SIMPLE FRICTION MODEL FOR SCISSOR-TYPE MOBILE STRUCTURES

By Charis Gantes, 1 Associate Member, ASCE, Jerome J. Connor, 2 Member, ASCE, and Robert D. Logcher, 3 Fellow, ASCE

ABSTRACT: Mobile or articulated structures have kinematic connections that permit large relative displacements between components that undergo small elastic deformations. Deployable structures are one example. They can be stored in a compact, folded configuration and can be deployed into a load-bearing, open form by simple articulation. The deployable structures investigated in this paper are assemblages of basic units consisting of two straight bars connected to each other by a pivot that are called scissor-like elements. These structures are stress free and stable in both their folded and deployed configurations, but exhibit a geometrically nonlinear behavior during deployment and dismantling. This behavior is influenced significantly by friction between the bars of each scissor-like element as they experience a relative rotation about the pivotal connection. Because of the highly nonlinear nature of the structural response, modeling of the problem with contact-type finite elements is not only very expensive, but also of doubtful convergence. Therefore, a simple model is proposed, based on simulating the effect of friction by adding nonlinear rotational springs at the pivotal connections. Numerical results obtained from this model were found to be in very good agreement with experimental measurements.

INTRODUCTION

Mobile or articulated structures are defined as structural systems with kinematic connections permitting large relative displacements and/or rotations between components that undergo small elastic deformations (Yoo and Haug 1986). One category of such mobile structural systems are the so-called deployable structures, that can be transformed from a closed compact configuration to a predetermined, expanded form in which they are stable and can carry loads. Deployable structures can be grouped into two categories, surface structures that consist of 2-D building modules, and strut structures in which the basic modules are 1-D bars with truss or beam action.

The present paper is concerned with strut structures that are stress free in their folded configuration, develop stresses and exhibit a geometrically nonlinear behavior during deployment, and experience a snap through that locks them in their deployed configuration where they are self standing and stress free except for dead-weight effects. These structures were introduced by Krishnapillai and Zalewski (1985) and have been investigated from a structural point of view by the writers (Gantes 1988, 1991; Gantes et al. 1989, 1990a, b, 1991a, b, 1992, 1993a, b; Logcher et al. 1989; Rosenfeld et al. 1988). A simple structure of this type is shown in Fig. 1. It has the plan

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2Prof. of Civ. Engrg., Head of Constructed Fac. Div., Massachusetts Inst. of Tech., Cambridge, MA 02139.
3Prof. of Civ. Engrg., Massachusetts Inst. of Tech., Cambridge, MA.

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view of a regular pentagon. More complex structures of flat or curved shapes are possible.

The basic module of these structures is the so-called scissor-like element (SLE) shown in Fig. 2. It consists of two bars connected to each other at an intermediate point through a pivotal connection. At their four end nodes,
the bars are hinged to end nodes of other SLEs to form larger units. The type of hubs used for the hinged connections is illustrated in Fig. 3, which shows a perspective view of the hub and a plan view of the connection between the hub and a bar.

The nonlinear deployment response of these structures has been simulated with a large-displacement, small-strain finite-element formulation using the program ADINA ("ADINA-IN" 1987; Bathe 1982). The automatic load-incrementation method (Bathe 1983) was used in order to follow the complete nonlinear load-displacement path. Details of the numerical modeling and analysis have been reported by Gantes et al. (1990a, b, 1991a), and Gantes (1991).

Despite a gradual refinement of the finite-element model, significant differences were observed between numerical and experimental results. A dismantling test was performed for the deployable unit shown in Fig. 1. Details regarding the properties of the model and the testing procedure have been reported by Gantes (1991). Fig. 4 shows the experimental and initial numerical load-displacement curves for a Pentagon.
numerical load-displacement curves obtained for this unit. The lower center node of the structure was fixed against horizontal and vertical motion, while the upper center node could only move vertically, and was subjected to a vertical concentrated load that is recorded on the vertical axis of the graph. The horizontal axis records the relative vertical displacement between the two center nodes. The beginning of the curve corresponds to the deployed configuration, and the end to the folded one.

