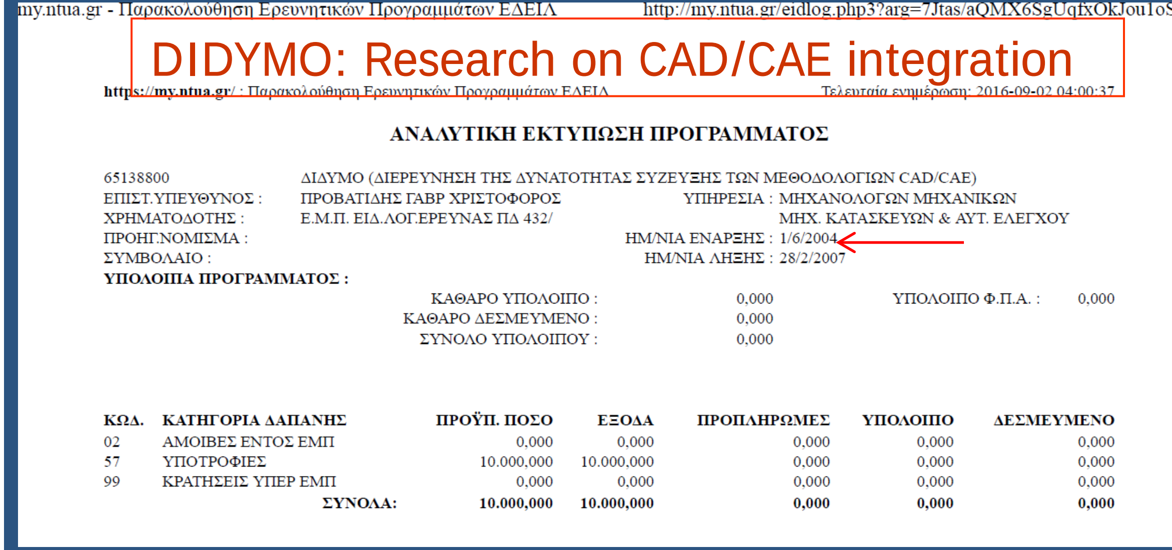
32

(Unlucky)

In 2002, a similar paper was submitted but ... they **could not find a willing Reviewer (!!!)**, so the paper was withdrawn.

(Lucky: Evaluated at a score 9,9 out of 10,00)

DIDYMO: Research on CAD/CAE integration



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ΕΠΙΣΤ.ΥΠΕΥΘΥΝΟΣ : ΠΡΟΒΑΤΙΔΗΣ ΓΑΒΡ ΧΡΙΣΤΟΦΟΡΟΣ ΥΠΗΡΕΣΙΑ : ΜΗΧΑΝΟΛΟΓΩΝ ΜΗΧΑΝΙΚΩΝ
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ΥΠΟΛΟΙΠΑ ΠΡΟΓΡΑΜΜΑΤΟΣ :
ΚΑΘΑΡΟ ΥΠΟΛΟΙΠΟ : 0,000 ΥΠΟΛΟΙΠΟ Φ.Π.Α. : 0,000
ΚΑΘΑΡΟ ΔΕΣΜΕΥΜΕΝΟ : 0,000
ΣΥΝΟΛΟ ΥΠΟΛΟΙΠΟΥ : 0,000

ΚΩΔ.	ΚΑΤΗΓΟΡΙΑ ΔΑΠΑΝΗΣ	ΠΡΟΫΠ. ΠΟΣΟ	ΕΞΟΔΑ	ΠΡΟΠΛΗΡΩΜΕΣ	ΥΠΟΛΟΙΠΟ	ΔΕΣΜΕΥΜΕΝΟ
02	ΑΜΟΙΒΕΣ ΕΝΤΟΣ ΕΜΠ	0,000	0,000	0,000	0,000	0,000
57	ΥΠΟΤΡΟΦΙΕΣ	10.000,000	10.000,000	0,000	0,000	0,000
99	ΚΡΑΤΗΣΕΙΣ ΥΠΕΡ ΕΜΠ	0,000	0,000	0,000	0,000	0,000
	ΣΥΝΟΛΑ:	**10.000,000**	**10.000,000**	**0,000**	**0,000**	**0,000**

COMMUNICATIONS IN NUMERICAL METHODS IN ENGINEERING
Commun. Numer. Meth. Engng 2004; **20**:521–533 (DOI: 10.1002/cnm.690)

Solution of two-dimensional Poisson problems in quadrilateral domains using transfinite Coons interpolation

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SUMMARY

This paper proposes a global approximation method to solve elliptic boundary value Poisson problems in arbitrary shaped 2-D domains. Using transfinite interpolation, a symmetric finite element formulation is derived for degrees of freedom arranged mostly along the boundary of the domain. In cases where both Dirichlet and Neumann boundary conditions occur, the numerical solution is based on bivariate Coons interpolation using the boundary only. Furthermore, in case of only Dirichlet boundary conditions and no existing axes of symmetry, it is proposed to use at least one internal point and apply transfinite interpolation. The theory is sustained by five numerical examples applied to domains of square, circular and elliptic shape. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: finite elements; macro elements; transfinite Coons interpolation; global approximation; Poisson problems

1. INTRODUCTION

The numerical solution of boundary value problems is an open task for over a century. Early attempts made by Ritz [1], Galerkin [2] and Trefftz [3] were based on *global* approximation of the solution within the whole domain. Later, finite difference methods [4] and finite element methods [5] suggested several *local* approximation schemes. In the sequence, the boundary element method [6] proposed a global approximation but it experienced difficulties with domain terms [7], a shortcoming that was finally treated with the higher order dual reciprocity method (DR/BEM) [6, 8]. After a better understanding of the unified character of all numerical methods, the invented term 'weighted-residual methods' [9] is still valid today. Also, within the last decade, meshless [10] and mesh-free [11] methods have been proposed as an alternative to traditional computational methods.

This paper is a contribution in the field of global approximation methods, and also of mesh-free techniques, as it is closely related to nodes mainly along the boundary (no mesh

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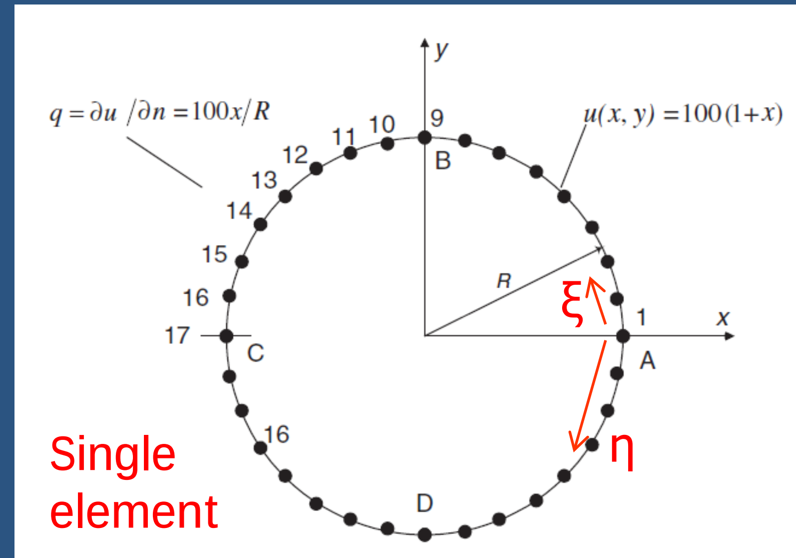

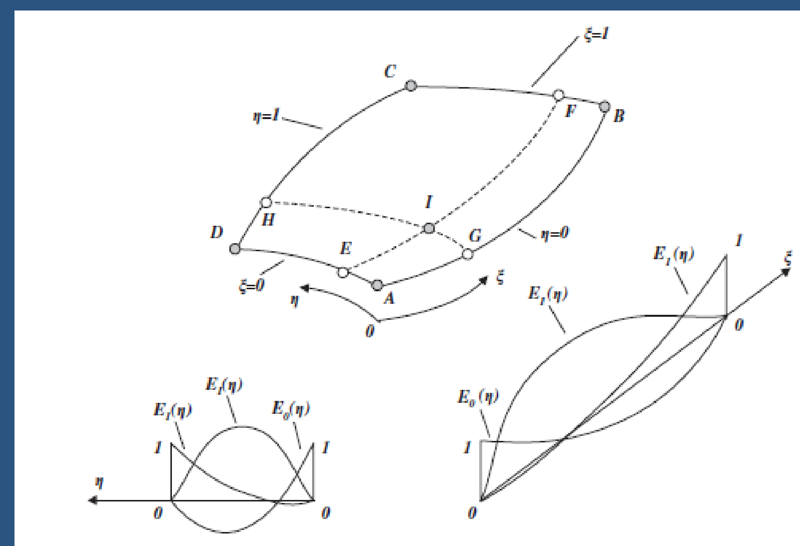

Figure 1. Transfinite Coons macroelement.

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Analysis of box-like structures using 3-D Coons' interpolation

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SUMMARY

This paper investigates the applicability of a recently proposed global functional set based on '3-D Coons' interpolation formula', in the static and dynamic analysis of box-like elastic structures. According to the proposed methodology, only the 12 edges of the entire hexahedral-like structure should be discretized into a small number of nodal points. In this way, the dimensionality of the problem is drastically reduced from three to one, in the sense that instead of the volume, only the control lines being absolutely necessary to define the geometric model should be considered in the analysis. Preliminary results obtained for paralleloidal and cylindrical beams in tension, bending and torsion, as well as an internally pressurized thick-walled cylinder were found to be encouraging for the static analysis. Moreover, the natural frequencies of a rectangular clamped beam were calculated with excellent accuracy. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: finite elements; macroelements; global approximation; Coons' interpolation; static and dynamic problems

1. INTRODUCTION

An overview on global and local approximation methods for the numerical solution of boundary value problems has been recently presented by the author [1]. Summarizing, early attempts by Ritz, Galerkin and Trefftz concerned with global basis functions within the entire domain, while finite difference methods and the finite element method [2] later proposed the local approximation of the unknown solution. Furthermore, aiming to avoid the mesh generation task, boundary element techniques [3] as well as a significant number of mesh-free [4] and meshless [5] methods have appeared.

Within the last few years the author has contributed in the field of global approximation methods by developing large planar (2-D and axisymmetric) Coons-patch macroelements with degrees of freedom (DOFs) along the boundary. From one point of view, these macroelements

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Three-dimensional Coons macroelements in Laplace and acoustic problems

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Abstract

This paper introduces a new global functional set for the FEM solution of three-dimensional boundary-value problems. The main idea is to construct large isoparametric finite elements based on the interpolation formula, which was developed in 1960s by S.A. Coons for the numerical representation of arbitrary solid CAD regions bounded by six curvilinear surfaces. In this way, besides the geometry, Coons interpolation formula is used here for the global interpolation of the unknown potential within the whole solid region (problem area), a procedure that leads to large elements, called "macroelements". For adequately smooth regions, the degrees of freedom appear only at the 12 boundary edges of the macroelement and can be used in the solution of both static (Laplace) and eigenvalue (acoustic) problems. The proposed approach is sustained by five numerical results where it is successfully compared with conventional finite elements and exact analytical solutions.

Keywords: Finite element; Macroelement; Global approximation; Coons interpolation; Potential problems; Acoustics

1. Introduction

It is well known that FEM suffers from the mesh generation process that is usually a time-consuming and—at the same time—an error prone task when CAE works within a CAD environment [1]. During the last four decades many scientists and engineers have proposed several techniques aiming to increase the size of the finite elements, which are based on various mathematical background—to mention a few—such as

- High-order polynomials [2] and macro-element adaptivity [3].
- Combination of analytical solutions with classical finite elements [4].
- Rational functions [5].
- Condensation and substructuring techniques [6–9].
- Reduction of degrees of freedom in the partial differential equation using Karhunen–Loeve Galerkin procedure [10] and other decomposition methods.
- CAD-based techniques [11–16].

Besides, Trefftz-based methods [17]—including the well-known Boundary Element Method (BEM) [18]—have achieved to reduce the dimensionality of the problem. In the particular case of three-dimensional

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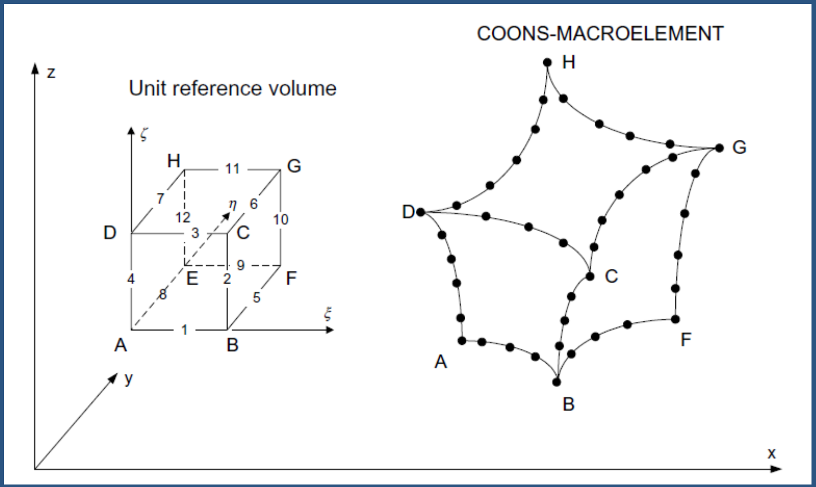

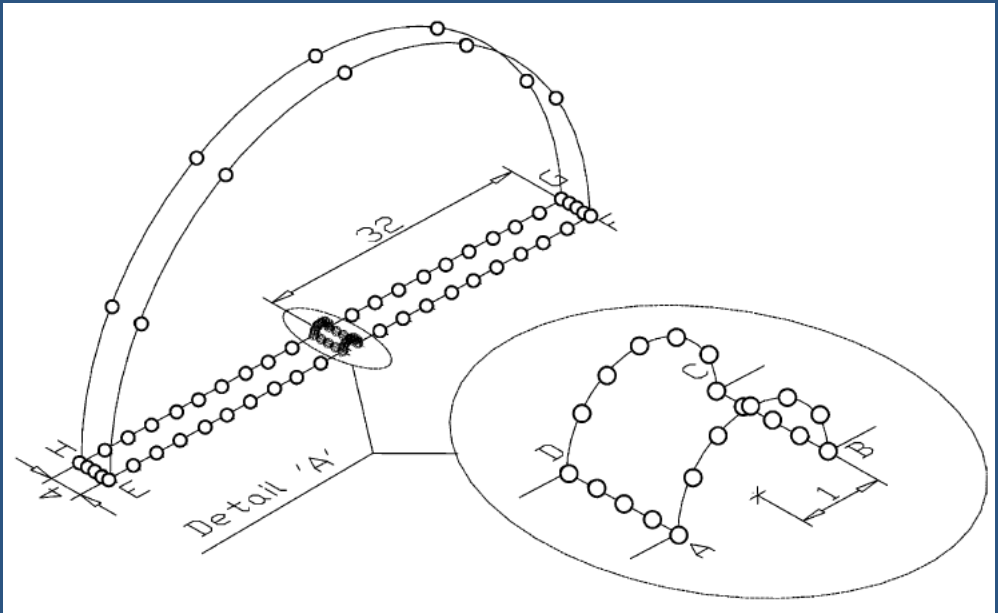

Fig. 2. Example 2: Half part of a cylindrical wall idealised with a 76-node macro-element.

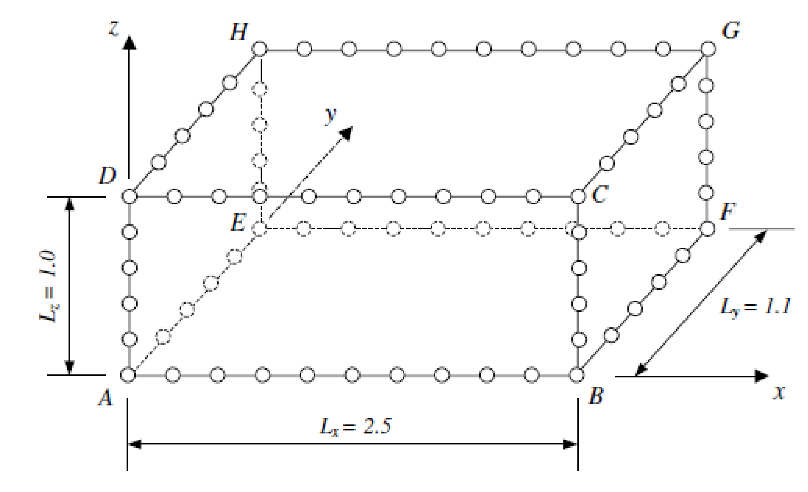

Fig. 10. Example 5: Definition of the 76-node macro-element.

