ANALYTICAL MODEL OF A PROMISING NOVEL MECHANISM

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SUMMARY

This paper presents the concept of a novel mechanism that achieves to produce a closed ∞-shaped curve in the three-dimensional space, along which two or more concentrated masses continuously move. In more details, the aforementioned path lies along the boundary (surface) of a sphere, thus possessing some remarkable properties. Besides, other configurations with different relationships between the angular velocities of the rolling components are discussed and relevant numerical results of the simulation are presented. In general, the findings of this work depict that this concept could inspire future designs of many rotating machines for either earth (energy save), marine, air or space applications.

1. INTRODUCTION

This preliminary study concerns the primitive mechanism shown in Figure 1. The mechanism consists of a conventional planetary system (spin gears S1 and S2, planet gears P1 and P2), in which concentrated masses, i.e. mass (a) and mass (b), are attached perpendicularly to the shafts of the spin gears S1 and S2, respectively.

Briefly, a motor drives the planet gear (P1) thus offering power transmission through P1-S1 towards the mass (a). Similarly, the rest half of the power produced by the motor is transmitted through P1-S2 towards the other mass (b).

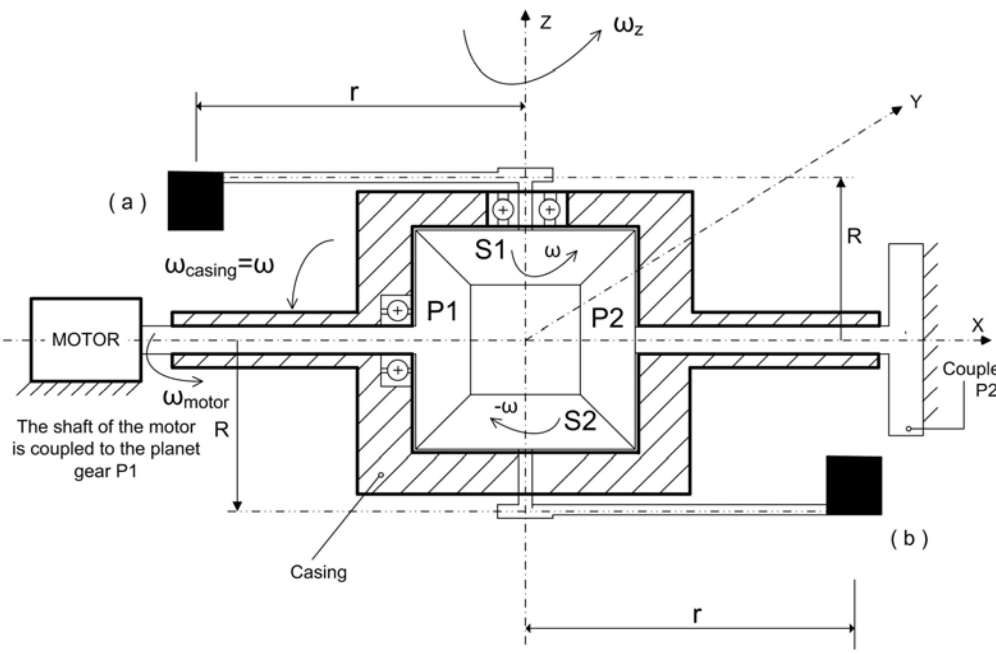

Figure 1: Sketch of the mechanism at the initial time instant ($t = 0$)

A characteristic of this mechanism is that the second planet gear, P2, is fixed thus causing rolling of the spin gears S1 and S2 on P2. Obviously, the rotation of the planet gear P1 enforces the spin gear S1 to rotate about its local axis (initially coinciding with the global *z*-axis) and also enforces the casing to rotate around *x*-axis. When assuming the same diameters of the four gears (P1, P2, S1 and S2), due to the aforementioned rolling at the interface between P2 and S1:

- the spin gear S1 has an angular velocity ω that is half that of the motor ($\omega = \omega_{motor}/2$)
- the spin gear S2 has the same angular velocity but of opposite sign, $-\omega$
- the casing rotates with the same angular velocity, ω.

Regarding the sign of the angles, the convention is that positive angles are considered those formatted as (OX,OY), (OY,OZ) and (OZ,OX).

