Design Strategies for Controlling Structural Instabilities

by

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Design Strategies for Controlling Structural Instabilities

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Abstract:. An overview of buckling behaviour is presented, with emphasis on its analysis and design implications and ways of incorporating these considerations in the design of spatial structures. In the first part of the paper the fundamental stability concepts are briefly reviewed. The major types of instability are presented and typical structures that are prone to fail in such modes are identified. The influence of initial imperfections is discussed, as well as the interaction between different failure modes. In the second part, practical issues pertaining to structural analysis and design are addressed. These include hints to identify potential instability modes and obtain approximate buckling loads by analytical considerations, guidelines for selection of appropriate analysis methods, as well as discussion of pertinent design approaches, code provisions and issues that are not covered by present codes.

1. INTRODUCTION

It is widely recognized that spatial structures are characterized by several features that distinguish their behaviour from that of other, more conventional, types of structures. Some of these features are not addressed by current codes, even though they can have implications in the analysis and design of spatial structures that may lead to unsafe solutions. Codes are mostly geared towards more traditional, frame-type constructions, which historically constitute the vast majority of structures. Large-span spatial structures are usually designed and constructed by specialized engineering firms that rely on previous expertise and follow up with research developments, thus avoiding pitfalls related to issues not covered by the codes. In recent years, however, there is an increased demand for such structures, which is combined with the progress in structural analysis software that "appears" to deal effectively with the problem, but may yield unreliable results, if it is used without sound knowledge of the theoretical background governing their response. This may soon lead to a situation where it will be imperative to propose design specifications for spatial structures.

One of the issues that distinguish the response of spatial structures from that of other structures is their stability behaviour. The continuous trend to design lighter and more slender structures, supported by progress in high-strength construction materials, manufacturing methods and analysis tools, has rendered instabilities of all types to gain importance as a potential failure mode in structural design, in comparison to strength and serviceability. Following this development, structural stability has attracted the attention of many researchers since its modern origins of Timoshenko's basic elastic stability theory [1] and Bleich's inelastic buckling theory [2]. The results of this work are reported in several books, for example those by Croll and Walker [3], Chen and Lui [4], Galambos [5], Bazant and Cedolin [6], as well as numerous journal papers and conference proceedings.

Many aspects of these findings have been incorporated in recent design codes, particularly those for metal structures, for which structural stability is a more relevant failure mode due to their increased slenderness. Examples of such codes are the American LRFD Manual [7] and the European EC3 [8].. The stability issues covered by these codes include primarily member and local buckling. Member buckling includes flexural buckling of compression members, based on buckling curves, interaction of axial compression and bending moment, flexural-torsional buckling (Trahair [9]), while local buckling is usually treated by classification of cross-sections into categories, depending on the slenderness of their constituent plates. In addition, the above codes address the issue of overall structural system buckling, but in a way that is more suitable for ordinary building structures, such as plane and space frames, towards which the codes are primarily geared.

There are, however, several instability phenomena that are either not treated at all, or are addressed very superficially by these codes, even though they may have significant impact on the structural response, particularly in the case of spatial structures. Examples are the potential need for geometrically nonlinear analysis to capture reliably global buckling modes, the lack of a systematic treatment of the influence of geometric and other imperfections, as well as the potential interaction of buckling modes and geometric and material nonlinearities, and the associated decrease of buckling loads and change of the nature of post-buckling behaviour. It is believed that future codes of practice for spatial structures should incorporate provisions for these issues.

In this paper it is attempted to provide an overview of buckling behaviour with an emphasis on analysis and design implications and suggestions for incorporating these considerations in structural design practice. It is believed that the initial stage of an effective stability design process is to identify in a qualitative manner all pertinent modes of potential instability, and appreciate their importance. The first part of the paper is devoted, accordingly, to a brief review of fundamental stability concepts. In this part the major types of instability are reviewed, their fundamental features are described, and typical structures that fail in such modes are identified. Then, the influence of initial imperfections is discussed for each one of these instability modes. The interaction between failure modes and particularly buckling modes is addressed next.

