

**SYMPOSIUM INTERNAZIONALE SUL CONTRIBUTO DELLA FOTOGRAMMETRIA
ALLA DOCUMENTAZIONE DEI CENTRI STORICI E DEI MONUMENTI**

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A T T I

Università degli Studi di Firenze
Dipartimento di Ingegneria Civile

ANALYTICAL RESTITUTION WITH STEREOCORD G-2

C.C.Ioannidis, C.A.Potsiou, E.E.Stambouloglou, J.G.Badekas

Laboratory of photogrammetry
National Technical University of Athens, Greece

Summary

The analytical restitution of a stereo-photo-pair is a method which has been developed and applied as a consequence of the recent development of computers. Using the Stereocord G-2, Direc 1, HP9845 S of the N.T.U. Photogrammetric Laboratory, a software for analytical restitution and plotting has been devised, written and tested. Some architectural terrestrial applications are presented and their results are discussed.

Résumé

La restitution analytique d'un couple de clichés est une méthode développée et appliquée récemment beaucoup, comme une suite normale du développement des Ordinateurs. En utilisant le système Stereocord G2+Direc 1+HP9845S du Laboratoire de Photogrammétrie de l'UTN, on a créé et contrôlé un "software", pour la restitution analytique et la représentation graphique. Quelques applications de Photogrammétrie Architecturale sont présentées, tandis qu'on discute leurs résultats.

Zusammenfassung

Die analytische stereophotogrammetrische Auswertung wurde, während der letzten Jahre, in steigendem Masse und im Rahmen der Computerentwicklung entwickelt und angewendet. Mit Benutzung des Systems Stereocord G-2, Direc 1, HP9845 S hat man im Photogrammetrischen Institut der N.T.U. Programmen analytischen Auswertung und graphischen Darstellung vorbereitet, mit deren Hilfe Anwendungen im Bereich der terrestrischen Photogrammetrie stattgefunden haben. Die Ergebnisse dienen als Grundlage für eine allgemeine Wertung der Methoden.

1. The existing software for analytical restitution with the Zeiss Stereocord-G2.

Zeiss (Oberkochen) medium accuracy analytical system Stereocord-G2, has the ability to measure the x, y coordinates of the left-hand image and the P_x -parallax of the right-hand image of a stereo-pair. These measured values are recorded onto a DIREC-1 and then transmitted to a desktop computer for the necessary calculations. The manufacturer supplies the system with a set of programmes suiting Hewlett Packard 9810 and 9830 computers.

The calculation of coordinates of points (also associated distances, slopes, areas, etc.) observed stereoscopically on the Stereocord, can be computed using the programmes mentioned above.

The procedure is as follows:

- a) removal of κ_1, κ_2 rotations by appropriate placing of the photos.
- b) relative orientation and calculation of ϕ_1, ϕ_2 and ω_2 rotations.
- c) absolute orientation.
- d) calculation of object coordinates (X, Y, Z) of any observed point by space intersection.

The mathematical model is a simplified one and thus several approximations have been incorporated.

The Laboratory of Photogrammetry of the National Technical University of Athens has used Stereocord in conjunction with an HP 9845 S desktop computer, since this computer is used for assisting other photogrammetric instruments too. It was then necessary to modify the programmes given by Zeiss for HP 9845 S. This closer examination and the use of them for several projects (see § 3), showed the need and the possibility for improved programmes capable of overcoming certain deficiencies such as, the low accuracy due to the use of approximate formulas and the limitations of graphic plotting and print-out.

2. Modification and improvement of analytical restitution.

A special programme based on the approximate mathematical model, which had been used by the manufacturer, was written for the HP 9845 S and given the name "RECORD". The program performs first a relative orientation based on 10 monoscopic observations. Then for the absolute orientation 4 control points are observed stereoscopically.

After that, the coordinates of each stereoscopically observed point, are calculated by using the same approximate formulas. The results of an investigation of the mathematical model of this programme and the results of its practical applications are given in section 3.