The characteristic dimensions of the mechanism are (Figure 1):

- the radius *r* of the level where the masses are attached and
- the radius *R* of the casing; more accurately it is the distance between the centroids of the masses (a) and (b)

As shown in Figure 1, the initial coordinates of the masses, at time *t* = 0, are:

- Mass (a): (*x*, *y*, *z*) = (-*r*, 0, +*R*)
- Mass (b): (*x*, *y*, *z*) = (+*r*, 0, -*R*)

2. EQUATIONS OF MOTION

The description of section 1 reveals that each spin gear, S1 and S2, undertakes two simultaneous rotations, both of the same angular velocity, ω. Instead of superimposing the corresponding inertial forces, which procedure is amenable to mistakes due to inattention, we prefer to express the global Cartesian co-ordinates (inertial system) as a function of the time *t*.

Superimposing the two abovementioned rotations in the usual way [1], it can be easily found that the path of the mass (a) with respect to the global inertial Cartesian co-ordinate system O*xyz*, are finally given by:

$$\begin{aligned} x_a(t) &= -r\cos\omega t \\ y_a(t) &= -\left(r\sin\omega t\cos\omega t + R\sin\omega t\right) \\ z_a(t) &= -r\sin^2\omega t + R\cos\omega t \end{aligned} \tag{1}$$

Furthermore, the corresponding velocities are given as:

$$\begin{aligned} \dot{x}_a(t) &= \omega r\sin\omega_1 t \\ \dot{y}_a(t) &= -\left(\omega r\cos 2\omega t + \omega R\cos\omega t\right), \\ \dot{z}_a(t) &= -\left(r\omega\sin 2\omega t + \omega R\,\mathrm{sins}\,\omega t\right) \end{aligned} \tag{2}$$

while the corresponding accelerations are given as:

$$\begin{aligned} \ddot{x}_a(t) &= \omega^2 r\cos\omega t \\ \ddot{y}_a(t) &= 2\omega^2 r\sin 2\omega t + \omega^2 R\sin 2t \\ \ddot{z}_a(t) &= -\left(2\omega^2 r\cos 2\omega t + \omega^2 R\cos\omega t\right) \end{aligned} \tag{3}$$

In a similar way, the path of the mass (b), as well as the velocity and accelerations components, is given by:

$$\begin{aligned} x_b(t) &= r\cos\omega t \\ y_b(t) &= -r\sin\omega t\cos\omega t + R\sin\omega t \\ z_b(t) &= -\left(r\sin^2\omega t + R\cos\omega t\right) \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{x}_b(t) &= -\omega^2 r\cos\omega t \\ \dot{y}_b(t) &= -\omega r\cos 2\omega t + \omega R\cos\omega t \\ \dot{z}_b(t) &= -\omega r\sin 2\omega t + \omega R\sin\omega t \end{aligned} \tag{5}$$

$$\begin{aligned} \ddot{x}_b(t) &= -\omega^2 r\cos\omega t \\ \ddot{y}_b(t) &= 2\omega^2 r\sin 2\omega t - \omega^2 R\sin\omega t \\ \ddot{z}_b(t) &= -\left(2\omega^2 r\cos 2\omega t - \omega^2 R\cos\omega t\right) \end{aligned} \tag{6}$$

3. GEOMETRIC PROPERTIES OF THE PATH

Concerning the properties of the path on which the masses (a) and (b) move, using Eq(1) and Eq(4), it can be proven that:

1. Both masses move on the *same* path, and each of them gives away to the other. For example, when the spin gear S1 rotates by 90 degrees ($\omega t = \pi/2$), the co-ordinates of the mass (a) become (x, y, z) = (+r, 0, -R), which means that it takes the initial position of (b) shown in Figure 1. In a similar way, the co-ordinates of the mass (b) become (x, y, z) = (-r, 0, +R), which means that it takes the initial position of (a) shown in Figure 1. The same happens after 90 degrees of further rotation of the casing around x-axis (and simultaneous rotation of S1 and S2 by also 90 degrees), and so on.

2. All points of the abovementioned path belong to a sphere of radius, i.e.:

$$x_a^2 + y_a^2 + z_a^2 = x_b^2 + y_b^2 + z_b^2 = r_{sphere}^2, \text{ with } r_{sphere} = \sqrt{r^2 + R^2} \tag{7}$$

3. The point, I, at which the patch intersects itself, is found at the place:

$$x_{\text{intersect}} = +R,\ \ y_{\text{intersect}} = 0,\ \ z_{\text{intersect}} = -r \tag{8}$$

It is worth-mentioning that in the hypothetical case that R = 0, the intersection I lies along the z-axis ($x_{\text{intersect}} = 0,\ y_{\text{intersect}} = 0$).