This is followed, in the second part of the paper by a discussion of practical, structural analysis and design issues. First, some considerations for the prediction of potential instability modes and their corresponding buckling loads by simple analytical models, are proposed. The importance of conceptual design and selection of a suitable structural system is emphasized. Next, alternative analysis methods are presented and guidelines for the selection of the most appropriate one, depending on the problem, are given. This is followed by a discussion of pertinent code provisions and issues that are not covered by present codes. Finally, some of these issues are demonstrated by means of an example of a simple truss structure.

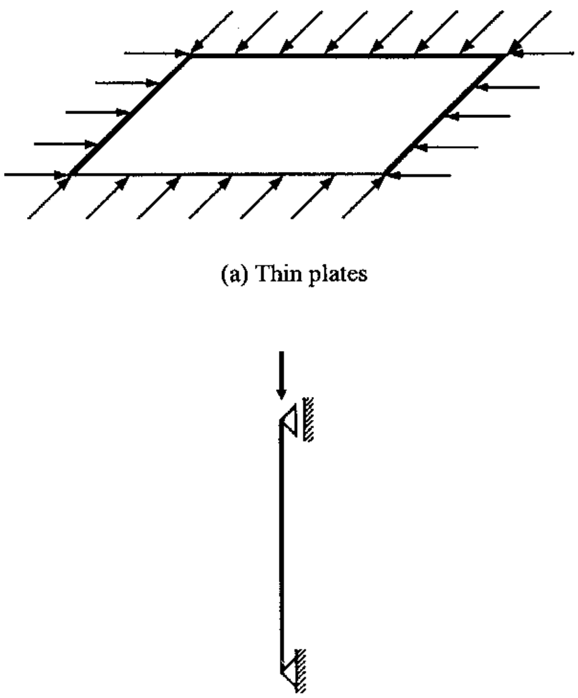
It should be mentioned that in an effort to provide a comprehensive compilation of most basic considerations that must be accounted for by the designer of a spatial structure, and due to space limitations, the paper had to sacrifice the depth of coverage. It has been attempted to provide sufficient references to both textbooks covering in detail the fundamental concepts of stability behaviour, and recent research papers including findings and recommendations for the state-of-the-art with respect to analysis and design issues. The reader is strongly encouraged to study these sources in parallel to the present paper.

2. FUNDAMENTAL CONCEPTS

Instability or buckling is defined as the sudden change in the geometry of a structure subjected to predominantly compressive stresses that results in the loss of its ability to resist loads. The equilibrium states immediately before and immediately after buckling correspond to the same value of the load, which is termed critical load. Instabilities may be associated with catastrophic structural failure; therefore, they must be taken into account during structural design, considered to be a particular case of strength limit state. Particularly sensitive to instability are slender structures, of which one characteristic dimension is much smaller than the other two, for example thin plates, or two characteristic dimensions are much smaller than the third, for example long columns (Fig. 1).

The buckling response of a structure is usually described by a load-deformation diagram, known as equilibrium path because all its points correspond to equilibrium states, in other words, they satisfy the equilibrium, compatibility and constitutive equations. An equilibrium state is called stable if an increase in load causes increase in deformation and unstable if an increase in load is associated with a decrease in deformation.

An immediate consequence of trying to detect two distinct configurations that correspond to the same value of load, and the fact that buckling is associated with a significant change in the geometry of the structure, is that the equilibrium equations must be



(a) Thin plates

(b) Long columns

Fig 1 Examples of structures prone to instability

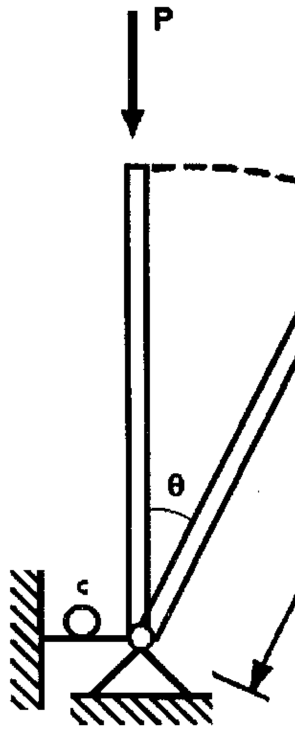


Fig 2 Rigid cantilever column supported by rotational spring

written in the deformed configuration. This is a first important difference between buckling analysis and classical structural analysis, where deformations are usually considered negligibly small, so that the equilibrium equations are written in the undeformed geometry.

