TIME DELAY AND SATURATION CAPACITY INTERACTION IN THE CONTROL OF STRUCTURES UNDER SEISMIC ACTIONS

Nikos G. Pnevmatikos¹, Charis J. Gantes¹

¹Metal Structures Laboratory, School of Civil Engineering, National Technical University of Athens, 9 Heroon Polytechniou, GR-15780 Zografou, Greece, Tel: +302107723440, Fax: +302107723442, Email: nikos_pnevmatikos@hotmail.com, chgantes@central.ntua.gr

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Abstract. For the last thirty years, many structural control concepts have been evolved for the reduction of structural response caused by earthquake excitations, and quite a few of them have been implemented in practice showing that control of structures has a lot of limitations and difficulties in practical implementation. In this paper we discuss the effectiveness of two practical issues, time delay and saturation effect, on the performance of control devices. Their influence, both separately and in interaction, to the response of structures controlled by a modified pole placement algorithm, developed by the authors, is investigated. The above approach is demonstrated by means of numerical examples, where one story building is analyzed under dynamic loads such as harmonic signals and actual seismic events. Conclusions regarding the choice of the control system based on its delay and the maximum capacity of the control devices are drawn.
1 INTRODUCTION

Structural control has received wide-spread attention in recent years. A strong trend in structural technology is to shift from conventional earthquake resistant structures, to structurally controlled buildings, which are designed to suppress the vibration itself. The research and application of control to civil engineering structures include analytical studies and experimental verifications. Over the past few decades various control algorithms and control devices have been developed, modified and investigated by various groups of researchers. Several well-established algorithms in control engineering have been introduced to control structures. While many of these structural control strategies have been successfully applied, technological problems and challenges relating to time delay, saturation capacity effects, the cost, the reliance on external power, and mechanical intricacy during the life of the structure have delayed their widespread use and relatively few structures are equipped with control systems.

In recent years the control of civil engineering structures has been investigated both theoretically and experimentally, and a lot of control systems have been installed in buildings, bridges and other structures [1]. The application of a control strategy to a structure involves, except efficient algorithms, hi-tech sensing and control devices, several other practical issues such as time delay and saturation of the control device.

Since the entire control process involves measuring response data, computing control forces by means of an appropriate algorithm, transmitting data and signals to actuators and activating the actuators to a specified level of force, time delays arise and cannot be avoided. The problem of time delay in the active control of structural systems has been investigated by many scientists and engineers [2–11]. In the work of Abdel-Rohman [2] it is shown how the stability of the structure could be lost due to time delay and two ways of time-delay compensation are suggested. In the first way the gain matrix is redesigned considering the presence of the time delay, while in the second low-pass filters are used to filter the velocity measurements from the frequency components of the high order modes. In the first case, the structure could remain unstable when using control moments as control actions and in the second, a number of vibrational modes can be controlled and compensated for time delay but the higher order modes remain uncontrolled. Sain et al. [3] compensate time delay with Pade approximations, while in the work of Agrawal et al. [4-6] the allowable time delay is related with natural period and feedback gain for a single degree of freedom system. The maximum allowable time delay is found to decrease with decrease in natural period of the structure as well as with increase in active damping, and a compensation of time delay by modeling it as transportation lag is suggested. Under earthquake excitations, simulation results for the response of multi degree of freedom structures indicate that the degradation of the control performance due to fixed time delay is significant when time delay is close to a critical value. It is further demonstrated that the time-delay problem is more serious for structures with closely spaced vibrational modes. In the work of Cai [7] an optimal control method for linear systems with time delay is developed. Time delay is considered at the very beginning of the control design, and no approximation and estimation are made in the control system. Thus, the system performance and stability are prone to be guaranteed. Instability in responses might occur only if a system with time delay is controlled by the optimal controller that was designed with no consideration of time delay. Furthermore, Pu [8] studied the influence of time delay to controlled base isolated structures. Through varied allocation of the controlled poles, the control system shows variable performance. However, the locations of the controlled pole pairs should therefore be carefully specified and checked according to the characteristics of the system. Analytical expressions for limited value of time delay, for single degree of freedom systems, are derived in the work of Connor [9], however such expressions are very difficult to obtain for
multi degree of freedom systems. In the work of Undwadia [10-12] a proportional control with positive feedback that uses intentional time delays, which may not necessarily be small compared with the natural periods of the structural system, is presented. All of these studies demonstrate how important the issue of time delay is in structural control and how it may result in a degradation of the control performance and may even render the controlled structure to be unstable. Most of the studies show that time delays are influence negatively the control system, therefore they should be kept small compared to the fundamental period of vibration of the system, and should, if possible, be eliminated and/or compensated.

