EQUIVALENT UNIFORM DAMPING RATIOS
for Irregular in Height Concrete / Steel Structural systems

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INTRODUCTION

Aim of this work is to deal with the seismic response of irregular structures, consisting of two parts, a lower part, called primary structure or substructure, founded on the ground, and an upper part, usually referred to as secondary structure or superstructure, resting on the primary one. The primary structure is usually denoted by the letter p and the secondary structure by the letter s. These two parts are characterized by different dynamic response. One reason for that can be the material distribution over the height of the structure, for example primary structure made of concrete and secondary one made of steel, leading to different damping properties of the two parts. Another reason could be the different lateral stiffness systems and/or energy dissipation mechanisms, e.g. a primary structure with bracings or bearing walls and a secondary structure with frames, or both. Thus, the overall dynamic response of the combined structural system, when it is subjected to earthquake excitation, can be very complex.

Such structures are, for example, frequently encountered in stadiums, where the spectators' seats are usually configured on concrete frames, used also to house auxiliary facilities beneath the seats, while cover of the seats is often provided by steel frames or trusses supported by the concrete substructure. Other possible applications include the addition of relatively light steel frames on top of existing reinforced concrete buildings, in an effort to reduce the additional dead weight or for speed of construction. In accordance with several practical applications, such as the ones mentioned above, the particular case of structures where the lower part is made of reinforced concrete and has a damping ratio equal to 5% (typical for reinforced concrete structures) and the upper part is made of steel with a damping ratio equal to 2% (typical for welded steel structures) is investigated here.

The seismic design of such structures is not satisfactorily covered by the analysis methods suggested by current design codes (IBC, EC8), especially when the mass of the secondary system is of the same order as the mass of the primary system. If one decides to carry out a full time history analysis of the irregular structure, a damping matrix that will account for the different levels of damping must be created. This kind of analysis is not usual for every day design practice and can not be handled by existing commercial software. Another alternative is to separate the structure in its eigenmodes and carry out a modal time history analysis or a modal spectral analysis but the resulting eigenmodes will be complex. None of the above alternatives is appealing for every day use, as they require significant computational cost as well as dealing with complex numbers, which is not generally implemented for design in civil engineering structures. Thus, a simple yet conservative approach is usually adopted in practice, namely that the structure has an overall damping ratio equal to 2%. This makes possible the use of a code-based response spectrum to evaluate the maxima of the design quantities, e.g. absolute accelerations and/or shear forces, or even the execution of a modal time history analysis without the drawback of complex eigenmodes.

Many investigators have proposed methods for simulating irregular damping distributions with more usual ones, mostly regarding cases of structures with added viscous dampers, due to their widespread implementation in buildings and the subsequent need for providing solutions for issues pertaining to their design. Perhaps the most commonly used method of that type is the so called modal strain energy method. The basic assumption of the method is that the actual complex mode
shapes are replaced by real ones that correspond to the undamped structure. Then, the ratio of the energy dissipated by the damping devices to the total strain energy of the structure is calculated for each mode shape and the equivalent damping for the specific mode is extracted.

Based on the modal strain energy method, other methods have been developed in order to enhance its results. Bilbao et al. [1] simulate the effect of the added dampers in structures with a Rayleigh damping matrix that becomes diagonal with modal transformation using real modes. Lee et al. [2] arrive at closed-form analytical solutions for the estimation of the equivalent damping of structures equipped with dampers. They make the state–space transformation of the equation of motion of the irregularly damped system and through a procedure of computations where the eigenmodes of the system are in complex form, they finally arrive at proposing real valued damping ratios for the structure, accounting also for the case of non-linear damping devices. Shen et al. [3] conduct scale experiments on multi-storey RC frames. After the frames are damaged by the ground motion they add dampers, monitor their new response characteristics and derive analytical solutions for the equivalent damping of the system with the added dampers by introducing a modification in the modal strain energy method. Similarly Chang et al. [4] study steel multi-storey frames experimentally and also propose a modification to the modal strain energy method to simulate the irregular damping distribution with equivalent modal damping. The aforementioned methods can be implemented in irregularly damped concrete/steel frames by appropriately simulating the damping of each storey with an equivalent damper and then use each proposal to evaluate the equivalent damping.

A different approach is attempted by Huang et al. [5]. They examine a multi degree of freedom (MDOF) structure comprised of two parts, each with different damping ratio and also seek to extract conclusions for the equivalent modal damping of the structure using a numerical and an analytical approach. In the numerical – exact one, they firstly simulate the structure with the exact damping distribution and then proceed with a new simulation where the damping is arbitrarily set to specific ratios. The error between the two analyses is then calculated and the equivalent modal damping is elected as the one that minimizes the error. In the analytical – approximate method, they simulate each of the two discrete parts of the MDOF system with their first mode oscillator and thus arrive at a 2-DOF system and they assume that the orthogonality condition applies for the damping matrix of the irregularly damped 2-DOF structure. From the normalized values of the damping matrix they extract analytical expressions for the equivalent modal damping of the simplified structure, and compare it to the results obtained by the numerical solution with satisfactory accuracy.