2.1 TYPES OF INSTABILITY

The classification of instability types proposed in this section is based on the nature of the structural response before and after buckling, and has been proposed extensively in the pertinent literature. The fundamental features of each instability type are presented here by means of simple structural models, for which analytical solutions exist. Emphasis is placed on behavioural characteristics that can be generalized for more complex structures, and not on derivation of individual equations. Detailed derivations of the equations governing the behaviour of these models can be found in several structural textbooks (for example those by Croll and Walker [3], Chen and Lui [4], Galambos [5], Bazant and Cedolin [6]),

The following cases are considered:

Stable symmetric bifurcation

Consider the idealized 1-DOF structural model of Fig. 2, consisting of a rigid bar of length ℓ, hinged at the bottom and supported by a linear rotational spring of stiffness c, subjected to a vertical concentrated load P at the top. Writing the moment equilibrium equation with respect to the bottom hinge, we obtain, in the general case of arbitrarily large deformations, the equations of primary and secondary equilibrium path as:

$$\theta = 0$$

$$P = \frac{c}{\ell}\frac{\theta}{\sin\theta} \tag{1}$$

while for small values of θ a linearization of the equation of the secondary path is possible by means of a Taylor expansion, yielding:

$$P = \frac{c}{\ell} \tag{2}$$

Plots of these equilibrium paths are shown in Fig. 3 for both the linear and nonlinear case. The primary path is the same for both solutions and corresponds to small values of the compressive load, for which the column

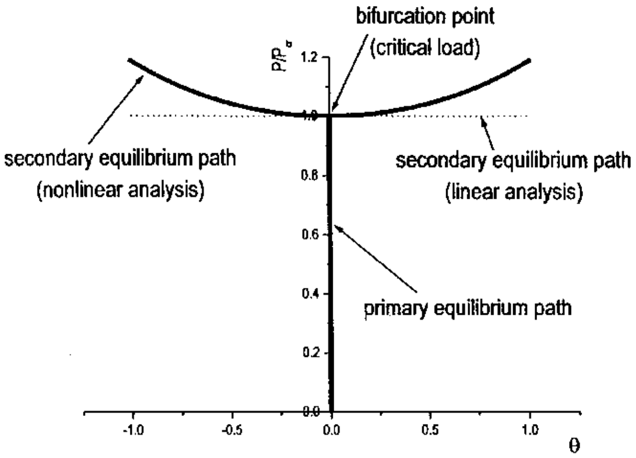


Fig 3 Equilibrium paths of rigid cantilever column supported by rotational spring

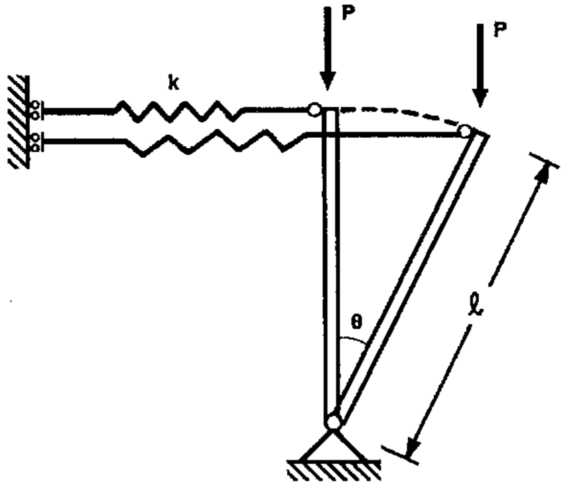


Fig 4 Rigid cantilever column supported by horizontal translational spring

remains in the vertical position and the rotational spring is undeformed. For a value of the load P equal to c/ℓ, the primary path intersects the secondary equilibrium path. This value of the load is the critical buckling load P_{cr}, and the point of intersection of primary and secondary path is called bifurcation point. The secondary path obtained with linear analysis is a straight horizontal line, while nonlinear analysis captures its ascending, therefore stable, nature. The secondary path is symmetric with respect to the vertical axis, indicating that the column may buckle either to the left (negative values of θ), or to the right (positive values of θ). The actual direction of buckling is determined in practice by the initial imperfections, as will be outlined next.

