Nonlinear in-plane behavior of circular steel arches with hollow circular cross-section

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Received 10 June 2007; accepted 9 January 2008

Abstract

In this work the nonlinear in-plane behavior of circular arches with hollow circular cross-section is investigated. The influence of a number of design parameters, such as the boundary conditions, the rise-to-span ratio, and the included angle on the strength is presented. Moreover, the effect of other behavior factors, such as the geometrical and material nonlinearities and the initial imperfections, is investigated. A criterion for the prediction of the type of nonlinear behavior of arches is given, and a formula for the determination of the nonlinear buckling load is proposed. It is found that the effect of initial imperfections on the strength depends largely on the arch slenderness and the imperfection magnitude in the case of shallow arches. When arches are deep this dependence becomes less significant. The effect of geometrical nonlinearity depends significantly on the shallowness and the slenderness of the arches. Stocky arches are less influenced by the rise-to-span ratio than slender ones. The effect of boundary conditions depends significantly on the shallowness of arches and the arch slenderness. The reduction of strength is larger in slender arches than in stocky ones.

Keywords: Circular arches; Nonlinear buckling; Strengths; Steel; Snap-through; Bifurcation

1. Introduction

Steel arches can exhibit interesting behavior characteristics when they are loaded with a variety of load patterns. The most common load case that is encountered in the bibliography concerns the uniformly distributed radial load. For this case linear buckling loads have been calculated by Timoshenko and Gere [1], while geometrically nonlinear buckling loads and load deflection curves for clamped arches have been given by Schreyer and Masur [2]. These researchers provided exact solutions for the nonlinear elastic equilibrium equations for clamped, circular, shallow arches with rectangular cross-section, together with a detailed analysis of the various buckling criteria that distinguish the different possible buckling types arches can exhibit. Moreover, they showed that, contrary to previous belief, the existence of an asymmetric buckling mode is not a sufficient condition for such instability to actually take place.

Besides the uniformly distributed radial load, arches may be subjected also to other kinds of loadings, both distributed and concentrated, symmetrical or asymmetrical, and they resist such loads by a combination of bending and axial compression effects. Pi and Trahair [3], using a finite element program for the nonlinear inelastic analysis, investigated the in-plane inelastic buckling and strength of circular steel I-section arches. The effects of initial imperfections, rise-to-span ratio, residual stresses, and dead load to total load ratio on the in-plane inelastic stability of steel arches in uniform compression and in combined compression and bending were addressed. It was found that the effect of a modified slenderness \( \lambda_f \), is important for the inelastic strength, which decreases as the modified slenderness increases. The effect of initial imperfections on the inelastic strength depends on whether bending or axial compression is the major action. When axial compression governs, the effect of the initial imperfections on the strength is significant, as in the case of arches in uniform axial compression or subjected to loads uniformly distributed along the horizontal projection of the entire arch. When bending is critical, as in the case of arches subjected to unsymmetrical distributed loads or to single concentrated loads, the effect of initial imperfections is less important. It was found here that the effect of initial imperfections on the
strength depends largely both on the slenderness $\lambda_T$ and the magnitude of the imperfections when arches are shallow. When arches are deep this dependence is becoming less significant. The effect of the rise-to-span ratio $f/L$ on the strength is not important for deep arches in uniform compression, but it is significant for shallow arches with a small rise-to-span ratio and small modified slenderness. The effect of the rise-to-span ratio on the strength is noticeable for slender arches subjected to symmetrical loads, but not important for arches subjected to unsymmetrical loads and for arches with a low modified slenderness. The strength reduction effect of residual stresses is important when compression is the major action, but less important when bending is the major action.

The nonlinear buckling and postbuckling of elastic I-section arches has been thoroughly investigated by Pi and Trahair [4]. However, this investigation was concerned with uniformly distributed radial load only, using a curved finite element model for the nonlinear analysis of elastic arches. It was found that the effects of prebuckling deformations on the buckling of shallow elastic arches are significant, that the existence of a linear bifurcation buckling load is not a sufficient condition for linear bifurcation buckling to occur, and that the nonlinear buckling loads of shallow arches may be much lower than the corresponding linear buckling loads.

