

Identification of Geometrical Shapes in Paintings and its Application to Demonstrate the Foundations of Geometry in 1650 B.C.

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Abstract—In this paper, an original general methodology is introduced to establish whether a handmade shape corresponds to a given geometrical prototype. Using this methodology, one can decide if an artist had the intention of drawing a specific mathematical prototype or not. This analysis is applied to the 1650 B.C. wall paintings from the prehistoric settlement on Thera, and inferences of great archaeological and historical importance are made. In particular, strong evidence is obtained suggesting that the spirals depicted on the wall paintings correspond to linear (Archimedes) spirals, certain shapes correspond to canonical 48-gon and 32-gon, while other shapes correspond to parts of ellipses. It seems that the presented wall paintings constitute the earliest archaeological findings on which these geometrical patterns appear with such remarkable accuracy.

Index Terms—Foundation of geometry via image processing, image processing on paintings, image shape analysis, prehistoric geometry via pattern extraction.

I. INTRODUCTION

THE excavations at Akrotiri on the Greek island of Thera (Santorini) brought to light the ruins of a prehistoric 17th century B.C. town. Numerous important findings were discovered in this archaeological site, among which include the magnificent wall paintings of outstanding importance for furthering human knowledge of the early Aegean world and beyond. According to prominent archaeologists, these wall paintings rank alongside the greatest archaeological discoveries. The late Professor S. Marinatos began the excavations, which are now continued by Professor C. Doulas. As with the treasures of Pompeii and Herculaneum, the wall paintings of Thera were preserved due to the pumice from the great eruption of a volcano that buried the settlement [1].

In this paper, it is demonstrated that the artist or artists of a number of wall paintings had an advanced sense of geometry, if not knowledge, and used it to develop geometrical methods for drawing. In order to confirm this statement, an original general methodology is presented in this paper together with a set of

original criteria to show that a specific shape in a painting has probably been drawn by means of a geometrical method.

In general, it is thought that geometrical shapes such as the ones appearing in the 1650 B.C. Thera wall paintings that are drawn with such accuracy, do not appear before Thales (600 B.C.), Euclid (350 B.C.), and the Greek School of Geometry of Classical times [2]–[4]. Moreover, the first accurate drawing of a linear spiral has been attributed so far to Archimedes and his colleagues at about 250 B.C.; to be specific, Pappus states that Conon discovered the curve now known as the spiral of Archimedes [5]. The methodology introduced in this paper demonstrates that the artist or artists in 1650 B.C. most probably knew how to draw Archimedes spiral, canonical, or regular polygons and ellipses. This was an extremely difficult task not only for people from this prehistoric era, but also for those from Classical times. In fact, according to archaeological findings known to date, and also the related analysis [2], [3], [6], the Egyptians' knowledge of geometry was limited to the calculation of the slope of lines and planes and of the area of various geometrical shapes, such as the triangle, the rectangular parallelogram, and the trapezoid. However, they erroneously calculated the area of an arbitrary quadrilateral, of the circle, etc. At the same time, namely after 1500 B.C., the Babylonians had a more advanced knowledge of mathematics than the Egyptians [2], [7], yet their knowledge of geometry was also limited to the calculation of areas, the practical application of the Pythagorean theorem, and, at a later period, the knowledge that the height of an isosceles triangle bisects its base.

However, in the prehistoric town of Akrotiri on Thera, a number of wall paintings have been found depicting a variety of geometrical shapes. These wall paintings and in particular those depicting spirals, myrtle leaves, cycloids, and so on, manifest an impressively advanced sense, application, and, perhaps, knowledge of geometry by the artist(s) of this prehistoric civilization.

Therefore, although relevant written evidence is not available from the excavations, based on the results of the present paper, one can assert that the foundations of geometry could very well be found in this prehistoric civilization, almost 1100 years before Thales and 1300 years before Euclid.

The wall paintings are frescos, which means that the artist would have had limited time in which to draw his theme, namely as long as the plaster on the wall was fresh. Therefore, the use of specific methods, tools, and stencils was essential to the artist(s) in order to finish a complicated drawing in a short space of time.

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Extensive pattern analysis led us to the conclusion that 3650 years ago the artist most probably used advanced geometrical methods in order to construct handmade “French curves” (stencils or templates) or other tools and employed these to draw certain figures.

It should be emphasized that the pattern recognition methods and related criteria introduced in this paper, and employed thus far only on the Thera wall paintings, can be applied to any painted shape. Moreover, we have noticed that research is being carried out in curve fittings lately. [8]–[11]. However, in this paper, a methodology is introduced for determining geometrical shapes or curves, which obey a given equation in paintings.

It can be seen that since relevant written evidence is not available at Akrotiri, the analysis presented here is an example of how image processing and pattern analysis can shed light on issues otherwise unattainable in the absence of written records.

II. CRITERIA FOR TESTING IF AN ARTIST DREW A SPECIFIC GEOMETRICAL PROTOTYPE

Examine the actual drawn pattern and imagine that one poses the question: “Does this pattern correspond to a geometrical prototype?” In other words, did the artist have the intention of drawing a specific geometrical prototype (model)? In the following analysis, a set of original criteria is proposed to answer this question, and, subsequently, these criteria are applied to patterns appearing on the prehistoric wall paintings at Akrotiri.

A. Defining the First Criterion

Consider a drawn shape and its digitized image as consisting of N^A pixels described by the sequence of vectors \vec{r}_i^A , $i = 1, 2, \dots, N^A$ starting at a reference center and pointing to each pixel center, where superscript A stands for actual. Then, suppose that one wants to test if this shape is the successful result of an artist’s attempt to draw a geometrical prototype, described by the parametric vector equation $\vec{r}^M(t|\Pi)$ where t is the independent variable and Π is the set of parameters for the curve. For example, for the ellipse polar parametric equation

$$\vec{r}^M(t) = (x_0 + a \cos(t))\vec{i} + (y_0 + b \cos(t))\vec{j} \quad (1)$$

where $t \in [0, 2\pi)$ is the independent variable (the polar angle) and $\Pi = \{x_0, y_0, a, b\}$ is the set of parameters for the ellipse.

Next, we compute the optimal set of parameters Π^O and the corresponding sequence of values of the independent variable t_i , $i = 1, 2, \dots, N_A$, so that $\vec{r}^M(t_i|\Pi^O)$ best fits \vec{r}_i^A according to a chosen norm L . The algorithms applied to achieve this are the well-known conjugate gradient and/or the easier to implement Nelder–Mead method [11] starting from a tentative set of values of Π and letting Π converge to Π^O so that L is minimized.

Finally, taking into consideration all the aforementioned statements, the following criterion is defined for deciding if the artist did indeed have the intention to draw the geometrical shape $\vec{r}^M(t)$.

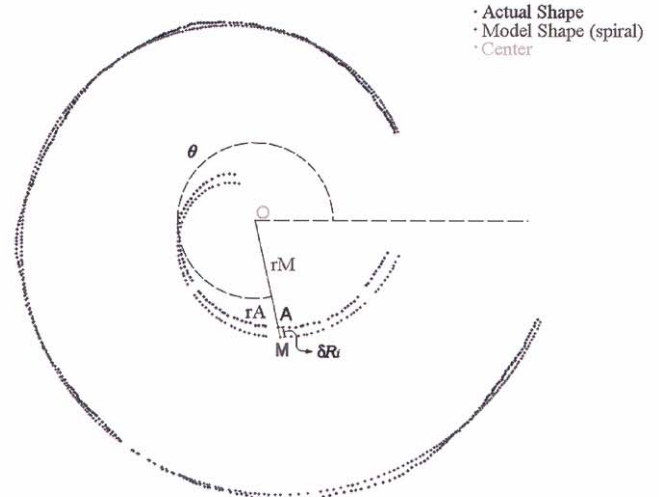


Fig. 1. Demonstration of criterion 1. Dark gray line: The actual drawn shape. Light gray line: The prototype shape (a spiral). Black dashed line: Indicates the independent variable θ . O : The origin (center).