The finite-element mesh used for the finite-element analysis is illustrated in Fig. 5. Only one-fifth of the unit was analyzed because of symmetry. Isoparametric 2-node beam elements or 9-node shell elements were used to model the diagonal scissor-like elements that are subjected to combined axial and flexural stresses. Beam elements provided satisfactory results for
As mentioned above, deployable structures consist of scissor-like elements that allow relative rotations of their two bars about the pivotal connection. The two bars of an SLE are theoretically straight lines that lie in a plane. In reality, however, the bars have cross sections with discrete widths, while their end nodes are forced to lie in a common plane, due to boundary conditions. This causes deformation of the bars as illustrated in Fig. 6, which shows a perspective and a top view of a scissor-like element. Because of this deformation, transverse forces between the two bars are generated. When there is a relative rotation between the two bars about the pivotal connection, these forces produce friction forces over the contact surface of the bars. These friction forces add up to a total friction moment that resists the modeling of members with nearly square cross sections, but led to convergence difficulties for members with thin cross sections in the presence of initial geometric imperfections because of an insufficient formulation of warping effects. This difficulty was overcome through the use of shell elements. The circumferential members are only subjected to tension, and were modeled with truss elements. The influence of discrete joint sizes has been taken into account in the finite-element analysis.

The difference between the two curves was attributed to the lack of modeling of friction effects in the numerical analysis. As could be observed from the behavior of the physical model, this effect is very important and has an increasing influence near the folded configuration of the structure, where, combined with the discrete width of cross sections of the members, it prevents the complete folding. The numerical curve, on the other hand, followed the theoretical behavior of this type of structure, characterized by a reversal in the sign of the load near the folded configuration. The present paper describes a simple friction model that was included in the numerical analysis resulting in a very close match with experimental measurements.

**Main Friction Mechanism**

As mentioned above, deployable structures consist of scissor-like elements that allow relative rotations of their two bars about the pivotal connection. The two bars of an SLE are theoretically straight lines that lie in a plane. In reality, however, the bars have cross sections with discrete widths, while their end nodes are forced to lie in a common plane, due to boundary conditions. This causes deformation of the bars as illustrated in Fig. 6, which shows a perspective and a top view of a scissor-like element. Because of this deformation, transverse forces between the two bars are generated. When there is a relative rotation between the two bars about the pivotal connection, these forces produce friction forces over the contact surface of the bars. These friction forces add up to a total friction moment that resists...
rotation. This happens with all SLEs of the structure during deployment and is the main mechanism of friction.

Solution of contact problems with and without friction is an active research area for many finite element investigators (Eterovic and Bathe 1991a, b; Fredriksson 1976, Fredriksson et al. 1984; Rothert et al. 1985; Torstenfelt 1983). The general idea is to require minimization of the total potential energy to formulate the contact problem so that the algorithm can be easily implemented in finite-element packages. Friction however, is a nonconservative force, therefore, an incremental approach has to be used, and the assumption of piecewise conservative friction forces should be made. In addition, iterations are required within each increment, since the contact area is not known in advance. The problem is further complicated when the structural behavior is nonlinear. Therefore, the available numerical algorithms for contact problems with friction suffer some important limitations, including high computational cost and convergence difficulties (Bathe, personal communication, 1990; Eterovic and Bathe 1991a, b).

Hence, and since our main interest was the macroscopic influence of friction on the structural behavior, we decided to adopt a simpler modeling approach that is illustrated in Fig. 7. It consists of adding at all pivotal connections, nonlinear rotational springs that simulate the effect of friction. Such an approach can be sufficient to describe the overall influence of friction on the structural response, although it cannot capture local effects. A more refined model based on improved contact algorithms could be used in the future to identify areas of local wear that require strengthening.

**PROPOSED FRICTION MODEL**

The overall approach to the problem consists of two steps. The first is the derivation of the frictional moments that resist rotation at the pivotal and hinged connections as functions of the constant geometry and material characteristics, and of the varying angle between the two bars of an SLE during deployment. The second step is the derivation of appropriate stiffness

![Fig. 7. Proposed Friction Model for SLE](image-url)
functions for a nonlinear rotational spring or a pair of nonlinear translational springs that can be included in the numerical model and produce resisting moments equal to those due to friction.