Accurate solutions and approximations for the nonlinear elastic symmetric and anti-symmetric buckling of both pin-ended and fixed shallow arches with arbitrary cross-section have been provided by Pi et al. [5]. Moreover, formulae for the in-plane anti-symmetric bifurcation buckling load of non-shallow arches were proposed, and criteria for making the distinction between shallow and non-shallow arches were also stated. Comparisons of the approximate solutions with finite element results demonstrated the validity of the paper’s analytical results. Furthermore, it was shown that classical buckling theory can correctly predict the in-plane anti-symmetric bifurcation buckling load of non-shallow arches, but overestimates the in-plane anti-symmetric bifurcation buckling load of shallow arches significantly.

In all of the aforementioned investigations supports were either pin-ended or fixed. An analytical investigation of elastically supported arches was presented by Pi et al. [6]. Using a virtual work formulation they proposed closed form solution for the asymmetric bifurcation buckling and for the snap-through buckling. The effect of the elastic supports was found to be significant.

In the aforementioned references, investigation was limited mainly to I-type sections and to uniformly distributed radial loading. In this paper an effort is made to extract the basic characteristics of the behavior of circular steel arches with a hollow circular cross-section subjected to uniformly distributed vertical load along the horizontal projection of the entire arch, a case frequently encountered in practice (Fig. 1). The methodology used in the aforementioned articles [4] and [5] is adopted.

Among others, a criterion for the prediction of the type of nonlinear behavior of arches is given and a formula for the determination of the nonlinear buckling load is proposed. Moreover, the effect of various factors, such as the initial imperfections, the rise-to-span ratio, the geometric nonlinearities and the boundary conditions, on the strength of the arches is investigated.

2. Methodology

For the purposes of this paper, the nonlinear finite element analysis program, ADINA [7], has been used for all numerical analyses. The arches are modeled with a sufficient number of straight Hermitian beam finite elements. Firstly, linear static analyses are carried out for an initial estimation of the compression and bending effects due to the transversal loading. Then, linearized buckling analyses are performed in order to calculate the first asymmetric buckling mode and the corresponding buckling load $q_{LB}$. A symmetric eigenvector is the critical one if arches are very shallow and stocky, e.g. $2\Theta \leq 10^\circ$ and $0.55/i \leq 40$. However, this possibility is ignored in the subsequent analyses. The calculated eigenvector is used as an initial imperfection for the subsequent nonlinear analyses.

The magnitude of the initial imperfection is considered equal to either $L/600$ or $L/50000$, where $L$ is the span length of the arch. The $L/600$ magnitude is in accordance to the provisions of EC3 [8] and is used in all analyses, except in the cases where the initial imperfection is used merely as an instability factor, to trigger possible anti-symmetric buckling. In these cases the initial imperfection’s magnitude is taken equal to $L/50000$. Nonlinear elastic analyses, using the aforementioned magnitude for the imperfections, are performed in order to investigate the geometrically nonlinear behavior of steel arches. The effect of the material nonlinearity is investigated by performing geometrically linear inelastic analyses. Then, nonlinear large displacement, inelastic analyses are carried out for the calculation of the strengths of the arches. Finally, for the estimation of the effect of initial imperfections geometrically and materially nonlinear analyses are performed with a varying initial imperfection $\xi$ from 0 to $L/300$.

For the solution of the nonlinear system of equations, the well-known load–displacement-control method (LDC) [7] is used, in combination with the full Newton–Raphson procedure and the use of line searches. An elastic isotropic material with modulus of elasticity $E = 210$ GPa and Poisson’s ratio $\nu = 0.30$ is used for the elastic analyses, and an inelastic bilinear material with yield limit $f_y = 275$ MPa and without any
hardening for the inelastic analyses (Fig. 2). A hollow circular cross-section with a ratio of external diameter \( D \) to thickness \( t \) sufficiently small to avoid local buckling effects is used throughout this paper (Fig. 1). For this type of cross-section it is appropriate to assume that there are no residual stresses \[9\].