Criterion 1: Consider, in the same plane, both the painted shape \vec{r}_i^A model shape $\vec{r}^M(t)$ that best fits it, placed in a position for the best possible match. For a specific example of this, see Fig. 1, where the actual painted curve is represented by a sequence of blue pixels, while the model curve is represented by a sequence of magenta pixels; O is the center of the model curve and \vec{r}_i^A is the i th vector starting at O and ending at the center of the i th pixel of the actual curve. Consequently, according to this criterion, the sequence

$$\delta r_i = \left| \vec{r}_i^A \right| - \left| \vec{r}^M(t_i|\Pi^O) \right|, \quad i = 1, \dots, N^A \quad (2)$$

is considered to be a random variable having a mean value $E(\delta r_i)$ equal to zero. Therefore, the hypothesis that the artist actually had the intention to draw the geometrical shape $\vec{r}^M(t)$ may be tested via the statistical hypothesis that $E(\delta r_i) = 0$.

It is quite evident that the aforementioned criterion constitutes a necessary condition for testing that the artist had the intention to draw a specific geometrical prototype, but this, in itself, is not sufficient. In fact, relatively large symmetric fluctuations of the painted shape around the prototype may satisfy criterion 1; therefore, the following criterion must be defined.

B. Defining the Second Criterion

If an artist has the intention of drawing an object following a specific geometrical pattern and succeeds in doing so, then one may expect that the drawn object will remain consistently close to that specific geometrical object. In other words, suppose that one makes the hypothesis that an existing drawn shape described by the set of points \vec{r}_i^A , $i = 1, 2, \dots, N^A$ is the result of the artist’s attempt to draw the geometrical shape described by the set of points $\vec{r}^M(t|\Pi)$. One may then intuitively expect that there will not be a connected part of the existing drawn shape that will have “a great distance” from the model shape. If a small part of the drawn object deviates from the model one,

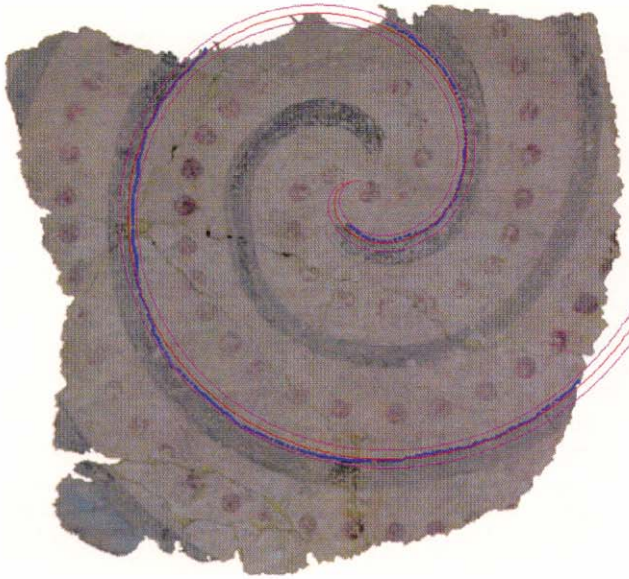


Fig. 2. 1) Ribbon-like set of points S_{RL} around the prototype shape (a linear spiral). 2) Demonstration of the fact that actual drawn spiral lies entirely in the ribbon like set S_{RL} of the prototype linear spiral. Blue dots: The actual drawn shape. Red line: The prototype shape (a linear spiral). Magenta lines: Indicate the borders of the ribbon like set S_{RL} .

one may assume that the artist made a statistically acceptable error in the drawing process and then he/she tried to correct it. On the other hand, if this deviation is considerably great, then one may assume that the artist intended to draw something else. In this section, and in the following one, two criteria will be set that will attempt to make the previous intuitively correct statements quantifiable.

In fact, consider the aforementioned digitized two-dimensional geometrical shape $\vec{r}^M(t|\Pi)$, expressed in a polar form

$$\begin{aligned} x^M &= x_0 + R^M(\theta) \cos(\alpha(\theta)) \\ y^M &= y_0 + R^M(\theta) \sin(\alpha(\theta)) \end{aligned} \quad (3)$$

where $R^M(\theta)$, $\alpha(\theta)$ are arbitrary real functions of the polar angle θ .

Moreover, consider the coplanar stripe-like (or ribbon-like) set of points P^S , where superscript S stands for stripe, defined as follows (see Figs. 2–4):

$$P^S = \left\{ \begin{array}{l} r^S(\theta) = x^S(\theta)\vec{i} + y^S(\theta)\vec{j} : \\ x^S(\theta) = x_0 + R^S(\theta) \cos(\alpha(\theta)) \\ y^S(\theta) = y_0 + R^S(\theta) \sin(\alpha(\theta)) \\ |R^S(\theta) - R^M(\theta)| < \epsilon \end{array} \right\}. \quad (4)$$

Following this, for a set of coplanar points $S^A = \{\vec{r}_i^A, i = 1, 2, \dots, N^A\}$ that may represent an actual drawn figure, let S_o be the subset of S^A consisting of its points lying outside the stripe-like set P^S ; clearly, S_o may be an empty set. Now, one may consider S_o as being the union of its maximally 8-connected disjointed subsets, i.e., one may express $S_o = \bigcup_i S_i^C$, where each S_i^C is maximally 8-connected in the sense that there is no 8-connected subset of S_o containing S_i^C

(see Fig. 3, where S_o consists of one maximally 8-connected subset).

We will call each S_i^C “a maximal area of discrepancy” (hereinafter abbreviated to MARD). The natural measure for the MARD S_i^C is defined as

$$\mu(S_i^C) = \text{the number of pixels forming } S_i^C \quad (5)$$

and, using it, one may order the MARD. Let us enumerate the S_i^C according to their measure, μ , starting from the one with the greatest measure, which is now denoted by S_1^{CO} , moving on to the second greatest measure, now symbolized as S_2^{CO} , and so on. Next, let us consider the set M_a^{CO} of ordered MARD consisting of the one with the greatest measure, together with those MARD that have a measure greater than or equal to a significant percentage of $\mu(S_1^{CO})$; i.e., consider this in the form of the equation

$$M_a^{CO} = \{S_j^{CO} : \mu(S_j^{CO}) \geq a\mu(S_1^{CO})\}. \quad (6)$$

Clearly, since the sets S_j^{CO} are disjoint, one may define a measure on M^{CO} , via the relation

$$\begin{aligned} \mu(M_a^{CO}) &= \sum_{j=1}^{N^{CO}} \mu(S_j^{CO}) \\ N^{CO} &= \text{the cardinal number of } M_a^{CO}. \end{aligned} \quad (7)$$

Obviously, if M^{CO} is an empty set, its measure is zero (0).

Then, one may expect that the greater the measure $\mu(M_a^{CO})$, the smaller the probability that the set of points S^A has been generated in the artist’s attempt to draw the model S^M . In other words, suppose that the artist wanted to draw the model S^M , in his/her attempt random deviations from S^M may occur, and, thus, drawing S^A may have been generated. The stripe-like set P^S represents precisely the area where these random deviations most probably belong. One may accept that perhaps the 8-connected discrepancies from P^S of small measure $\mu(S_i^C)$ occur as a result of the random failure of the drawing process. If, however, there is a set of large 8-connected parts lying outside P^S , then one would be inclined to accept that the artist intended on drawing some other model or no model at all. Clearly, the validity of the previous statements can be strengthened by the proper choice of value for the percentage a , which is related to the capabilities and intentions of specific artists. However, we generally believe that there are no essential differences in the results, as long as a remains close to 1. In the other drawings that were examined, where it was evident that the artist intended to draw a known model S_M , e.g., a circle, (see Fig. 4), the subsequent analysis showed an optimum value of $a = 2/3$. For simplicity, the symbols M^{CO} and $M_{2/3}^{CO}$ will be considered to be equivalent.

The question that arises now is what is the acceptable random deviation of S^A from S^M resulting from the erratic failures of the drawing process. In order to answer this question, we used the following method.