In trying to avoid systematic errors due to the approximate formulas of the mathematical model of this particular programme, two new solutions were developed, which used "full" formulas. The first, allows only monoscopic observations on the Stereocord although the second has the flexibility of allowing stereoscopic

observations as well. Two corresponding programmes were written for HP 9845 S and named "RACOPL" (Relative-Absolute-Coplanarity) and "SDAOE" (Simultaneous Determination of All Orientation Elements). The programme which uses monoscopic observations is structured as follows:

- a) The five parameters of relative orientation ($\omega_2, \phi_2, \kappa_2, b_y, b_z$) are calculated by observing more than five points, first on the left-hand and then on the right-hand photo. Using the coplanarity condition as observation-equation a least square system of equations is formed. Its solution gives the unknown orientation parameters from which model coordinates are calculated.
- b) The absolute orientation is achieved by adjusting the model to the ground system, by applying a similarity transformation to at least three control points according to the observation equations:

$$\text{axis X: } X - X_0 - x = 0 \cdot d\Omega + z \cdot d\Phi + (-y) \cdot d\kappa + x \cdot d\mu + 1 \cdot dx_0 + 0 \cdot dy_0 + 0 \cdot dz_0$$

$$\text{axis Y: } Y - Y_0 - y = (-z) \cdot d\Omega + 0 \cdot d\Phi + x \cdot d\kappa + y \cdot d\mu + 0 \cdot dx_0 + 1 \cdot dy_0 + 0 \cdot dz_0$$

$$\text{axis Z: } Z - Z_0 - z = y \cdot d\Omega + (-x) \cdot d\Phi + 0 \cdot d\kappa + z \cdot d\mu + 0 \cdot dx_0 + 0 \cdot dy_0 + 1 \cdot dz_0$$

where x, y, z the model-coordinates

X, Y, Z the ground-coordinates

- c) Now, the ground-coordinates of each monoscopically observed point in both photos, are calculated using the collinearity equation.

With this programme, the question of low accuracy of "RECORD" due to the use of simplified equations was adequately answered. However, the lack of recording y-parallaxes (or P_y) onto a forth display on DIREC-1, deprived the system of the information of y-image-coordinate of the right-hand photo and consequently "RACOPL" of the ability to use a stereoscopic observation. As a result of this, plotting took much longer than it would otherwise since the observation of the homologue images of each point had to be done twice; one on the left-hand and one on the right-hand photo.

This deficiency was in fact overcome by the second approach adopted for the programme "SDAOE" which has the following characteristics:

- a) a space resection is solved using monoscopic observations on the images of at least 3 control points. A simultaneous computation of all 12 parameters of exterior orientation is performed by least squares using the collinearity equation.
- b) Ground-coordinates of each stereoscopically observed point are now computed using the collinearity equation. For each unknown observed point 3 equations are created one for each measured value of x^{left} , y^{left} and x^{right} with 3 unknowns, the ground-coordinates:

$$\alpha'_{17} \cdot dx + \alpha'_{18} \cdot dy + \alpha'_{19} \cdot dz = x^{\text{left}} - c \cdot \frac{m'_{11}(X' - X_{01}) + m'_{12}(Y' - Y_{01}) + m'_{13}(Z' - Z_{01})}{m'_{31}(X' - X_{01}) + m'_{32}(Y' - Y_{01}) + m'_{33}(Z' - Z_{01})}$$

$$\alpha'_{27} \cdot dX + \alpha'_{28} \cdot dY + \alpha'_{29} \cdot dZ = y^{\text{left}} - c \cdot \frac{m_{21}^1 (X' - X_{01}) + m_{22}^1 (Y' - Y_{01}) + m_{23}^1 (Z' - Z_{01})}{m_{31}^1 (X' - X_{01}) + m_{32}^1 (Y' - Y_{01}) + m_{33}^1 (Z' - Z_{01})}$$

$$\alpha''_{17} \cdot dX + \alpha''_{18} \cdot dY + \alpha''_{19} \cdot dZ = x^{\text{right}} - c \cdot \frac{m_{11}^r (X' - X_{02}) + m_{12}^r (Y' - Y_{02}) + m_{13}^r (Z' - Z_{02})}{m_{31}^r (X' - X_{02}) + m_{32}^r (Y' - Y_{02}) + m_{33}^r (Z' - Z_{02})}$$

where X' , Y' , Z' are the approximate values of the unknowns X , Y , Z respectively.

These approximate values are calculated by an initial solution where the unknown element is considered to be:

$$y^{\text{right}} = y^{\text{left}} + \frac{\sum_{i=1}^n (y^{\text{right}} - y^{\text{left}})_i}{\sum_{i=1}^n (i)}$$

where n = the number of control points used.

The development of the system for analytical plotting was considered as another area where some improvements could be made. The capability of the graphic display on the HP 9845 S, allowed the writing of a number of programmes for the graphic representation of the results.

In on-line plotting, special subroutines have been made for plotting, coding and storing the coordinates of the points which have been computed analytically. These subroutines can also be joined with "RECORD" and "SDAOE".