In the present work, an attempt is made to simulate the irregularly damped structure not with modal equivalent damping ratios, but with a uniform one instead. The reason for this is that the implementation of a unique damping ratio in every day practice is more convenient than the implementation of a modal damping ratio, which requires the evaluation of more than one different response spectra, one for each modal damping ratio.

1 EQUIVALENT DAMPING RATIOS

A procedure similar to the one of Huang et al. [5] is attempted in the present work. Simple, elastic 2-DOF structures are considered, as the one shown in Fig. 1, where \( M_i, C_i \) and \( K_i \) are the mass, damping coefficient and stiffness of each part and \( \ddot{x}_g \) is the input ground motion. The damping distribution is irregular, with the primary and the secondary parts having damping ratios equal to \( \zeta_p = 5\% \) and \( \zeta_s = 2\% \), respectively. The eigenfrequency of each substructure is then expressed as:

\[
\omega_i = \sqrt{K_i / M_i}, \quad i = s, p.
\]

Accordingly, the damping matrix coefficients are calculated as

\[
C_i = 2M_i\omega_i\zeta_i, \quad i = s, p.
\]
In order to characterize the response of the system depending on the properties of the two parts, the eigenfrequency $R_\omega$ and the mass ratio $R_m$ of secondary to primary part are defined:

$$R_\omega = \frac{\omega_s}{\omega_p}, \quad R_m = \frac{M_s}{M_p}$$  \hspace{1cm} (1)

For the numerical analyses, the primary system is selected to have an eigenperiod equal to 0.1 sec and a mass equal to 1000Mgr. A wide range of eigenfrequency and mass ratios is examined and for each ratio pair, and given the characteristics of the primary system, the complete 2 – DOF structure can be formed.

![2-DOF structure](image)

**Fig. 1. 2 – DOF irregular structure**

The structure is analyzed as a whole with the ground excitation induced at its base:

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = -[M]\{r\} \ddot{x}_g$$  \hspace{1cm} (2)

where $[M]$, $[C]$, and $[K]$ are the mass, damping and stiffness matrices of the structure, $\{y\}$ is the vector of relative displacements of the degrees of freedom of the structure with respect to its base and $r = (1,...,1)^T$, for the general case of an n degree of freedom structure. The ground motion $\ddot{x}_g$ is selected to be harmonic in resonance with the first mode of the 2 – DOF structure. The results obtained are the total accelerations at each level e.g. $\{\ddot{y}\} = \{\ddot{y}\} + \{r\} \ddot{x}_g$.

For each pair of eigenfrequency and mass ratios, two analyses are carried out: (i) an exact one, with the correct distribution of damping over the height of the structure, and (ii) an approximate one, in which the entire structure is assumed to have one uniform equivalent damping ratio. The damping matrix of the exact analysis is equal to:

$$C_{ex} = \begin{bmatrix} 2M_p\omega_p\zeta_p + 2M_s\omega_s\zeta_s & -2M_s\omega_s\zeta_s \\ -2M_s\omega_s\zeta_s & 2M_s\omega_s\zeta_s \end{bmatrix}$$  \hspace{1cm} (3)

while the damping matrix of the approximate analysis with damping ratio equal to $\zeta_{eq}$, is equal to:

$$C_{app} = \begin{bmatrix} 2M_p\omega_p\zeta_{eq} + 2M_s\omega_s\zeta_{eq} & -2M_s\omega_s\zeta_{eq} \\ -2M_s\omega_s\zeta_{eq} & 2M_s\omega_s\zeta_{eq} \end{bmatrix}$$  \hspace{1cm} (4)

The maxima of the two responses, exact and approximate, are obtained and the error at each level between the two analyses is calculated as follows:
Minimizing this error is the base for the decision of the proposed equivalent damping, meaning that the equivalent damping ratio that will yield less error will be the optimum one for the specific eigenfrequency and mass ratios. Apart from the errors at each level of the structure, the error measures expressed as the sum of individual errors and as their Euclidean norm are also computed. For each type of error, the resulting equivalent damping ratios over the \((R_\omega - R_m)\) plane is plotted, as shown in Fig. 2, where the case of the sum of errors at each floor is presented.