4. The co-ordinates of the centroid of the couple (a,b) are:

$$\begin{aligned} x_{cm}(t) &= \frac{1}{2}\left[x_a(t) + x_b(t)\right] \equiv 0 \\ y_{cm}(t) &= \frac{1}{2}\left[y_a(t) + y_b(t)\right] = -r\sin\omega t\cos\omega t = -\frac{1}{2}r\sin 2\omega t \\ z_{cm}(t) &= \frac{1}{2}\left[z_a(t) + z_b(t)\right] = -r\sin^2\omega t \end{aligned} \tag{9}$$

Obviously, since it holds that:

$$x_{cm}^2(t)+y_{cm}^2(t)+\left[z_{cm}(t)+\frac{r}{2}\right]^2=\left(\frac{r}{2}\right)^2, \tag{10}$$

it is evident that the centroid of the two masses moves on the yz-plane along the circumference of a circle of radius $\left(\frac{r}{2}\right)$, centered at the point $\left(0,0,-\frac{r}{2}\right)$.

4. INDUCED INERTIAL FORCES

Using Eq(1)-Eq(6) in combination with Newton's Second Law [2], the inertial force components at each makeweight are given by:

$$\begin{aligned} F_{a,x} &= -m\ddot{x}_a(t) = -m\omega^2 r\cos\omega t \\ F_{a,y} &= -m\ddot{y}_a(t) = -m\left(2\omega^2 r\sin 2\omega t+\omega^2 R\sin 2t\right) \\ F_{a,z} &= -m\ddot{z}_a(t) = +m\left(2\omega^2 r\cos 2\omega t+\omega^2 R\cos\omega t\right) \end{aligned} \tag{11}$$

and

$$\begin{aligned} F_{b,x} &= -m\ddot{x}_b(t) = +m\omega^2 r\cos\omega t \\ F_{b,y} &= -m\ddot{y}_b(t) = -m\left(2\omega^2 r\sin 2\omega t-\omega^2 R\sin\omega t\right) \\ F_{b,z} &= -m\ddot{z}_b(t) = +m\left(2\omega^2 r\cos 2\omega t-\omega^2 R\cos\omega t\right) \end{aligned} \tag{12}$$

Therefore, the resultant inertial force components in the entire mechanism are given by:

$$\begin{aligned} F_x &= F_{a,x}+F_{b,x}\equiv 0 \\ F_y &= F_{a,y}+F_{b,y} = -4m\omega^2 r\sin 2\omega t \\ F_z &= F_{a,z}+F_{b,z} = +4m\omega^2 r\cos 2\omega t \end{aligned} \tag{13}$$

We notice that while in the x-direction there is no resultant force, in contrast, harmonic components exist in the y- and z-directions. Most interesting, the vertical z-component is of amplitude $4m\omega^2 r$ and appears a maximum upward value at $\omega t=0,\pi,2\pi,3\pi,\ldots$, while it appears the *same* downward value at $\omega t=\pi/2,3\pi/2,5\pi/2,\ldots$

5. MOMENTS AND POWER

With respect to the x-axis the induced moments of the inertial forces at the makeweights are found as:

$$M_x = M_{a,x}+M_{b,x} = -2m\omega^2 r^2\sin 2\omega t \tag{14}$$

Also, the power spent by the electric motor is calculated using the classical consideration of the inertial forces through the formula

$$P=-m\left(\ddot{x}_a\dot{x}_a+\ddot{y}_a\dot{y}_a+\ddot{z}_a\dot{z}_a+\ddot{x}_b\dot{x}_b+\ddot{y}_b\dot{y}_b+\ddot{z}_b\dot{z}_b\right) \tag{15}$$

which, in virtue of Eqs(1-6), leads to:

$$P(t)=-m\omega^3 r^2\sin 2\omega t \tag{16}$$

6. KINETIC AND POTENTIAL ENERGY

Kinetic, $E_{kinetic}$, and potential, $E_{potential}$, energy of the couple of masses (a) and (b) is given by:

$$E_{kinetic} = \frac{1}{2} m \left[\left(\dot{x}_a^2 + \dot{y}_a^2 + \dot{z}_a^2 \right) + \left(\dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2 \right) \right] = \\ = m\omega^2 \left[R^2 + r^2 \left(1 + \sin^2 \omega t \right) \right] \tag{17}$$

and

$$E_{potential} = -2mgr \sin^2 \omega t \tag{18}$$

7. WORK OF INERTIAL FORCES

Integrating Eq(16), the work spent from $t = 0$ until to every other time t is given by:

$$W(t) = -\int_0^t P(\tau)\, d\tau = 2m\omega^2 r^2 \sin^2 \omega t \tag{19}$$

One can easily validate that the work given by Eq(19) equals to the change of the kinetic energy, as always happens. The sign (-) in Eq(19) was put in order to depict that the real resultant is $m_a \ddot{\vec{r}}$, while the inertial force was taken as $-m_a \ddot{\vec{r}}$.