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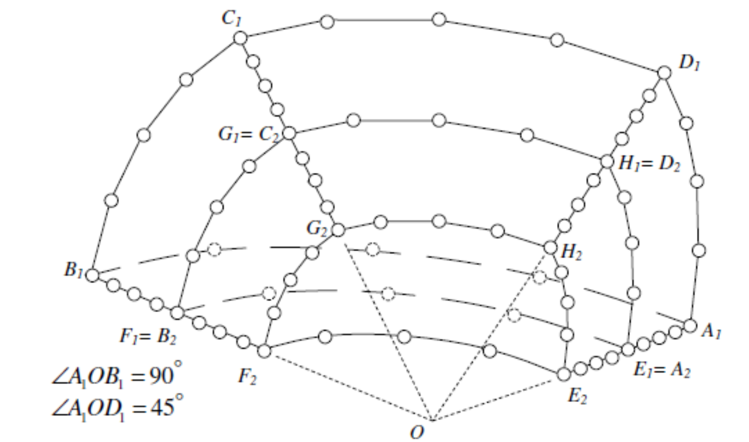
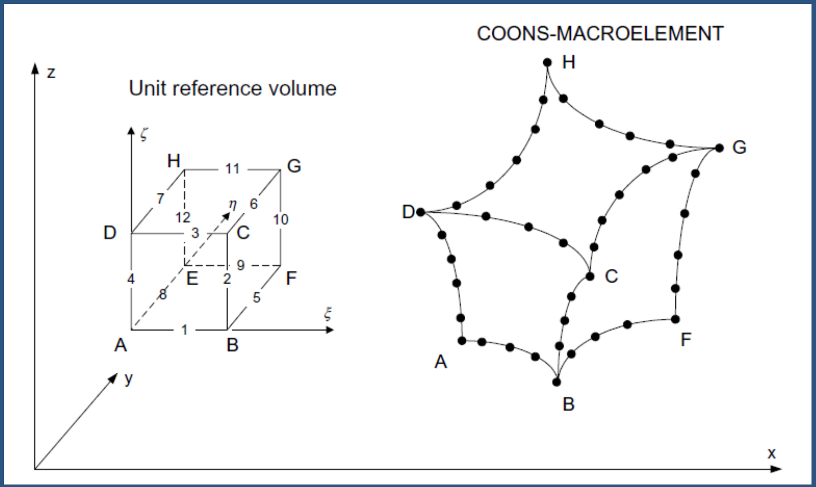


Fig. 8. Example 4: View of the two macroelements.



U(ξ,η,ζ)

$$
\begin{aligned}
\mathbf{x}(\xi,\eta,\zeta) = {} & E_0(\eta)E_0(\zeta)\mathbf{x}(\xi,0,0) + E_1(\xi)E_0(\eta)\mathbf{x}(1,0,\zeta) \\
& + E_0(\eta)E_1(\zeta)\mathbf{x}(\xi,0,1) + E_0(\xi)E_0(\eta)\mathbf{x}(0,0,\zeta) \\
& + E_1(\xi)E_0(\zeta)\mathbf{x}(1,\eta,0) + E_1(\xi)E_1(\zeta)\mathbf{x}(1,\eta,1) \\
& + E_0(\xi)E_1(\zeta)\mathbf{x}(0,\eta,1) + E_0(\xi)E_0(\zeta)\mathbf{x}(0,\eta,0) \\
& + E_1(\eta)E_0(\zeta)\mathbf{x}(\xi,1,0) + E_1(\xi)E_1(\eta)\mathbf{x}(1,1,\zeta) \\
& + E_1(\eta)E_1(\zeta)\mathbf{x}(\xi,1,1) + E_0(\xi)E_1(\eta)\mathbf{x}(0,1,\zeta) \\
& - 2[E_0(\xi)E_0(\eta)E_0(\zeta)\mathbf{x}(0,0,0) + E_1(\xi)E_0(\eta)E_0(\zeta)\mathbf{x}(1,0,0) \\
& + E_1(\xi)E_0(\eta)E_1(\zeta)\mathbf{x}(1,0,1) + E_0(\xi)E_0(\eta)E_1(\zeta)\mathbf{x}(0,0,1)
\end{aligned}
$$

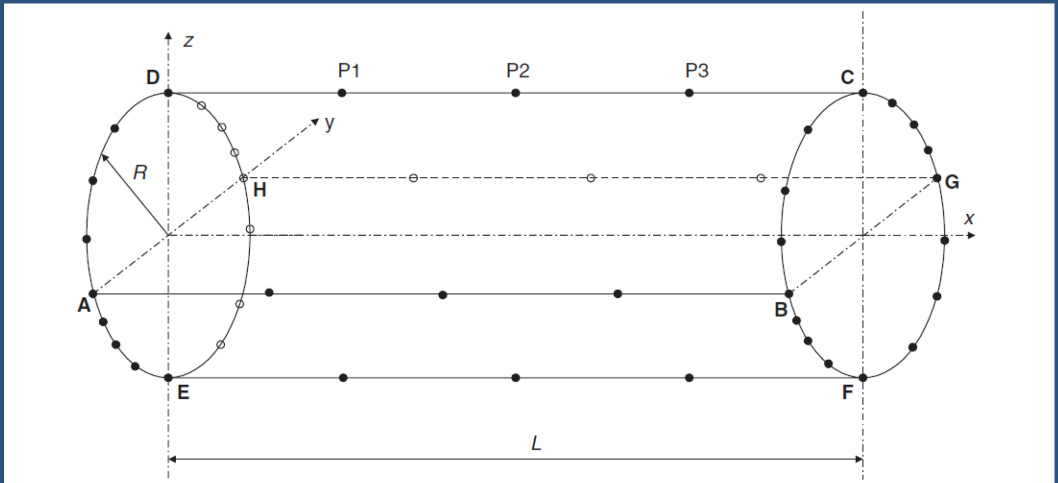


Figure 4. Example 4: cylinder of length $L = 10$ mm and radius $R = 1$ mm subject to torsion using one 3-D Coons macroelement (44 nodes).

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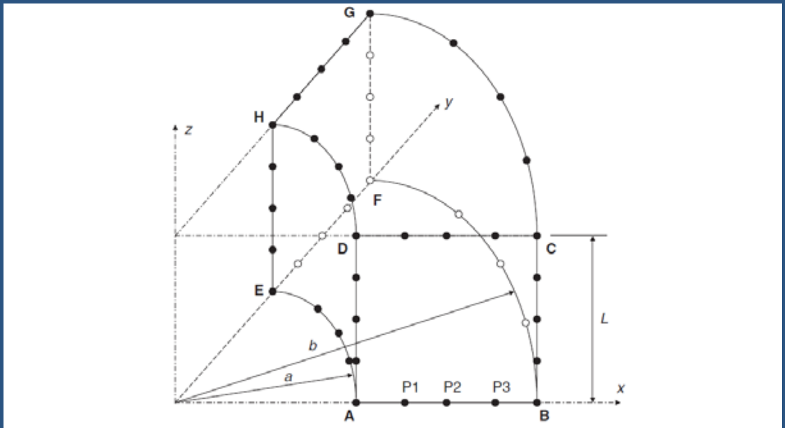

Figure 5. Example 4: thick cylinder of radii $a = 10$ mm, $b = 20$ mm subject to internal pressure $P = 20$ MPa. The model consists of one 3-D Coons macroelement of 44 nodes.

Pre-IGA: C-element

October 2005: IGA Appears!

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Three-dimensional Coons macroelements: application to eigenvalue and scalar wave propagation problems

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SUMMARY

This paper discusses the matter of using higher order approximations in three-dimensional problems through Coons macroelements. Recently, we have proposed a global functional set based on 'Coons interpolation formula' for the construction of large two-dimensional macroelements with degrees of freedom appearing at the boundaries only of the domain. After successive application in many engineering problems, this paper extends the methodology to large three-dimensional hexahedral macroelements with the degrees of freedom appearing at the 12 edges of the entire domain in case of smooth box-like structures. Closed-form expressions of the global shape functions are presented for the first time. It is shown that these global shape functions can be automatically constructed in a systematic way by arbitrarily choosing univariate approximations such as piecewise-linear, cubic B-splines, Lagrange polynomials, etc., along the control lines. Moreover, the mechanism of adding facial and internal nodes is presented. Relationships with higher order p-methods are discussed. Following to excellent results previously derived for the solution of the Laplace equation as well as static and eigenvalue extraction analysis of structures, the paper investigates the performance of Coons macroelements in 3-D eigenvalue and scalar wave propagation problems by implementing standard time-integration schemes. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: finite element; hexahedral macroelement; global approximation; Coons–Gordon interpolation; CAD/CAE integration; higher order approximation; scalar wave propagation

1. INTRODUCTION

After the establishment of small-size finite elements [1], a significant number of researchers have been investigating the possibility of reducing mesh-generation cost by developing competitive methods such as the boundary element method [2], rational polygonal elements [3, 4] as well as

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Coons-patch macroelements in two-dimensional parabolic problems

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Abstract

Having recently obtained encouraging results in elliptic and hyperbolic problems, this paper summarizes previous work and further investigates the performance of large isoparametric finite elements based on the Coons–Gordon interpolation formula in the analysis of two-dimensional parabolic potential problems. The latter formula allows the global interpolation of the potential within the whole problem domain and leads to the so-called Coons-patch-macroelements (CPM), where the degrees of freedom appear primarily at the element boundaries but in the general case it is also possible to use any desirable number of internal nodes. Mathematical and numerical aspects such as the relationship between boundary-only Coons-patch macroelements and Serendipity type elements, the systematic and straightforward way of adding internal nodes, the procedure of merging dissimilar domains and, finally, efficient numerical integration schemes are discussed. Numerical results on typical static (Laplace) and time-dependent thermal problems sustain the proposed method, which is successfully compared with conventional bilinear finite elements and exact analytical solutions.

Keywords: FEM; CAD/CAE; Parabolic problems; Macroelements; Mortar methods

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Free vibration analysis of two-dimensional structures using Coons-patch macroelements

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Abstract

Recently, Coons' interpolation was used for the construction of large finite elements with degrees of freedom appearing mostly along the boundaries of a structure. In the regime of elasticity problems, these so-called "Coons-patch macroelements" were successfully applied to the static analysis of plane structures [C.G. Provatidis, Analysis of axisymmetric structures using Coons' interpolation, Finite Elem. Anal. Des. 39 (2003) 535–558.] while this paper continues the research by investigating their performance in the extraction of natural frequencies and mode shapes. Apart from the piecewise-linear and cubic B-splines interpolation previously used, the performance of Lagrange polynomials and the role of additional internal nodes is studied here. Relationships with classical Serendipity and Lagrangian type elements are discussed. Moreover, the capability of Coons-patch macroelements to couple with conventional finite elements is investigated. The proposed method was applied to three illustrative examples and it was successfully compared with conventional bilinear finite elements.

Keywords: Large elements; Macroelements; Coons interpolation; CAD/CAE; Natural frequencies; Two-dimensional structures

1. Introduction

A lot of attempts have been made within the last years in order to replace conventional finite element methods with other methods such as the boundary element method (BEM) [1] or mesh-free and meshless techniques [2–5]. Essentially, the main practical need that justifies the relevant research activity is to minimize data preparation cost (related to the time-consuming mesh generation task) and to increase the accuracy in calculations by a simultaneous reduction of analysis effort. However, so far BEM did not achieve to replace FEM in the regime of dynamic analysis since its original formulation suffers from frequency-dependent fundamental solutions. This fact leads to a nonalgebraic problem while its alternative dual reciprocity formulation (DR/BEM) [6] highly depends on the choice of the radial basis functions or requires internal nodes [7–9]. Moreover, the mesh-free techniques are not always shortcomings-free due to difficulties related to the inversion of the matrix of coefficients [10]. Therefore, the need of a robust and effective computational technique is still timely.

During the last six years, the above thoughts have motivated the author to develop a new method for the construction of large finite elements with the nodal points along the boundaries only. The background of the method is Coons' interpolation, a formula established in CAD-surface theory that was applied to the automotive industry of USA since the middle 1960s. In the framework of engineering analysis, this method has been successfully applied mainly to potential [11–14] and recently to static elasticity problems [15].

Since not adequate experience exists regarding the behavior of Coons-patch macroelements (CPM) in elastodynamics, this paper aims to further investigate their capability of solving structural free vibration problems (eigenfrequency and mode shape extraction) and compare with conventional FEM solutions of the same boundary discretization. So far, the CPM approach has been applied in conjunction with piecewise-linear and cubic B-spline interpolation along the boundary of an elastic structure. In addition to that, this paper extends the latter interpolation to also Lagrange polynomials of which numerical integration aspects and size limitations are discussed.

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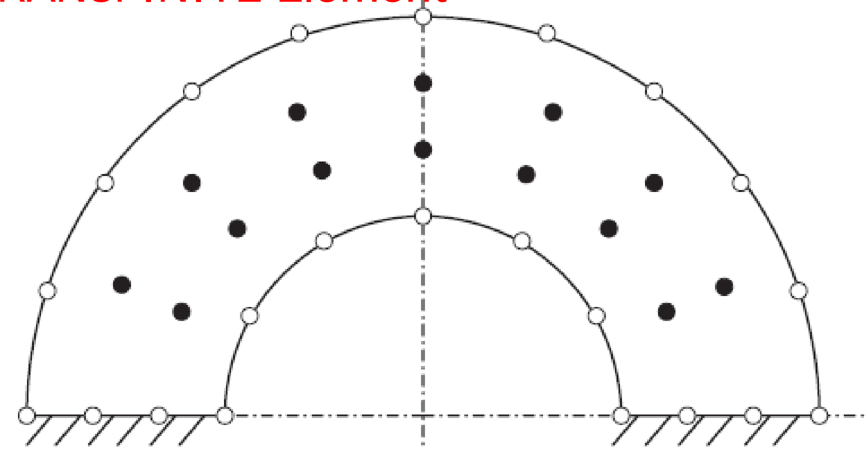
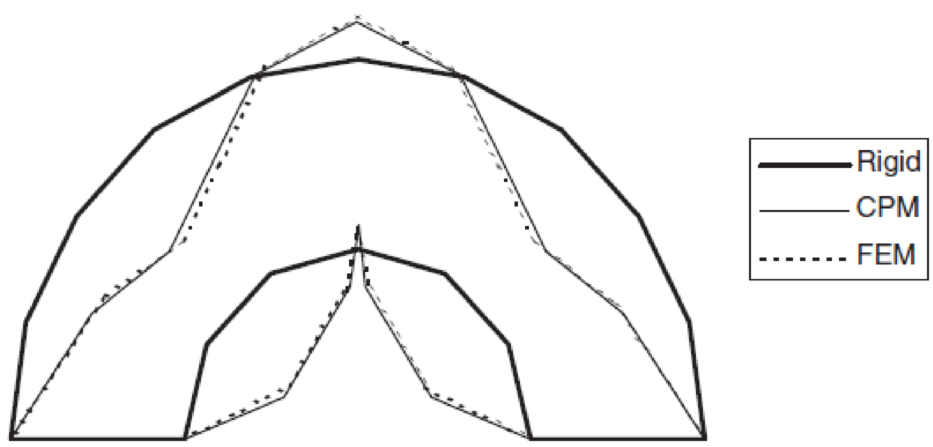


MODE 4



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Transient elastodynamic analysis of two-dimensional structures using Coons-patch macroelements

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Abstract

Recently, Coons' interpolation was used for the construction of large finite elements with degrees of freedom appearing mostly along the boundaries of a structure. So far, these so-called Coons-patch macroelements were successfully applied to the analysis of two-dimensional and axisymmetric elastic structures as well as potential problems including Poisson equation and acoustics. Now, this paper continues the research by investigating their applicability and performance in calculating the propagation of elastic waves within continua due to sudden loads. Explicit (central difference) and implicit (θ-Wilson) time-integration schemes have been successfully applied to four typical model problems in conjunction with the proposed Coons-patch macroelements—without and with substructuring—and the results are successfully compared with conventional finite elements having the same number of nodes along the boundary. Finally, theoretical issues between the proposed global technique and well-established computational methods are discussed.

Keywords: Large finite elements; Coons-patch macroelement; *p*-methods; Elastodynamics; Transient analysis; Wave propagation; Time-integration schemes

1. Introduction

The numerical solution of elasticity problems is an open task for over a century. Early attempts made by Ritz (1908) and Trefftz (1926) were based on global approximation of the displacement within the whole structure. Later, finite element methods (FEM) suggested several local approximation schemes (Zienkiewicz, 1977); within the context of isoparametric assumptions, Taig's (1961) work should be mentioned. However, due to high manual effort required to data preparation (related to the mesh generation task) as well as further needs for increased accuracy in calculations, a lot of attempts have been made so far in order to replace conventional

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(2006c)

is assumed. Then, if this element is *not* rectangular, it is trivial to validate that the shape function associated with the node A does *not* vanish along BC and CD although they do not pass through it. A similar notice has been also made for meshfree techniques (as was above mentioned), where the lack of Kronecker delta function properties and related difficulties in imposing boundary conditions has been commented (Liu and Gu, 2004, p. 476). In contrast, when working with the reference square, the standard methodology of the isoparametric element does not induce such difficulties but this is paid in computer effort to calculate domain integrals (stiffness and mass matrices).

6.3. A note in recent progress

The structure of Eqs. (7)–(11) is also valid for three-dimensional analysis but it requires substantial effort to achieve it. So far, the CPM method has been successfully applied on static potential as well as static and dynamic (eigenvalue extraction) analysis of structures; the goal was to deal with nodes arranged only along the twelve edges of a box-like structure, thus reducing the dimensionality (Provatidis, 2005a,b, 2006b). A similar mesh-reduction achievement has been previously reported in the analysis of 3-D solid structures using the two-dimensional Coons interpolation for each boundary patch in conjunction with the BEM (Provatidis, 2001).

Finally, upon finishing this paper, we feel obliged to mention that the 'disadvantage' of the time-consuming domain integration that characterizes the proposed CPM method has been now reduced by properly applying a global collocation method, which is supported on well-known background in standard finite element analysis (Carey and Oden, 1983, p. 170). Preliminary results have reported by Provatidis (2005d).

To come to an end, the proposed method is comparable with other ones in transient elastodynamic analysis and possesses some advantages. Perhaps the advantage of the proposed method is more clear in three-dimensional modeling where it makes both geometric and analysis (affine) models to coincide, thus minimizing the possibility of data transfer (CAD/CAE) errors during the product design cycle in an industrial environment (Provatidis, 2005a,b; Provatidis, 2006b). Also, due to the small size (reduced model) of the proposed CPM technique, it is anticipated to be efficient in shape optimization loops but further research has to be conducted for validation.