Although some more sophisticated friction theories exist (Curnier 1984), the simple Coulomb theory (Bowden and Tabor 1967) is used for the sake of simplicity. Following this assumption, the transverse forces between the two bars—due to the deformed shapes they are forced to assume because of their discrete width—are calculated first. To accomplish this, some further assumptions must be made about these deformed shapes. Let us first take a look at a detail of a hinged connection as illustrated in Fig. 3. The width of the bar is $b$, while the slot of the hub is a little larger, having a width of $b_h$. Assuming that the connecting pin also has a small margin, some rotation of the bar takes place until its sides are in contact with the walls of the slot of the hub.

It was further assumed that at the pivotal connection there is a small gap.
between the bars caused by a tiny margin in the length of the pin and by
the extensibility of its material. In other words, the pin, which is under
tension, elongates slightly so that there is no contact between the two bars
at the connection. This led us to the deformed configuration of one of the
bars of an SLE illustrated in Fig. 8. The bar can be thought of as consisting
of six segments. The first and the sixth are in the slots of the end hubs and
are in contact with these hubs only at their end points. Segments 2 and 3
and segments 4 and 5 are separated by the contact points with the other
bar of the SLE. Segments 3 and 4 are separated by the pin. Because of the
elongation of the pin, there is no continuous contact surface between the
two bars. There is only contact along the lines (AB), (BC), (CD), and
(DA), shown in the more detailed Fig. 9.

Following these assumptions, three sources of friction could be identified:
friction between bars 1 and 2 in the form of line forces at the interface;
friction between the bars and the hubs, also as line forces along the contact
lines; friction between the bars and the pins at both pivotal and hinged
connections. All three sources of friction create moments that resist relative
rotation of the bars during deployment. The lever arm for the friction forces
between bars and pins is much smaller than those corresponding to the first
two sources. In addition, the friction coefficient for the third friction source
can be easier minimized through lubrication. Therefore, the third friction
mechanism was neglected and only friction between the two bars and be-
tween bars and hubs was taken into account.

Fig. 10 shows in more detail the assumed deformed configuration of a
bar due to the discrete cross section width. Let \((X, Y)\) be a global Cartesian
coordinate system, while \((X_i, Y_i), i = 1, \ldots, 6\) are local systems for each
of the six segments. Note that

\[
v_0 = \frac{b_h - b}{2} \tag{1}
\]

\[
v_1 = \frac{b}{2} = 0.5b \tag{2}
\]

\[
v_2 = \kappa b \tag{3}
\]

where \(\kappa\) = numerical factor slightly larger than 0.5 that depends on the
extensibility of the pin at the pivotal connection. For (2) and (3), it is

![FIG. 10. Details of Deformation of Bars Due to their Discrete Width](image.png)
inherently assumed that the two bars are of approximately the same length and the same stiffness.

It can further be assumed that the six segments that constitute the bar have cubic shapes, expressed in the local coordinate systems as

\[ v(X_i) = a_iX_i^3 + b_iX_i^2 + c_iX_i + d_i; \quad i = 1, \ldots, 6 \]  \hspace{1cm} (4)

Then the slope is given by

\[ v'(X_i) = 3a_iX_i^2 + 2b_iX_i + c_i; \quad i = 1, \ldots, 6 \]  \hspace{1cm} (5)

while the bending moment distribution is

\[ M(X_i) = EI(6a_iX_i + 2b_i); \quad i = 1, \ldots, 6 \]  \hspace{1cm} (6)

The 24 unknown coefficients \( a_i, b_i, c_i, d_i, i = 1, \ldots, 6 \) can be calculated from boundary conditions: known end displacements at each segment; moment and slope continuity at interfaces between segments; and zero moment at the two ends due to the hinged connections.