8. APPLICATIONS

8.1 Typical paths

For time instances so that $0 \le \omega t \le \pi/2$, a typical path followed by the masses (a) and (b) is illustrated in Figure 2. One can notice that the path is a continuous ∞-shaped curve of unequal loops. Clearly, when the casing and the spin gear rotate by 90 degrees, the mass (a) completes the blue line while the mass (b) completes the red line. For $\pi/2 \le \omega t \le \pi$, the mass (a) follows exactly the path already completed by the mass (b), while the mass (b) follows exactly the path already completed by the mass (a), and so on!

In other words, the masses (a) and (b) *mutually offer space to each other*.

In order to have a better feeling of the path, its *asymmetry* and its unique *intersection* point I, as well as the circle followed by the centroid, are shown in Figure 3.

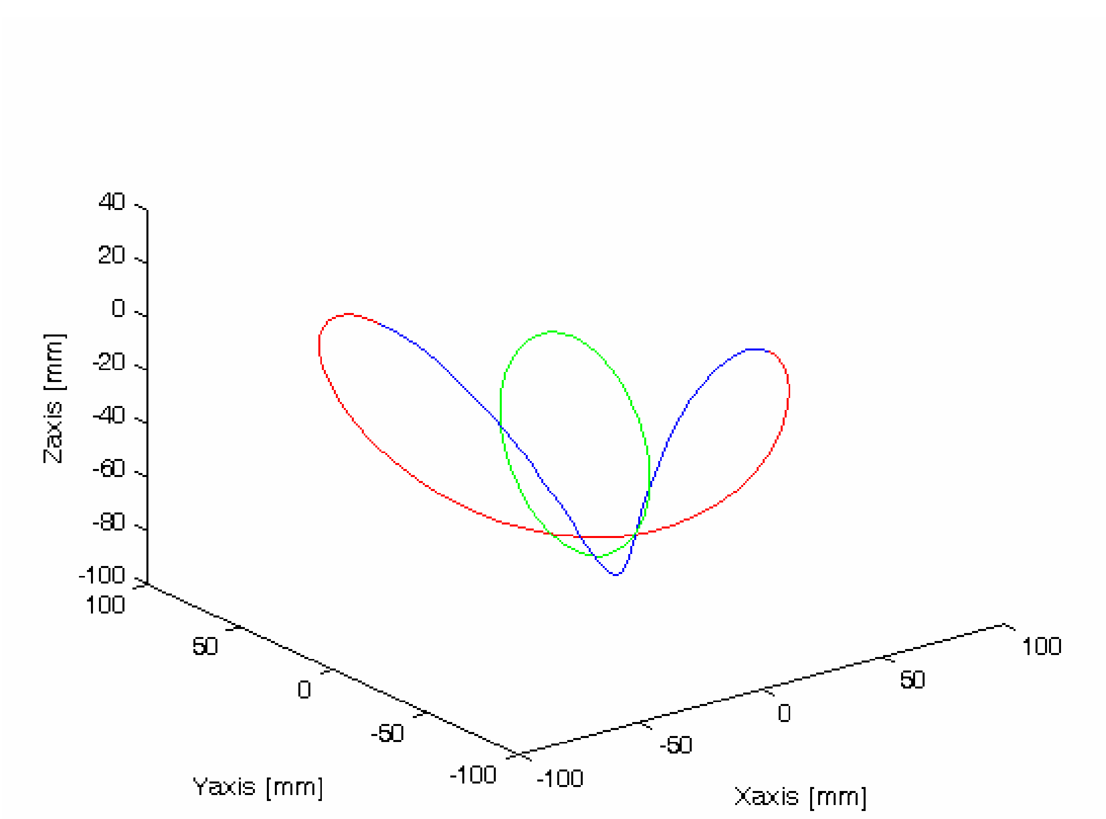

Figure 2: Patch of the masses and their centroid (r =80mm, R=25mm). The blue line corresponds to the mass (a), the red one to the mass (b), while the green corresponds to the centroid.