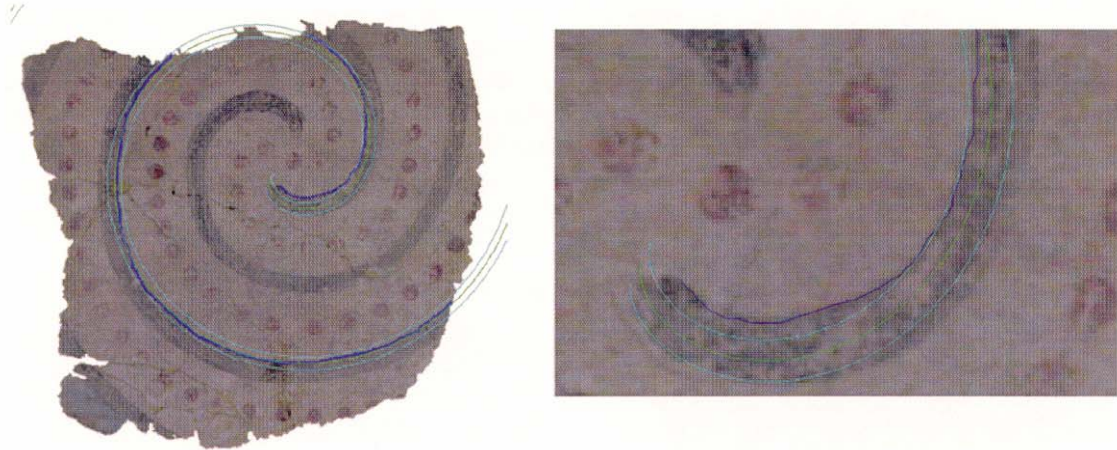


Fig. 3. 1) The ribbon-like set of points S_{RU} around the prototype shape (an involute of a circle). Demonstration of the fact that parts of the actual drawn spiral lie outside the ribbon like set S_{RU} of the prototype spiral. 2) Demonstration of one maximally 8-connected subset lying outside S_{RU} . Blue line: The actual drawn shape. Green line: the prototype shape (i.e., a spiral generated by unwrapping a thread around a peg). Cyan lines: Indicate the borders of the ribbon like set S_{RU} .



Fig. 4. 1) The ribbon-like set of points S_{RL} around the prototype linear spiral. 2) The circle that best fits to the contour of an actual drawn object, used for the determination of μ_{UP} . Notice that the very narrow ribbon around the circle is not shown to avoid confusion in the figure.

- Suppose that the model S^M is described by the generic set of (3) and consider the nearest circle K of $R^M(\theta)$ at an arbitrary point of the existing painted shape corresponding to the angle θ_0 . It is well known that, independently of the exact form of $R^M(\theta)$, circle K best approximates $R^M(\theta)$ in the least square sense near θ_0 , with a maximum error of order $O((\theta_f - \theta_0)^3)$, where θ_f is the angle of the points under consideration with the greatest distance from θ_0 .
- Next, consider the n first points of the actual drawing S^A that are a realization of a corresponding part of the model S^M , where n is small relative to the total length of S^A . Consider, moreover, the circular arc K_α that best fits these n points and the distance $d_i^{K_\alpha}$ of each one of these points from K_α . The maximum distances $d_i^{K_\alpha}$ of these are taken to be the width $\varepsilon_l(\theta_0)$ of the ribbon at this particular point θ_0 .



Fig. 5. Demonstration of the accuracy with which the linear (Archimedes) spiral matches the actual drawn one. The asterisk indicates the spiral center.



Fig. 6. Involute of a circle, i.e., the spiral generated by unwrapping a thread around a peg does not match the actual drawn spiral.

- The width of the stripe-like area is chosen to be $\varepsilon = 2.4 \text{mean} \left\{ \varepsilon(\theta_i) : \vec{r}^A(\theta_i) \in S^A \right\}$; we have used the coefficient 2.4, since 99% of the standard normal distribution population is less than this number.

Using the aforementioned approach, the following additional criterion is proposed for deciding if an artist that has actually drawn the shape S^A had the intention of drawing the theoretical model S^M .

Criterion 2: Consider a theoretical model curve S^M and the ribbon-like area P^S around it as is defined above.

- 1) If measure $\mu(M_a^{CO}) = 0$ [see (7)], then, clearly, S^A lies entirely inside P^S . Thus, according to criterion 2, the hypothesis that the artist drew S^A in his/her attempt to draw S^M is accepted.
- 2) If measure $\mu(M_a^{CO}) > 0$, then, in order to accept or reject this hypothesis, knowledge of the artist's drawing capabilities is required. A good estimate of his/her capabilities can be worked out by directly computing $\mu(M_a^{CO})$ in the instances where the artist's drawing intentions are evident or known.

For the present case, we have estimated the upper limit of $\mu_{UP} = \mu(M_a^{CO})$ by directly computing its value, in the case of drawn circles (see Fig. 4), since, in this instance, the artist's intention is evident. Therefore, if $\mu_{UP} < \mu(M_a^{CO})$, then the hypothesis that the artist wanted to draw S^M is rejected.

C. Defining Two More Criteria

Subsequently, one can calculate the value of the norm $\varepsilon_L = \left\| \vec{r}_i^A - \vec{r}^M(t_i | \Pi^O) \right\|$ and use the ε_L value to accept or reject the appropriate statistical hypothesis that the artist successfully drew the geometrical shape described by \vec{r}^M . If this hypothesis is accepted, one may use a suitable measure of distance between \vec{r}_i^A and $\vec{r}^M(t_i | \Pi^O)$ to compute the probability that the artist did indeed intend to draw a geometrical shape $\vec{r}^M(t)$. Thus, following Pearson [12], one may choose such a measure to be the quantity

$$S^P = \sum_{i=1}^{N^A} \frac{\left(\left| \vec{r}^M(t_i | \Pi^O) \right| - \left| \vec{r}_i^A \right| \right)^2}{\vec{r}^M(t_i | \Pi^O)}. \quad (8)$$

N^A constitutes the number of points of the actual drawn shape, which follows a chi square distribution with $N^A - n - 1$ degrees of freedom, where n is the number of independent parameters in Π . One may then set the following criterion that will give an estimate of the probability that the actual drawn shape \vec{r}_i^A is a (successful) random realization of the prototype shape $\vec{r}^M(t)$.

Criterion 3: Consider the size of population of the chi square distribution with n degrees of freedom, lying at all points that are greater than S^P ; in other words, consider the probability

$$P = P(x \geq S^P). \quad (9)$$

This probability is the chosen measure of goodness of fit between the actual painted shape \vec{r}_i^A , and the geometrical prototype $\vec{r}^M(t)$.

We will employ the methodology introduced above to demonstrate that the artist(s) who painted the wall paintings that were excavated at Akrotiri on Thera had an extraordinary sense of geometry for that era.

Criterion 4: A Criterion for Plausibility: This criterion is not a mathematical one, but, rather, it has to do with reasoning based on archaeological and historical evidence and the potential for geometric development within specific eras. In fact, in Section III, evidence will be given indicating that the artist(s) at Akrotiri knew how to construct a linear spiral and canonical 32-gon and 48-gon. These two types of canonical polygons can be constructed using a ruler and a pair of compasses by means of an innovative method for that era (the Bronze Age), and even for Classical times, yet it was not *a priori* impossible from an archaeological and historical point of view. On the other hand, as a clear-cut counter example, we cannot accept that the artist in 1650 B.C. knew how to construct a canonical 17-gon, as this requires a tremendous mathematical background, as is verified by the fact that this problem was solved with difficulty by the great mathematician Gauss in 1798.

III. REMARKABLE SENSE AND APPLICATION OF GEOMETRY BY THE ARTIST(S) OF THE WALL PAINTINGS AT AKROTIRI

We have tested various color image segmentation techniques [14], [15] in order to extract the contours of various objects, as can be seen in most of the figures. We have also used an original color image segmentation algorithm that can be applied more suitably to the images in hand and which is referred to in a separate paper by the present authors, currently under review. In this way, many shapes are obtained to which the methodology introduced in Section II has been applied, and which can be compared with various geometrical prototypes.