In off-line plotting, a programme by the name "REPLOTT" was made, which has the capability of replotting an initially plotted portion, either by "RECORD" or by "SDAOE", without any operator's assistance.

To perform a correction or a completion of a previous plotting, a programme by the name of "CORCOM" (Corrections-Completions), which runs in combination with "REPLOTT", was written. This programme consists of two main parts. The first, purges, by digitizing on the C.R.T. all the unwanted portions of the plotted object and stores them, while the second, makes and stores the necessary completions.

Finally, the manipulation and the classification of data on the tape cartridge is achieved by a new programme named "GREADY" ("Graphics are ready"), which runs before "REPLOTT". Like "REPLOTT", "GREADY" also runs without any operator's assistance.

The use of these three programmes for plotting, allows the storage of the object-coordinates determined analytically which might be useful for future use.

3. Applications and results of the analytical restitution programmes.

To calculate the systematic errors, which are due to the use of the approximate mathematical model, of "RECORD", a theoretical test field, of 25 points of known coordinates, was considered. Changing the heights of the field points and the rotations ω and ϕ of simulated images several stereopairs with different characteristics were obtained.

Basic data of this experiment were: camera constant $c=100$ mm, base to object distance ratio : $B = \frac{1}{\bar{Y}}$, image scale $\bar{Y} = 1,5$, 1:100. For the distribution of the 4 control points on the left-hand image see Fig. 1.

Ground-coordinates of the 21 check points were calculated by "RECORD" for each case. This way, a large number of diagrams for the observed discrepancies were plotted. From these diagrams it may be said that an increase of more than a few grads in rotations results in a considerable increase in the discrepancies of object-coordinates. Part of the results of the investigation are given in the Table 1,2 and 3 (page 10). Specifically the maximum values of discrepancies at the 21 check points are included.

Besides the systematic errors due to the simplified mathematical model, accidental errors due to the instrument, to the observer and to other sources, interfere as well. To calculate the scale of these errors, terrestrial stereopairs, close enough to the normal case in order to eliminate errors from the mathematical model, were used. The computed ground-coordinates of the check-points had the following errors (on image scale of about 1:100):

	ground (cm)	image (μm)
r.m.s. X	0,62	62
r.m.s. Y	0,33	33
r.m.s. Z	0.62	62

In the same way, a digital restitution of a stereopair of a neoclassical building in Pireus was undertaken to test the accuracy of the new programmes: "RACOPL" (for monoscopic observation) and "SDAOE" (for stereoscopic observation) with the "full" mathematical models.

This particular stereopair deviates from normal case since a general Ω -rotation of about 16^g exists. The results for image scale 1:120 are the following:

	Programme "RACOPL"		Programme "SDAOE"	
	Ground (cm)	Image (μm)	Ground (cm)	Image (μm)
r.m.s. X	1,49	124	0,55	46
r.m.s. Y	0,57	48	0,61	50
r.m.s. Z	0,48	40	0.42	35

It may be seen that the errors are smaller than 50 μm on image scale except for the error on the X-axis in programme "RACOPL".

In addition, the plotting capability of the points, by means of the graphic mode of the C.R.T. of 9845 S, was tested. For this purpose, the coordinates of a net of 72 points were measured on Stereocord. The distance between the points was 2 cm. This net had been plotted by means of the graphic mode of the HP 9845 S.

The standard error of all measurements, along x and y axes, was equal to 0.19 mm, which is the graphic accuracy of the HP 9845 S.

Finally, a detailed restitution and plotting of an Athenian neoclassical house was undertaken, on a scale of 1:20. The photogrammetric stereopair had been taken with a galileo B stereo-camera with a photo scale of about 1:100. The programme "RECORD" was used for its analytical restitution. This programme is suitable only for a normal case (such as this). A total of 18400 points were observed and used for the plotting.

In order to produce the plotting-image on the computer C.R.T. whose useful dimensions are 18.5x15 cm², the total area of the model was divided into 9 portions. Each one was displayed on the C.R.T. and printed on the thermal printer of the 9845 S. The complete output of this plotting, properly reduced is given in this paper (page 11).

The errors of the final plotting were calculated by checking the distances between the control points. The expected accuracy was a function of:

- errors due to the computation of coordinates by programme "RECORD" ($\sigma_R=63 \mu\text{m}$ on the photo scale).
- errors due to the graphic representation of the points using graphic mode ($\sigma_g=0.19 \text{ mm}$ on the plotting scale).
- errors due to the measuring procedure of the distances on the final plotting ($\sigma_\mu=0.125 \text{ mm}$ on the plotting scale).