3. First-order actions

Circular arches subjected to a uniformly distributed load on the horizontal projection of the entire span resist in a combination of compression and bending actions (Fig. 3). To investigate the characteristics of the response, three groups of arches with \( 0.5S/i = 40, 100, 170 \) are used, where \( S \) is the length of the arch and \( i \) the in-plane radius of inertia of the cross-section, \( i = \sqrt{I/A} \), with \( I \) the second moment of area and \( A \) the cross-section area. In Fig. 4 the dimensionless compression action \( (N_{\text{max}}/N_Y)/(N_{\text{max}}/N_Y + M_{\text{max}}/M_p) \) is plotted on the vertical axis, with respect to the rise-to-span ratio \( f/L \), plotted on the horizontal axis, where \( N_Y = Af_y \) is the squash load of the cross-section, \( M_p \) is the plastic moment of the cross-section, and \( N_{\text{max}}, M_{\text{max}} \) are the maximum absolute values of compression and bending actions, respectively.

Three different cases of arch slenderness are considered, corresponding to \( \lambda_T = 1.958, 1.152, 0.461 \). The slenderness \( \lambda_T \) of the arch is defined as:

\[
\lambda_T = \sqrt{\frac{N_Y}{N_o}}
\]

where \( N_o \) is a nominal elastic critical load, defined as:

\[
N_o = \frac{\pi^2 EI}{(0.5S)^2}
\]

so that

\[
\lambda_T = \frac{0.5S}{i} \sqrt{\frac{f_y}{(\pi^2E)}}
\]

It can be seen that very shallow arches resist mainly with compression actions, while deep arches with bending actions. Moreover, it is observed that there is a value of the rise-to-span ratio, depending on the slenderness, for which the compression action obtains its maximum value. Finally, the more slender an arch is, the larger the bending actions are in the case of deep arches, and the larger the compression actions are in the case of shallow arches.

In order to gain further insight on the effect of slenderness on the first-order actions of arches, five groups of arches with subtended angles \( 2\Theta = 10^\circ, 50^\circ, 90^\circ, 140^\circ, 180^\circ \) have been used. The dimensionless maximum compression \( (N_{\text{max}}/N_Y)/(N_{\text{max}}/N_Y + M_{\text{max}}/M_p) \) is plotted in Fig. 5 versus slenderness \( \lambda_T \). It is observed that there is a qualitative difference between the way in which shallow and non-shallow arches respond to this type of loading. In shallow
arches compression is the major action, while in deep arches bending governs. The more slender a shallow arch is, the more significant is the compression action. On the other hand, the more slender a deep arch is the less significant is the compression action. In intermediate cases, i.e. in arches with subtended angle $2\Theta = 50^\circ$, the compression action is insensitive to the slenderness $\lambda T$.

4. Linearized buckling

In Figs. 6 and 7 the first anti-symmetric and the first symmetric linear buckling eigenvectors of a shallow and a non-shallow arch, respectively, are given. The critical eigenvector is nearly always the anti-symmetric one. However, it is possible for a symmetrical eigenvector to be the critical one, if the arch is shallow and stocky enough, e.g. $2\Theta \leq 10^\circ$ and $0.5S/i \leq 40$.

Fig. 6. First anti-symmetric and symmetric eigenvectors of a relatively shallow arch.

Fig. 7. First anti-symmetric and symmetric eigenvectors of a high arch.