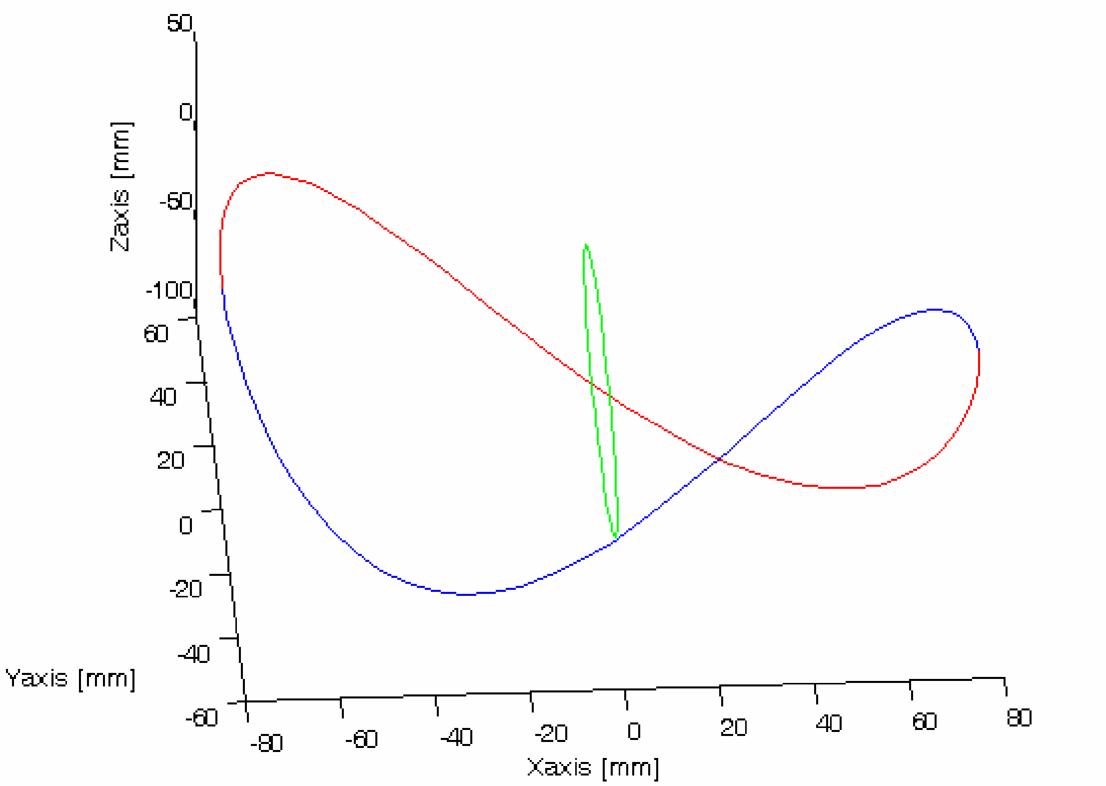

Figure 3: A different perspective view for the conditions of Figure 2.

8.2 Simulation of forces and power

The resultant of the vertical force components are shown in Figure 4, where the *x*-, *y*- and *z*-components are illustrated in blue, red and green colour, respectively. The *z*-force component (F_z), which is in green colour, is of major importance as it is related to the ability of the mechanism to move upwards. One can notice in Figure 4 an alternating time history, thus causing no upward impulse.

In more details, when the masses are found exactly as in Figure 4 (at initial time: t = 0, Angle = 0 degrees), the resultant vertical force is *positive* (upwards) and it is very similar for both masses (identical only when R = 0), while the ***same*** value but of *negative* sign appears at the angle of 90 degrees.

Moreover, concerning the power transmitted generated by the motor and then transmitted to the masses is shown in Figure 5, in which the friction has been neglected.