These boundary conditions result in a system of the 24 linear equations for the 24 unknowns. The system can be solved with Gauss elimination. Then, the bending moment and shear-force diagrams can be obtained, and the concentrated transverse forces can be calculated as shown in Fig. 11, where

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**FIG. 11. Deformation, Moment, Shear, and Free Body Diagrams of Bar**
\( V_i = 6a_i EI; \quad i = 1, \ldots, 6 \) ................................................ (7)

\( F_i = 6(a_i - a_{i-1}) EI; \quad i = 1, \ldots, 7 \) ................................................ (8)

where,

\( a_0 = a_7 = 0 \) ................................................ (9)

It should be noted however, that the lengths \( L_i, i = 1, \ldots, 6 \) are not constant along the height of the beam. Referring to Fig. 9, the lines of contact between the two beams of the SLE are not perpendicular to the longitudinal axes of the beams. Therefore, we should either use some average values for the lengths \( L_i \), or calculate the exact values of \( L_i \) referring to a specific longitudinal fiber of the beam, and then integrate along the height \( h \) of the beam. Both approaches were carried out, and gave what are called an approximate and an exact model, respectively, for the desired stiffness of the rotational spring.

**Exact Model for Pivotal Connection between Two Bars**

In the exact model, some known lengths, depending on the geometry of the beam under investigation and the hubs used, are defined in Fig. 12. \( B \) and \( C \) are the lengths between the three pivots of the beam. \( A \) is the distance between the pivots at the hinges and the ends of the beam. Finally, \( D \) is the distance between the pivots at the hinges and the edges of the hubs. Then, the lengths \( L_i \) for a fiber of width \( dy_3 \) illustrated in Fig. 9 can be calculated as follows:

\[
x_3 = \frac{h}{2 \sin \omega_3} = \frac{h}{2 \cos(\omega_3 - 90^\circ)} .............................. (10)
\]

Then

\[
L_1 = A + D ............................................................... (11)
\]

\[
L_2 = B - D - x_3 \cos[(180^\circ - \phi) - \omega_3] = B - D + \frac{h \cos(\phi + \omega_3)}{2 \sin \omega_3} .......................... (12)
\]

\[
L_3 = \frac{h}{2 \sin \phi} ............................................................. (13)
\]

\[
L_4 = \frac{h}{2 \sin \phi} ............................................................. (14)
\]

\[
L_5 = C - D - \left\{ \frac{h}{\sin \phi} - x_3 \cos[(180^\circ - \phi) - \omega_3] \right\} = C - D
\]

![FIG. 12. Definition of Characteristic Beam Dimensions](image-url)
\[
- \frac{h}{\sin \phi} - \frac{h \cos(\phi + \omega)}{2 \sin \omega} \quad \text{.................................. (15)}
\]

\[
L_6 = A + D \quad \text{........................................... (16)}
\]

Now, we can proceed assuming that the coefficients \(a_i, b_i, c_i, d_i, i = 1, \ldots, 6\) of (4)-(6), and hence, the forces \(F_i, i = 1, \ldots, 7\) of (8), are known for that particular fiber. Then, corresponding friction forces are obtained by multiplying with the friction coefficient \(\mu\).

\[
dT_3 = \mu \, dF_3 = \frac{1}{2} \, \mu b^3 E (a_3 - a_2) \, dy_3 \quad \text{........................................... (17)}
\]

And, since

\[
dy_3 = - \frac{x_3}{\cos(\phi + \omega)} \, d\omega_3 \quad \text{........................................... (18)}
\]

follows:

\[
dT_3 = - \frac{1}{2} \, \mu b^3 E (a_3 - a_2) \, \frac{x_3}{\cos(\phi + \omega)} \, d\omega_3 \quad \text{........................................... (19)}
\]

Then

\[
dM_3 = 2x_3 \, dT_3 = \frac{1}{4} \, \mu b^3 h^2 E \, \frac{a_2 - a_3}{\sin^2 \omega_3 \cos(\phi + \omega)} \, d\omega_3 \quad \text{........................................... (20)}
\]

The exact value of the total resisting moment due to friction is

\[
M_3^{\text{ex}} = \int_{\omega_3^{\text{1}}}^{\omega_3^{\text{2}}} dM_3 = \frac{1}{4} \, \mu b^3 h^2 E \int_{\omega_3^{\text{1}}}^{\omega_3^{\text{2}}} \frac{a_2 - a_3}{\sin^2 \omega_3 \cos(\phi + \omega)} \, d\omega_3 \quad \text{........................................... (21)}
\]

where

\[
\omega_3^{\text{1}} = 90^\circ - \frac{\phi}{2} \quad \text{........................................... (22)}
\]

\[
\omega_3^{\text{2}} = 180^\circ - \frac{\phi}{2} \quad \text{........................................... (23)}
\]

FIG. 13. Detail of Hinged Connection
Since no analytical expressions are available for \( a_2 \) and \( a_3 \), numerical integration is required for the calculation of the moment \( M_x^e \). The values of \( a_2 \) and \( a_3 \) are calculated with Gauss elimination at each step of the numerical integration.