For the first time, we proposed the CAD-based Global Collocation Method

7. Conclusions

In this paper it was shown that Coons' interpolation formula could be used to construct large finite elements, with any number of nodes along each side of a four-sided (quadrilateral) patch that represents the structure under plane-stress or plane-strain conditions. This formula can be combined with either piecewise-linear, cubic B-splines or Lagrange polynomials along each of the sides of the structural patch under consideration. In general, in the absence of discontinuities (smooth fields) it was found that the proposed boundary-only method is effective in small problems with simple problem domains. Otherwise it is suggested either to subdivide the domain into some concave macroelements or use some additional internal nodes. The proposed Coons-patch macroelements are generally applicable to transient elastodynamic analysis in conjunction with only consistent mass matrix and standard time-integration schemes such as the explicit central-difference and the implicit θ-Wilson. Apart from the advantage of using a smaller number of DOFs, for a specified accuracy, compared to the conventional finite elements, in all cases tested the transient dynamic behavior of the Coons-patch macroelements was excellent. Particularly, when using the robust cubic B-splines model the proposed methodology was found to be more accurate than the FEM solution, while in more complex geometries the results were of similar quality, of course with the same number of nodes along the boundary.

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ORIGINAL

Christopher G. Provatidis

Free vibration analysis of elastic rods using global collocation

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Abstract This paper investigates the performance of a novel global collocation method for the eigenvalue analysis of freely vibrated elastic structures when either basis or shape functions are used to approximate the displacement field. Although the methodology is generally applicable, numerical results are presented only for rods in which one-dimensional basis functions in the form of a power series, as well as equivalent Lagrange, Bernstein or Chebyshev polynomials are used. The new feature of the proposed methodology is that it can deal with any type of boundary conditions; therefore, the cases of two Dirichlet as well as one Dirichlet and one Neumann condition were successfully treated. The basic finding of this work is that all these polynomials lead to results identical to those obtained by the power series expansion; therefore, the solution depends on the position of the collocation points only.

Keywords Collocation · Spectral methods · Finite element · Eigenvalue problem · Elastodynamics

1 Introduction

In systems of great complexity, frequency determination from the differential equation often becomes so complicated that it is practically impossible [6]. Historically, it was Rayleigh who first proposed a generalized energy method based on an assumed shape for the lowest natural frequency [20]. Later, Ritz generalized this procedure to more than one parameter. The major drawback of the Rayleigh–Ritz method is the difficulty in constructing a set of admissible functions, particularly for a compound structure. This difficulty can be overcome by using the finite-element method [24], which provides an automatic means of constructing such functions.

However, the aforementioned finite-element methods often require more degrees of freedom (DOF) for a specified accuracy than a classical Ritz procedure would, thus causing a considerable delay in design problems that require repeated eigenvalue computations during iterations [14]. Since computational effort increases approximately as the order of the problem cubed, many attempts have been made to reduce the number of degrees of freedom (system order). Among these methods, it is worth mentioning the reduction of the degrees of freedom directly by retaining master DOFs [9,12], the use of substructures [4], and the higher-order *p*-methods [23].

Moreover, closely related to the well-known *p*-method [23], an alternative method has been proposed for dynamic analysis [17–19]. The latter is based on computer-aided design (CAD) considerations, primarily on the Coons–Gordon interpolation [15]. In these works, the standard Galerkin–Ritz procedure was applied, thus

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A GLOBAL COLLOCATION METHOD FOR TWO-DIMENSIONAL RECTANGULAR DOMAINS

CHRISTOPHER G. PROVATIDIS

This paper proposes the use of a global collocation procedure in conjunction with a previously developed functional set suitable for the numerical solution of Poisson's equation in rectangular domains. We propose to expand the unknown variable in a bivariate series of monomials $x^i y^j$ that exist in Pascal's triangle. We also propose the use of the bivariate Gordon–Coons interpolation, apart from previous intuitive choices of the aforementioned monomials. The theory is sustained by two numerical examples of Dirichlet boundary conditions, in which we find that the approximate solution monotonically converges towards the exact solution.

1. Introduction

The use of collocation methods for the solution of partial differential equations (PDE) has become a subject of intensive interest in the past [Lanczos 1938; Ronto 1971; Russell and Shampine 1972; Finlayson 1972; De Boor and Swartz 1973; Diaz 1977; Houstis 1978; Botha and Pinder 1983]. While most research has focused on the local interpolation of a variable, not many works have appeared concerning the global collocation in two-dimensional problems [Frind and Pinder 1979; Hayes 1980; Van Blerk and Botha 1993]. In contrast, global approximations have been successfully applied in conjunction with the standard Galerkin–Ritz procedure either using the Gordon–Coons bivariate interpolation [Cavendish et al. 1976] or higher order *p*-methods [Szabó and Babuška 1991].

Moreover, closely related to the abovementioned *p*-methods [Szabó and Babuška 1991], an alternative Gordon–Coons method has been proposed for the static and dynamic analysis of structures [Provatidis 2006a; 2006b; 2006c; 2006d]. In these works, the standard Galerkin–Ritz procedure was applied, thus leading to stiffness and mass matrices in the form of integrals over the entire domain. This method has been called the *Coons patch macroelement* (CPM) approach [Provatidis 2006a; 2006b; 2006c; 2006d]. Although CPM works perfectly well, and can deal with even Π-shaped domains [Provatidis 2006a; 2006d], one could say that it has the disadvantage of fully populated matrices particularly when Lagrange polynomials (with no compact support) are used.

This paper contributes to overcoming the abovementioned shortcoming and modifies the previous CPM methodology first by preserving the global functional space, and second by moving from the Galerkin–Ritz to a *collocation* procedure in which no integral is needed. Quite recently, as a pilot study, this idea has been successfully applied to one-dimensional eigenvalue problems [Provatidis 2007].

It should be noted that the proposed collocation method could be formulated in terms of the global cardinal shape functions of the CPM approach, which are applicable even to curvilinear domains [Provatidis 2006a; 2006b; 2006d]. However, the aforementioned shape functions are derived through a numerical

Keywords: global collocation, Coons interpolation, Poisson's equation.

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(2008b)

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ORIGINAL

Christopher G. Provatidis

Time- and frequency-domain analysis using lumped mass global collocation

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Abstract Quite recently, a novel global collocation method for the eigenvalue analysis of freely vibrated elastic structures was proposed (Archive of Applied Mechanics: DOI: 10.1007/s00419-007-0159-4). This paper extends the latter methodology on several levels, in both the time and frequency domain. Firstly the formulation is updated so that it can also deal with rods of variable cross section. Then, the fully populated mass matrices of the previous formulation are properly replaced by lumped masses, thus saving still more computer effort. Subsequently, a new general formulation for the transient response analysis is proposed. Finally, a novel procedure for the coupling of two neighboring collinear rods is presented. The theory is supported by six test cases concerning elastic rods of constant and variable cross sections. Among these, transient analysis refers to the response of a single rod due to a Heaviside-type loading as well as to the impact between two collinear rods of different cross sections.

Keywords Collocation · Finite element · Eigenvalues · Transient response · Impact

1 Introduction

Static as well as eigenvalue and transient response analysis of one-, two-, and three-dimensional structures is usually performed using the well-known finite-element method in conjunction with approximating polynomials of low degree [1,15]. Besides, higher-order *p*-methods have been used since the 1970s [14]; in general, for a certain number of *n* subdivisions the *p*-version (polynomial of *n*th degree) has better performance than the *h*-version (*n* linear finite elements) [14]. An alternative way to create higher-order elements is based on the use of Coons–Gordon interpolation, which is well known in computer-aided design (CAD) theory, and allows for the automatic derivation of global shape functions for any discretization of the boundary and the interior of the structure [8–10]; the thus obtained finite elements have been called Coons macroelements. In the context of CAD-oriented techniques, Bézier and nonuniform rational B-spline (NURBS) interpolation [7] have been also applied in engineering analysis [3,5,6,13].

Although the aforementioned Coons macroelements can be applied in conjunction with piecewise-linear and piecewise-quadratic interpolation, thus achieving compact support, numerical experience has shown that they are more accurate and converge faster when applied in conjunction with higher-order Lagrange polynomials, for example up to the eighth or tenth degree [8–10]. In such a case, the obtained matrices become fully populated and the computer effort may be substantial [10].

As a remedy to the aforementioned shortcoming of Coons macroelements, it was recently proposed to preserve the same global shape functions and substitute Galerkin–Ritz by a novel global collocation scheme

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Integration-free Coons macroelements for the solution of 2D Poisson problems

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SUMMARY

Large isoparametric macroelements with closed-form cardinal global shape functions under the label 'Coons-patch macroelements' (CPM) have been previously proposed and used in conjunction with the finite element method and the boundary element method. This paper continues the research on the performance of CPM in conjunction with the collocation method. In contrast to the previous CPM that was based on a Galerkin/Ritz formulation, no domain integration is now required, a fact that justifies the name 'integration-free Coons macroelements'. Therefore, in addition to avoiding mesh generation, and saving human effort, the proposed technique has the additional advantage of further reducing the computer effort. The theory is supported by five test cases concerning Poisson and Laplace problems within 2D smooth quadrilateral domains. Copyright © 2008 John Wiley & Sons, Ltd.

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KEY WORDS: finite elements; transfinite interpolation; global approximation; boundary-value problems; global collocation

1. INTRODUCTION

The numerical solution of boundary-value problems (BVP) has been an open issue for over a century. The *global* approximation character of the early Rayleigh/Ritz methods [1] was later replaced by several finite element schemes applicable to arbitrary domains thanks to their *local* approximation ability [2]. Despite that, the latter advantage created the need for a high manual

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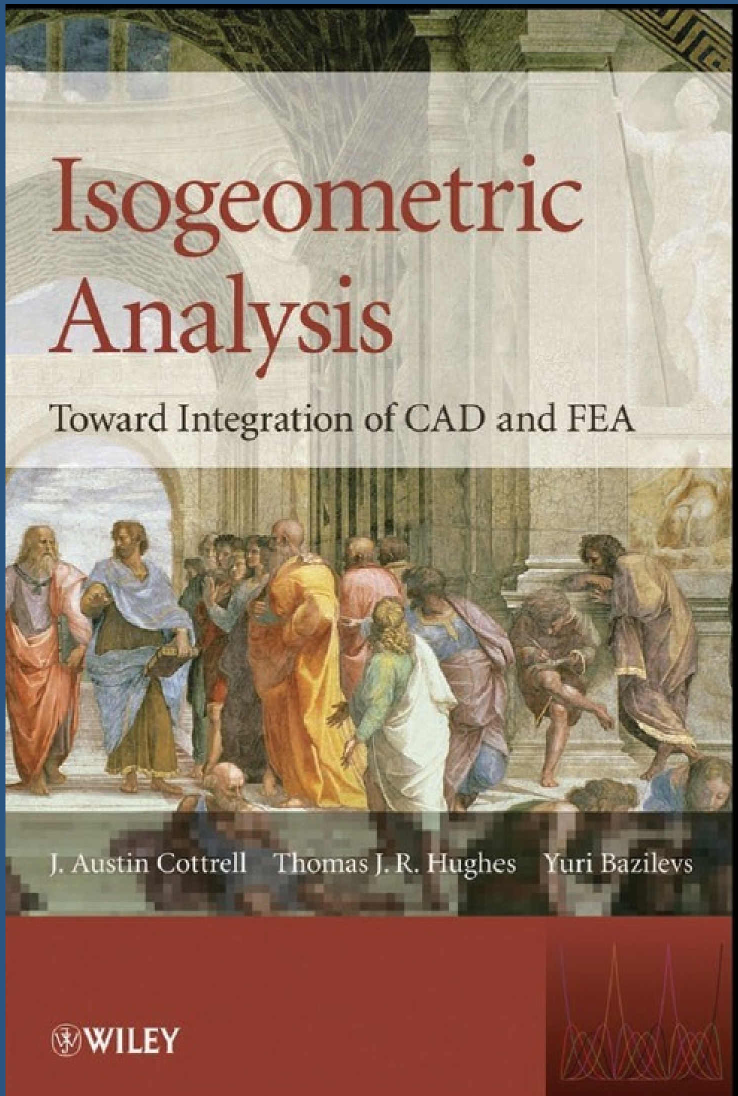


(2009)

8 Isogeometric Analysis: Toward Integration of CAD and FEA

important role in isogeometric technology. Subdivision solids have been studied by Bajaj *et al.*, 2002.

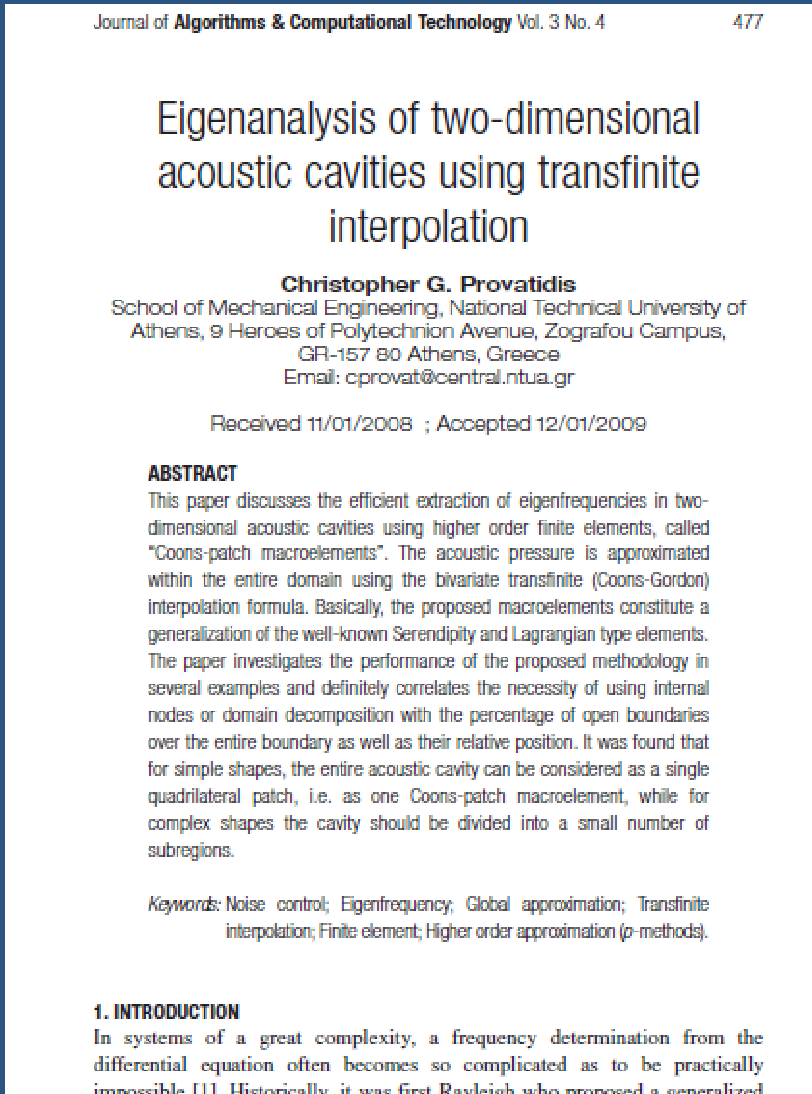
Other geometric technologies that may play a role in the future of isogeometric analysis include Gordon patches (Gordon, 1969), Gregory patches (Gregory, 1983), S-patches (Loop and DeRose, 1989), and A-patches (Bajaj *et al.*, 1995). Provatidis has recently solved a number of problems using Coons patches (see Provatidis, 2009, and references therein). Others may be invented specifically with the intent of fostering the isogeometric concept, namely, to use the surface design model directly in analysis. This would only suffice if analysis only requires the surface geometry, such as in the stress or buckling analysis of a shell. In many cases, the surface will enclose a volume and an analysis model will need to be created for the volume. The basic problem is to develop a three-dimensional (trivariate) representation of the solid in such a way that the surface representation is preserved. This is far from a trivial problem. Surface differential and computational geometry and topology are now fairly well understood, but the three-dimensional problem is still open (the Thurston conjecture characterizing its solution remains to be proven, see Thurston, 1982, 1997). The hope is that through the use of new technologies, such as, for example, Ricci flows and polycube splines (see Gu and Yau, 2008), progress will be forthcoming.

...Not so late, since 20 papers had been published earlier than 2009 in refereed journals!

NOTE

- Before 2009: **20** Journal papers
  - (10 of them before 2005 where IGA appeared for the first time in CMAME)

- After 2009 : **15** papers


Journal of **Algorithms & Computational Technology** Vol. 3 No. 4 477

Eigenanalysis of two-dimensional acoustic cavities using transfinite interpolation

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ABSTRACT
This paper discusses the efficient extraction of eigenfrequencies in two-dimensional acoustic cavities using higher order finite elements, called "Coons-patch macroelements". The acoustic pressure is approximated within the entire domain using the bivariate transfinite (Coons-Gordon) interpolation formula. Basically, the proposed macroelements constitute a generalization of the well-known Serendipity and Lagrangian type elements. The paper investigates the performance of the proposed methodology in several examples and definitely correlates the necessity of using internal nodes or domain decomposition with the percentage of open boundaries over the entire boundary as well as their relative position. It was found that for simple shapes, the entire acoustic cavity can be considered as a single quadrilateral patch, i.e. as one Coons-patch macroelement, while for complex shapes the cavity should be divided into a small number of subregions.

Keywords: Noise control; Eigenfrequency; Global approximation; Transfinite interpolation; Finite element; Higher order approximation (p-methods).