So the total error is:

$$\sigma_S = \pm \frac{1}{S} \sqrt{\Delta X^2 (\sigma_{x_1}^2 + \sigma_{x_2}^2) + \Delta Y^2 (\sigma_{y_1}^2 + \sigma_{y_2}^2)} \approx \pm \sqrt{\sigma_x^2 + \sigma_y^2} \quad (S = \sqrt{\Delta X^2 + \Delta Y^2})$$

$$\sigma_x^2 = (\sigma_R)_x^2 + (\sigma_g)_x^2 + (\sigma_\mu)_x^2$$

where:

$$\sigma_y^2 = (\sigma_R)_y^2 + (\sigma_g)_y^2 + (\sigma_\mu)_y^2$$

$$: \sigma_x \approx \sigma_y$$

Finally, the expected and achieved accuracies derived from measurements taken from plottings both in scale 1:20 and 1:50 which was also produced off-line, are:

σ_S (mm)		
	expected errors	achieved errors
1:20	10.99	13.45
1:50	18.38	18.28

Their differences can be considered to be negligible.

4. Comparisons of methods and results.

By comparing the three detailed programmes, for a computation of coordinates of individual points, the following conclusions may be drawn: the method of mono or stereo-observation of points counts most in the whole function of each programme, because this determines the way of placing the photos, the computer time, the accuracy and, above all, the usefulness of each programme. The programme "RECORD" based on the mathematical model suggested by Zeiss, has the advantages of Stereoscopic observation, it requires a moderate working time and can be use for plotting. There exist some shortcomings, however. A special preparation of the photos is needed before their placing on the photo-carriage which dissipates almost all the advantage of moderate working time. The more a stereopair deviates from normal case, the less accuracy is achieved. This second characteristic drastically reduces the number of applications for "RECORD".

On the other hand, the programme "RACOPL" has all the necessary facilities enabling it to be used in any steropair orientation; assuming that all the points which have to be determined are either clearly distinguishable natural points or presignaled ones.

Admittedly, since only monoscopic observation can be made, no plotting can be achieved by "RACOPL".

No special preparation is needed for the placing of the photos which could contribute errors to the final results (in contradiction to "RECORD" where the special preparation affects the restitution's accuracy). It takes only a few minutes for orientation and the achieved accuracy reaches the maximum of Stereocord.

Finally, the programme "SDAOE" not only can be used for any

stereopair orientation, as "RACOPL", but it allows stereoscopic observation as well. The achieved accuracy is better than 50 μm at the image scale (section 3). The remaining probable sources of errors are:

- a) the observation system (diameter of the flying mark =100 μm , magnification system) and the measuring system of Stereocord G2.
- b) the inevitable use of photo paper-prints and
- c) the observer's experience and capability.

The Zeiss serial Stereocord system has no graphic ability. Yet, the programmes "REPLOTT", "CORCOM" and "GREADY" which are mentioned above make graphic representation of an analytically restituted object readily available.

The main characteristics of graphic restitutions are:

- Their easy realization due to the simplicity of the instruments and the easiness of using the programmes.
- The limited working time, resulting from the analytical methods which are used in combination with the speed of the computer.
- The great flexibility of the programmes.
- The off-line plotting capability, since the plotting is stored in a tape-cartridge.
- The ability of producing the output of the plotting at any required scale.

Finally, the accuracy that can be achieved in a graphic restitution depends on the programme used and on the scale of both photos and plotting; with regard to its applied application to the neo-classical house the accuracy is quite sufficient (approximately 1 cm on the ground, for a plotting scale 1:20), and the r.m.s. is given generally by the equation (in programme "SDAOE"):

$$\sigma^2 = 0.05^2 \cdot K_{im}^2 + 0.19^2 \cdot K_{pl} \text{ (mm)}$$

where: K_{im} - the scale of the photos

K_{pl} - the scale of the plotting

5. Conclusions.

The analytical medium accuracy instrument, Zeiss Stereocord G2 equipped with a desktop computer of type HP 9845 S, is a very powerful system for many applications.

The new software developed above can reach the maximum accuracy of the instrument for metric photographs. Nevertheless non-metric photographs can also be processed with a little modification to the existing programmes, in order to exclude distortion.

Improvements of the system can be:

- a) the use of a special photo-carriage for negative or diapositive plates or film (provided by the manufacturer).
- b) the improvement of the observation system concerning magnification and flying mark
- c) the connection of the computer to a line-plotter in order to

- increase direct plotting capabilities.
- d) the addition of a measuring device for the P parallax. This could establish the stereocord as a medium Y accuracy stereo-comparator for on-line and off-line analytical restitution and plotting.
- e) an unlimited number of programmes and improvements could be added to the existing software of the system such as, corrections for distortions, perspective and axonometric views and plotting.