Another important practical problem is the saturation of the control force. Actuator saturation occurs when the actuator is given a demand requiring an output greater than its designed peak output. Failure to account for this nonlinear effect can drive the structure unstable. Most control algorithms are linear, assuming that there is no limit in the magnitude of the control force. However, maximum capacity of the control devices is limited. Therefore, designing controllers to account for bounded nature of the devices is desirable. Many researchers have considered bounded controllers for control of civil engineering structures. Clipped optimal control derived from $H_2$/LQG [13], a polynomial controller to represent the bounded controller [14], modified bang bang controller [15], saturation control based on matrix inequalities [16], continuous and robust bounded controllers for active control in structures [17-19], probabilistic bounded non-linear control [20-21], saturation control of hysteretic structures [22-23] and evolutionary control of damaged systems using a rehabilitative algorithm [24] are some algorithms and techniques which are reported in the above studies.

From the above studies it is concluded that the two issues of time delay and saturation of the control device are in most cases considered and studied separately. However, in the application of real control systems these two issues act simultaneously. With this fact in mind, in this paper the interaction of the nonlinear phenomena of bounded capacity of the actuators and time delay of the system acting simultaneously during the control process is investigated. Their influence to the structural response is obtained and limits for time delay and saturation capacity are proposed, that can be used in the design process of controlled structures. The controlled algorithm that is used in the numerical simulations is a modified pole placement algorithm, developed by the authors [25-26].

2 CONTROL OF STRUCTURES ACCOUNTING FOR TIME DELAY AND LIMITED SATURATION CAPACITY

2.1 Time delay

The equation of motion of a structural system with n degrees of freedom controlled by m forces and subjected to an earthquake excitation $a_g$, without considering the time delay and limited saturation control capacity, is:

$$M\ddot{x}(t) + C\dot{x}(t) + Ku(t) = -ME_g(t) + E_f F(t)$$

(1)

where $M$, $C$, $K$ denote the mass, damping and stiffness matrices of the structure, respectively, $F$ is the control force matrix and $E_g$, $E$ are the location matrices for the earthquake and the control forces on the structure, respectively.

In the state space approach the above equation (1) can be written as follows.

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B_g a_g(t) + B_f F(t)$$

(2)

The matrices $X$, $A$, $B_g$, $B_f$ are given by:
Using a linear state feedback, the control force $F$ is given by

$$ F(t) = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} U(t) \\ \dot{U}(t) \end{bmatrix} = -K_f X(t) $$

(4)

$K_f$ is the feedback gain matrix, which is calculated according to a control algorithm. $k_1$ and $k_2$ are the sub-matrices of $K_f$ related to the displacement and velocity of the system, respectively. The control force, $F$, should be applied to the structure in a direct or indirect way, depending on the control device that is used.

Since the above control process involves measuring response data, $X(t)$, computing the feedback matrix, $K_f$, and control forces from the algorithm, transmitting data and signals to the actuators and activating the actuators to a specified level of force, a time delay $t_d$ arises that cannot be avoided. Accounting for time delay the control force $F$, instead of equation (4), is now given by

$$ F(t) = -K_f X(t - t_d) $$

(5)

From a structural point of view, the influence of time delay is to change the response of the controlled structure. From a mathematical point of view, time delay brings in additional terms in the eigenvalues of matrix $A$. This may cause the real part of an eigenvalue to become positive, consequently the system will be unstable.

For single degree of freedom systems analytical expressions for time delay, beyond which the system becomes unstable, can be obtained, [9]. For multi degree of freedom systems numerical simulations are needed in order to quantify the influence of time delay on the response of the controlled structure.

If $u_{max}$ is the maximum response of the uncontrolled system and $u_{max,td}$ is the maximum response of the controlled structure accounting for time delay, it is well known from the literature [4-7], that the variation of the ratio $u_{max,td}/u_{max}$ with respect to time delay $t_d$ is qualitatively described by a curve like the one shown in figure 1. In that figure it is shown that when an ideally controlled system is analyzed without considering time delay the maximum response is $u_{max,td=0}$, which is the lowest response that can be achieved. As time delay increases $u_{max,td}$ also increases. There is an upper bound of time delay, $t_{d,max}$, for which the response of the controlled system becomes equal to the response of the uncontrolled system. In order for the control to be meaningful, time delay should be much lower than $t_{d,max}$.