A. Related Spiral Types

The general equation for a spiral is

$$\begin{aligned} x(\theta) &= x_0 + R(\theta - \phi_0) \cos(\alpha(\theta - \phi_0)) \\ y(\theta) &= y_0 + R(\theta - \phi_0) \sin(\alpha(\theta - \phi_0)) \end{aligned} \quad (10)$$

where x_0, y_0 are the coordinates of the spiral center, $R(\theta)$ is any increasing function of θ , and $\alpha(\theta - \phi_0)$ is any function of θ , while ϕ_0 accounts for a probable rotation of the spiral. The most well-known spirals are the following ones, the equations of which are written so as to incorporate the instances where one has to deal with parts of the corresponding spiral that does not necessarily start at $\theta = 0$.

The Archimedes spiral, namely the one with

$$R(\theta) = k(\theta - \theta_0), \text{ and } \alpha(\theta) = (\theta - \theta_0) - \phi_0 \quad (11)$$

where α, β are constants.

The involute of a circle with radius r_0 , namely the spiral generated when a thread wrapped around a peg is unwrapped

$$\begin{aligned} R(\theta) &= r_0 \sqrt{1 + (\theta - \theta_0)^2}, \\ \alpha(\theta) &= (\theta - \theta_0) - \arctan(\theta - \theta_0) - \phi_0. \end{aligned} \quad (12)$$

This spiral seems to be the simpler one to draw.

The logarithmic spiral satisfying

$$R(\theta) = ae^{\beta(\theta-\theta_0)}, \quad \alpha(\theta) = (\theta - \theta_0) - \phi_0 \quad (13)$$

where α , β are constants and, once more, ϕ_0 accounts for a probable rotation of the spiral. Notice that most spirals encountered in nature, such as those found on seashells, are logarithmic spirals.

We have extracted the line boundaries of all the parts of the spirals that are depicted in the available excavated fresco fragments. In this paper, between the two line boundaries of each part of the spiral, we have mainly chosen the one where the external region of the spiral is locally convex (see Fig. 2); we will call this “the internal boundary of the spiral.” Notice that the same results are obtained if one chooses the “external boundary,” namely the boundary for which the external region of the spiral is locally nonconvex.

B. Rejecting the Hypothesis That the Spiral Parts Where Drawn by Unwrapping a Thread Around a Peg

Applying, once more, the methodology introduced in Section II, we want to test the hypothesis that the artist drew the spirals by unwrapping a thread around a peg, i.e., that the spirals are involutes of a circle. In order to do so, one must repeat the steps of the previous section.

- 1) First, one observes that the set of parameters for this model spiral given by (12) is $\Pi_U = \{x_0, y_0, r_0, \theta_0, \varphi_0\}$. Thus, if one considers any actual painted part of the spirals S^A represented in a digital image by the set of points (pixels) \vec{r}_i^A , $i = 1, 2, \dots, N^A$, one may then find the linear spiral S^U , where U stands for unwrapping, with optimal parameters Π_U^O that best fits S^A , as is described in Section II-A.
- 2) Consider S^U placed in the position that best fits S^A ; one then applies criterion 1, namely expressing the hypothesis that $E(\delta r_i) = 0$, where, $\delta r_i = \left| \vec{r}_i^A \right| - \left| \vec{r}_i^M(\theta_i | \Pi_U^O) \right|$, $i = 1, \dots, N^A$, θ_i as is shown in Figs. 3 and 6. The hypothesis is rejected with a confidence level of 99.9% for 63% of all the available parts of the spirals as is shown in Fig. 6.
- 3) Although criterion 2 rejected the hypothesis that the parts of the spirals are involutes of a circle, we, nevertheless, applied criterion 3. Thus, consider, once more, S^U placed in the position that best fits S^A , one then defines the ribbon-like set P^S as is described in Section II-B. Such a ribbon-like area is shown in Fig. 3 for a specific part of a spiral. Next, one determines the connected subsets of S^A that lie outside P^S and checks if criterion 2 is satisfied. In other words, one examines the value of $\mu(SM_a^{CO})$ and its relation to μ_{UP} . The related analysis shows that, for all the parts of the spirals, $\mu(SM_a^{CO}) > 0$ holds, while $\mu(M_a^{CO}) > \mu_{UP}$ in all cases, as is shown in Table II. Thus, the hypothesis that the parts of the spirals were drawn by unwrapping a thread around a peg is once more rejected.
- 4) Since criteria 1) and 2) rejected the aforementioned hypothesis, we did not proceed to applying criterion 3).

Notice that the actual drawn parts of the spirals S^A correspond to a logarithmic spiral and they too have been tested. Testing this hypothesis is also essential because, as has already been mentioned, this type of spiral exists in nature. By applying the series of steps referred to above, a piecewise approximation of the parts of the spiral against logarithmic ones is possible, but a good approximation of the big parts of the spirals ($\theta - \theta_0 > 2\pi$) that are frequently drawn by the artist, is not possible.

C. Knowledge of Drawing Archimedes Spiral in 1650 B.C.

Applying the methodology introduced in Section II, we want to find out if there is a theoretical model that best fits these painted spiral patterns, which would have been generated by the artist(s) with the means we presume to have been available to them then. In fact, we will give strong evidence to demonstrate that the artist(s) 3650 years ago had the intention to draw a linear–Archimedes spiral. In order to do so, we will proceed as follows.

- 1) First, one will observe that the set of parameters for a linear spiral given by (11) is $\Pi_L = \{x_0, y_0, \kappa, \theta_0, \varphi_0\}$. Thus, if one considers any actual painted part of a spiral S^A represented in a digital image by the set of points (pixels) \vec{r}_i^A , $i = 1, 2, \dots, N^A$, then one can calculate the linear spiral S^L with optimal parameters Π_L^O that best fits S^A , as described in Section II-A.
- 2) Consider S^L placed in the position that best fits S^A ; one then applies criterion 1, namely expressing the hypothesis that $E(\delta r_i) = 0$, where, $\delta r_i = \left| \vec{r}_i^A \right| - \left| \vec{r}_i^M(\theta_i | \Pi_L^O) \right|$, $i = 1, \dots, N^A$, θ_i , as shown in Fig. 1. The hypothesis is fully accepted for all available parts of the spirals as is shown in Table I.
- 3) Once more, consider S^L placed in the position that best fits S^A ; one then defines the ribbon-like set P^S as described in Section II-B. Such ribbon-like areas are shown in Figs. 2 and 4 for specific parts of the spirals. Next, one can check if criterion 2 is satisfied, using the connected subsets of S^A that lie outside P^S . In other words, one examines the value of $\mu(M_a^{CO})$ and its relation to μ_{UP} . The related analysis shows that an overwhelming number of the parts of a spiral $\mu(M_a^{CO})$ equals zero (0), while, in all cases, they remain essentially smaller than μ_{UP} , as shown in Table II. Thus, the hypothesis that all the parts of the spirals were drawn so as to follow the Archimedes spiral pattern is always accepted on the basis of criterion 2.
- 4) The probability that the drawn spiral part S^A is actually a random realization of S^L is explicitly calculated by means of criterion 3. We emphasize that this probability is almost 1 for all the available parts of the spirals, i.e., it is almost certain that the artist intended to draw a linear spiral (see Figs. 2, 4, and 5).