**Exact Model for Hinged Connections between Bars and Hubs**

To derive an expression for the friction moment between bar and hub, consider Fig. 13. In this case, we can make the assumption that the contact line between the bar and the hub is always perpendicular to the bar. Then, the moment can be calculated as follows:

\[
M_x^e = \int_{\omega_1}^{\omega_1'} x_1 \, dT_1 + \int_{\omega_2}^{\omega_2'} x_2 \, dT_2 
\]

(24)

where

\[
\omega_1 = 0 
\]

(25)

\[
\omega_1^2 = 2 \tan^{-1} \left( \frac{h}{2A} \right) 
\]

(26)

\[
\omega_2 = 0 
\]

(27)

\[
\omega_2^2 = 2 \tan^{-1} \left( \frac{h}{2D} \right) 
\]

(28)

Following the same reasoning, we obtain

\[
M_x^e = \frac{1}{2} \mu E b^3 \left[ A^2 \int_{\omega_1}^{\omega_1'} \frac{a_1}{\cos^3 \left( \frac{1}{2} \omega_1^2 - \omega_1 \right)} \, d\omega_1 
\]

\[
+ D^2 \int_{\omega_2}^{\omega_2'} \frac{a_1 - a_2}{\cos^3 \left( \frac{1}{2} \omega_2^2 - \omega_2 \right)} \, d\omega_2 \right] 
\]

(29)

**Approximate Model for Pivotal Connection**

Using the approximate model, the numerical integration can be avoided if we assume that the contact line at the pivotal connection between the two bars is \((A'B')\) instead of \((AB)\) (Fig. 9), and that the friction force is parallel to \((A'B')\) and constant along the height of the beam. Then, the lengths \( L_i \) are a function of the known lengths \( A, B, C, D, h, \) and the variable angle \( \phi \)

\[
L_1 = A + D 
\]

(30)

\[
L_2 = B - D - \frac{h}{2 \sin \phi} 
\]

(31)

\[
L_3 = \frac{h}{2 \sin \phi} 
\]

(32)
Moment due to friction [lb.in]

'exact' model

approximate model

FIG. 14. Variation of Friction Moment During Dismantling

\[ L_4 = \frac{h}{2 \sin \phi} \] ............................................... (33)

\[ L_5 = C - D - \frac{h}{2 \sin \phi} \] ............................................... (34)

\[ L_6 = A + D \] ............................................... (35)

In that case

\[ T_3 = \mu F_3 = \frac{1}{2} \mu h b^3 E (a_3 - a_2) \] ............................................... (36)

The approximate expression for the friction moment at the pivot is

\[ M_{\text{approx}} = 2T_3 \frac{h}{2 \sin \phi} = \frac{T_3 h}{\sin \phi} = \frac{1}{2} \mu h^2 b^3 E \frac{a_3 - a_2}{\sin \phi} \] ............................................... (37)
Approximate Model for Hinged Connections

The frictional moment at the connection between a bar and a hub can be calculated in a similar manner

\[
M_{\text{approx}} = T_1A + T_2D = \mu(F_1A + F_2D) = \frac{1}{2} \mu hb^3E[a_1A + (a_1 - a_2)D]
\]

\[.................................(38)\]

IMPLEMENTATION OF PROPOSED MODEL

In the previous section, simple expressions for the resisting moments that are produced at the pivotal and hinged connections of deployable structures due to friction during the deployment and dismantling processes were derived. Calculation of these moments as a function of the angle \(\phi\) between the bars of the diagonal SLE for the pentagonal unit of Fig. 1 produced some very interesting results. The friction at the hinges between bars and hubs gave a moment that was smaller than 5% compared to the moment at the pivot. Therefore, the friction at the hubs was neglected for the rest of the calculations. The variation of the moment at the pivot during dismantling is shown in Fig. 14. A reasonably good agreement between exact and approximate model can be observed. As expected, the moment is minimum for \(\phi = 90^\circ\), when the contact area between the two bars is minimum. The moment increases rapidly near the folded configuration, when the dis-
The concrete width of the bars causes large transverse forces between them, and prevents them from folding completely.