It is well known from linear theory [1] that the linear buckling load of circular arches under uniformly distributed radial loads is equal to the buckling load of the second eigenvector of a pin-ended column:

$$q_0 = \frac{N_o}{R} = \frac{1}{R} \left( \frac{\pi^2 EI}{(S/2)^2} \right).$$  \hspace{1cm} (4)

When pin-ended circular arches are subjected to uniformly distributed vertical loads, the linear buckling load $q_{LB}$ demonstrates a deviation from the corresponding load of arches under uniformly distributed radial loads $q_0$, as shown in Fig. 8. Linear buckling loads $q_{LB}$ are larger than $q_0$ in the case of shallow arches and smaller in the case of deep arches. Medium height arches have linear buckling loads that are practically equal to $q_0$. Linear buckling loads of the first anti-symmetric eigenvector seem to be highly influenced by the arch slenderness $0.5S/i$ only in the case of very shallow arches. The less slender the arch is, the larger is the deviation from $q_0$.

5. Nonlinear elastic buckling and postbuckling

For the investigation of the geometrically nonlinear buckling and postbuckling behavior of elastic steel arches, two groups of arches are used. The first group (Group A) of arches is characterized by a constant slenderness $0.5S/i = 100$, while the second one is characterized by a common radius of curvature $R = 100$ m. In order for anti-symmetric buckling to be initiated, an initial, small imperfection is used, with the shape of the first anti-symmetric buckling eigenvector and magnitude equal to $L/50 000$.

In Fig. 9, the dimensionless load–deflection curves of the arch crown are given for both the arches of Group A and Group B. The load $q$ is nondimensionalized with respect to the linear elastic critical buckling load $q_{LB}$, while the vertical displacement at the crown $v_c$ is nondimensionalized with respect to $f$. A different behavior is observed between shallow and non-shallow arches. As pointed out in [4] for the case of radial load and confirmed by our analyses for vertical loads, arches can exhibit one of a total of five types of nonlinear behavior:

(i) The arches bifurcate anti-symmetrically (Fig. 9(b) $2\Theta = 150^\circ$-Group A, $180^\circ$-Group A) and Fig. 9(d) ($2\Theta = 160^\circ$-Group B, $180^\circ$-Group B)). After buckling, the load carrying capacity is increased, sometimes considerably, as in the case of arches with subtended angle $2\Theta = 180^\circ$. Thus, the maximum load is larger than the buckling load.

(ii) The arches undergo anti-symmetric snap-through as previously, however, the load carrying capacity is then decreased (Fig. 9(a) ($2\Theta = 18^\circ$-Group A), Fig. 9(c) ($2\Theta = 30^\circ$-Group B)).

(iii) The arches buckle by symmetric snap-through, and then, on the descending branch of the load–deflection curve they bifurcate anti-symmetrically (Fig. 9(a) ($2\Theta = 10^\circ$-Group A), Fig. 9(c) ($2\Theta = 20^\circ$-Group B)).

(iv) The arches buckle by symmetric snap-through without any bifurcation (Fig. 9(a) ($2\Theta = 7^\circ$-Group A), Fig. 9(c) ($2\Theta = 16^\circ$-Group B)).
The arches undergo no buckling, just an initial softening followed by a subsequent hardening (Fig. 9(a) \(2\Theta = 4^\circ\)-Group A), Fig. 9(c) \(2\Theta = 13^\circ\)-Group B).

From this investigation, it turns out that anti-symmetric buckling appears when \(2\Theta \geq 10^\circ\) for Group A arches and \(2\Theta > 20^\circ\) for Group B arches. Symmetric snap-through followed by anti-symmetric bifurcation on the descending branch of the load–deflection curve appears when \(9^\circ < 2\Theta < 10^\circ\) for Group A arches and \(19^\circ \leq 2\Theta \leq 20^\circ\) for Group B arches. Arches show snapping with no anti-symmetric buckling when \(5^\circ \leq 2\Theta < 9^\circ\) for Group A arches and \(14^\circ \leq 2\Theta \leq 19^\circ\) for Group B arches. Finally, there is no buckling when \(2\Theta < 5^\circ\) for the Group A arches and \(2\Theta < 14^\circ\) for the Group B arches. The corresponding limits between different types of nonlinear behavior of the ratio \(f/i\) of the rise to the radius of gyration are \(4.361, 3.925, 2.181\) for the Group A arches and \(4.406, 3.977, 2.162\) for the Group B arches. It is observed that these limits are approximately the same. Thus, a single criterion for the determination of the type of nonlinear behavior, for circular arches subjected to uniformly distributed vertical loading on the entire span, can be established with the use of the ratio \(f/i\). This criterion is based on the work already reported in [4], which was concerned with radial distributed loading instead. It is found that these limit values are a little different from the corresponding ones of the uniform radial load case found in [4].