D. A Probable Method for Drawing the Archimedes Spiral 3650 Years Ago

It is clear that one cannot be sure of the method that the artist employed to draw a linear spiral, unless related evidence can be brought to light by the excavations. However, by logical deduction and the necessary supporting evidence, one may put

TABLE I
RESULTS OF THE APPLICATION OF CRITERION 1 TO ALL THE AVAILABLE PARTS OF THE SPIRALS

Spiral part	x dim (cm)	y dim (cm)	Linear				Involute			
			Mean (Rm Diff)	Tinv (0.9995)	Rm_t	Accepted - Rejected	Mean (Rm Diff)	Tinv (0.9995)	Rm_t	Accepted - Rejected
#1	9,6	11,1	-1,006E-02	3,5099	-1,0303	Accepted	-4,436E-02	3,5099	-3,707	Rejected
#2	10,9	9,0	-3,666E-03	3,4394	-0,4929	Accepted	1,435E-02	3,4394	0,7382	Accepted
#3	9,7	10,4	-1,011E-03	3,515	-0,0666	Accepted	-1,113E-02	3,515	-0,6509	Accepted
#4	5,1	8,6	8,053E-03	3,6896	0,6546	Accepted	-1,613E-02	3,6896	-1,0553	Accepted
#5	13,2	9,3	4,927E-03	3,3167	1,8407	Accepted	1,232E-02	3,3167	3,65862	Rejected
#6	16,8	16,5	2,110E-02	3,3152	3,0777	Accepted	-8,243E-02	3,3152	-5,7968	Rejected
#7	20,9	20,5	-1,081E-02	3,3179	-1,0604	Accepted	-1,343E-02	3,3179	-0,843	Accepted
#8	19,4	16,6	4,034E-02	3,3416	2,4822	Accepted	-9,302E-02	3,3416	-4,5988	Rejected
#9	21,5	13,3	7,373E-03	3,3328	0,903	Accepted	-9,255E-02	3,3328	-53,587	Rejected
#10	25,2	26,7	-1,547E-02	3,3361	-1,431	Accepted	-8,197E-02	3,3361	-5,3262	Rejected
#11	29,1	26,6	3,462E-02	3,3135	3,234	Accepted	-9,020E-02	3,3135	-6,837	Rejected

TABLE II
RESULTS OF THE APPLICATION OF CRITERION 2 TO ALL THE AVAILABLE PARTS OF THE SPIRALS $\mu_{UP} = 20$

Spiral part	ϵ	Linear			Involute			
		$\mu(S_1^{CO})$	$\mu(SM_a^{CO})$	Accepted - rejected	$\mu(S_1^{CO})$	$\mu(S_2^{CO})$	$\mu(SM_a^{CO})$	Accepted - rejected
#1	5.1	0	0	Accepted	24	19	43	Rejected
#2	5.9	0	0	Accepted	45	0	45	Rejected
#3	6.2	9	9	Accepted	37	0	37	Rejected
#4	5.4	0	0	Accepted	24	18	42	Rejected
#5	5.8	10	10	Accepted	30	22	52	Rejected
#6	5.2	0	0	Accepted	70	0	70	Rejected
#7	5.8	0	0	Accepted	78	0	78	Rejected
#8	6.5	17	17	Accepted	29	21	50	Rejected
#9	5.5	0	0	Accepted	22	20	42	Rejected
#10	4.9	0	0	Accepted	47	0	47	Rejected
#11	6.3	12	12	Accepted	66	0	66	Rejected

forward a hypothesis regarding the method used by the artist to draw these parts of spirals. In fact, according to criterion 4, the proposed method must be relative to the means we presume would have been available to them at that time. In any case, it should be mentioned that the method used by the artist was extremely innovative for the era in question. The simplest method we can think of, which is still considered to be extraordinary for this period, is the following one (see Fig. 7).

- 1) The artist drew a large number of homocentric circles with the center O , where two consecutive circles had a fixed radius difference.
- 2) The artist divided the 2π entire angle into N equal angles and drew the corresponding radii starting at the same center O .
- 3) The sequence of points of intersection of consecutive radii and the homocentric circles lies on an Archimedes spiral.

It should be emphasized that, although this method may seem simple to us nowadays, it is, in fact, extremely innovative for 1650 B.C., and even for Classical times. In fact, accomplishing the task of dividing the 2π radians angle into N equal angles, presupposes and incorporates advanced sense, knowledge, and the ability to achieve the following:

- 1) the construction of an initial canonical polygon, e.g., a square. Euclid was the first to write about canonical schemes at about 350 B.C.;

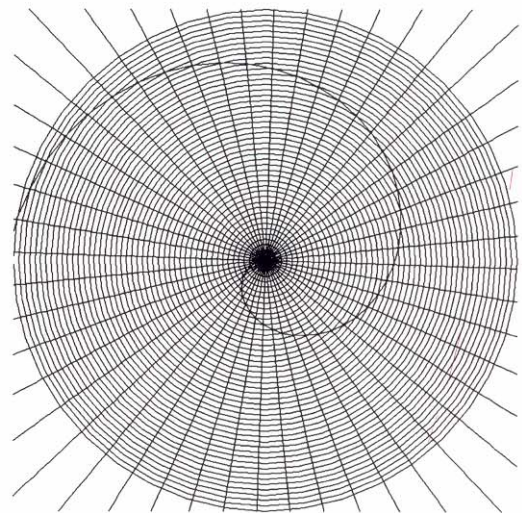


Fig. 7. Method for constructing Archimedes spiral. The circles are concentric and all pairs of consecutive straight semi lines are isogonal.

- 2) finding the middle of a given line segment;
- 3) creating a line vertical to a given one;
- 4) the perpendicular at the center of the chord of a circle that must dichotomize the corresponding epicenter angle, i.e., it bisects the corresponding angle with the vertex at the center of the circle;

- 5) the proper repeated application of the above four tasks to lead to canonical polygons with a continually increasing number of equal epicenter angles.

This innovation and the difficulty involved in achieving the aforementioned, is further highlighted when taking into account the following historical beliefs.

- 1) Oenopides is considered to be the first to have divided a given angle into two equal parts at approximately 450 B.C., i.e., 1200 years later.
- 2) So far, Archimedes is considered to be the first to have proposed the method for constructing a linear spiral together with his colleagues; he is thought to be one of the greatest mathematicians of all times.

Additional evidence demonstrating that the artist may have used this method for drawing linear spirals is offered in Section III-E.

E. Knowledge of Drawing Regular Polygons

At this point, the idea emerged that, if the artist employed the method described in Section III-D for drawing the spirals, then he probably used the equiangular straight lines shown in Fig. 7 for drawing the red dots, too. Thus, we attempted to test this hypothesis by applying criteria 1, 3, and 4 of Section II.

In order to test if the red spots lie on straight semi lines that divide the 2π radians angle into n equal angles, we proceed as follows.

- 1) After the proper segmentation of the image showing the wall painting fragment with the red spots is carried out, the contour of each of the spots is extracted. The circle that best approximates this contour is employed and the center coordinates of this circle are stored, which will be considered to be the red spot center in the following.
- 2) We choose an arbitrary point $O = (x_0, y_0)$ and consider it to be a tentative center of the corresponding n -gon. We connect O with the center of the N^R red spots, thus forming a sequence of straight semi lines ε_j , $j = 1, 2, \dots, N^R$, N^R being the number of red spots under consideration.
- 3) Let θ_j^a be the actual angle of the two semi lines ε_j and ε_{j+1} , $j = 1, 2, \dots, N^R$. Moreover, let $\theta_i^A = \sum_{n=1}^i \theta_n^a$.
- 4) We define θ^x as the angle so that $\theta^x = 2\pi/x$, where x is an arbitrary real number, as well as the sequence of angles $\theta_i^x = \theta_0^x + (i-1)\theta^x$, where θ_0^x is an arbitrary initial angle.
- 5) We consider θ_i^x as being a prototype sequence with a set of parameters $\Pi = \{x_0, y_0, x, \theta_0^x\}$, and, in addition, the norm $E_\theta = \sum_{i=1}^{N^R} |\theta_i^x - \theta_i^A|$. We minimize E_θ in the parameter space by means of an algorithm that uses a version of the conjugate gradient method. In this way, we obtain a set of optimal parameters $\Pi^0 = \{x_0^0, y_0^0, x^0, \theta_0^0\}$. Point (x_0^0, y_0^0) will be called “canonical x center.”