It is observed that the optimum values of uniform equivalent damping are influenced more by the frequency ratio \(R_\omega\) than by the mass ratio \(R_m\). In the area of high \(R_\omega\) ratios, where the superstructure is much stiffer than the substructure, the resulting damping ratio is close to the one of the substructure. The system is stiff enough to follow the motion of the substructure, and there is little relative displacement between the two parts. Only the primary structure exhibits significant relative deformations to its base and, therefore, the overall damping is close to the one of the substructure. In the other extremity of the \(R_\omega\) ratios, the superstructure is very flexible and its relative deformations are much larger than the ones of the supporting substructure, therefore, the resulting equivalent damping approaches the value of 2%.

\[
e = \left[ \frac{\max(\| \mathbf{y}_{\text{appr}} \|) - \max(\| \mathbf{y}_{\text{ex}} \|)}{\max(\| \mathbf{y}_{\text{ex}} \|)} \right]
\]  

(5)

2 APPLICATION TO REAL STRUCTURES

The proposed uniform equivalent damping values are next applied to a real two – storey, one - bay frame with realistic dimensions, shown in Fig. 3, in order to first examine the accuracy of the proposed approach in a very simple structure. The frame has a concrete lower storey with damping ratio equal to 5% and a steel upper storey with damping ratio equal to 2%. The concrete’s Young’s modulus is assumed to be equal to 27.5MPa, and for the steel part the corresponding modulus is 2.1GPa. The concrete columns have a rectangular cross – section of 50cm by 50cm, and the steel column has a HEB320 cross - section. The slabs are considered sufficiently stiff to ensure diaphragm action at both levels. The mass of the p – system is 20Mgr and the one of the s – system is 16Mgr. The dynamic characteristics of the structure are shown in Table 1 and the deformed shapes of each mode are shown in Fig. 4 and Fig. 5. These values correspond to a mass ratio equal to 0.8 and an eigenfrequency ratio of 0.75. According to Fig. 2, the resulting equivalent damping ratio is 3.5%.
Table 1. Dynamic characteristics of frame structure

<table>
<thead>
<tr>
<th>mode</th>
<th>Period (sec)</th>
<th>modal mass participation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.84</td>
</tr>
<tr>
<td>2</td>
<td>0.06</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Fig. 3. Frame structure under consideration  
Fig. 4. 1st mode deformed shape  
Fig. 5. 2nd mode deformed shape

For the structure described above, subjected to a sinusoidal excitation in resonance with its first mode, an exact time history analysis is carried out, accounting for the real distribution of the damping, as well as three approximate ones, one with the use of the resulting uniform damping of 3.5%, one with the use of the conservative damping consideration of 2% and one considering an overall damping equal to 5%.

From each analysis the absolute maxima of the total accelerations at each level of the structure were obtained. The error that each of the analyses with a uniform damping ratio exhibited in comparison to the analysis with the actual damping distribution is calculated as indicated next, in Eq. (6), and the resulting errors for each level of the structure are listed in Table 2.

\[ e^* = \frac{\max(\|\tilde{\gamma}_{ex}\|) - \max(\|\tilde{\gamma}_{appr}\|)}{\max(\|\tilde{\gamma}_{ex}\|)} \]  

(6)

The differences between the exact analysis and the ones with the arbitrarily selected damping confirm that for the structure under consideration the uniform damping ratio equal to 3.5% gives response properties that are much closer to the ones of the same structure with the exact damping distribution. Furthermore, as expected, the assumption of a 2% damping ratio gives conservative results, while 5% underestimates the response at both levels. The use of the harmonic excitation in resonance with the first mode that has a quite large participation ratio, as shown in Table 1, is rather conservative, which means that for an actual seismic record the resulting errors will be smaller. Thus, an upper limit for the estimation of the error occurring by the use of a uniform damping ratio is provided.

Table 2. Resulting errors

\[ e^* \]
3 SUMMARY

Structures that have irregular damping configuration over their height are examined. For a wide range of dynamic characteristics arbitrarily selected damping ratios are tested and the ones that yield less error, as far as the response characteristics are concerned, are selected as the most suitable to be used as equivalent uniform ones. An example of a real two-storey, one-bay frame with different damping ratios over its height is analyzed with its exact damping distribution, with the proposed one and with the damping ratios corresponding to each part. The proposed uniform ratio gave response characteristics that were close to the ones of the actual structure while the other two damping ratios gave significant errors.

Future extension of this work will include (i) evaluation of the proposed uniform equivalent damping ratios for multi-story frame structures, as well as more complex structural systems, (ii) evaluation for the case of seismic input motion, using actual accelerograms, and (iii) application of the proposed damping values in the framework of response spectrum, as opposed to time history, dynamic analyses.

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REFERENCES