Unstable symmetric bifurcation

Consider now the idealized 1-DOF structural model of Fig. 4, consisting of a rigid bar of length ℓ, hinged at the bottom and supported by a linear translational spring of stiffness k at the top, where it is also subjected to a vertical concentrated load P. Similarly, we obtain the equations of primary and secondary equilibrium path as:

$$\theta = 0$$
$$P = k\ell\cos\theta \tag{3}$$

while the linearized expression for the secondary path is:

$$P = k\ell \tag{4}$$

Plots of these equilibrium paths are shown in Fig. 5 for both the linear and nonlinear case. The primary path is again the same for both solutions and corresponds to

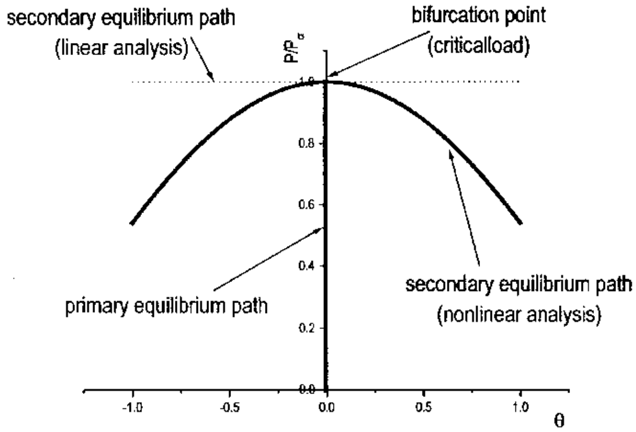


Fig 5 Equilibrium paths of rigid cantilever column supported by horizontal translational spring

small values of the compressive load, associated with the situation before buckling, for which the column remains in the vertical position and the translational spring is undeformed. For a value of the load P equal to the critical buckling load $P_{cr} = k\ell$, the primary path intersects the secondary equilibrium path at the bifurcation point. The secondary path obtained with linear analysis is again a straight horizontal line, while nonlinear analysis is required to capture its descending, therefore unstable, nature. The secondary path is also symmetric with respect to the vertical axis, indicating that the column may buckle either to the left (negative values of θ), or to the right (positive values of θ), depending on the initial imperfections.

Asymmetric bifurcation

Consider now the idealized 1-DOF structural model of Fig. 6, consisting of a rigid bar of length ℓ, hinged at

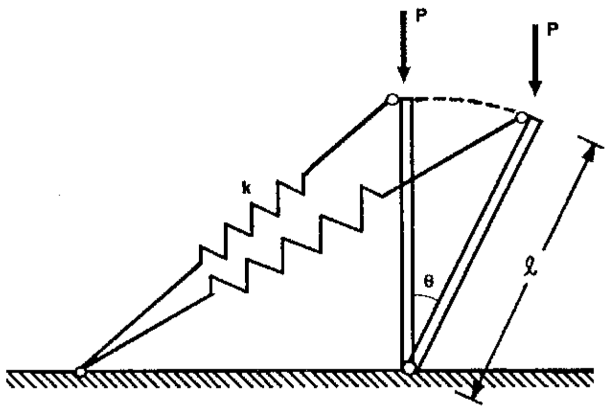


Fig 6 Rigid cantilever column supported by inclined translational spring

the bottom and supported by an inclined by 45° linear translational spring of stiffness k at the top, where it is also subjected to a vertical concentrated load P. In the same way, the equations of primary and secondary equilibrium path are obtained as:

$$\theta = 0$$

$$P = k\ell\frac{\left(\sqrt{1+\sin\theta}-1\right)\sqrt{1-\sin\theta}}{\sin\theta} \tag{5}$$

while the linearized expression for the secondary path is:

$$P = \frac{k\ell}{2} \tag{6}$$

Plots of these equilibrium paths are shown in Fig. 7 for both the linear and nonlinear case. The primary path is once again the same for both solutions and corresponds to small values of the compressive load, for which the column remains in the vertical position and the translational spring is undeformed. For a value of the load P equal to the critical buckling load $P_{cr} = k\ell/2$, the primary path intersects the secondary equilibrium path at the bifurcation point. The secondary path obtained with linear analysis is again a straight horizontal line, while nonlinear analysis is required to capture its real nature, which is descending, therefore unstable, on the right (positive values of θ), and ascending, therefore stable, on the left (negative values of θ). The secondary path is now asymmetric with respect to the vertical axis. The column may buckle either to the left or to the right, depending on the initial imperfections, but the buckling response of the perfect system is considered unstable, which is the worst-case scenario.

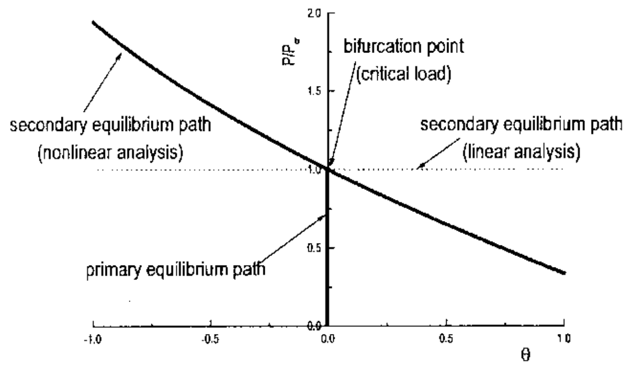


Fig 7 Equilibrium paths of rigid cantilever column supported by inclined translational spring

Snap-through buckling

Finally, consider the two-bar truss with the geometry shown in Fig. 8, with bars of axial stiffness EA, subjected to a vertical concentrated load P at the top, where, for the sake of clarity, only vertical displacements are allowed (Pecknold et al. [10]). This truss is also known as von Mises truss. Combining the equilibrium, compatibility and constitutive equations in the deformed configuration, the equilibrium path of this structure is obtained as:

$$P = 2EA(H-\delta)\left[\frac{1}{\sqrt{L^2+(H-\delta)^2}}-\frac{1}{\sqrt{L^2+H^2}}\right] \tag{7}$$

A plot of this equilibrium path is shown in Fig. 9a in nondimensionalized form. It can be seen that there is now only one equilibrium path that is curved from the onset of loading, exhibiting a gradually decreasing stiffness until reaching a maximum. The maximum load is now the critical buckling load and the corresponding point of the equilibrium path is called limit point. In this case there is no point in linearizing the response, as the linear analysis will not yield any meaningful results.

From Fig. 9a one can see that the equilibrium path consists of stable and unstable parts. The entire path can only be traced, either experimentally or numerically, if displacements are controlled instead of

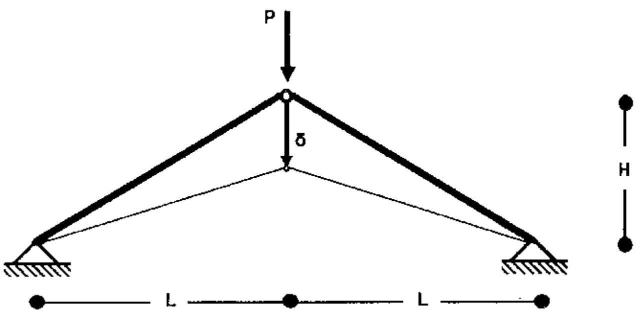
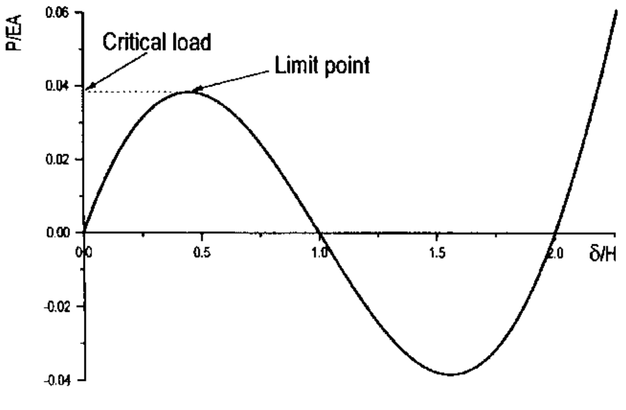
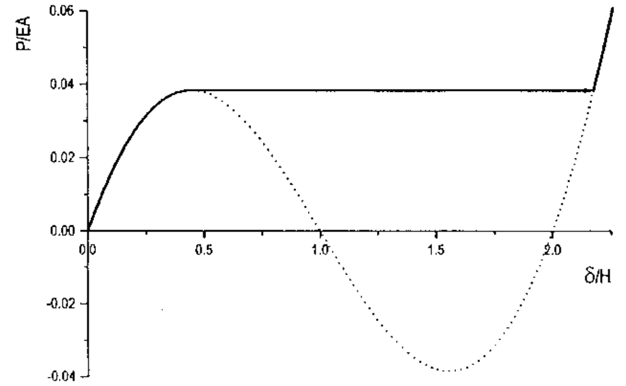