We have applied this method to a class of red spots sequence depicted in all available fragments, together with the corresponding parts of the spiral (see Fig. 8). In this way, we have obtained values of x between 47.95 and 47.97, a fact indicating that the artist 3650 years ago was most probably capable of drawing a canonical 48-gon. Fig. 8 strongly supports the validity of this suggestion. In order to formally confirm this statement, we have applied criteria 1 and 3 using the set of parameters $\Pi^0 = \{x_0^0, y_0^0, 48, \theta_0^0\}$. Indeed, criterion 1

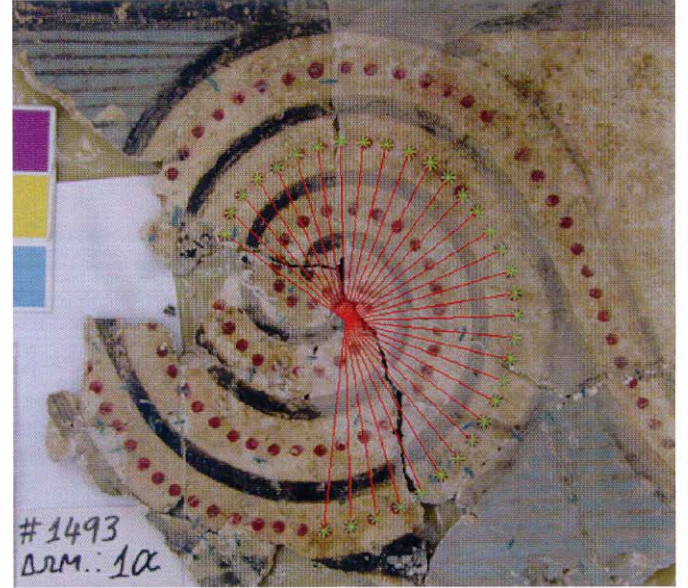


Fig. 8. Strong evidence that the artist approximately 3650 years ago was capable of drawing a canonical 48-gon.

confirmed the hypothesis that the centers of two successive red spots lie on semi lines forming an angle $2\pi/48$. Criterion 3 with a measure of goodness of fit $S^A = \sum_{i=1}^{N^R} \left(\frac{(\theta_i^{48} - \theta_i^A)^2}{\theta_i^{48}} \right)$ shows that the corresponding probability is practically 1. In other words, Criterion 3 shows that the centers of the red spots almost certainly lie on semi lines passing through the vertices of a canonical 48-gon.

It is stressed that the aforementioned procedure has also been applied to other canonical polygons that have n number of vertices different to 48. The results that were obtained indicate that, for $n < 45$ and $n > 51$, there is no n -canonical center that “sees” the centers of the red spots, as is shown in Fig. 8. Notice that such an n -canonical center does not exist, not even in a subset of at least 24 red spots of the ones shown in Fig. 8. For $45 \leq n \leq 51$, one can find n -canonical centers; however, with greater error. In fact, for $n = 45$, the minimum error (E_θ/N^R) is $2.48 \cdot 10^{-2}$ rad ($\cong 1.42^\circ$), while, for $n = 48$, it is $1.13 \cdot 10^{-2}$ rad ($\cong 0.64^\circ$). Moreover, the canonical 48-gon can be constructed geometrically starting from a canonical hexagon and then applying a successive bisection of the epicenter angle until one arrives at $n = 48$. This method of construction although innovative for the era can be accepted from an archaeological and historical point of view; in other words, it is compatible with criterion 4.

It is worthwhile noticing that, for all the parts of the spirals that were examined, the first 16–20 centers of the red spots lay on a canonical 48-gon, whose center was slightly moved with respect to the one shown in Fig. 8. The same is also true for the last 15 red spots. One cannot rule out the possibility that the artist made this translation of the centers to make the red spots sequence agree with his aesthetic criteria.

We have considered the probability that the artist(s) used their knowledge of constructing equiangular straight lines, to make other decorative shapes, as well. Thus, a thorough examination of the wall painting fragments indicates that in the class of spirals such as the ones shown in Fig. 9, the acanthus-like vertices possibly lie on equiangular straight lines. In order to test this,

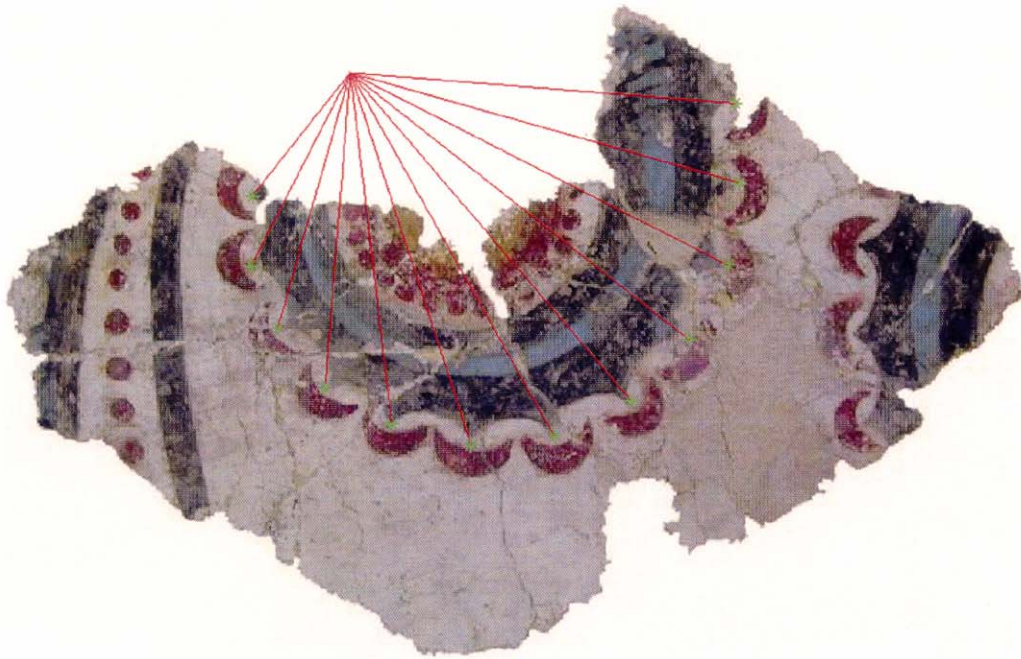


Fig. 9. Notice, that, as in the case of the canonical 48-gon, the center of the canonical 32-gon lies quite close to the start of the linear spiral.

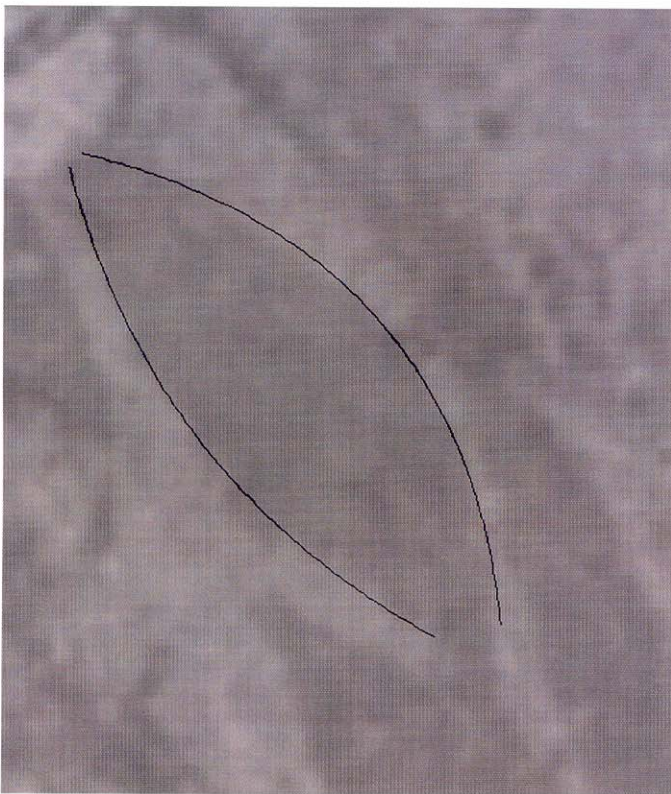


Fig. 10. Demonstration of the fact that both sides of a myrtle leaf fit an ellipse with great accuracy.