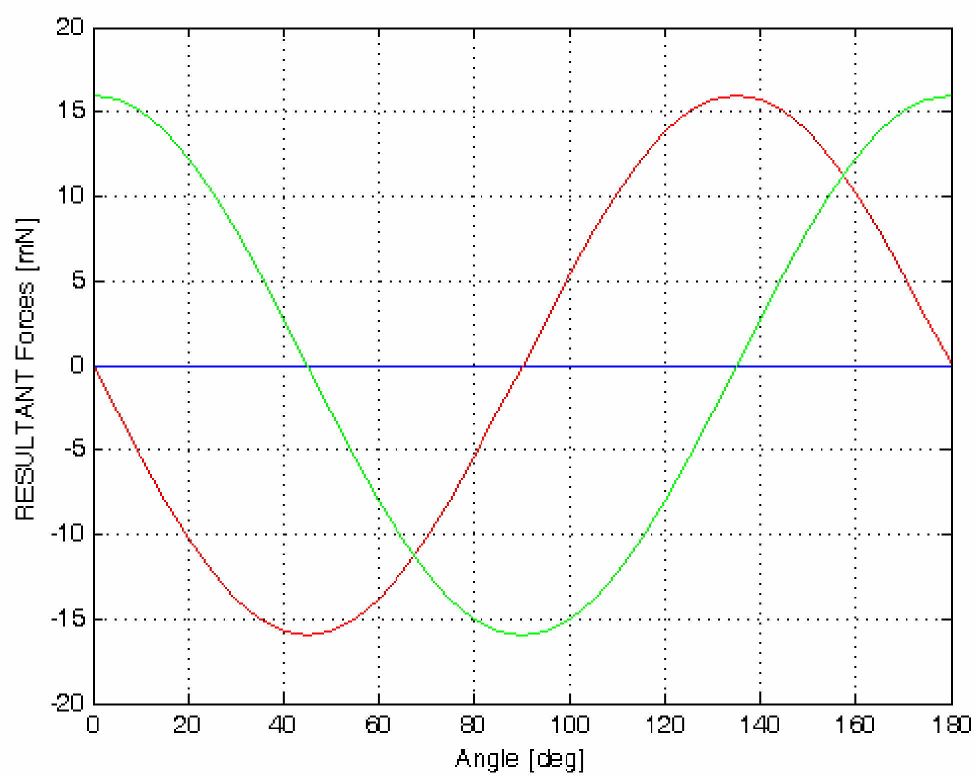

Figure 4: Centrifugal force components (blue: Fx, red: Fy, green: Fz). Data: (r =80mm, R=25mm).

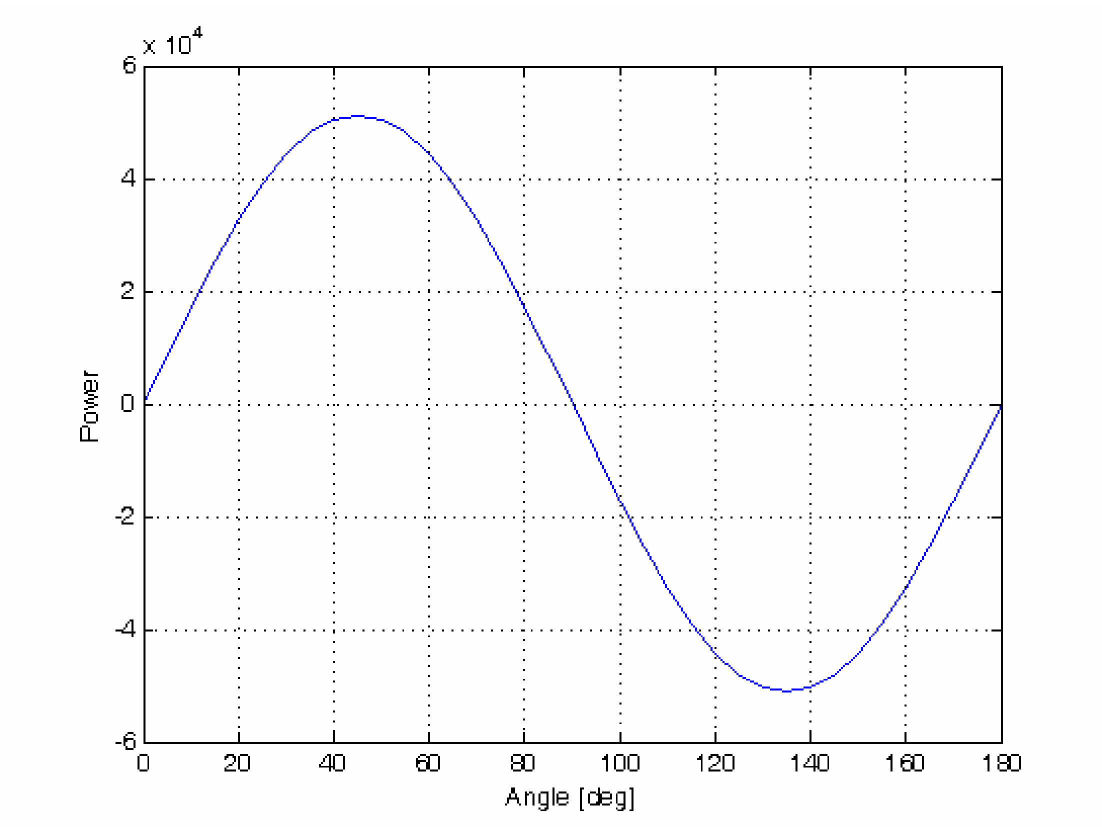

Figure 5: Time history of the motor power (r =80mm, R=25mm).