Fig 8 Two-bar von Mises truss



(a) for displacement control



(b) for load control

Fig 9 Equilibrium path of von Mises truss

loads. The response for the imposed, gradually increasing loads is observed in Fig. 9b. When the load reaches the maximum, the structure "jumps" to the opposite branch, seeking a stable equilibrium configuration corresponding to the increased load. This phenomenon is called snap-through buckling, and is often of dynamic nature, involving a considerable exchange of energy.

2.2 Influence of imperfections

The description of instability types in the previous section restricted itself to perfect systems. In reality, however, structures are characterized by geometric imperfections, material variability, load eccentricities etc., and this has a strong influence on their buckling response. Some qualitative characteristics of this influence of imperfections will now be demonstrated by means of the same simple models of section 2.1.

Structures that buckle via stable symmetric bifurcation

Consider first the rigid column of Fig. 2 with an initial imperfection modelled as a deviation of the unloaded

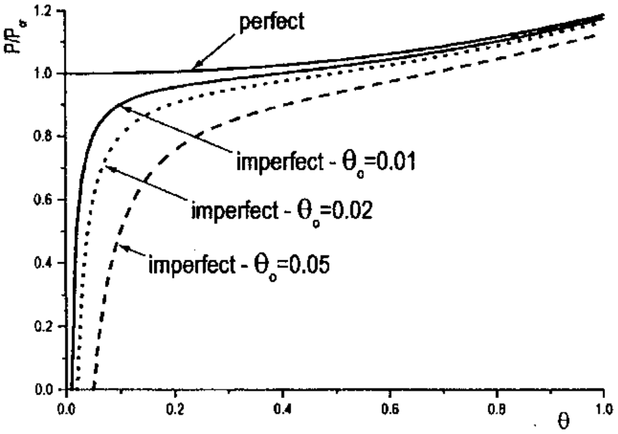


Fig 10 Influence of imperfections in cases of stable symmetric bifurcation

column from the vertical direction by an angle θ_o. Then, the moment equilibrium equation has a unique solution:

$$P = \frac{k}{\ell}\frac{\theta - \theta_o}{\sin\theta} \tag{8}$$

plotted in Fig. 10 for several magnitudes of the initial imperfection. It is observed that the primary and secondary equilibrium paths are replaced by a single, nonlinear equilibrium path that retains its stable character. There is no more bifurcation buckling but rather a sharp decrease of structural stiffness as the load approaches the critical load of the perfect system. The direction of deformation is now uniquely determined by the direction of the imperfection.

Structures that buckle via unstable symmetric bifurcation

Consider next the rigid bar of Fig. 4 with a similar initial imperfection θ_o. Then, the moment equilibrium equation has again a unique solution

$$P = k\ell\frac{\sin\theta - \sin\theta_o}{\tan\theta} \tag{9}$$

plotted in Fig. 11 for several magnitudes of the initial imperfection. It is again observed that the primary and secondary equilibrium paths are replaced by a single, nonlinear equilibrium path that retains its unstable character in the post-buckling region. The bifurcation buckling of the perfect structure is now replaced by snap-through buckling, and the bifurcation point by a limit point. The direction of deformation is uniquely determined by the direction of the imperfection.