(1) For \(f/i > 4.40\) arches bifurcate anti-symmetrically (first and second types).
(2) For \(3.95 < f/i < 4.40\) arches buckle by symmetric snap-through initially, while then they bifurcate anti-symmetrically on the descending branch of the load–deflection curve (third type).
(3) For \(2.17 < f/i < 3.95\) arches buckle by symmetric snap-through without bifurcation (fourth type).
(4) For \(f/i < 2.17\) arches undergo no buckling (fifth type).

In Fig. 10 the nonlinear elastic buckling load \(q_{GNA}\) defined as the maximum value of load on the geometrically nonlinear elastic load–displacement curve, nondimensionalized with respect to the linear elastic buckling load \(q_{L,R}\) is plotted with respect to the subtended angle \(2\Theta\) for the arches of group A and B. Although the analyses include initial imperfections, these are too small and they are used to trigger the anti-symmetric bifurcation wherever it is crucial, therefore the

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critical load is denoted as $q_{GNIA}$ instead of $q_{GNIA}$. It can be seen that the nonlinear elastic buckling loads decrease dramatically when the subtended angle $2\Theta$ is less than $50^\circ$. For very shallow arches no buckling takes place. Finally, for deep arches geometrically nonlinear buckling loads are practically the same with the corresponding linear buckling loads.

On the basis of the above analyses, the following approximate expression for the geometrically nonlinear buckling load of elastic arches is proposed:

$$q_{GNIA} = \frac{EJ}{L^2}. \quad (5)$$

The nonlinear buckling factor $\gamma$ can be approximated by the following equation, obtained with the aid of curve fitting tools in Matlab [10]

$$\gamma = 49.22 \sin \left( 5.147 \frac{f}{L} - 0.04396 \right) + 6.651 \sin \left( 12.47 \frac{f}{L} + 6.011 \right). \quad (6)$$

The function $f/L > 0.012$.

In Fig. 11 the proposed approximate nonlinear buckling factor $\gamma$ is compared to values from finite element results.

6. Geometry and material nonlinear analyses

In the previous section the geometrically nonlinear behavior of circular arches has been investigated. However, in order to make a reliable prediction of the bearing capacity of arches, the material nonlinearity must also be included in the analyses.

In Fig. 12(a)–(d) a set of results for the geometrically and material nonlinear (GMNIA) responses of circular arches is given and compared with the corresponding geometrically only nonlinear response (GNIA). The shape of the first anti-symmetric eigenvector is used as an initial imperfection with a maximum magnitude of $L/600$. It is seen that the two types of response are nearly the same in the case of extremely shallow arches ($2\Theta < 10^\circ$). That is because high stresses do not develop, until significant displacements take place. On the other hand, the deeper the arch is, the more significant the impact of the material nonlinearity on the arch bearing capacity is. The geometrically and material nonlinear response of deep arches has the same postbuckling characteristics, namely the corresponding load–displacement curves exhibit a descending branch in all cases. In deep arches the ignition of section yielding is almost coincident with the arch bearing capacity.