The influence of time delay depends also on the dynamic characteristics of the structure to be controlled. In the literature it is stated that, the larger the eigenperiod is, the higher margin of time delay exists to achieve the same reduction in the response of the structure. In this paper, the upper bound of time delay with respect to the fundamental period, for a system to have the same response reduction as the corresponding uncontrolled system subjected to sinusoidal and earthquake excitation, is going to be determined.

### 2.2 Saturation capacity of control force

One other parameter that influences also the response of the controlled structure is the maximum capacity of the control devices, $F_{sat}$. It may very often be the case that the control
algorithm calculates a control force that is higher than the maximum capacity of the control device. In that case the control force which will finally be applied to the structure will be the maximum capacity of the control device, $F_{\text{sat}}$. This phenomenon should be considered in the numerical simulation. Accounting for the maximum capacity of the device, the saturated control force, $\text{sat}F(t)$, is given by:

$$
\text{sat}F(t) = \begin{cases} 
F(t), & F(t) < F_{\text{sat}} \\
F_{\text{sat}}, & F(t) \geq F_{\text{sat}} 
\end{cases}
$$

(6)

Figure 1: The influence of time delay to the response of the controlled structure.

Figure 2. The influence of maximum capacity of the control devises, $F_{\text{sat}}$ on the response of the controlled structure.

Replacing the above expression for the control force into equation (2) the following equation is obtained

$$
\ddot{X}(t) = AX(t) + B_\gamma \dot{g}(t) + B_{\text{sat}} \text{sat}F(t)
$$

(7)
Solving the above equations numerically for different levels of saturation capacity, $F_{\text{sat}}$ it is found that, if $u_{\text{max,sat}}$ is the maximum response of the controlled structure accounting for force saturation then the variation of the ratio $u_{\text{max,sat}}/u_{\text{max}}$ with respect to the level of saturation capacity, $F_{\text{sat}}$ is described qualitatively by a curve like the one of figure 2. The lower the saturation capacity level $F_{\text{sat}}$ is, the larger the response of the controlled system becomes. There is an upper bound in saturation capacity of the device, $F_{\text{sat,max}}$, beyond which the performance of the system does not improve any more. The response of the controlled system, $u_{\text{max,sat,min}}$, in this case is the lower value that can be achieved. As a result, there is no need for the control device to have the ability to provide more force than the limit value $F_{\text{sat,max}}$.

### 2.3 Coupling of time delay and saturation capacity of control force

If the above two parameters, time delay and saturation capacity, are considered simultaneously, then equation (7) is highly nonlinear and their influence on the response of the uncontrolled structure can not be considered as linear superposition of each one acting separately. The coupling of time delay with saturation effect for structures subjected to earthquake action cannot be studied by means of analytical expressions for the solution of the equation (7).

In order to study the influence of those two parameters acting together, equation (7) is thus solved numerically, by means of a software program developed in MATLAB environment. Pole placement algorithm was chosen as a control algorithm for the analysis. The locations of poles of the controlled structure are based on the frequency content of the signal excitation. The main files, their function and the simulink model are shown in figure 3. The parameters of the system, mass, stiffness and damping matrices, the time delay, the maximum saturation control capacity, initial conditions and some other parameters related to the signal are given in the Control on line.m file. The state space formulation is also established in this file. As the signal arrives, it is analyzed for every small time interval by FFT process, and its spectrum is obtained. A transformation of the signal to the complex plane and the choice of the new location of poles are performed in the Selection of poles.m Matlab file. Then, going back to the main program, Control on line.m file, the calculation of feedback matrix based on pole placement algorithm is performed and the new locations of poles are obtained. After that, in the simulink file pole_place_mdof_on_line.mdl, a dynamic time history control analysis is performed and the response of the system for this time interval is calculated, the results are stored and the final state of the system is used as initial conditions for the next time period of earthquake signal. The procedure is repeated until the end of the signal. The above analysis is performed for a wide range of values of time delay and saturation capacity. Details about the above dynamic control analysis and the choice of the location of poles of the controlled structure can be found in the work of Pnevmatikos and Gantes [25-26].