Structures that buckle via asymmetric bifurcation

Consider now the rigid bar of Fig. 6 with the same initial imperfection angle θ_o. Then, the moment

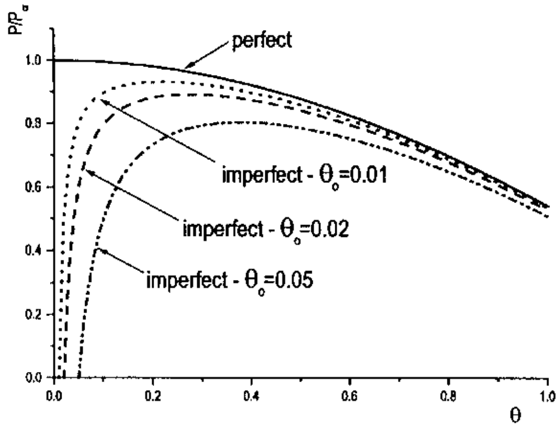


Fig 11 Influence of imperfections in cases of unstable symmetric bifurcation

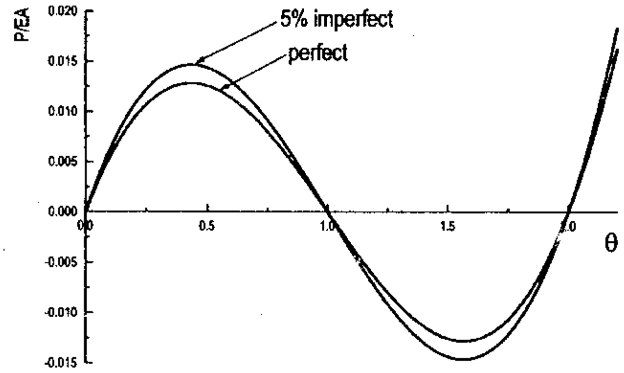


Fig 13 Influence of imperfections in cases of snap-through buckling

equilibrium equation has the unique solution

$$P = k\ell \frac{\left(\sqrt{1+\sin\theta} - \sqrt{1+\sin\theta_o}\right)\sqrt{1-\sin\theta}}{\sin\theta} \tag{10}$$

plotted in Fig. 12 for several magnitudes of the initial imperfection. It is observed that the primary and secondary equilibrium paths are once again replaced by a single, nonlinear equilibrium path, which, however, may be stable or unstable in the post-buckling region, depending on the direction of the imperfection.

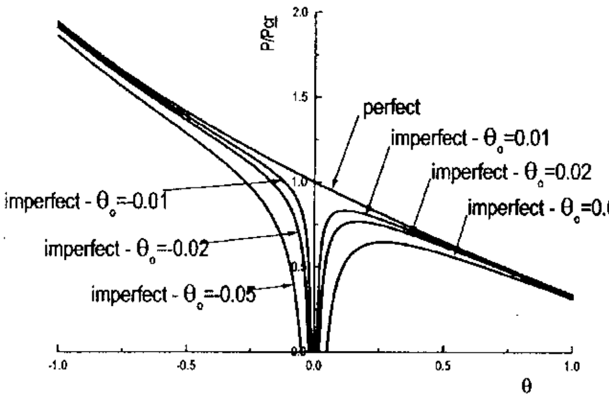


Fig 12 Influence of imperfections in cases of asymmetric bifurcation

Structures that buckle via snap-through

Consider finally the two-bar von Mises truss of Fig. 8 with an imperfection modelled as a deviation of the initial height H from its theoretical value by Ho. The equilibrium Eqn (7) remains the same, replacing H by ($H-H_0$). The influence of a 5% initial imperfection is plotted in Fig. 13. It is observed that the nature of the response does not change and the value of the critical load is not affected considerably.

2.3 Interaction between failure modes

The discussion on buckling has so far been restricted to the case of linearly elastic material behaviour and primarily 1-DOF systems that have only one buckling mode. In real cases, however, the interactions between buckling and material nonlinearity, as well as between different buckling modes must be accounted for. This will be illustrated by means of some simple examples.