1. INTRODUCTION
In systems of a great complexity, a frequency determination from the differential equation often becomes so complicated as to be practically impossible [1]. Historically, it was first Rayleigh who proposed a generalized

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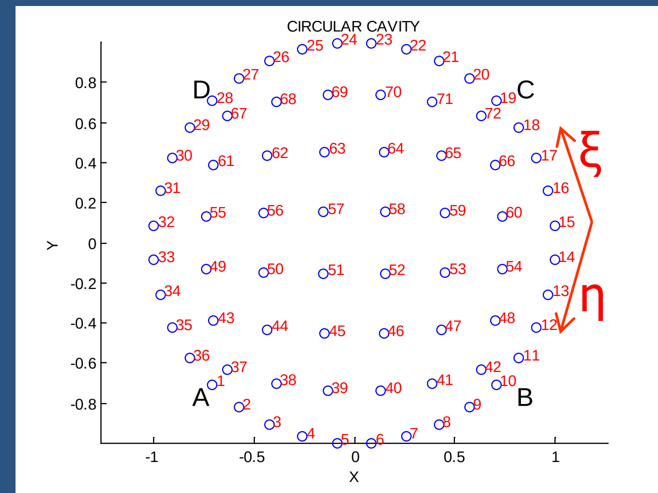
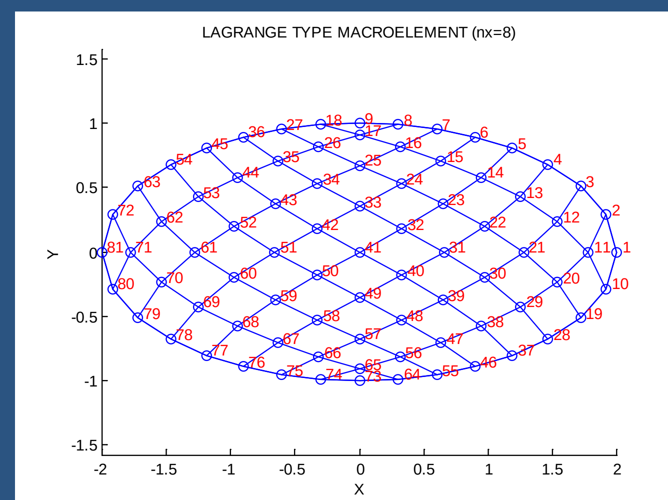
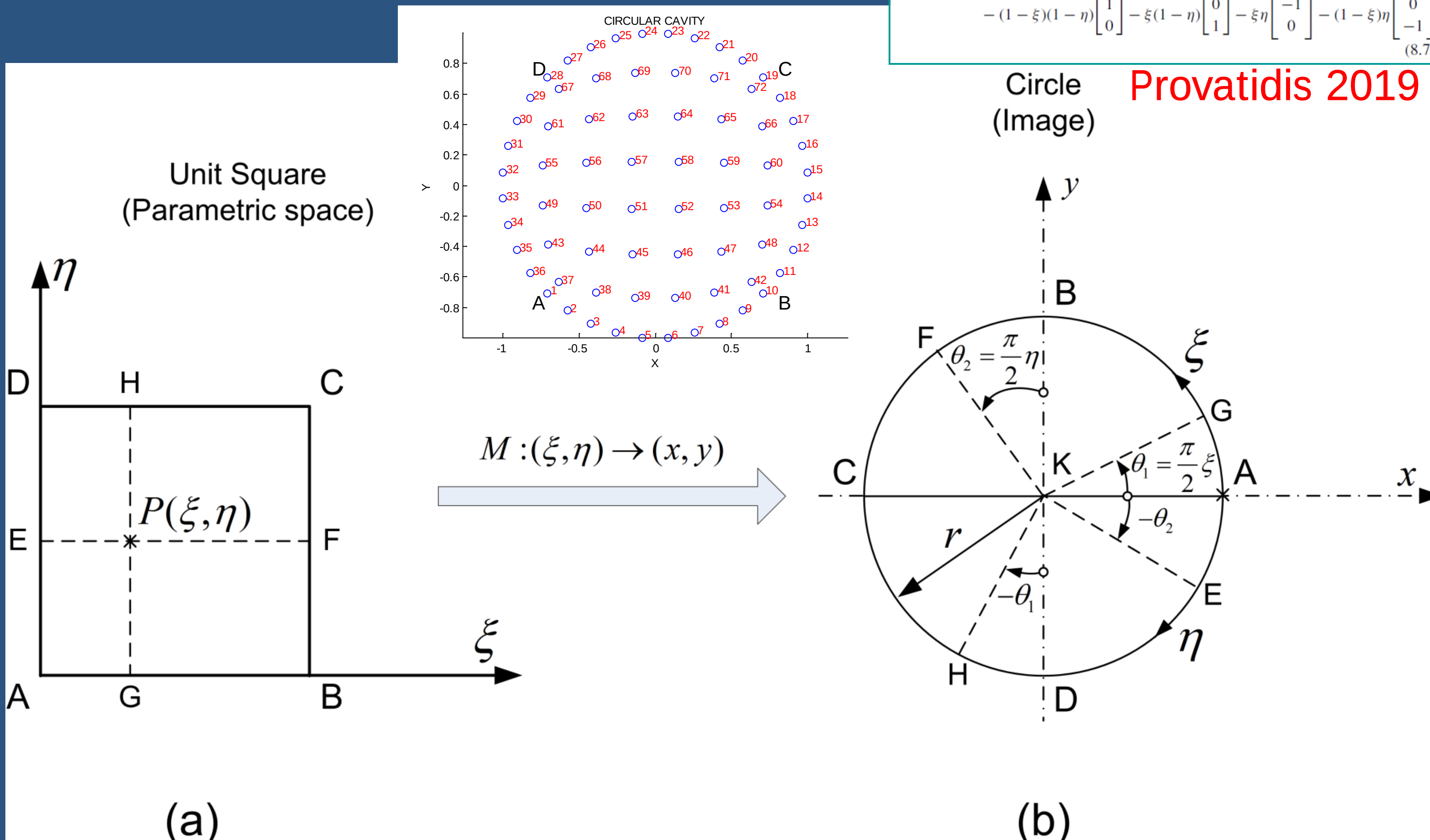
A whole circle and a whole ellipse were treated as single (say) Lagrange element





COONS MAPPING

$$\begin{bmatrix} x(\xi,\eta) \\ y(\xi,\eta) \end{bmatrix} = (1-\xi)\begin{bmatrix} \cos(\frac{\pi}{2}\eta) \\ -\sin(\frac{\pi}{2}\eta) \end{bmatrix} + \xi \begin{bmatrix} -\sin(\frac{\pi}{2}\eta) \\ \cos(\frac{\pi}{2}\eta) \end{bmatrix} + (1-\eta)\begin{bmatrix} \cos(\frac{\pi}{2}\xi) \\ \sin(\frac{\pi}{2}\xi) \end{bmatrix} + \eta \begin{bmatrix} -\sin(\frac{\pi}{2}\xi) \\ -\cos(\frac{\pi}{2}\xi) \end{bmatrix} - (1-\xi)(1-\eta)\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \xi(1-\eta)\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \xi\eta\begin{bmatrix} -1 \\ 0 \end{bmatrix} - (1-\xi)\eta\begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad (8.7)$$



(2009)

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ORIGINAL

C. G. Provatidis · K. S. Ioannou

Static analysis of two-dimensional elastic structures using global collocation

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Abstract Based on previous findings concerning the numerical solution of one-dimensional elastodynamical problems [Provatidis in Arch Appl Mech 78(4):241–250, 2008] this paper extends the methodology to the static analysis of two-dimensional problems in quadrilateral domains. This target is achieved by replacing the Galerkin/Ritz procedure involved in Lagrangian (or Gordon–Coons) type finite elements by a global collocation scheme. In brief, the boundary conditions are fulfilled at all boundary nodes, while the governing equation is fulfilled at internal points. The theory is supported by four test cases concerning rectangular and curvilinear structures under plane-stress or plane-strain conditions, where the convergence rate is successfully compared with that of conventional bilinear finite elements with the same mesh density.

Keywords Collocation · Least squares · Finite element · Elastostatics

1 Introduction

Static analysis is usually performed using the well-known finite element method in conjunction with polynomials of low degree [1, 19]. Besides, higher order *p*-methods using polynomials up to the seventh degree have been used since 1970s [17]. Alternatively, higher order elements can be created using a CAD-based interpolation such as Coons–Gordon [2–4] from one side as well as Bézier and Non Uniform Rational B-splines (NURBS) on the other side; these interpolations lead to the so-called ‘*Coons-patch macroelements*’ (CPM) [7–13] and ‘*isogeometric*’ elements [5, 6], respectively. So far, in static analysis, the aforementioned elements have been developed in conjunction with the Galerkin/Ritz procedure, thus leading to a stiffness matrix, whose elements are domain integrals; in dynamic analysis, the same holds for the mass matrix [11, 13].

Focusing particularly on the CPM implemented in conjunction with Lagrange interpolation, which is rather the most accurate relevant choice, it was observed that the obtained stiffness matrix becomes fully populated thus requiring substantial computer effort [11]. This observation was the motivation to seek for a competitive *global collocation method*, in which, however, the same global approximation is preserved (i.e. Coons–Gordon as in the previous Galerkin/Ritz scheme). To this direction, pilot studies in one-dimensional problems revealed an excellent overall quality of the numerical solution [14, 15]; also, another pilot study in rectangular domains governed by Poisson’s equation (field problem) and Dirichlet boundary conditions led to encouraging results when a special set of Cartesian basis functions was used in the form $x^i y^j$ [16].

Based on the aforementioned encouraging findings, this paper investigates the applicability of the global collocation method on the static analysis of two-dimensional elastic structures, for the first time. Moreover,

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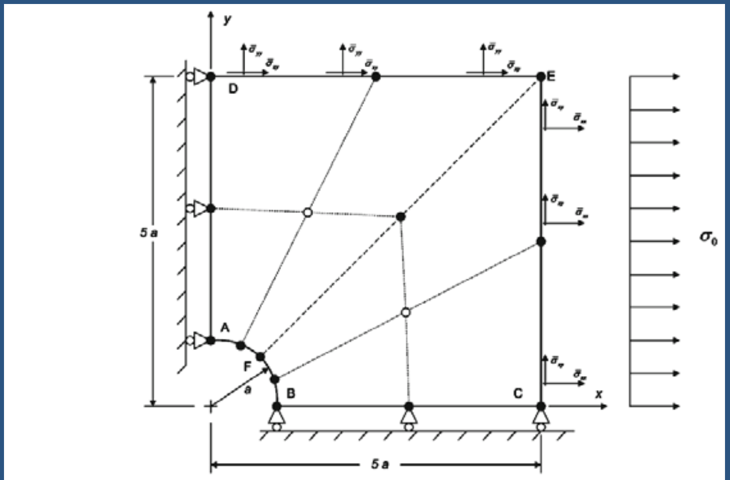
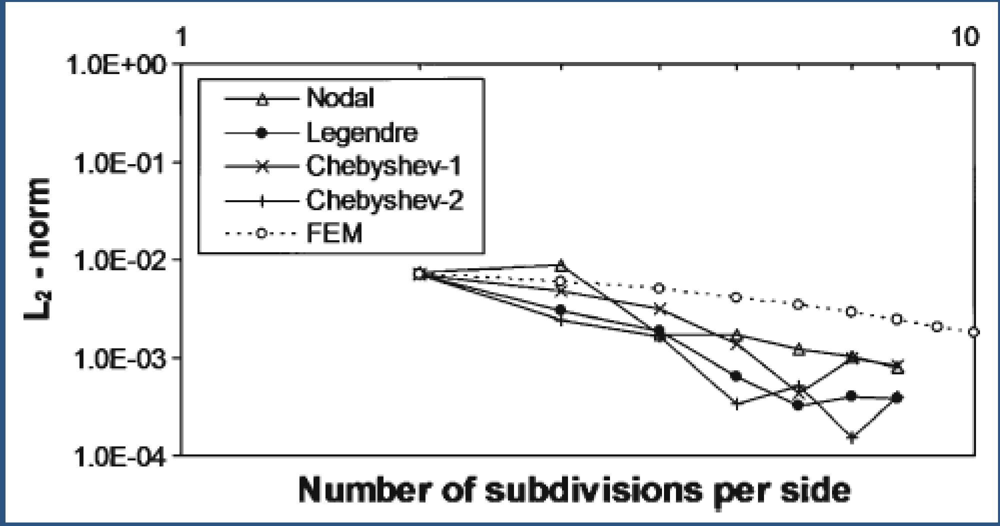


Fig. 9 Example 4. Infinite plate with a hole subject to a uniform stress σ_0 at infinity



(2010)

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DOI: 10. 5923/j.am.20110101.02

Equivalent Finite Element Formulations for the Calculation of Eigenvalues Using Higher-Order Polynomials

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Abstract This paper investigates higher-order approximations in order to extract Sturm-Liouville eigenvalues in one-dimensional vibration problems in continuum mechanics. Several alternative global approximations of polynomial form such as Lagrange, Bernstein, Legendre as well as Chebyshev of first and second kind are discussed. In an instructive way, closed form analytical formulas are derived for the stiffness and mass matrices up to the quartic degree. A rigorous proof for the transformation of the matrices, when the basis changes, is given. Also, a theoretical explanation is provided for the fact that all the aforementioned alternative pairs of matrices lead to identical eigenvalues. The theory is sustained by one numerical example under three types of boundary conditions.

Keywords Finite Elements, Galerkin/Ritz, Global Approximation, P-Methods, Eigenvalue Analysis

1. Introduction

In the framework of the standard Galerkin/Ritz method for the solution of one-dimensional boundary value problems governed by a differential equation within the domain [0,L], the usual procedure consists of subdividing [0,L] into a certain number of finite elements for which piece-wise-linear (i.e., local) interpolation is assumed[1]. In general, the shortest the elements are the more accurate the numerical solution is (h-version). Alternatively, higher order p-methods[2] suggest the introduction of nodeless basis functions based on differences of Legendre polynomials (up to the seventh degree) that cooperate with the two linear shape functions, i.e., $N_1(x)=1-x/L, N_2(x)=x/L$, the latter associated to the ends x = 0 and x = L. A literature survey suggests that for a certain discretization of the domain, the corresponding p-version is generally more accurate than the h-version[3-6].

The matter of using higher order approximations through computer-aided-design (CAD) based Coons-Gordon macroelements has been recently discussed for two- and three-dimensional problems[7-11]. In those works some similarities and differences of the so-called 'Coons macro-elements' with respect to the 'higher order p-method' have been reported in detail. Moreover, alternative CAD based NURBS or/and Bézier techniques have been proposed within the last eighteen years[12-16].

In this context, this paper continues the investigation on the eigenvalue problem by moving from 2-D and 3-D to 1-D eigenvalue problems, and seeks for any similarities or essential differences between five alternative methods. The study includes classical Lagrange polynomials and extends to the Bernstein polynomials that are inherent in the definition of Bézier CAD curves[17], as well as to Chebyshev polynomials that have been previously used in spectral and collocation methods (e.g.[18]). In this paper it was found that all the aforementioned polynomials are equivalent in the sense that (after the proper transformation) they symbolically coincide with the classical higher order p-method (or p-version)[2] as well as with the class $\{x^n\}$ (Taylor series).

2. Galerkin/Ritz Formulation

2.1. General

A general Sturm-Liouville problem can be written in the following differential equation

$$\frac{d}{dx}\left(p(x)\frac{dU}{dx}\right)+\left(r(x)\lambda-q_1(x)\right)U=0$$

It can be reduced to a study of the canonical Liouville normal form

$$U''+(\lambda-q(x))U=0 \quad (1)$$

Without loss of generality, in this paper we deal with the particular case that $q(x)=0$, for which Eq(1) degenerates to the well-known 'Helmholtz equation':

$$U''(x)+\lambda U(x)=0, \quad x\in[0,L] \quad (2)$$

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DOI 10.1007/s11012-011-9489-y

Two-dimensional elastostatic analysis using Coons-Gordon interpolation

Christopher G. Provatidis

2005...2012

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Abstract During the last years blending-function (Coons') interpolation has been utilized for the construction of large 2D and 3D finite elements with degrees of freedom appearing along the boundaries of the domain. In the particular case of elasticity problems, these so-called "boundary-only Coons macroelements" have been applied to the analysis of simple structures in which adequate accuracy was remarked. This paper continues the research investigating, for the first time, the role of internal nodes in the accuracy of the numerical solution using various trial functions along the boundary in conjunction with various blending functions (piecewise-linear, cubic B-splines and Lagrange polynomials). The performance and limits of the proposed Coons-Gordon macroelements are tested in typical 2D elastostatic examples, where they are also compared with conventional four-node bilinear finite elements of the same mesh density. It was definitely found that although the 'boundary-only formulation' of the proposed Coons macroelements successfully pass some well-established patch tests and may be very accurate in some simple test cases, in general, it must be substituted by the 'trans-finite formulation' (Coons-Gordon) where a sufficient number of internal nodes is necessary to ensure convergence.

Keywords Elasticity · Finite elements · Global approximation · Gordon-Coons interpolation

1 Introduction

It is well known that the majority of the approximate numerical solutions for the unknown displacement vector are based on series expansion of trial functions multiplied by unknown coefficients which are found by stationarity of the total energy [1–3]. The conventional finite element method (FEM) uses standard shape functions [1, 4] but it often requires more degrees of freedom (DOF) for a specified accuracy than might a classical Ritz procedure [2, 3], thus causing a considerable delay in design problems that require repeated computations [5]. In order to reduce the number of DOFs, competitive methods such as the boundary element method [6, 7] and closely related Trefftz [8, 9] or energy-based combined methods [10], rational polygonal elements [11, 12], radial basis function (RBF)-methods [13] as well as mesh-less and mesh-free methods [14–16], among others, have appeared.

Within the narrow context of FEM, the use of polynomials of progressively increasing degree (p-version) on a fixed finite element mesh could be more

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Equivalent Finite Element Formulations for the Calculation of Eigenvalues Using Higher-Order Polynomials

C. G. Provatidis

Department of Mechanical Engineering, National Technical University of Athens, Athens, GR-157 80, Greece

Abstract This paper investigates higher-order approximations in order to extract Sturm-Liouville eigenvalues in one-dimensional vibration problems in continuum mechanics. Several alternative global approximations of polynomial form such as Lagrange, Bernstein, Legendre as well as Chebyshev of first and second kind are discussed. In an instructive way, closed form analytical formulas are derived for the stiffness and mass matrices up to the quartic degree. A rigorous proof for the transformation of the matrices, when the basis changes, is given. Also, a theoretical explanation is provided for the fact that all the aforementioned alternative pairs of matrices lead to identical eigenvalues. The theory is sustained by one numerical example under three types of boundary conditions.