In Figs. 13 and 14 the dimensionless load $q_{GMNIA}/q_{LB}$ with respect to the included angle $2\Theta$ is given, for $0.5S/i = 180, 40$ respectively. In the same figures the dimensionless loads $q_{GNIA}/q_{LB}$ and $q_{MNIA}/q_{LB}$ are also plotted for comparison purposes. The imperfection of the GNIA and MNIA analyses are the same as for the GMNIA analyses. It can be seen that the interaction of the geometrically and materially nonlinearities is very important in the case of slender arches. For this case, $q_{GMNIA} \approx q_{GNIA}$ for very shallow arches ($2\Theta \leq 20^\circ$) and $q_{GMNIA} \approx q_{MNIA}$ for very deep arches ($2\Theta \geq 160^\circ$). For very stocky arches it can be seen that $q_{GMNIA} \approx q_{MNIA}$ irrespectively of the arch included angle. In Figs. 15 and 16 the corresponding results for the case of fixed arches are given. The same conclusions as for the pinned-ended arches hold in the case of fixed arches.

7. Effect of magnitude of initial imperfections on the strength

For the determination of the effect of initial imperfections on the strength of circular inelastic arches subjected to a uniformly distributed load on the horizontal projection of the entire span, arches with subtended angles $10^\circ, 50^\circ, 90^\circ, 180^\circ$ and slenderness $0.5S/i$ in the range from 40 to 170 are used. The first anti-symmetric eigenvector is used as the shape of initial imperfection. The maximum magnitude $\xi$ of the initial imperfections varies from 0 to $L/300$. This is in accordance with the provisions of EC3 for steel bridges [8].

In Fig. 17(a)–(d) the variations of the dimensionless ultimate load $q/q_{LB}$ obtained with GMNIA analyses with respect to the dimensionless vertical displacement of the crown $v_c/f$ are given, for arches with different subtended angles $2\Theta$. It can be observed that the effect of initial imperfections depends on the
shallowness of the arch. The shallower the arch is, the larger the influence of initial imperfections is.

The effect of the magnitude of initial imperfections on the strengths of inelastic arches for different values of the subtended angles is given in Figs. 18–20 where the dimensionless load \( q_{\text{GMNIA}} / q_{\text{GMNA}} \) is plotted against the slenderness \( \lambda_T \), where \( q_{\text{GMNIA}} \) and \( q_{\text{GMNA}} \) are the ultimate carrying capacities of a geometrically nonlinear inelastic analysis with and without initial imperfections, respectively.

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is verified that the initial imperfections have a more pronounced effect when arches are shallower. Furthermore, it can be seen that this effect becomes less important when the slenderness of the arch is smaller, i.e. when the arch is stiffer. As far as the strength \( q_{GMNIA} \) is concerned, it is noted that in shallow arches the strength, in comparison to the ultimate carrying capacity of the perfect arch, decreases with an increase of the slenderness \( \lambda_T \). Thus, when an arch of the same subtended angle \( 2\theta \) is more slender, the initial imperfections have a more serious impact on its strength. On the other hand, in deep arches, the initial imperfections have about the same impact on their strengths for any size of the slenderness \( \lambda_T \).

8. Effect of the rise-to-span ratio on the strength

For the investigation of the effect of the rise-to-span ratio on the strength of arches, arches with an initial imperfection of magnitude \( L/600 \) and rise-to-span ratio in the range 0.02–0.5 are used. The variations of the dimensionless load bearing capacity \( q_{GMNIA}/(4\pi^2EI/S^2) \) with the rise-to-span ratio \( f/L \) for different values of the slenderness are given in Fig. 21. It can be seen that the rise-to-span ratio affects the slender arches more, i.e. arches with \( \lambda_T = 0.922, 1.152, 1.613, 1.958 \). The maximum strength for arches with the same slenderness \( \lambda_T \) is observed neither in very shallow arches, nor in very deep arches but rather for values of the rise-to-span ratio in the range 0.18–0.24.