8.3 Modification of the mechanism

Using a different configuration, it could be possible to modify the angular velocity of the casing, so that it obtains, for example, half the value of the spin gear S1 ($\omega_{casing}=\omega_{spin\ gear}/2$). In such as case, the corresponding paths of the two masses ***do not coincide*** but they are quite distinct as shown in Figure 6. In more details, the masses (a) and (b) follow the blue and red lines, respectively, while their centroid forms an exotic shape of a *four-leaf* (tetrafilon) flower!

It is also worth-mentioning that in the abovementioned case, not one (I) but two intersections of the path appear.

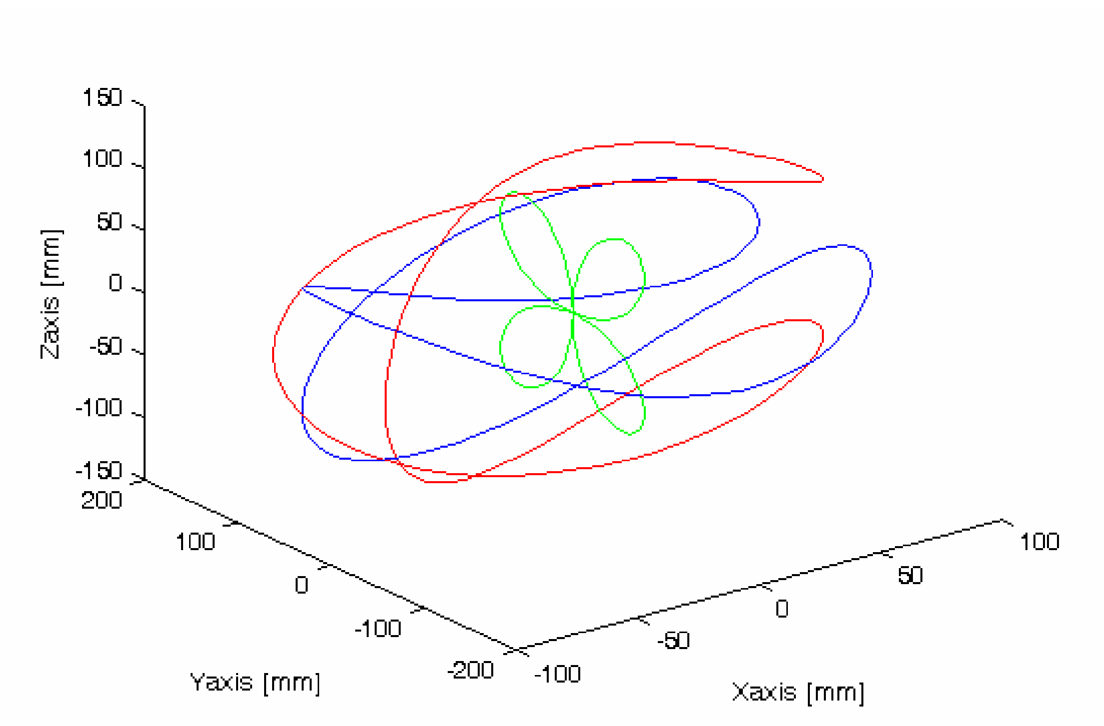

Figure 6: Paths of the masses. The blue line corresponds to the mass (a), the red to the mass (b) while the green to their centroid. (r =100mm, R=60mm, $\omega_{casing}=\omega_{spin\ gear}/2$)

A more close investigation reveals that the motion sequence of the centroid along the four leaves (tetrafilon) is according to Figure 7 (1→2→3→4→5→6→7→8→9≡1).