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1. Introduction

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Received: 16 February 2011 / Accepted: 6 October 2011 / Published online: 8 November 2011

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Keywords Elasticity · Finite elements · Global approximation · Gordon-Coons interpolation

1 Introduction

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Solution of One-dimensional Hyperbolic Problems Using Cubic B-Splines Collocation

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(Abstract) In this paper we extend the cubic B-splines collocation method to enable it to solve one-dimensional hyperbolic (eigenvalue and wave propagation) problems under arbitrary boundary conditions. This is achieved by analogy with the finite element method, introducing a 'collocation mass matrix' that cooperates with the previously known system matrix, now called 'collocation stiffness matrix'. In agreement with earlier findings on elliptic problems, we found that in time-dependent problems it is again sufficient to use double internal knots (C^1-continuity) in conjunction with two collocation points between successive breakpoints. In this way, the number of unknowns becomes equal to the number of equations, which is twice the number of breakpoints. We paid particular attention to the handling of Neumann-type boundary conditions, where we found it necessary to properly eliminate a column in both mass and stiffness matrices. For the first time, we found that the cubic B-splines collocation procedure (with C^1-continuity) leads to identical results with those obtained using piecewise Hermite collocation. The numerical examples show an excellent quality of the numerical solution, which is far superior to that of the conventional finite element method, for the same number of nodal points.

Keywords: B-splines; Collocation; Eigenvalues; Hermite polynomials; MATLAB™; Transient analysis.

1. INTRODUCTION

In the framework of global interpolation methods based on Lagrange polynomials, it has been recently shown that the computation effort may be substantially reduced when applying the collocation instead of the Galerkin-Ritz formulation [1]. However, using spline curves, or piecewise polynomials, is more effective in representing the solution to the differential equation than pure polynomials [2]. The volume of Ascher et al. [3] provides a treatise on spline bases, collocation theory, and spline collocation for application to the numerical solution of boundary-value-problem (BVP) for ordinary differential equations (ODE). Fairweather and Meade [4] give an extensive review (273 papers covering the period 1934–1989) of collocation methods and various implementations. They describe the most common forms of collocation, including nodal, orthogonal, and collocation/Galerkin. An early work towards the solution of eigenvalue problems, however based on Schoenberg's formulation [5], is [6].

In spite of the abovementioned work, so far most part of the relevant research focuses on mathematical topics and reduces mainly to elliptic problems. For example, the FORTRAN codes of [2] have been implemented also in the MATLAB software (formerly under the name *splines tool*) [7], but they operate 'as is' for the solution of linear and nonlinear elliptic problems only. This happens because (i) the eigenvalue and transient analysis require the construction of mass and stiffness collocation matrices, and (ii) although the implementation of Dirichlet type boundary conditions is a trivial task, the same does not hold for Neumann-type ones that require a special treatment. As for the state-of-the-art, the implementation of the function 'spcol' in time-dependent analysis has been made in very few biometrics [8] and chemical engineering applications [9].

In this context, the primary aim of this paper is to investigate the applicability and performance of deBoor's methodology [2] in the numerical solution of hyperbolic problems (eigenvalue and transient). The main novel feature of this work is the development of *mass* and *stiffness* collocation matrices analogous to the finite element method. Consequently, standard eigenvalue analysis based on the QR algorithm, and standard time-integration techniques such as the central difference method will be applied.

In addition to the abovementioned *global* cubic B-splines collocation, *piecewise*-Hermite polynomials (without upwind features) that act between adjacent breakpoints [10] will be compared for the first time. The finding of coincidence is the secondary novel feature of this work.

The theory is sustained by four one-dimensional numerical examples from the field of applied mechanics.

2. FORMULATION

Below is the formulation of typical static (thermal) and dynamic problems and then follows the global B-spline and piecewise Hermite interpolation.

2.1 Thermal Analysis

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Performance of Coons' Macroelements in Plate Bending Analysis

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This paper proposes a general procedure to develop arbitrary noded-plate macroelements with C^1-continuity based on Coons' interpolation formula. The novelty of the proposed approach lies in the fact that the degrees of freedom appear only along the boundaries of the quadrilateral patch and are used for the successful calculation of displacements and relevant stresses. The boundary nodes are being interpolated using univariate Hermitian polynomials, thus allowing bivariate global shape functions to be automatically produced. The proposed macroelement, which will be hereafter called COPE (Coons' Plate Element), provides comparable results with closed-form analytical solutions in six benchmark plate-bending cases.

Keywords Plate bending, Finite elements, Coons' interpolation, Global approximation, *P*-methods

1. INTRODUCTION

Today, most of the computational methods are based on local rather than global approximation schemes. The first category includes the conventional finite element methods [1], as well as finite difference [2] and meshless/meshfree techniques [3, 4]. The second category includes the boundary element method [5] and its Trefftz-like variations [6], as well as a great number of spectral methods. In between these two categories, higher-order *p* methods [7] have a strong potential, but they currently appear to have some difficulties in problems requiring C^1-continuity [8].

Within the last decade, the first author has followed a different method in computational mechanics. The solution $w = W(x,y)$ of a two-dimensional problem, governed by the general partial differential equation $D(w) = 0$, has been considered as a curved surface in the Wxy space and, therefore, it has been approximated using Coons' or Coons-Gordon's interpolation [9–14]. Clearly, Coons' interpolation formula deals only with boundary data [9–12], while Coons-Gordon's (transfinite) interpolation deals with both boundary and internal nodes [13,14]. The method has been extended in three-dimensional problems, as well [15–17].

Plate bending appears to have particular interest for structural engineers. Historically, it was Melosh [18] who first proposed rectangular non-conforming elements with 12 nodal parameters $(w, \partial w/\partial x, \partial w/\partial y)$ at each of the four nodal points–thus leading to advantageous closed-form expressions for the stiffness matrix but to discontinuous slopes. As Zienkiewicz and Cheung [19] reported similar findings, this element is usually called the MZC rectangle.

Later, rectangular conforming four-noded elements with four degrees of freedom per node $(w, \partial w/\partial x, \partial w/\partial y, \partial^2 w/\partial x \partial y)$, a tot of 16 DOF, were proposed by Bogner et al. [20] (called BFS element). This element is entirely conforming and satisfies the criterion of constant strain, while the same authors also proposed 24- and 36-parameter elements. Moreover, a degeneration of the hexahedral solid finite element to serve as a plate or shell element by making one dimension considerably small compared to the other two was proposed by Ahmad et al. [21], and applies for both thick and thin shells. The latter quadrilateral eight-noded element (briefly, PBQ8) belongs to the serendipity family. Zienkiewicz et al. [22] found that its accuracy increases when applying a 2×2 reduced integration. In addition to the aforementioned serendipity element, other quadrilaterals for plate bending are of interest. For example, the straight-sided bilinear displacement quadrilateral, proposed by Hughes et al. [23], is simpler to use but, at the same time, insufficient to be adapted to general shells or curved boundaries. The Lagrange [24] and heterosis elements [25] both have interior nodes and require several numerical integration points to provide satisfactory accuracy. After these pioneering contributions, within the last 30 years an extremely large number of new plate elements have appeared, but due to lack of space, it is impossible to mention them all here.

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Bézier versus Lagrange polynomials-based finite element analysis of 2-D potential problems

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ABSTRACT

In this paper two types of tensor product finite macro-elements are contrasted, the former being the well known Lagrange type and the latter the Bézier (Bernstein) type elements. Although they have a different mathematical origin and seemingly are irrelevant, they both are based on complete polynomials thus sharing the same functional space, i.e. the classes $\{x^n\}$ and $\{y^n\}$. Therefore, from the theoretical point of view it is anticipated that they should lead to numerically identical results in both static and dynamic analysis. For both types of elements details are provided concerning the main computer programming steps, while selective parts of a typical MATLAB® code are presented. Numerical application includes static (Laplace, Poisson), eigenvalue (acoustics) and transient (heat conduction) problems of rectangular, circular and elliptic shapes, which were treated as a single macroelement. In agreement to the theory, in all six examples the results obtained using Bézier and Lagrange polynomials were found to be identical and of exceptional accuracy.

1. Introduction

During the last 45 years, the CAD technology has passed through five main stations, the last being the NURBS representation of surfaces and volumes [1,2]. The first three stations are determined by the names of their founders, which are Coons [3], Gordon [4], and Bézier [5], whereas the fourth is called B-splines [6,7]. For a detailed handbook the reader is referred to Farin et al. [8].

The strong relationship between CAD and CAE was early understood in 1973 by Gordon and Hall [9] and can be also found as a comment in the classical textbook of Zienkiewicz in 1977 [10, pp. 181–182] (“… *quite independently the exercises of devising various practical methods of generating curved surfaces for purposes of engineering design led to the establishment of similar definitions by Coons, and indeed today the subjects of surface definitions activity…*”). In the decade of 1980s some researchers started dealing with the development of large finite elements based on Coons–Gordon interpolation; for a detailed review the reader is referred to [11–13] and papers therein. Also, the use of B-splines and the most recent NURBS in engineering analysis are reviewed in the textbooks of Höllig [14] and Cottrell et al. [15], respectively.

In the particular case of isogeometric elements, useful software aspects have recently appeared [16,17].

Despite the abovementioned progress, there are still some unclear points, from both the theoretical and the programming point of view. For example, it is well known that tensor product Lagrange type finite elements have been extensively used and can be found in all textbooks such as those by Zienkiewicz [10] and Bathe [18]. Nevertheless, it is not perhaps widely known that the aforementioned tensor product based on Lagrange polynomials is a special case in which the Gordon–Coons interpolation degenerates [12, pp. 327–328]. Except of the aforementioned relationship of CAD/CAE character, there is a second one as follows. It is well known that Bézier surfaces are based on tensor products of Bernstein polynomials of the form $B_{i,n}(x) = \frac{n!}{i!(n-i)!} x^i (1-x)^{n-i}$. Obviously, each basis function is a polynomial of *n*-degree. It differs from the Lagrange polynomial in the sense that the first does not possess the property of cardinality $B_{i,n}(x_j) \neq \delta_{ij}$ and does not include the nodal values but nodeless coefficients.

Recently it has been shown that in 1-D problems the Lagrange and Bézier polynomials, as well as the popular p-method [19] lead to the same eigenvalues [20,21], and it is only a matter of basis change [18].

In this context, this paper continues the investigation in 2-D quadrilaterals in both static and time dependent problems. The study reduces to the Galerkin–Ritz formulation.

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B-SPLINES COLLOCATION EIGENANALYSIS OF 2D ACOUSTIC PROBLEMS

CHRISTOPHER G. PROVATIDIS

We continue our research on the performance of CAD-based global approximation to the analysis of 2D acoustic problems. In addition to previous “boundary-only” Coons and transfinite Gordon–Coons interpolations, we now investigate the quality of the solution when utilizing “tensor product B-splines” interpolation. For the latter, we propose a global collocation method that is successfully compared with the well known Galerkin–Ritz formulation. Particular attention is paid to the handling of Neumann boundary conditions as well as to the role of multiplicity of internal knots. The theory is supported by two numerical examples, one for a rectangular and the other for a circular acoustic cavity in which the approximate solution rapidly converges towards the exact solution.

1. Introduction

The tendency in contemporary computer methods in applied mechanics and engineering is to integrate solid modeling (computer-aided-design or CAD) with analysis (computer-aided-engineering or CAE) using NURBS interpolation, in such a way that both the geometry and the mechanical variables (displacement, temperature, etc.) are mathematically expressed in a similar manner (global approximation) [Cottrell et al. 2009]. In fact, though the nonuniform B-splines (NURBS) of today is, chronologically speaking, the fifth important formulation applied to the mathematical description of CAD models, the same integration can be achieved with using any of the previous formulations. The first bivariate formula was proposed in 1964–1967 by Coons [1967], the second by Gordon [1971] and the third in 1966–1971 by Bézier [1971]. Furthermore, B-splines are chronologically the fourth formula in CAD practice. Although older mathematical formulations of splines were first published by Schoenberg [1946], they became very popular only after 1972 when de Boor [1972] proposed his computationally efficient algorithms. Finally, B-splines were later modified on the basis of weighting coefficients, thus producing the popular NURBS of today, which are fully controlled sculptured surfaces [Piegl 1991; Piegl and Tiller 1995]. For a detailed review we refer to [Farin et al. 2002].

Concerning mechanical analysis in problems of solids and structures including acoustics, it is well known that there are three main methodologies: the popular finite element method (FEM), the boundary element method (BEM), and the promising global collocation method ([Provatidis 2008b; 2009b; Provatidis and Ioannou 2010] and about 300 references therein). For the sake of brevity, finite volume, finite difference, mesh-less and mesh-free methodologies are not commented on. So far, FEM [Höllig 2003] and BEM [Cabral et al. 1990; 1991] have been applied in conjunction with tensor product B-splines in several engineering problems. Also, Coons–Gordon transfinite interpolation has been extensively used in conjunction with the Galerkin–Ritz formulation; for an overview we refer to [Provatidis 2012] and

Keywords: B-splines, Galerkin–Ritz, global collocation, eigenvalues, CAD/CAE.

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Research Article

Finite Element Analysis of Structures Using C^1-Continuous Cubic B-Splines or Equivalent Hermite Elements

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We compare contemporary practices of global approximation using cubic B-splines in conjunction with double multiplicity of inner knots (C^1-continuous) with older ideas of utilizing local Hermite interpolation of third degree. The study is conducted within the context of the Galerkin-Ritz formulation, which forms the background of the finite element structural analysis. Numerical results, concerning static and eigenvalue analysis of rectangular elastic structures in plane stress conditions, show that both interpolations lead to identical results, a finding that supports the view that they are mathematically equivalent.

1. Introduction

Structural analysis is usually performed using commercial codes that include finite elements of low (usually first or second) degree, where the accuracy of the calculations increases by mesh refinement (h-version). Alternatively, keeping the number and the position of the nodal points unaltered, the numerical solution improves using polynomials of higher degree (p-version) [1].

As an extension of the above p-version "philosophy," tensor-product Lagrange polynomials as well as CAD-based (Gordon-Coons) macroelements—based on several interpolations—have been used [2–4]. The aforementioned macroelements integrate the solid modelling (CAD: computer-aided-design) with the analysis (CAE: computer-aided-engineering). In more detail, these macroelements use the same global approximation for both the geometry and the displacement vector. In order to avoid the undesired numerical oscillations caused by Lagrange polynomials of high degree, the next generation of CAD-based macroelements replaced them with tensor-product B-splines [5]. Since 2005, the nonuniform-rational-B-splines (NURBS) interpolation has started to prevail [6].

A careful study of literature reveals that most of recent papers referring to the so-called *isogeometric* analysis (IGA) start with some essentials on the definition of B-splines and relevant recursive formulas due to de Boor [7]. It should be recalled that NURBS is an extension of B-splines (nonuniform and rational) modified on the basis of weighting coefficients, thus producing fully controlled sculptured surfaces [8, 9]. In a B-splines expansion, the multiplicity of the inner knots plays a significant role in the continuity of the variables. In general, the multiplicity of λ inner knots per breakpoint in combination with a piecewise polynomial of degree p ensures $C^{p-\lambda}$-continuity of the variable (here: displacement components) [7, 9]. Thus considering cubic B-splines ($p = 3$) in conjunction with double inner knots ($\lambda = 2$), C^1-continuity is ensured. Höllig [5, page 93] has solved plane stress problems using B-splines of degree $n = 2, 3, 4$, and 5, but his study is not a complete investigation on the influence of the multiplicity and corresponding continuity of variables involved.

On the other point of view, tensor-product Hermite elements of third degree have been proposed for the solution of fourth-order problems, such as plate-bending problems, using Galerkin-Ritz formulation. The need for smoother (C^1) global basis functions is also encountered in second-order problems when collocation finite element methods are utilized [10, page 66].

With these situations in mind, we next examine the relationship between particular Hermite elements of third degree

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Lumped mass acoustic and membrane eigenanalysis using the global collocation method

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Additional information is available at the end of the article

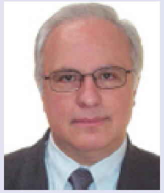
Abstract: The paper proposes a direct way to build lumped masses for performing eigenvalue analysis using the global collocation method in conjunction with tensor product Lagrange polynomials. Although the computational mesh is structured, it has a non-uniform density, in such a way that the internal nodes are located at the position of Gaussian points or the images of the roots of Chebyshev polynomials of second kind. As a result, the mass matrix degenerates to the identity matrix. In this particular nodal collocation procedure, no complex eigenvalue appears. The theory is successfully applied to rectangular and circular acoustic cavities and membranes.

Subjects: Acoustical Engineering, Computer Science, Design, Engineering & Technology, Mathematics & Statistics for Engineers, Mechanical Engineering, Technology

Keywords: global collocation, CAD/CAE integration, acoustics, membranes

1. Introduction

Transfinite elements were inspired in early 1970s (Gordon & Hall, 1973) for the purposes of CAD/CAE integration, but only much later were applied for the numerical solution of static and dynamic engineering problems using natural cubic B-splines (Kanarachos & Deriziotis, 1989; Kanarachos,



ABOUT THE AUTHOR

Christopher Provatidis is a full professor of Mechanical Engineering at National Technical University of Athens (NTUA), Greece. He obtained his PhD from the School of Mechanical Engineering at NTUA in 1987. His research work is in the general areas of computational mechanics (finite elements, boundary elements, finite volumes, etc.) and mechanical design using integrated CAD/CAM/CAE systems. In particular, he has performed computer modeling in biomechanics (orthopedics, orthodontics, prosthodontics), acoustics, and elastodynamics using novel boundary element methods, static-eigenvalue-transient analysis using novel CAD-based isoparametric macroelements, fracture mechanics, textile micromechanics, active noise control, inverse problems for NDT using a variety of optimization algorithms and neural networks, shape and motion reconstruction, structural size-shape-topology optimization, and wave propagation in thin silica films and laser cleaning of paintings. Recently, he has contributed to inertial propulsion, mathematics (Goldbach's Conjecture), and physics. He has published over 350 refereed publications.