9. Effect of the geometric nonlinearity on the strength

In this section, the effect of geometric nonlinearity on the strength of arches is investigated. For this purpose, arches with subtended angles \( 2\theta = 10^\circ, 50^\circ, 90^\circ, 140^\circ, 180^\circ \) and \( 0.5L/i \) in the range from 40 to 170 are analyzed. It is assumed that arches have initial imperfections of magnitude \( L/600 \). In Fig. 22 the variations of the dimensionless load \( q_{GMNIA}/q_{MNIA} \) with the slenderness \( \lambda_T \) are shown, where \( q_{MNIA} \) is the load bearing capacity of a geometrically linear inelastic analysis with initial imperfections.

It is concluded that the load bearing capacity of arches is influenced in a more serious manner by the geometric nonlinearity when the arches are shallower. In deep arches, i.e. arches with subtended angles \( 2\theta = 140^\circ, 180^\circ \), this effect on their strength is comparatively small. The divergence of the results of a MNIA analysis with respect to the corresponding results of a GMNIA analysis would be of magnitude less than 20% for arches with subtended angle 180° and less than 30% for arches with subtended angle 140°. On the other hand, the load bearing capacity of the shallow arches is influenced dramatically. In arches with subtended angle \( \theta = 10^\circ \), this divergence would be almost 80%.

In addition, the effect of the geometric nonlinearity on the strength of arches is influenced by the magnitude of the slenderness \( \lambda_T \). It can be concluded that, in spite of some exceptions, the more slender the arch is, the more significant this effect on its strength is. In deep arches, the load bearing capacity is influenced mainly from the inelastic nature of the material.

10. Effect of the boundary conditions on the strength

For the investigation of the effect of boundary conditions on the strength of steel arches, three groups of arches with \( 0.5L/i = 40, 100, 170 \) (\( \lambda_T = 0.461, 1.152, 1.958 \)) are used. In Fig. 23, the variations of the dimensionless load bearing capacity \( q_{p(GMNIA)}/q_{f(GMNIA)} \) with the subtended angle \( 2\theta \) of the arches, is given, where \( q_{p(GMNIA)} \) is the load bearing capacity of pin-ended arches and \( q_{f(GMNIA)} \) is the load bearing capacity of fixed arches.

It can be seen that fixed arches are, as expected, stronger than the corresponding pin-ended arches, due to the higher degree of indeterminacy, which makes it possible for a more extensive redistribution of the bending moments to take place. Moreover, the more slender the arches are, the more important is the influence of the boundary conditions on their strengths. The reduction of strength, if pin-ended supports are used, can be up to approximately 45%, 45%, 33% for arches having 0.5L/i equal to 170, 100 and 40 respectively.

11. Conclusions

In this paper an investigation of the nonlinear behavior of steel circular arches with circular hollow cross-section was...
Fig. 17. Effect of initial imperfections for shallow and non-shallow arches.

Fig. 18. Effect of initial imperfections on the strength of arches with $2\Theta = 10^\circ$.

Fig. 19. Effect of initial imperfections on the strength of arches with $2\Theta = 90^\circ$.

carried out. The influence of a number of design parameters, such as the boundary conditions, the rise-to-span ratio, the included angle, on the strength was investigated. In addition, the effect of other factors influencing the structural behavior, such as geometrical and material nonlinearity and initial imperfections was sought as well.
It was observed that a single criterion can be used for the determination of the type of nonlinear behavior of arches in mind. It was found that these limit values are a little different from the corresponding ones of the uniform radial loading case. An appropriate expression for the determination of the elastic nonlinear buckling load was also proposed. As far as the effect of initial imperfections on the strength of arches is concerned, it is by far more important for shallow arches than for deep ones. The rise-to-span ratio has a greater impact on slender arches. The maximum strength for arches with the same slenderness \( \lambda \) was observed neither in very shallow nor in very deep arches, but for values of the rise-to-span ratio in the range 0.18–0.24. Finally, geometric nonlinearities affect the strength of shallow arches more significantly than the strength of deep arches. The more slender an arch is, the more significant is the effect of the geometrical nonlinearities on the arch strength. In deep arches the load bearing capacity is influenced mainly from the inelastic nature of the material.

References