It is obvious that this system is among the first of a new era in which computer technology and electronics are rapidly speeding all aspects of life and consequently Photogrammetry, too.

Bibliography

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Position of points on left-hand image

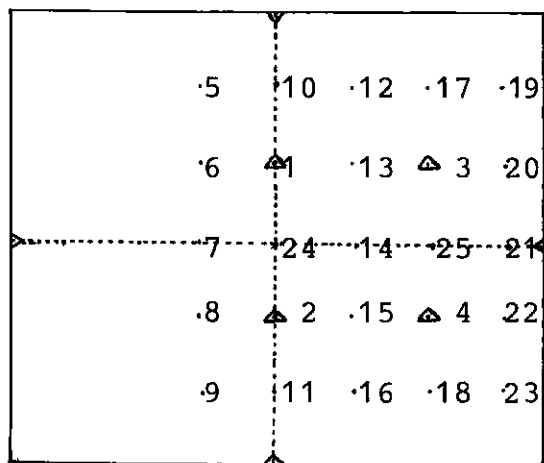


fig. 1

- ▲ control point
• check point

Terrain relative height differences

Am (m)	0 %															10 %															20 %														
	0	1	2	3	4	5	10	15	0	1	2	3	4	5	10	15	0	1	2	3	4	5	10	15																					
0	0	3	14	31	54	81	127	489	1034	0	75	153	260	363	474	1147	1927	0	179	363	552	748	949	2046	3250																				
1	0	0	0	0	0	0	0	0	0	0	159	329	511	704	909	2112	3612	0	243	703	1081	1478	1883	4266	7180																				
2	0	0	0	0	0	0	0	0	0	0	75	153	222	316	398	858	1402	0	159	323	491	665	844	1831	3016																				
3	71	135	200	280	371	497	1017	129	145	184	233	281	330	383	1580	242	351	483	583	683	708	833	1522	2289																					
4	48	86	137	196	261	332	1270	310	203	374	583	803	1035	1264	3994	66	303	723	1130	1556	1989	4508	7574	Y																					
5	10	55	101	158	196	247	551	1098	78	107	187	269	350	438	982	1578	66	223	379	535	692	848	1891	3139	Z																				
6	89	108	208	403	533	674	1587	2973	310	348	356	498	502	577	1148	2195	551	587	628	673	710	837	1323	2156	X																				
7	49	108	174	257	351	454	1075	1859	97	205	477	676	924	1182	2042	4018	158	495	822	1204	1698	2130	4782	8013	Y																				
8	40	133	228	325	428	534	1179	2160	104	147	227	319	413	507	1051	1752	171	302	462	631	787	956	1941	3255	Z																				
9	260	342	495	661	841	1035	2264	4084	546	625	735	848	975	1116	2100	3781	827	848	967	1005	1052	1141	1767	3259	X																				
10	112	198	293	398	521	661	1476	2466	184	323	407	525	1067	1352	2950	4884	278	625	922	1318	1795	2287	5087	8450	Y																				
11	89	234	382	536	696	864	1889	3475	200	388	487	688	855	1051	2276	3781	318	416	537	720	893	1085	1980	3364	Z																				
12	356	548	752	971	1209	1456	2033	3419	836	965	1147	1325	1520	1733	3182	5661	1373	1461	1555	1665	1825	1987	3269	5978	X																				
13	200	315	439	573	716	882	1894	3039	299	520	762	1002	1248	1548	3288	5394	427	803	1179	1557	1938	2470	5428	9034	Y																				
14	157	387	564	778	1002	1237	2689	4875	325	489	628	779	950	1185	2386	4655	508	590	673	821	998	1175	2147	4225	Z																				
15	356	800	1059	1333	1625	1928	3291	5926	1180	1393	1822	1870	2138	2432	4407	7839	1890	2059	2248	2460	2659	2859	5004	9220	X																				
16	316	459	612	775	949	1133	2329	3806	441	690	945	1207	1474	1768	3657	5848	607	999	1395	1792	2192	2680	5798	9615	Y																				
17	242	564	773	1053	1347	1657	3589	6687	481	682	915	1151	1403	1672	3519	6832	744	903	1078	1366	1477	1714	3512	7332	Z																				

Table 1.

Discrepancies of object coordinates (µm on image scale)

Table 2.

Table 3.