PUBLIC INTEREST STATEMENT

Physical phenomena are usually of time-dependent character. Therefore, adequate computer effort is required to simulate their progress. One of the most important information needed to be extracted by an analyst (or a designer) is the resonance frequency. This frequency must be avoided because it is harmful or extremely annoying. This paper proposes a fast method for the numerical extraction of the resonant frequencies, particularly those of acoustic cavities (such as rooms, theaters, music halls, and musical instruments) and those of vibrating elastic membranes. The method eventually bypasses the computation of the mass matrix, and uses only the stiffness matrix. In this way, the total computation (CPU) time is highly reduced. In terms of mathematics, the proposed procedure is a global collocation method based on tensor product Lagrange polynomials. More generally, the same idea can be applied to the numerical solution of wave propagation problems such as acoustics and elastodynamics.

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Numerical Determination of Eigenfrequencies in Two-Dimensional Acoustic Cavities Using a Global Collocation Method

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Abstract: - Based on a previous global collocation concept concerning the numerical solution of two-dimensional potential and elastostatic problems, this paper investigates its applicability to the eigenvalue analysis of acoustic cavities. The key point is to consider global approximation of the acoustic pressure within the entire cavity, the former based either on conventional (tensor product) Lagrangian or on generalized Gordon-Coons interpolation. The procedure is supported by several test cases concerning circular and rectangular acoustic cavities under Neumann and Dirichlet boundary conditions. The proposed method is successfully compared with analytical solutions concerning both eigenvalues and eigenvectors, as well as with a commercial finite element program.

Keywords: - global collocation, Coons-patch elements, eigenfrequency analysis

1. INTRODUCTION

In systems with a very large amount of degrees of freedom, frequency determination from the differential equation often becomes so complicated that it is practically impossible to calculate [1]. Historically, it was Rayleigh who first proposed a generalized energy method based on an assumed shape for the lowest natural frequency [2]. Later, Ritz generalized this procedure to more than one parameter. The major drawback of the Rayleigh–Ritz method is the difficulty in constructing a set of admissible functions, particularly for a compound structure. This difficulty can be overcome by using the finite-element method [3], which provides an automatic means of constructing such functions. Alternatively, a boundary element method (BEM)-based variational method has been proposed [4].

However, most of the known finite-element methods often require more degrees of freedom (DOF) for a specified accuracy than a classical Ritz procedure would, thus causing a considerable delay in design problems that require repeated eigenvalue computations during iterations [5]. Since computational effort increases proportionally to the third power of the problem (system) order, many attempts have been made to reduce the number of degrees of freedom. Among them, the "Coons-Patch Macroelement" (CPM) method has shown an excellent performance for both two- and three-dimensional problems [6-8], especially when applied in conjunction with the Lagrange interpolation [9]. However, the obtained stiffness matrix becomes fully populated thus requiring additional computer effort.

In this context, in order to reduce the computational effort due to the domain integration, a global collocation method has been recently proposed to replace the time consuming Galerkin/Ritz procedure. Initially the former method was applied to one-dimensional problems [10,11] and then to two-dimensional Poisson equation [12] and elastostatics [13].

This paper extends the above ideas and investigates the numerical performance of the global collocation methodology to the solution of eigenvalue problems for acoustic cavities. Although the proposed methodology is applicable to arbitrary shaped quadrilaterals, this study deals only with rectangular and circular cavities for which closed form analytical solutions are available in literature.

2. GENERAL FORMULATION

2.1. The proposed Global Collocation method

We consider a two dimensional acoustic cavity which is discretized in $n_x \times n_y$ intervals, i.e. $n = (n_x+1)\times(n_y+1)$ nodes [13]. From these nodes, $n_b = 2(n_x+n_y)$ nodes belong to the

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LUMPED MASS COLLOCATION METHOD FOR 2D ELASTODYNAMIC ANALYSIS

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ABSTRACT: **This paper modifies a previous global approximation method for the free vibration analysis of elastic structures [*Finite Element in Analysis and Design Vol. 42(6), 2006, 518-531*] by proposing a novel nodal collocation method in which the mass matrix degenerates to the identity matrix. This is accomplished by positioning the internal nodes at the positions of Gaussian points, among others. The proposed method is implemented to 2D problems using global approximation through tensor-product Lagrange polynomials or, more generally, through Gordon-Coons (i.e. transfinite) interpolation. In addition to eigenvalue analysis, transient response analysis is performed as well. The theory is sustained by numerical examples on rectangular and curvilinear domains under several boundary or/and loading conditions, in which the approximate solution rapidly converges towards the exact solution.**

KEYWORDS: **Transfinite interpolation; Macroelement; Global collocation; Eigenvalues; Transient response; Lumped mass.**

1. INTRODUCTION: It is well known that, in early 1970s, an industrial team under Gordon's leadership, based on the ideas put forward in (1), used transfinite blending function methods to interpolate the geometry and the variable as well, thus producing some interesting element families [(2), (3)]. This moment was the beginning of CAD/CAE integration.

One decade later, closed-form expressions for the involved global shape functions were derived by others [(4)-(6)]. The first computational results in conjunction with Galerkin-Ritz formulation concerned with 2D potential problems (7) and 2D elasticity (8). These ideas were later extended to axisymmetric potential problems (9), 2D free vibrational analysis (10), axisymmetric elasticity (11), and 3D problems (see (12), among others). For a detailed review (of over 160 references) on the use of CAD-based macroelements the reader is referred to (13).

The computational experience has shown that simple domains such as rectangles, circles, ellipses or hollow half circles can be successfully treated in terms of the Galerkin-Ritz formulation using a single macroelement, which constitutes the so-called Coons Patch Macroelement (CPM) method (10). For domains of complex shape, decomposition in a certain number of large convex macroelements is proposed. However, since the net computer effort devoted to the CPM solution is comparable with that of standard finite elements, current needs demand further reduction of the corresponding computational time. Therefore, in order to make the global approximation method more attractive, since 2005 it has been proposed (see [14, p. 6704]) to preserve the global interpolation functions and replace the Galerkin-Ritz procedure with a global collocation scheme. So far, this idea performed well in 2D static analysis (15), 2D potential problems [(16), (17)] and 1D elastodynamics [(18), (19)]. This paper extends these ideas to the eigenvalue and transient analysis of 2D elastic structures using the global collocation method and particularly its novel lumped mass version.

Initial findings in the eigenvalue extraction of 2D structures, in conjunction with tensor-product Lagrange polynomials, have revealed increased errors as well as the appearance of complex eigenvalues when the collocation points are chosen at the internal nodal points of a uniform mesh (*nodal* collocation). In order to overcome this drawback, *orthogonal* collocation has been proposed and successfully applied in conjunction with the Gaussian points or the roots of Chebyshev polynomials of 2nd kind (20). However, the latter two choices lead to fully-populated mass matrices, a fact that somehow diminishes the advantage of using the global collocation method.

Under these circumstances it is imperative to investigate whether there are suitable positions to put the internal nodes of the computational mesh and then perform a nodal collocation at them.

The paper is structured as follows. First, we propose the orthogonal global collocation method in full elastodynamics formulation (eigenvalue and transient analysis). Second, we choose the nodal points to coincide with those of the Gaussian points. In this way the mass matrix degenerates to the identity matrix, thus no computational cost is spent for it (it is sufficient to estimate the eigenvalues of the stiffness matrix only). Third, we theoretically show that, for a single Coons-Gordon macroelement under Dirichlet-type boundary conditions, the calculated

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CAD-based collocation eigenanalysis of 2-D elastic structures

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ABSTRACT

This paper investigates the performance of the global collocation method for the numerical eigenfrequency extraction of 2-D elastic structures. The method is applied to CAD-based macroelements, starting from the older blending function Coons-Gordon interpolation (based on Lagrange polynomials) and extending to tensor product Bézier and B-splines. Numerical findings show equivalence between Lagrangian and Bézierian macroelements, while a mass lumping procedure is proposed for the former ones. Concerning B-splines, the influence of multiplicity of inner knots and the position of collocation points is thoroughly investigated. The theory is supported by 2-D numerical examples on rectangular and curvilinear structures of simple shape under plane stress conditions, in which the approximate solution rapidly converges towards the exact solution faster than that of the conventional finite element of similar mesh density.

1. Introduction

The use of CAD-based global approximation for the numerical solution of partial differential equations (CAE: computer-aided-engineering) is as old as the theory of computer-aided-design (CAD) itself. It is well known that an industrial team (at General Motors) under Gordon's leadership, in early 1970s, used blending function methods, based on the ideas put forward in [1], to produce some interesting element families [2,3]. Although this team presented the mathematical background for the common description between the geometric model and the unknown variable (CAD/CAE integration), unknown reasons (perhaps the high computational cost) prevented further dissemination of this excellent idea.

One decade later, El-Zafrany and Cookson [4,5] also used Coons' and Barnhill's ideas for quadrilateral and triangular patches, respectively, whereas Zhaobei and Zhiqiang proposed the use of Coons' interpolation for the analysis of plates and shells [6].

Nevertheless computational results concerning CAD-based isoparametric macroelements (occupying a Coons patch ABCD) were presented for the first time by Kanarachos, Deriziotis and Provatidis [7,8] in 2D potential and elasticity (static and dynamic) problems, where the so-called "C"-elements were successfully compared with conventional finite elements and boundary elements of similar mesh density. For a detailed review (of over 160 references) on the use of CAD-based macroelements the reader is referred to [9].

Summarizing some of the most important previous findings concerning macroelements that occupy a 2D quadrilateral patch ABCD or a 3D hexahedral block ABCDEFGH, in chronological correspondence with the progress in CAD-theory (Coons, Gordon, Bézier, B-splines and NURBS) (see, for instance, [10]), it has been reported that:

(i) (Boundary-only) *Coons* interpolation is capable of creating a broad family of arbitrary-noded elements that may be equivalent to that of Serendipity type. For example, the conventional 4- up to 8-noded 2D elements, as well as the 8- and 20-noded 3D elements can be directly derived applying Coons interpolation [11–14].

(ii) *Gordon-Coons* (transfinite blending function) interpolation when applied to a *structured* macroelement of which the boundary and internal nodal points lay at the same normalized (ξ, η) positions, degenerates to the classical Lagrangian type finite element [14].

(iii) Coons-Gordon interpolation allows for dealing with a (relatively) *unstructured* mesh of internal nodes. Using a single quadrilateral macroelement, not only simple shapes such as a rectangular or a circle can be treated, but also it was possible to perform analysis until the complexity of a pi-shaped domain (see, [14–16], among others). For more complex shapes, domain decomposition using large macroelements becomes necessary.

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B-SPLINES COLLOCATION FOR PLATE BENDING EIGENANALYSIS

CHRISTOPHER G. PROVATIDIS

Following the recent encouraging findings in the area of 2-D acoustics, this paper investigates the performance of a B-spline collocation method in the extraction of natural frequencies (eigenvalue analysis) of thin plates in bending. Numerical formulation and associated results refer to uniformly discretized rectangular and circular plates, for which closed-form analytical or approximate solutions are available in the literature. The computational results show that the proposed B-spline collocation method is of higher quality than the previously known cubic B-splines Galerkin–Ritz formulation; both of them converge more rapidly to the accurate solution than what the conventional finite element method does for the same mesh density.

1. Introduction

Engineering analysis of arbitrarily shaped or arbitrarily loaded structures is usually performed using the well-known finite element method (FEM) [Bathe 1996]. Particularly in mechanical engineering, where the structural components generally consist of free shaped boundaries produced by a CAD system, it is more convenient to deal with their B-splines representation [de Boor 1972; Farin et al. 2002; Piegl and Tiller 1995]. In addition to a CAD model, computational engineering analysis (CAE) can be performed on the basis of either B-splines [Höllig 2003] or NURBS [Cottrell et al. 2009]. For a detailed review on the CAD/CAE integration, the interested reader may consult [Provatidis 2013].

B-splines based finite elements have been extensively used in the finite element praxis. In more detail, structural engineering applications cover static, dynamic and stability analyses [Peng-Cheng et al. 1987; Akhras and Li 2011]; an older survey is [Grigorenko and Kryukov 1995]. A great number of papers on B-splines finite element models applied to plates and shells have been published in the last twenty years. These include isotropic [Antes 1974; Gupta et al. 1991; Fan and Luah 1995], orthotropic [Cheng and Dade 1990], cross- and angle-ply multilayered laminated [Patlashenko and Weller 1995; Dawe and Wang 1995; Kolli and Chandrashekhara 1997; Reddy and Palaninathan 1999; Park et al. 2008; Kapoor and Kapania 2012; Golmakani and Mehrabian 2014], functional graded materials (FGM) [Valizadeh et al. 2013; Tran et al. 2013] and shell [Echter et al. 2013] structures, among others. A tendency of the last few years is to combine B-splines with wavelet ideas [Han et al. 2007; Zhang et al. 2010; Li and Chen 2014].

Despite the aforementioned progress, it has been reported that the computer effort required to estimate the matrices of these CAD-based macroelements (in potential and structural problems) is relatively high [Provatidis 2004; 2012]. As a remedy to this shortcoming, in 2005 the author proposed preserving the global CAD-based interpolation but substituting the Galerkin–Ritz formulation (which needs domain integration to estimate the mass and stiffness matrices) by a global collocation scheme [Provatidis 2006,

Keywords: B-splines, collocation method, finite element method, CAD/CAE.

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SPECIAL

C. G. Provatidis

Engineering analysis with CAD-based macroelements

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Abstract Older and contemporary CAD-based interpolations, either for surfaces or for volume blocks, are capable of creating sets of basis functions on which finite-element (Galerkin–Ritz) and global collocation procedures can be supported. For some of these interpolations, this paper investigates the quality of the relevant numerical solution in several 2D and 3D engineering problems. It is shown that the global character of all these CAD interpolations ensures excellent numerical solution, although somewhere the boundary may be slightly violated. The study deals with several benchmark tests that span a large part in the spectrum of engineering analysis, from potential problems (Poisson equation-electrostatics and acoustics) to elasticity ones (beam in torsion, plate bending: statics and dynamics).

Keywords Finite element · Collocation · CAD · Isogeometric · Boundary value problem · Eigenanalysis

1 Introduction

Within the last decade, *'isogeometric analysis'* (IGA) has been presented as the approach that bridges CAD (computer-aided design) with CAE (computer-aided engineering) [1]. In fact, there is no doubt that the involved NURBS (Non-Uniform Rational B-Splines) is the state-of-the-art interpolation in all modern CAD systems, and it is capable of accurately representing all conics (i.e. circles, ellipses, hyperbolas and parabolas) as well as any rotationally symmetric analogue (i.e. spheres, ellipsoids, hyperboloids and paraboloids). Therefore, it is widely believed that IGA is a quite novel technique that achieves the CAD/CAE integration through the accurate approximation of the geometric model. The overall impression is that the excellent performance of IGA, particularly in the eigenvalue extraction [2], has to be attributed to the superiority of NURBS interpolation over piecewise approximations.

Nevertheless, it should be reported that before the acronym NURBS was even known in CAD community, the author participated in a project at the National Technical University of Athens (NTUA) investigating the integration of "Coons interpolation" [3] (i.e. chronologically the first mathematical expression in CAD history) with the finite-element analysis (FEA). This happened earlier than 1984, i.e. 20 years before IGA appeared. The first two relevant papers on that CAD/CAE bridging are [4,5], whereas a review paper concerning the overall attempt is [6].

Before going on with the CAD-based computational methods that may be used for the solution of the boundary value problem and the eigenvalue analysis, we briefly report on the most "important" CAD interpolations. In the chronological evolution of CAD theory, the first (cornerstone) station is due to Steven Coons (1912–1979), who (for the first time in 1964 and later in 1967) proposed the so-called Coons interpolation formula within curvilinear surface patches ABCD and volume blocks ABCDEFGH [3]. Drafty speaking, Coons'

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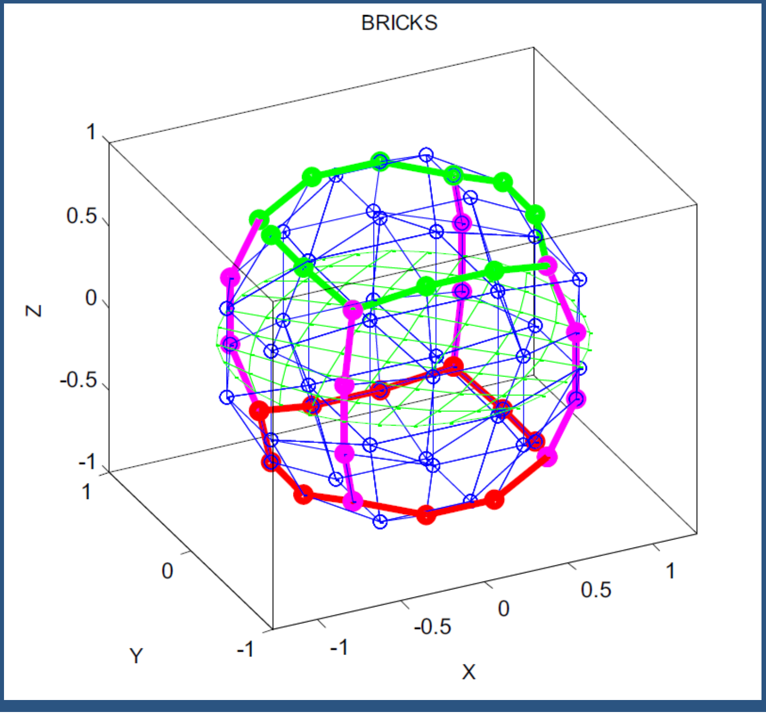

Table 5 Example 6 (Poisson's equation) convergence rate of calculated potential u and the volume V

Provatidis (2017)

Element type	Calculated errors	Number of subdivisions per edge (n_s)			
		3	5	7	9
Lagrangian	Error of u: L_u (in%)	15.89	0.60	0.06	0.004
	Error (in%) of V	1.3538	−0.0809	0.0093	2.34×10^{-4}
Brick	Error of u: L_u (in%)	13.34%	5.59%	3.00	1.85
	Error (in%) of V	−9.03	−3.29	−1.69	−1.02

Boundary-Element (BEM) vs. Galerkin-Ritz

- The fundamental solution (GREEN's function) operates as a weighting function in the Galerkin's method.
- Easiness when handling external problems (e.g. acoustics).
- Difficulties when handling non-homogeneous problems
- Eigenfrequency analysis is a non-algebraic problem unless DR/BEM is applied.