Next, this friction model was implemented into the finite-element model of the pentagonal unit structure. The initial idea was to attach a nonlinear rotational spring connecting the two nodes used to model the pivotal connection. The stiffness of that spring could be calculated from

\[ k_{\text{rot}}(\phi) = \frac{M_3}{\phi - \phi_0} \]

where \( \phi_0 = \) angle between the two bars of the SLE in the deployed configuration.

This approach however, was not possible, because of lack of availability of nonlinear rotational spring elements in ADINA. Therefore, an alternative solution was employed. A pair of nonlinear translational springs attached between the points \((RS)\) and \((UT)\) were used to simulate the rotational spring, as illustrated in Fig. 15. This requires the modeling of the beams with shell elements, so that nodal points existed at \(R, S, T,\) and \(U\). For reasons of conformity with the acceptable input mode of ADINA, the spring...
stiffness had to be described in terms of its stress-strain law instead of its force-elongation relation. The derivation of an appropriate stress-strain law for the springs had to take into account the rate in the change of distance $d_{RS}$ between $R$ and $S$, in other words, the change of the length of the spring, as a function of $\phi$. In addition, the change in the distance $d_{(RS)-(UT)}$ between the two springs ($RS$) and ($UT$) had to be expressed in terms of $\phi$. These quantities are as follows:

$$d_{(RS)-(UT)} = h \cos \frac{\phi}{2}, \quad d_{RS} = h \sin \frac{\phi}{2} \quad \cdots \quad (40)$$

The desired variation of the spring force during dismantling is given by

$$F_{RS} = \frac{M_3}{d_{(RS)-(UT)}} \quad \cdots \quad (41)$$

and the spring stiffness is
Finally, since the required input format of the nonlinear spring in the finite-element model is through a stress-strain diagram, the stress $\sigma_{eq}$ and strain $\varepsilon_{eq}$ of the spring ($RS$) were expressed as functions of $\phi$, and plotted as shown in Fig. 16

\[
\sigma_{eq} = \frac{F_{RS}}{A_{spr}}, \quad \varepsilon_{eq} = \frac{\Delta d_{RS}}{d_{RS_0}} = \frac{\sin \frac{\phi}{2}}{\sin \frac{\phi_0}{2}} - 1 \quad \cdots (43)
\]

where $A_{spr}$ is the spring cross-sectional area.

**Verification of Proposed Numerical Model**

To verify the proposed model, a numerical analysis of the pentagonal unit that had been tested earlier was carried out. The members of this unit were made of high density polyethylene (HDPE). The friction coefficient between these members was obtained experimentally and found equal to $\mu = 0.36$. For input into the finite-element model, the stress-strain curve was approximated by a multilinear curve as illustrated in Fig. 16. Implementing this friction model into the finite-element model gave a load-displacement curve that is in excellent agreement with the experimental results as shown in Fig. 17.

**Summary and Conclusions**

Friction between the members is a major concern during the folding and unfolding process of deployable structures consisting of scissor-like elements. The nonlinear character of the response makes the application of contact finite-element analysis very troublesome for this problem. This led to the development of a simple, approximate model that simulates the macroscopic effects of friction through the addition of nonlinear rotational springs or pairs of nonlinear translational springs at all pivotal connections. The results provided by this model for the analysis of a simple deployable structure were found to be in very good agreement with experimental measurements. It is believed that the proposed model can be used for any type of mobile structures consisting of scissor-like elements.