In order to study the influence of time delay, saturation capacity and their coupling, a large number of numerical simulations were performed using the above software, and the results are described in the next session.

### 3 EXAMPLES AND NUMERICAL EXPERIMENTS

The above dynamic control strategy has been applied to a single, degree-of-freedom system, modeling buildings with the properties shown in figure 4, subjected first to sinusoidal and then to seismic actions.

A sinusoidal loading is first applied to the single degree of freedom system. The period of the loading is equal to the eigenperiod of the system. The response of the controlled system is calculated for a wide range of time delay, neglecting first the issue of saturation. Calculating
the ratio of the maximum response of the controlled system, $u_{\text{max,td}}$, to the maximum response of the uncontrolled one, $u_{\text{max}}$, with respect to time delay, $t_d$, figure 5 is obtained, verifying the negative influence of time delay. For low values of time delay the controlled response is also at low levels compared to the uncontrolled response. As time delay increases the response of the controlled system is also increasing until becoming equal or even larger than the response of the uncontrolled system. The maximum response of the controlled system without considering time delay, $u_{\text{max,td}=0}$, is at 2% of the response of the uncontrolled one (98% reduction). In this figure is also shown that, while time delay is between 1 ms and 13 ms, $(t_d,\text{min}=13\text{ms})$, the response is always situated to the minimum level (98% reduction). For $t_d$ larger than 13 ms the response starts increasing rapidly, up to a value of $t_d,\text{max}$, which is equal to 45 ms, where the response of the controlled system becomes equal to the response of the uncontrolled one. Beyond that value of time delay, $t_d,\text{max}$, the influence of control to the response is detrimental.

Figure 3: The main files, their function and the simulink model of the software accounting for time delay and saturation effects.

Figure 4: The simulation models and their dynamic characteristics.
Figure 5: The ratio of the maximum response of the one story controlled building subjected to sinusoidal excitation to the corresponding maximum response of the uncontrolled building with respect to time delay.

The same single degree of freedom system is then subjected to the Athens 1999 earthquake record shown in figure 6 and figure 7 is obtained. In this case, the initial reduction in the response without considering time delay is 85%. The value of time delay, $t_{d,min}$, where the response is kept at a minimum level equal to the initial one (85% reduction) is 11 ms. The value of time delay, $t_{d,max}$, for which the response of the controlled system becomes higher than the response of the uncontrolled one, is 28 ms. The earthquake excitation gives lower limits for time delay than the sinusoidal one.

Figure 6: Athens 1999 earthquake excitation.

During the design of the control system one requirement that should be checked is the value of existing time delay that the system itself has. This should be at least lower than the maximum value, $t_{d,max}$, so that the controlled system has lower response than the uncontrolled one. Depending on the desired level of performance of the controlled building, $u_{max,td}/u_{max}$, the existing time delay should not exceed a specific level of time delay, $td$, which is obtained by
the numerical simulation. The maximum performance of the controlled building can be achieved when the existing time delay, \( t_d \), is less than the value of \( t_{d,\text{min}} \).

\[
\frac{u_{\text{max},t_d}}{u_{\text{max}}} \quad \frac{u_{\text{max},t_d=0}}{u_{\text{max}}}
\]

\( t_d \) (sec)

Figure 7: The ratio of the maximum response of the one story control building to the maximum response of the uncontrolled building with respect to time delay subjected to Athens 1999 earthquake excitation.

In order to study the influence of time delay from the dynamic characteristics of the system, namely its eigenperiod, a sinusoidal excitation which is always in resonance with the system is applied. The results of these simulations are shown in figure 8. In this figure the required time delay, \( t_d \), with respect to eigenperiod, \( T \), for different systems which have the same reduction ratio \( \frac{u_{\text{max},t_d}}{u_{\text{max}}} \) equal to 30\% is presented. It is concluded that the higher the eigenperiod is, the longer the time delay becomes for the same level of performance. In other words, the acceptable time delay in tall buildings, which are in resonance with the excitation, is larger than in low buildings which are also in resonance with the excitation, for the same desired reduction of the response.

If the excitation is not in resonance with the system then figure 9 is obtained. In that case a sinusoidal signal with a period of 0.6 sec is applied on a system with period ranging from 0.1 sec to 1 sec. It is observed that the higher value of the acceptable time delay, for the same response reduction ratio, is when the system is in resonance, while when the system is out of resonance the acceptable time delay is lower. Applying the Athens 1999 earthquake a similar behavior is observed (figure 10).