Our Contribution

- Coons interpolation was applied in conjunction with BEM in Diploma-Works I supervised, first in elastostatics (Tzanakis 1991) and then in acoustics (Mastorakis 1992).
- Official publications in Conferences only:
  - 2001: *4^{th} Europ. Conf. Noise Control* (C21)
  - 2001: *ASME-Greek Confe*rence (C26)
  - 2002: *4^{th} GRACM Congress on Computational Mech.*
  - 2002: *Acoustics 2002 Conf.* (C40)
  - 2003: *5^{th} Europ. Solid Mech. Conf.* (C55, C56)

A typical BEM Coons-based paper

ASME - GREEK SECTION, First Nat. Conf. on Recent Advances in Mech. Eng., September 17-20, 2001, Patras, Greece

Proceedings of ASME – Greek Section
September 17-20, Patras, Greece

Paper ANG1/P129

STRESS ANALYSIS OF 3D SOLID STRUCTURES USING LARGE BOUNDARY ELEMENTS DERIVED FROM 2D-*COONS*' INTERPOLATION

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ABSTRACT

This paper proposes a new technique to improve the efficiency of the Boundary Element Method so that to become capable of drastically reducing the number of collocation points involved. The method refers to an elastic solid structure of arbitrary shape, consisting of several curvilinear boundary patches. For each patch the new method applies the well-known Coons' interpolation formula, which is the simplest mathematical representation of a surface in Computational Geometry. By using Coons' formula, all three: geometry, boundary displacements and tractions are interpolated in terms of their values along the edges of the patch in which they belong. As a result, no degrees of freedom appear within the patches excepting their edges. Since the involved geometrical entities can be the absolutely necessary quantities that built up the CAD-model, the proposed method seems to "marry" CAD with CAE. The efficiency of the method is elucidated with three numerical examples.

KEYWORDS

Boundary Element Method, CAD, CAE, Coons' interpolation, Stress analysis.

INTRODUCTION

Thirty-four years after the first practical application of the Boundary Element Method (BEM) in stress analysis by Rizzo [1], it is now a well-established technique. The key ingredient is to apply the historical Somigliana's [2] integral equations in conjunction with constant, linear or quadratic interpolation of both the displacements and tractions along the boundary of a solid structure [3, 4]. The advantage of BEM is that it reduces the dimensionality of the problem by one; from 3D (volume) to 2D (surface). This paper investigates the possibility of using as less boundary information as possible and achieves to reduce the dimensionality of the problem by one more; from 3D (structure's volume) to 1D (lines of the patches).

NOMENCLATURE

V	volume of the structure
K_{tp}	number of nodes in the whole structure
N_l	number of points on patch boundary
N_n	total number of nodes in the whole structure
N_p	number of patches in the structure
b_i	body force
ip	ascending number of a patch
p_k	traction
u_k	boundary displacement
u^*_{lk}	fundamental solution
Γ	boundary of the structure
Φ_k	global shape functions

ASME - GREEK SECTION, First Nat. Conf. on Recent Advances in Mech. Eng., September 17-20, 2001, Patras, Greece

replaced by $\left|G(\xi,\eta)\cdot G'(\xi',\eta')\right| d\xi' d\eta'$, which requires a trivial (e.g., 2×2, 3×3, 4×4) Gaussian quadrature.

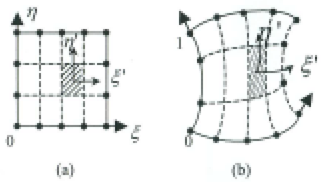

Fig. 2 (a) Unit reference and (b) Real patch geometry.

So, the final algebraic system obtains the form

$$\mathbf{C}\,\mathbf{U} + \sum_{ip=1}^{Np} \mathbf{H}^{ip}\,\mathbf{U}^{ip} = \sum_{ip=1}^{Np} \mathbf{G}^{ip}\,\mathbf{P}^{ip} \qquad (15)$$

where $\mathbf{U}$ is the displacement vector of all nodes on the boundary of the structure (along the patch edges), $\mathbf{U}^{ip}$ and $\mathbf{P}^{ip}$ are displacement and traction vectors referring to the *ip*-th patch. Also, the matrices $\mathbf{H}^{ip}$ and $\mathbf{G}^{ip}$ are of order $3N_v \times 3K_{ip}$, where N_v is the number all nodes of the whole structure and K_{ip} is the number of the *ip*-th patch. Their elements $\mathbf{H}^{ip}_{ij}$ and $\mathbf{G}^{ip}_{ij}$, each of order 3x3, relate the *i*-th geometrical node of the structure with the *j*-th node of the *ip*-th patch. The $\mathbf{C}$-matrix is a diagonal one of order $3N_v \times 3N_v$.

It is here reminded that apart of the particular case of an ideal smooth boundary, in most cases the number of the geometry nodes is smaller than the number N_m of traction points [4]. So, Eq. (15) finally becomes

$$\mathbf{C}\,\mathbf{U} + \hat{\mathbf{H}}\,\mathbf{U} = \mathbf{G}\,\mathbf{P} \qquad (16)$$

where

- $\mathbf{C}$: diagonal matrix ($3N_v \times 3N_v$)
- $\mathbf{U}$: displacement vector ($3N_v \times 1$)
- $\mathbf{P}$: traction vector ($3N_m \times 1$)
- $\hat{\mathbf{H}}$: total displacement –influence matrix ($3N_v \times 3N_v$)
- $\mathbf{G}$: total traction-influence matrix ($3N_v \times 3N_m$)

Again, the final displacement–influence matrix ($\hat{\mathbf{H}}$) is square while the traction–influence one ($\mathbf{G}$) will be nonsquare possessing more columns than rows.

With respect to the diagonal terms of the matrix $\mathbf{H} = \mathbf{C} + \hat{\mathbf{H}}$, these can be easily calculated as in the conventional BEM [3, 4] on the basis of rigid body considerations. In this work, no special attention was given to the singular $\mathbf{G}_{ii}$-terms.

NUMERICAL EXAMPLES

The proposed method is sustained by three examples.

Example 1: Cube in tension

A cube of unit length is fixed at its one surface (x_1=0) while the opposite side is uniformly loaded in tension (P_x=1). Elastic modulus and Poisson's ratio are: E=1, ν=0.

This problem was solved for three different uniform meshes of twenty, forty-four and sixty-four geometry (displacement) nodes, respectively, as shown in Fig. 3.

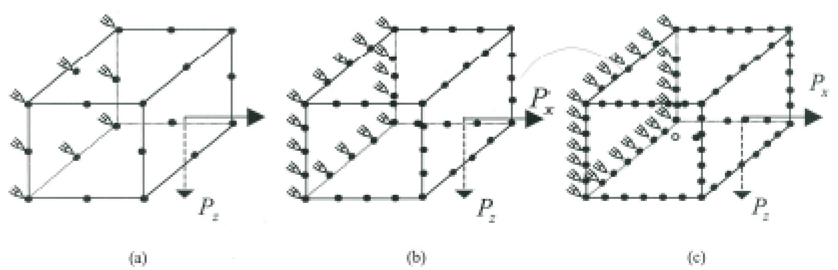

Fig. 3 Clamped unit cube model using (a) twenty, (b) forty-four and (c) sixty-four nodes, uniformly distributed

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Each case was also solved for three different Gaussian quadratures: 2x2, 3x3 and 4x4 per cell. Results are presented in Table 1.

Table 1 Calculated displacement at the loaded surface: Tension

Number of nodes	Gauss points per cell			Exact solution
	2x2	3x3	4x4	
20	0.939	0.972	0.984	1.000
44	0.978	0.989	0.994	1.000
68	0.989	0.994	0.997	1.000

Example 2: Cube in bending

A cube of unit length is fixed at its one surface (x_1=0) while the opposite side is loaded by a uniform shear traction in bending (P_z=3.2). Elastic modulus and Poisson's ratio are: E=1, ν=0.2. This problem was solved for a uniform mesh of forty-four geometry (displacement) nodes (Fig. 3b) using three different Gaussian quadrature schemes: 2×2, 3×3 and 4×4 per cell. As only approximate closed formulas exist for this example, now the results shown in Table 2 are compared with the commercial FEM-code ALGOR (for the same number of divisions).

Table 2 Calculated displacement at the loaded surface: Bending

Displacement	Gauss points per cell			ALGOR
	2×2	3×3	4×4	
u	9.53	10.06	10.25	10.13
v	0.00	0.00	0.00	0.00
w	20.18	21.14	21.49	21.35

Example 3: Thick hollow cylinder under internal pressure.

A thick hollow cylinder (R_i=10mm, R_o=20mm) and height L=20 mm is subjected to a uniform internal pressure P=20 MPa. The model consists of one-fourth (90 degrees) circumferentially where lower and upper sides do not move along the axis of revolution but they only roll on the plane (x_1,x_2) shown in Fig. 4. The material is isotropic and linear elastic (E=210000 MPa, ν=0.3).

The numerical model consists of forty-four geometry (displacement) nodal points. In other words it consists of six patches with sixteen nodes per patch. Numerical results are presented at boundary nodes (Tables 3 and 4) as well as at internal points. In all cases these are compared with the analytical solution given as

$$\sigma_\theta = \frac{a^2 P}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right),\quad \sigma_r = \frac{a^2 P}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

$$E\,\varepsilon_\theta = \sigma_\theta - \nu\sigma_r,\quad \varepsilon_\theta = \frac{u}{r},\quad u = \frac{r}{E}(\sigma_\theta - \nu\sigma_r)$$

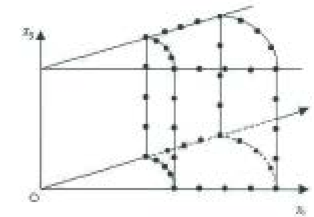

Fig. 4 One-fourth of a thick cylinder

Boundary nodes

Table 3 Radial displacement ($u_r \times 10^3$) on the internal boundary

2×2	3×3	4×4	Exact
1.79	1.83	1.85	1.87

Table 4 Circumferential stresses σ_θ (MPa) on the boundary

2×2	3×3	4×4	Exact
-36.10	-34.58	-34.28	-33.33

Internal nodes

Table 5 Radial displacement ($u_r \times 10^3$) in the interior at x_3=L/2=10

Radius (R)	Gauss points per cell			Exact
	2×2	3×3	4×4	
12.5	1.60	1.56	1.57	1.59
15.0	1.38	1.39	1.41	1.43
17.5	1.31	1.32	1.31	1.33

OVERVIEW OF PUBLICATIONS

TABLE OF CAD/CAE INTEGRATION PUBLICATIONS

GALERKIN-RITZ	COLLOCATION	BEM
J119, J114, J111, J108, J92, J69 J47, J44, J42, J41, J37, J36, J29, J28, J24, J19, J17	J129, J128, J126, J125, J121, J117, J99, J93, J70, J67, J63, J60, J59	
JOURNAL: 13	JOURNAL: 17	JOURNAL: 0
C236, C192, C191, C173, C136, C108, C96, C89, C78, C57, C56, C55, C53, C48, C25, C22, C19, C12, C4	C193, C172, C135, C122	C56, C55, C36, C35, C26
CONFERENCE: 19	CONFERENCE: 4	CONFERENCE: 5
SUBTOTAL: 32	SUBTOTAL: 21	SUBTOTAL: 5
GENERAL SUM: 58 PAPERS +		

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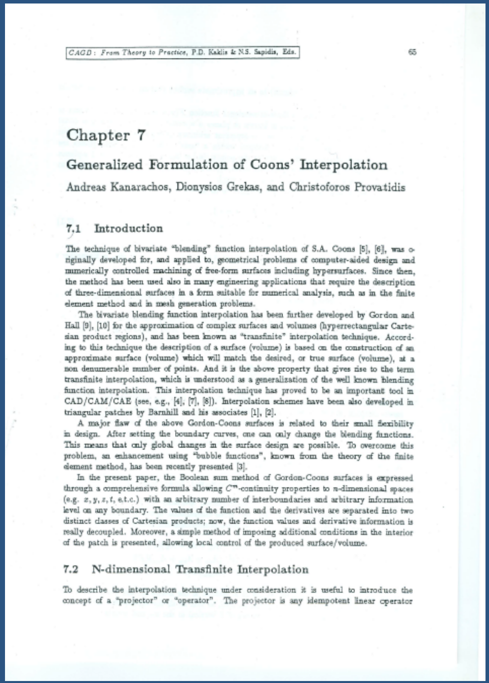
DOMAIN Decomposition


CAGD: From Theory to Practice, P.D. Kaklis & N.S. Sapidis, Eds. 65

Chapter 7

Generalized Formulation of Coons' Interpolation

Andreas Kanarachos, Dionysios Grekas, and Christoforos Provatidis

7.1 Introduction

The technique of bivariate "blending" function interpolation of S.A. Coons [5], [6], was originally developed for, and applied to, geometrical problems of computer-aided design and numerically controlled machining of free-form surfaces including hypersurfaces. Since then, the method has been used also in many engineering applications that require the description of three-dimensional surfaces in a form suitable for numerical analysis, such as in the finite element method and in mesh generation problems.

The bivariate blending function interpolation has been further developed by Gordon and Hall [9], [10] for the approximation of complex surfaces and volumes (hyperrectangular Cartesian product regions), and has been known as "transfinite" interpolation technique. According to this technique the description of a surface (volume) is based on the construction of an approximate surface (volume) which will match the desired, or true surface (volume), at a non denumerable number of points. And it is the above property that gives rise to the term transfinite interpolation, which is understood as a generalization of the well known blending function interpolation. This interpolation technique has proved to be an important tool in CAD/CAM/CAE (see, e.g., [4], [7], [8]). Interpolation schemes have been also developed in triangular patches by Barnhill and his associates [1], [2].

A major flaw of the above Gordon-Coons surfaces is related to their small flexibility in design. After setting the boundary curves, one can only change the blending functions. This means that only global changes in the surface design are possible. To overcome this problem, an enhancement using "bubble functions", known from the theory of the finite element method, has been recently presented [3].

In the present paper, the Boolean sum method of Gordon-Coons surfaces is expressed through a comprehensive formula allowing C^m-continuity properties to n-dimensional spaces (e.g. x, y, z, t, e.t.c.) with an arbitrary number of interboundaries and arbitrary information level on any boundary. The values of the function and the derivatives are separated into two distinct classes of Cartesian products; now, the function values and derivative information is really decoupled. Moreover, a simple method of imposing additional conditions in the interior of the patch is presented, allowing local control of the produced surface/volume.

7.2 N-dimensional Transfinite Interpolation

To describe the interpolation technique under consideration it is useful to introduce the concept of a "projector" or "operator". The projector is any idempotent linear operator

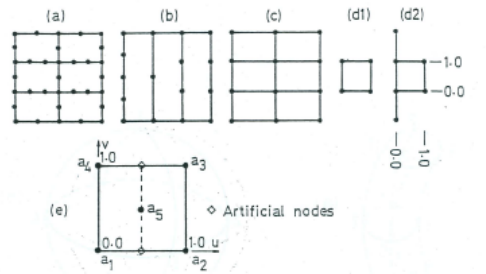


Figure 7.2: Characteristic cases of Coons interpolation: (a) full formula, (b) blending in u-direction, (c) Cartesian product, (d1) patch to match with c-patch, (d2) modification of d1-patch, (e) artificial nodes.