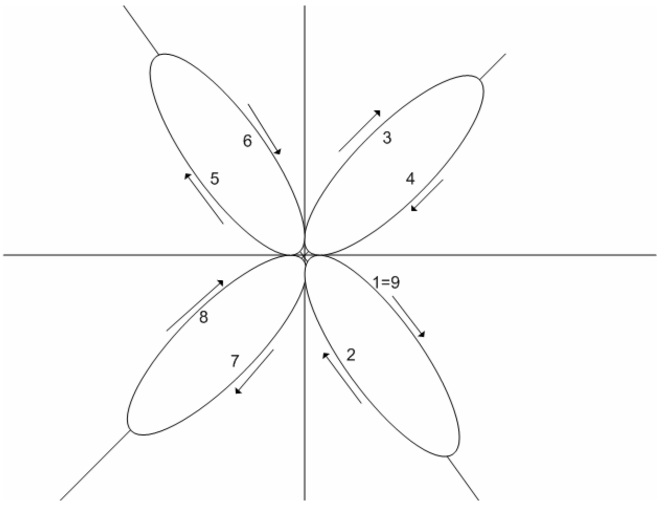

Figure 7: Motion sequence of the centroid along the four leaves (shown in Figure 6)

Remark: Preserving the diameter of the casing and increasing the radius of the concentrated mass, for example, choosing r = 500mm, R = 60mm, the paths become as those shown in Figure 8. In other words, the longer the radius r becomes the larger the extreme loops become, by also increase of the four-leaf shape.

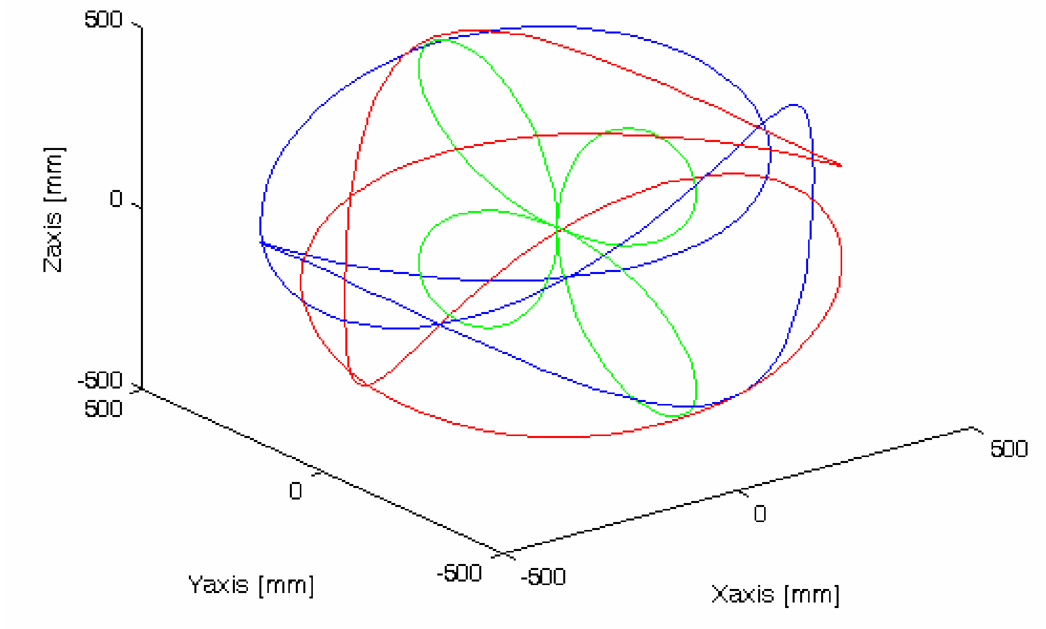

Figure 8: Paths of the makeweights and their centroid (r = 500mm, R = 60mm).

9. DISCUSSION – CONCLUSION - QUESTIONS

The principle of a novel mechanism has been presented in full detail. Not only its operation but also closed-form analytical expressions of kinematics and dynamics have been reported.

For the sake of briefness, in this preliminary report the case $\omega_z \neq 0$ has been omitted.

Instead of a usual conclusion, we would prefer to pose some questions:

- What would happen if, besides the abovementioned two rotations (ω), the entire mechanism **rotates** at a **high** angular velocity ω_z about the z-axis?
- Do the three force components remain constant when the ω_z is introduced?
- Which of the three force components are altered by ω_z?
- Do antigravity 'components' appear in the mechanism?
- Does the sign of $\pm\omega_z$ play any role?
- Are other peculiar inertial phenomena anticipated?
- How could this primitive mechanism be improved?

10. Acknowledgement

I acknowledge the inventor, Mr. Theodore Tsiriggakis [3], with whom I have cooperated since 1981, for the fruitful discussions we had since then about the concept elaborated in this paper. Particularly, I thank him, as well as his son Mr. Vassilis Tsiriggakis, for our continuous discussions we had during the last two years where the novel mechanism was developed in a more systematic way and was manufactured by them in some options, one of which is illustrated in Figure 1.

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and more particularly:

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