In the above examples the force capacity was unlimited. Performing simulations with constant value of time delay but changing the level of control force capacity figures 11 and 12 are obtained. The values of the parameters \( F_{\text{sat,max}} \), normalized to the weight of building \( W \), and the ratio \( \frac{u_{\text{max,sat,min}}}{u_{\text{max}}} \) are summarizing in table 1. For these simulations it is clear that the lower the force capacity is the higher the response becomes. However, there is a limit of the saturation capacity, \( F_{\text{sat,max}} \), beyond which the response is not further decreased.
Figure 8: The required time delay, $t_d$, with respect to eigenperiod, $T$, in order, always, the one story building has the same reduction response ratio $u_{\text{max, } t_d}/u_{\text{max}}$, and be resonance with sinusoidal excitation.

Figure 9: The required time delay, $t_d$, with respect to eigenperiod, $T$, in order, always, the one story building has the same reduction response ratio $u_{\text{max, } t_d}/u_{\text{max}}$, not in resonance with sinusoidal excitation.

Figure 10: The required time delay, $t_d$, with respect to eigenperiod, $T$, in order, always, the one story building has the same reduction response ratio $u_{\text{max, } t_d}/u_{\text{max}}$, subjected to Athens earthquake excitation.
Figure 11: The reduction response ratio, $u_{\text{max,sat}}/u_{\text{max}}$, with respect to saturation capacity, $F_{\text{sat}}$, for one story building subjected to sinusoidal excitation.  

Figure 12: The reduction response ratio, $u_{\text{max,sat}}/u_{\text{max}}$, with respect to saturation capacity, $F_{\text{sat}}$, for one story building, subjected to Athens earthquake excitation.  

<table>
<thead>
<tr>
<th>One story building</th>
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<tbody>
<tr>
<td>Sinusoidal</td>
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<tr>
<td>$u_{\text{max,sat,min}}/u_{\text{max}}$</td>
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<tr>
<td>$F_{\text{sat, max}}/W$</td>
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Table 1: The values of parameters, $F_{\text{sat, max}}/W$, and the ratio $u_{\text{max,sat,min}}/u_{\text{max}}$. 

The value of the parameter $F_{\text{sat, max}}$ is 22% of the weight of the building, and the corresponding maximum performance of the controlled building is 28% of the response of the uncontrolled building. 

In real control systems, time delay and saturation of control force capacity exist simultaneously and influence each other. Simulations have been performed for a wide range of val-
ues of those two parameters and the ratio of the maximum response of the controlled system, $u_{\text{max,con}}$, to the maximum response of the uncontrolled one, $u_{\text{max}}$, was obtained. The results of those simulations for the single degree of freedom system subjected to Athens 1999 earthquake excitation are shown in figure 13. In this figure the influence to the response of coupling of these parameters is shown. It is noticed that there is a region $\Omega$, where the response is kept at low values compared to the response of the uncontrolled system. The values of time delay and saturation capacity in this region can be considered as the design specifications of the control devices and system that is going to be used. The border of this region depends on the desired performance level of the controlled structure. From this figure it is also shown that even though when the saturation capacity of the device is high and low response is expected, the simultaneous existence of high time delay causes instability to the buildings.

![Graph](image)

Figure 13: The influence to the response of the controlled one story building, for every couple of time delay and force capacity, subjected to Athens earthquake excitation.

These examples show the need of performing numerical simulations before installing the control system in the building in order to identify the limits of time delay and saturation capacity of the control devices which keep the building stable.

4 SUMMARY AND CONCLUSIONS

The influence of time delay and saturation capacity to the response of the controlled structure has been examined. Since there are no close forms for the response due to time delay and saturation capacity interaction, numerical simulations, before the installation of a control system, should be done accounting for those two important parameters. The numerical simulations will give limits of time delay and saturation capacity that should not be exceed, in order the response of controlled system will be lower than the uncontrolled one. From the numerical example that was executed, it was obtained that the influence on the response is different when time delay and saturation capacity are acting separately than when they acting together. This influence depends on the dynamic characteristic of the building and the characteristic of excitation. In the case where a coupling between time delay and saturation capacity exists, there is a region of values that can be used as a design specification for the control devices and control system that is going to be chosen to install in the building.
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