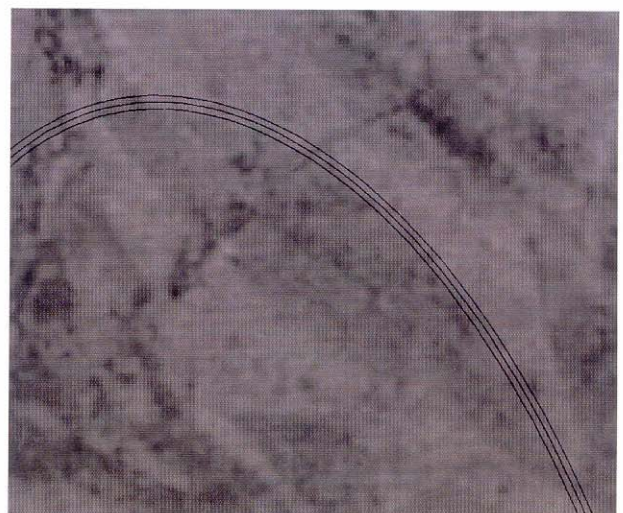


Fig. 11. Ribbon-like area around an ellipse best fitting to a single myrtle leaf contour.

we have proceeded in the same way as in the case with the red spots; in this instance, the minimization process lead to the epicenter angle of a canonical 32-gon. In other words, in all these spiral patterns, there is always a point lying in the proximity of the spiral end that, with any two such consecutive vertices, forms the angle $2\pi/32$ up to ϵ , where $\epsilon \leq 2 \cdot 10^{-2}$. After spotting this point with the aforementioned minimization process,

we applied criteria 1 and 3. Therefore, as before, let θ_j^a be the actual angle the found center forms with two consecutive vertices; also, let $\theta_i^A = \sum_{n=1}^i \theta_n^a$ and $\theta_i^{32} = \theta_0^{32} + (i - 1)\theta^{32}$. The hypothesis then that $E(\theta_i^A - \theta_i^{32}) = 0$ is verified for all such patterns that are available, i.e., criterion 1 is satisfied. Moreover, criterion 3 applied with $S^A = \sum_{i=1}^{N^R} \left((\theta_i^{32} - \theta_i^A)^2 / \theta_i^{32} \right)$ confirms that the probability that $\theta_j^a = (2\pi/32)$ is equal to 1.

It should be noted that the centers of the red spots and the acanthus-like vertices do not lie on the circumference of a circle. Therefore, one cannot claim that the artist started drawing equidistant points and accidentally formed canonical 48-gon and 32-gon. On the contrary, the distance between the centers of two consecutive red spots or two consecutive vertices greatly varies from point to point. The related statistical hypothesis that the distance of two consecutive points is fixed is clearly rejected.

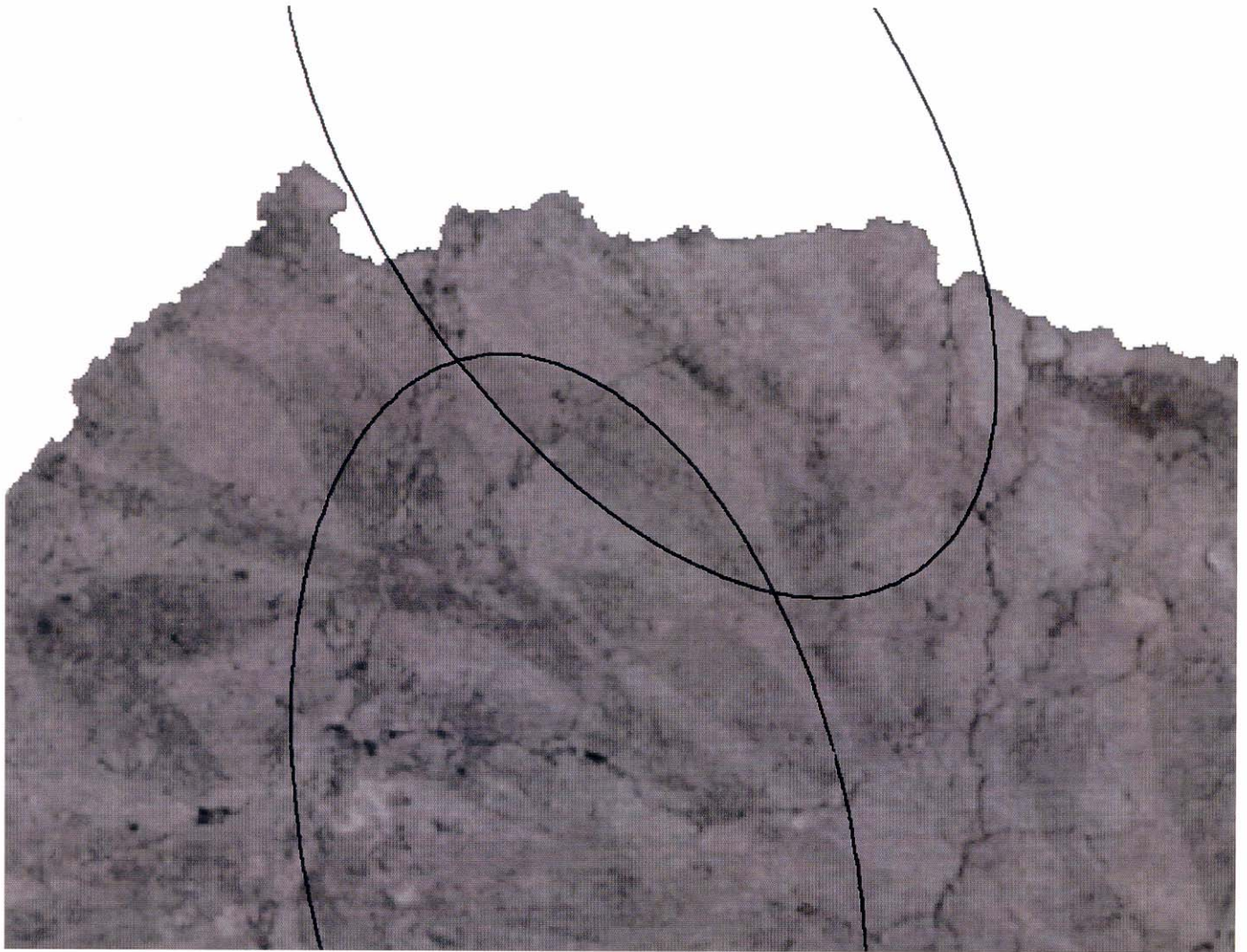


Fig. 12. Demonstration of the fact that the same ellipse stencil matches both sides of a myrtle leaf with great accuracy.

F. Knowledge of Ellipse Drawing

An important wall painting that was excavated and preserved in a fragmentary state, depicts myrtles. A simple inspection of the myrtle leaves shows that their borders are smooth, well defined and with no trembling; This fact suggests that a tool may have been used to draw them. Since the previous analysis shows that the artist could use a number of different geometrical methods to create the drawings, the idea to look for probable geometrical methods for drawing the borders of the myrtle leaves emerged. After extensive analysis and a related search, the most probable candidate for the drawing of the leaves seemed to be the ellipse. To test this, we have moved along the same lines as was introduced in Section II.

The general equation for a conic is

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0$$

$$A, B, C, D, E, F \in \mathbf{R}. \quad (14)$$

However, the general equation for an ellipse with one of its axes parallel to the x axis and with its center at (x_0, y_0) is

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} - 1 = 0, \quad a, b \in \mathbf{R}. \quad (15)$$

Equation (15) is a direct consequence of (14) after a rotation of θ radians, in the sense that if transformation

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (16)$$

is applied to (14), where

$$\cot \theta = \frac{A - B}{C} \quad (17)$$

then one obtains (15) with (18), shown at the bottom of the next page.