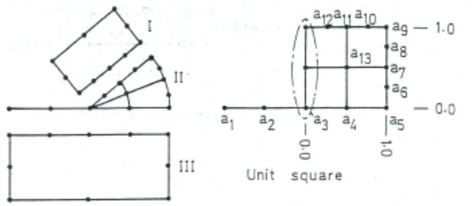
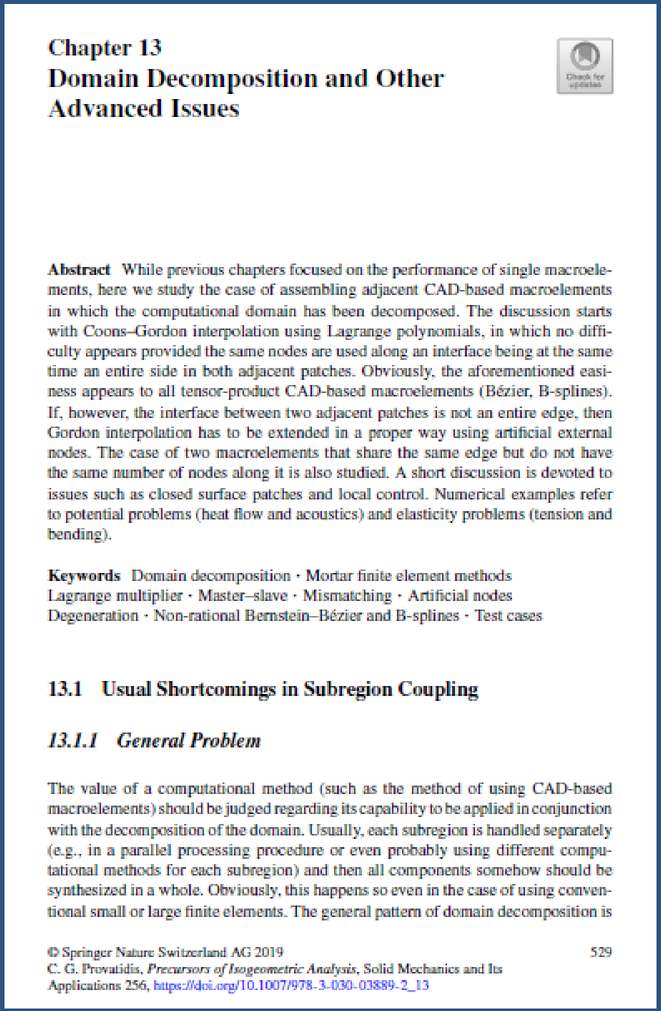


Figure 7.3: Matching non-similar patches (I) and (III).

1995: Workshop to the honor of G. Farin
(similarities with T-spline)


Chapter 13
Domain Decomposition and Other Advanced Issues

Abstract While previous chapters focused on the performance of single macroelements, here we study the case of assembling adjacent CAD-based macroelements in which the computational domain has been decomposed. The discussion starts with Coons–Gordon interpolation using Lagrange polynomials, in which no difficulty appears provided the same nodes are used along an interface being at the same time an entire side in both adjacent patches. Obviously, the aforementioned easiness appears to all tensor-product CAD-based macroelements (Bézier, B-splines). If, however, the interface between two adjacent patches is not an entire edge, then Gordon interpolation has to be extended in a proper way using artificial external nodes. The case of two macroelements that share the same edge but do not have the same number of nodes along it is also studied. A short discussion is devoted to issues such as closed surface patches and local control. Numerical examples refer to potential problems (heat flow and acoustics) and elasticity problems (tension and bending).

Keywords Domain decomposition · Mortar finite element methods · Lagrange multiplier · Master–slave · Mismatching · Artificial nodes · Degeneration · Non-rational Bernstein–Bézier and B-splines · Test cases

13.1 Usual Shortcomings in Subregion Coupling

13.1.1 General Problem

The value of a computational method (such as the method of using CAD-based macroelements) should be judged regarding its capability to be applied in conjunction with the decomposition of the domain. Usually, each subregion is handled separately (e.g., in a parallel processing procedure or even probably using different computational methods for each subregion) and then all components somehow should be synthesized in a whole. Obviously, this happens so even in the case of using conventional small or large finite elements. The general pattern of domain decomposition is

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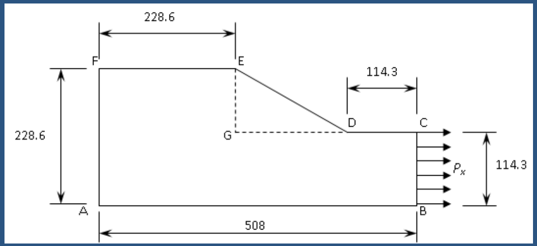
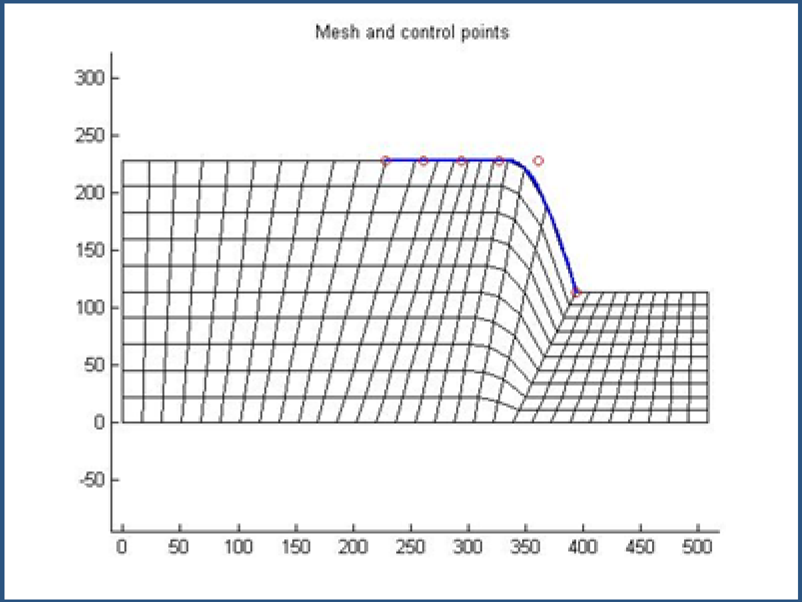
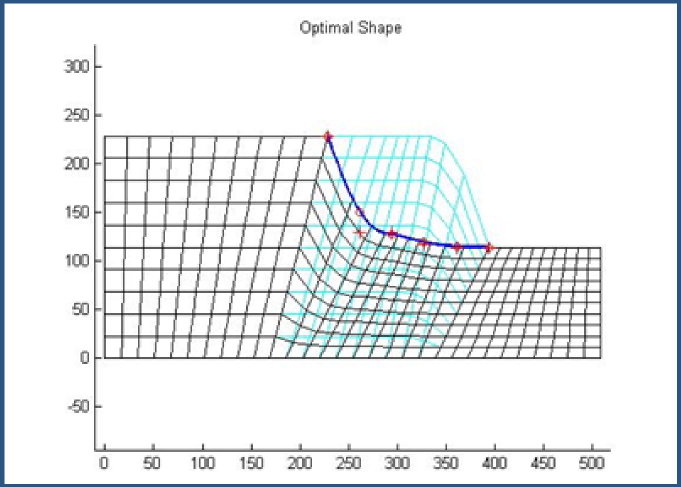
Provatidis, Springer, 2019

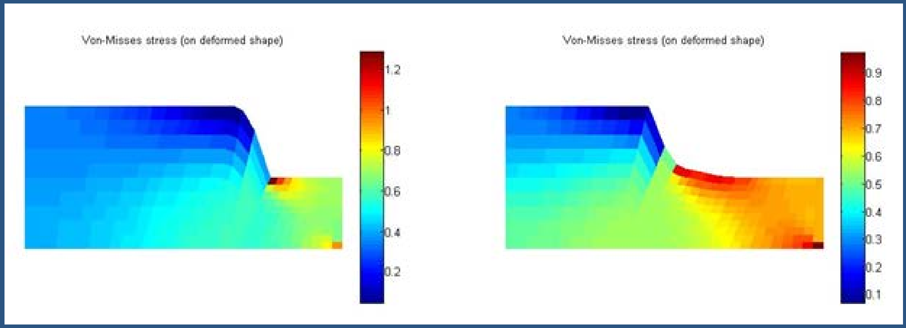
Shape optimization

B-spline tensor product









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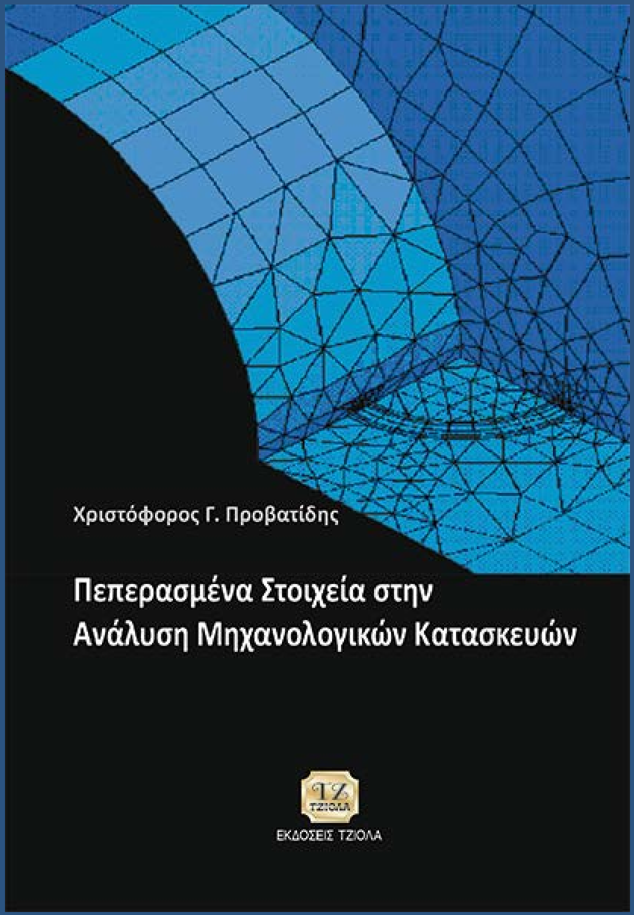

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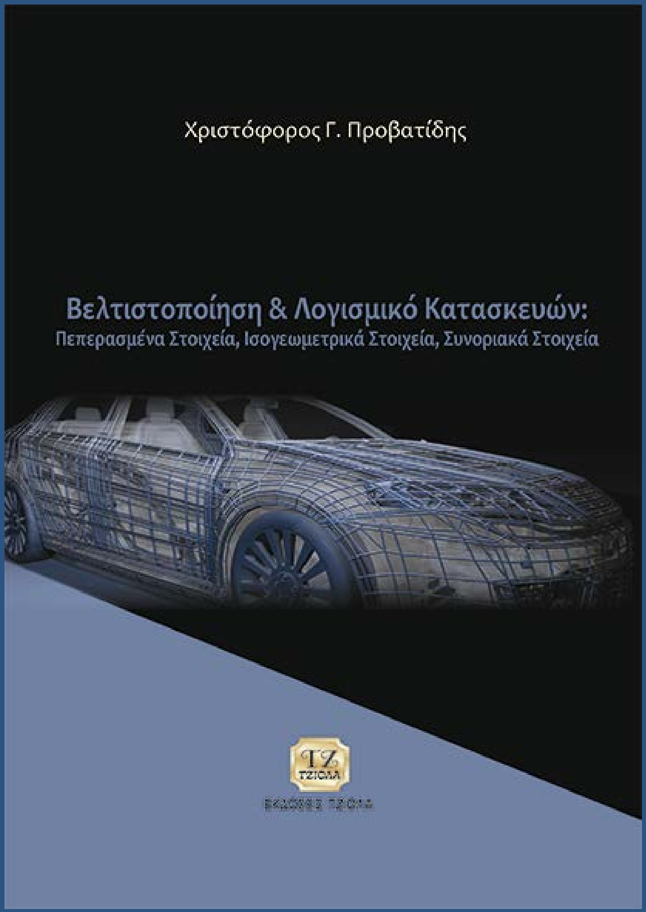
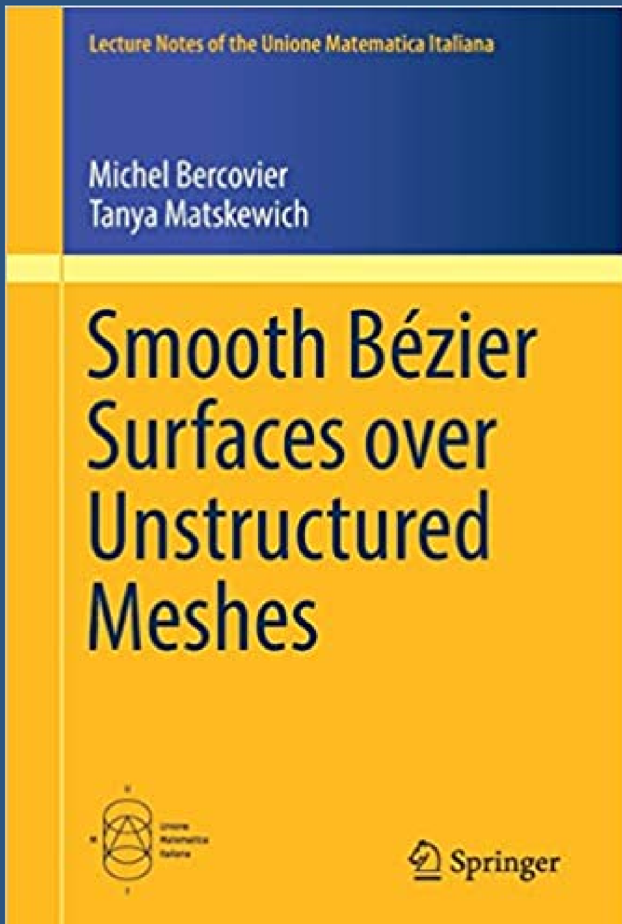


960 pp.

Bézier related Book



2017: forwarded by TJR. Hughes

Bercovier M., *Design of a Mesh Generator as a EUCLID Application*, Matra-Datavision report, Orsay, 1986

Burn JM., Bercovier M., *Solid Modeling Based CAD Systems Lead to the Integration of Finite Element Modeling*. First Wld Conference on Computational Mechanics, Austin, Texas, Sept. 1986

JM Brun: developer of EUCLID https://en.wikipedia.org/wiki/Euclid_(computer_program)

Covers the period about 2000

CONCLUSIONS (1/4)

- Five CAD-based interpolation formulas, established well before NURBS, have become the vehicle to develop higher-order FEM macro-elements of high accuracy.
- The above macroelements are applicable in conjunction with ***Galerkin-Ritz*** , Global ***Collocation*** Method, and ***BEM***.
- The whole procedure is rather automatic.
- We were introduced in 1982 but we published with delay
- Approx. eight Diploma Works-MSc Theses (conducted from time-to-time)
- **GALERKIN-RITZ** "Pre-IGA" was published first in 1989.
- **BEM**-"Pre-IGA" was developed in 1992-1993, documented in a few official student Theses, but was published in early 2000.
- **COLLOCATION**-"Pre-IGA" was published in 2006 and then in 2008.

CONCLUSIONS (2/4)

- Classical Serendipity elements are COONS-elements using Lagrange interpolation (trial functions) per side.
- Classical Tensor-Lagrange elements are GORDON's-transfinite elements using Lagrange blending and Lagrange trial functions.
- Transfinite elements "live" between Serendipity and Lagrangian elements.
- Tensor-product (non-rational) Bézierian elements are mathematically equivalent with Lagrangian elements.
- CORDON and COONS are applicable in conjunction with several blending and trial functions, such as cardinal B-splines, cosine-like, etc.

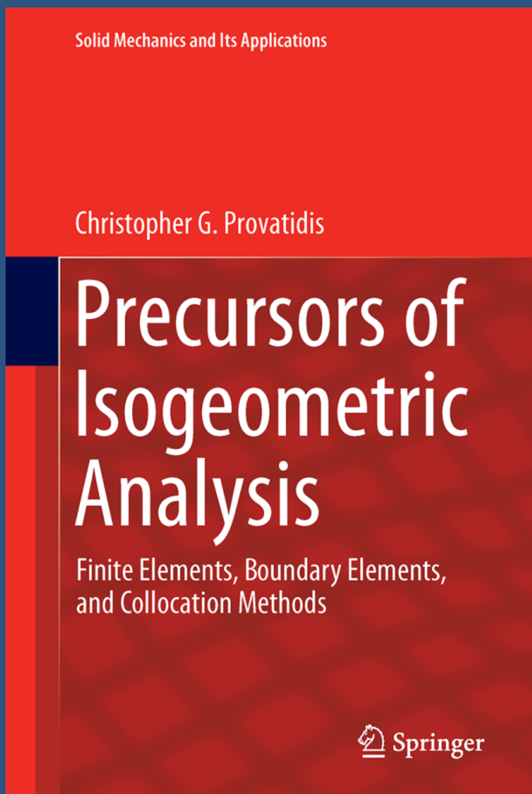
CONCLUSIONS (3/4)

- Tensor-product B-spline and NURBS are simple extensions of our older transfinite elements.
- Domain-decomposition and remeshing schemes have been somehow studied but not to a sufficient extent.
- For example, we have implemented ***transfinite techniques*** (to be published) to deal with **hanging nodes** and also, following Bernd Simeon’s (Jonathan Jahnke’s programming style for 1D THB-Spline-Finite Element), we saw that for 2D problems were solved using G+Smo facilities in C++)
- In-house or a widely accepted free CAD-tool (G+Smo + ParaView + ?)?
- C++ or Python?

CONCLUSIONS (4/4)

- Overall in pre-IGA: ~60 papers (30 journals, 30 conferences), about 15% of team’s resumé.
- Total fund: ***10,000 Euros*** only (in 2014-2016)!
- In the lack of funding, since 1998, seven-to-eight PhD candidates discontinued their theses (NL C-elements, Bézier-elements, B-splines, NURBS)

Summary & Details



14.3.2 Author's "Odyssey" and "Tutankhamun's Curse"

Most of you are surely aware about the meaning of "Odyssey" (one of two major ancient Greek epic poems attributed to Homer concerning the adventures of Odysseus to return at home after the Trojan War). On the other point of view, the reader may be also aware about the meaning of "Tutankhamun's Curse". Pharaoh's tomb was opened on November 29, 1922. Everyone who was present at the tomb's opening died after a short period. The same is hypothesized to have occurred to the grave robbers (https://en.wikipedia.org/wiki/Curse_of_the_pharaohs).

Sometimes I have the feeling that my attempt at NTUA to develop and promote the idea of CAD-based macroelements was a personal Odyssey in the course toward Ithaca, as will be exposed below. In more details, it should be reported that any student who undertook to deal with the subject of CAD-based macroelements "failed". Not even the first Ph.D. student did *not* complete and did not defend his thesis (officially started in January 1985) because in 1990 he succeeded as an owner of a private company in mechanical engineering, but also the second one (D. G.) stopped very

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