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**Appendix I. References**


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APPENDIX II. NOTATION

The following symbols are used in this paper:

\[ A = \text{distance between end of member and pin for hinged connection;} \]
\[ A_{spr} = \text{cross-sectional area of translational spring;} \]
\[ a_i = \text{coefficient of cubic term in deformation polynomial;} \]
\[ B = \text{distance between pin for hinged connection at one end and pin for pivotal connection;} \]
\[ b = \text{width of member cross section;} \]
\[ b_h = \text{width of slot of hub at hinged connection;} \]
\[ b_i = \text{coefficient of quadratic term in deformation polynomial;} \]
\[ C = \text{distance between pin for hinged connection at other end and pin for pivotal connection;} \]
\[ c_i = \text{coefficient of linear term in deformation polynomial;} \]
\[ D = \text{distance between end of hub and pin for hinged connection;} \]
\[ d_i = \text{constant term in deformation polynomial;} \]
\[ d_{RS} = \text{length of translational spring;} \]
\[ d_{RS_0} = \text{initial length of translational spring;} \]
\[ d_{(RS)-(UT)} = \text{distance between the two translational springs;} \]
\[ d_y = \text{width of the fiber under consideration at pivotal connection;} \]
\[ E = \text{Young's modulus of members;} \]
\[ F_i = \text{transverse forces between members due to deformation caused by discrete width;} \]
\[ F_{RS} = \text{force in translational spring;} \]
\[ h = \text{height of member cross section;} \]
\[ I = \text{moment of inertia of member cross section for deformations out of SLE plane;} \]
\[ k_{eq}^{rot} = \text{stiffness of equivalent rotational spring;} \]
\[ k_{eq}^{tr} = \text{stiffness of equivalent translational spring;} \]
\[ L_i = \text{lengths of the six segments constituting member;} \]
\[ M = \text{bending moment in members due to deformation caused by discrete width;} \]
\[ M_{12}^{pp} = \text{friction moment at hinged connection according to approximate model;} \]
\[ M_{12}^{eq} = \text{friction moment at hinged connection according to exact model;} \]
\[ M_j = \text{friction moment at pivotal connection;} \]
\[ M_{3}^{pp} = \text{friction moment at pivotal connection according to approximate model;} \]
\[ M_3^{eq} = \text{friction moment at pivotal connection according to exact model;} \]
\[ T_i = \text{friction force at pivotal connection (end of member;)} \]
\[ T_2 = \text{friction force at hinged connection (end of hub;)} \]
\[ T_3 = \text{friction force at pivotal connection;} \]
\[ V = \text{shear force in members due to deformation caused by discrete width;} \]
\[ v = \text{deflection of members due to their discrete width;} \]
\[ v_i = \text{values of } v \text{ at interfaces between the six segments;} \]
(X, Y) = global cartesian coordinate system;
(X_i, Y_i) = local Cartesian coordinate systems;

x_1 = lever arm for friction force between member and hub at end of member;

x_2 = lever arm for friction force between member and hub at end of hub;

x_3 = lever arm for friction force at pivotal connection;

\( \varepsilon_{\text{eq}} \) = strain of equivalent translational spring;

\( \kappa \) = numerical factor slightly larger than 0.5 depending on relative stiffnesses of members and pins, indicating distance between two bars at pivotal connection;

\( \mu \) = friction coefficient between bars of SLE;

\( \sigma_{\text{eq}} \) = stress of equivalent translational spring;

\( \phi \) = angle between two bars of scissor-like element;

\( \phi_0 \) = angle between two bars of scissor-like element in deployed configuration;

\( \omega_1 \) = critical integration angle for friction between member and hub at end of member;

\( \omega_2 \) = critical integration angle for friction between member and hub at end of hub;

\( \omega_3 \) = critical integration angle for friction between members at pivotal connection;

\( \omega_1^\prime \) = minimum value of angle \( \omega_1 \) for given angle \( \phi \);

\( \omega_1^\prime \) = maximum value of angle \( \omega_1 \) for given angle \( \phi \);

\( \omega_2^\prime \) = minimum value of angle \( \omega_2 \) for given angle \( \phi \);

\( \omega_2^\prime \) = maximum value of angle \( \omega_2 \) for given angle \( \phi \);

\( \omega_3^\prime \) = minimum value of angle \( \omega_3 \) for given angle \( \phi \); and

\( \omega_3^\prime \) = maximum value of angle \( \omega_3 \) for given angle \( \phi \).