At this point, we pick $t = x$ to be the independent variable of each myrtle contour, we define $\Pi = \{A, B, C, D, E, F\}$ to be the parameter space of the ellipse defined via (14) and we let

$$\overrightarrow{r^M}(x|\Pi) = x\vec{i} + y\vec{j} \quad (19)$$

be the parametric equation of the ellipse whose dependent variable is y .

Subsequently, one applies the methodology introduced in Section II to check if the artist drew the two contours of each of the myrtle leaves by using a geometrical method for drawing

an ellipse or by means of an appropriate stencil (a handmade geometric template). Thus, we define

$$\vec{r}_m^A = x_m^A \vec{i} + y_m^A \vec{j}, \quad m = 1, 2, \dots, N^A \quad (20)$$

to be the sequence of vectors corresponding to pixels forming the digital image of an actual myrtle leaf contour. Moreover, we define the parametric equation for an ellipse that is assumed to approximate \vec{r}_m^A to be

$$\vec{r}_m^M(x_m|\Pi) = x_m^M \vec{i} + y_{(x_m|\Pi)}^M \vec{j} \quad (21)$$

and the norm of difference between the actual painted \vec{r}_m^A and the model

$$E_{EL}(x|\Pi) = \sum_{m=1}^{N^A} (|y_m^A - y^M(x_m|\Pi)|). \quad (22)$$

- 1) We minimize E_{EL} in the parameter space and we obtain the optimal set of parameters $\Pi^O = \{A^O, B^O, C^O, D^O, E^O, F^O\}$ for which the ellipse $\vec{r}_m^M(x_m|\Pi)$ best approximates the actual drawn leaf contour \vec{r}_m^A . Let S^E be the ellipse that best fits the leaf contour S^A (see Fig. 10).
- 2) At this point, criterion 1 has been applied and the hypothesis that $E(y_i^M - y_i^A) = 0$ has been confirmed for all 29 two-sided myrtle leaf contour. Therefore, this is the first strong piece of evidence we have that the artist(s) may have painted the leaves by drawing an ellipse via a geometrical method.
- 3) Let us consider S^E that best fits S^A and define the ribbon-like set P^S in a manner analogous to the one described in Section II-B. In fact, using the polar form of the ellipse we define the set S^E as follows:

$$S^E = \left\{ (x, y) \in \mathbf{R}^2 \left[\begin{array}{l} x = x_0 + a_R, \quad a - \varepsilon \leq a_R \\ y = y_0 + b_R, \quad b - \varepsilon \leq b_R \end{array} \right] \right\} \quad (23)$$

where $\varepsilon > 0$ is defined as in Section II-B. Such a ribbon-like area is shown in Fig. 11 for a specific leaf contour. Next, one determines the connected subsets of S^A that lie outside S_{RE} and checks if criterion 2 is satisfied. Namely, one checks the value of $\mu(M_a^{CO})$ and its relation to μ_{UP} . The related analysis shows that the contour of all the leaves $\mu(M_a^{CO})$ equal zero. Thus, we are presented with further strong evidence indicating that the contours of the leaves are formed from the parts of an ellipse.

- 4) Finally, the probability that the drawn contours of the leaves S^A are actually a random realization of S^E is calculated by means of criterion 3. This probability P^E is very close to 1, actually $1 - 10^{-9} < P^E \leq 1$ for all the available leaf contours.

It should be noted that there is evidence supporting the hypothesis that the artist used a very limited number of ellipse stencils to draw myrtle leaves (see Fig. 12 where both sides of the myrtle leaves match the same ellipse stencil with a great degree of accuracy. This observation, as well as the analogous one concerning the linear spirals, makes it possible to match fragments with their corresponding prototype. This is a very important process in archaeology, which will be the subject of another paper.

IV. CONCLUSION

The aim of this paper is to introduce an original general methodology to determine whether a handmade shape corresponds to a given geometrical prototype. To achieve this, three mathematical criteria are introduced, two of them being of statistical nature and the other one being based on fuzzy logic. The application of these criteria to the very important Late Bronze age wall paintings, decorating the internal walls of an edifice excavated at Akrotiri, Thera, shows that the spirals depicted on these wall paintings correspond to linear (Archimedes) spirals with exceptional accuracy. In addition, it is shown that there are many sets of decorative elements placed, with remarkable precision, along radii of various canonical polygons (48-gon and 32-gon). Moreover, a number of other shapes are determined that correspond to parts of ellipses.

REFERENCES

- [1] C. Doumas, "The wall-paintings of Thera," in *The Thera Foundation Petros Nomikos*. Athens, Greece: Kapon, 1999.
- [2] L. N. H. Bunt, P. S. Jones, and J. D. Bedient, *The Historical Roots of Elementary Mathematics*. Upper Saddle River, NJ: Prentice-Hall, 1981.
- [3] S. T. Heath, *A History of Greek Mathematics—Volume 1: From Thales to Euclid*. New York: Dover, 1981.
- [4] E. Spandagos, R. Spandagou, and D. Travlou, *Ancient Greece Mathematicians*. Athens, Greece: Aethra, 2000.
- [5] S. T. Heath, *A History of Greek Mathematics—Volume 2*. New York: Dover, 1981.
- [6] A. Szabo, *The Beginnings of Greek Mathematics*. Dordrecht, The Netherlands: Reidel, 1968.
- [7] E. Robson, *Mesopotamian Mathematics 2100-1600 BC Technical Constants in Bureaucracy and Education*. Oxford, U.K.: Clarendon, 1999.
- [8] M. Sarfraz, "Fitting curve to planar digital data," in *Proc. 6th Int. Conf. Information Visualization*, London, U.K., Jul. 2002, pp. 633-633.
- [9] T. Prahasto and S. Bedi, "Optimization of knots for the multi curve B-spline approximation," in *Proc. Geometric Modeling and Processing*, Hong Kong, Apr. 2000, pp. 150-150.

$$\begin{aligned} x_0 &= -\frac{D \cos \theta + E \sin \theta}{2(A \cos^2 \theta + B \sin^2 \theta + C \cos \theta \sin \theta)} \\ y_0 &= -\frac{E^p \cos \theta - D^p \sin \theta}{2(B \cos^2 \theta + A \sin^2 \theta - C \cos \theta \sin \theta)} \\ a &= \frac{(A \cos^2 \theta + B \sin^2 \theta + C \cos \theta \sin \theta) x_0^2 + (B \cos^2 \theta + A \sin^2 \theta - C \cos \theta \sin \theta) x_0^2 - 1}{(A \cos^2 \theta + B \sin^2 \theta + C \cos \theta \sin \theta)^{(1/2)}} \\ b &= \frac{(A \cos^2 \theta + B \sin^2 \theta + C \cos \theta \sin \theta) x_0^2 + (B \cos^2 \theta + A \sin^2 \theta - C \cos \theta \sin \theta) x_0^2 - 1}{(B \cos^2 \theta + A \sin^2 \theta - C \cos \theta \sin \theta)^{(1/2)}} \end{aligned} \quad (18)$$

- [10] S. J. Ahn, W. Rauh, H. S. Cho, and H. J. Warnecke, "Orthogonal distance fitting of implicit curves and surfaces," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 24, no. 5, pp. 620–638, May 2002.
- [11] K. Sengupta and P. Burman, "A curve fitting problem and its application in modeling objects in monocular image sequences," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 24, no. 5, pp. 674–686, May 2002.
- [12] E. Pearson, R. D'Agostino, and K. Bowman, "Tests for departure from normality: Comparison of powers," *Biometrika*, vol. 64, pp. 231–246, 1977.
- [13] J. A. Nelder and R. Mead, "A simplex method for function minimization," *Comput. J.*, vol. 7, pp. 308–313, 1965.
- [14] T. Vlachos and A. G. Constantinides, "Graph-theoretical approach to color picture segmentation and contour classification," *Proc. Inst. Elect. Eng.*, vol. 140, pp. 36–45, 1993.
- [15] Adam, Gillian, Xiaoyi, J. Patrick, Horst, B. Dmitry, Kevin, W. David, Andrew, and B. Robert, "An experimental comparison of range image segmentation algorithms," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 18, no. 7, pp. 673–689, Jul. 1996.



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