Yielding, local buckling and global buckling of von Mises truss

Consider first the two-bar von Mises truss of Fig. 8. Eqn (7) expresses the equilibrium path, which is characterized by snap-through buckling or global buckling of the whole structure, as discussed in section 2.1, and shown again in Fig. 14a for two different span to height ratios. There are, however, two further failure modes that may occur, namely member yielding and local member buckling. Assuming elastic-perfectly plastic material behaviour, member yielding occurs if the axial member stress becomes equal to the yield stress f_y. This condition can be rewritten to obtain the yielding failure envelope, relating external load P and deflection δ:

$$P = 2Af_y \frac{H-\delta}{\sqrt{L^2 + (H-\delta)^2}} \tag{11}$$

Member buckling occurs if the axial force in one of the bars becomes equal to the Euler buckling load. This condition leads to:

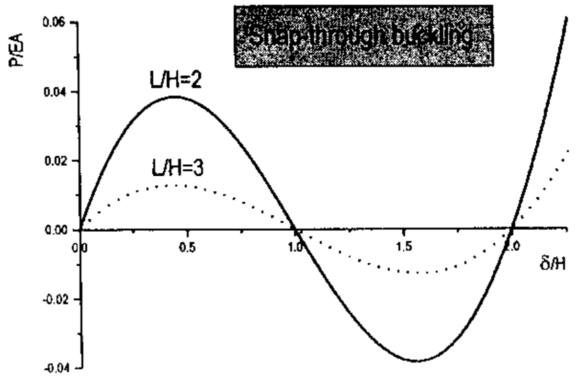
$$P = 2\pi^2 EI \frac{H-\delta}{\left[L^2 + (H-\delta)^2\right]^{\frac{3}{2}}} \tag{12}$$

These relations are plotted in Figs 14b,c. Any of the above three failure modes may be critical, depending on the geometric layout of the truss and the inertia characteristics of its members. Three possible failure scenarios are shown in Figs 15,b,c.

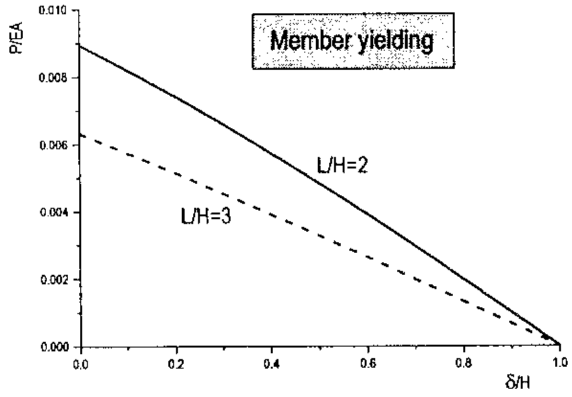
This example alerts us to the fact that there are several failure modes that must be accounted for, including material degradation and global and local buckling modes. However, it has been treated in a simplified way that fails to identify potential interactions between failure modes. Such interactions do exist, and can play a significant role in the overall structural response. In accordance to this reasoning, the member yielding and member elastic buckling limit states should actually be replaced by one limit state of elastic or inelastic buckling. This will be outlined further with the next examples.

Yielding and buckling of simple columns

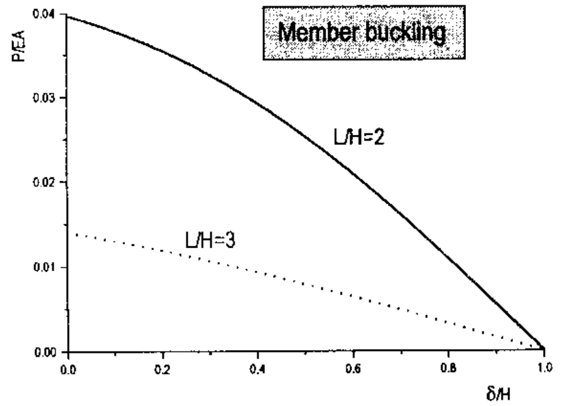
Let us now consider the rigid column of Fig. 2 and



(a) global buckling

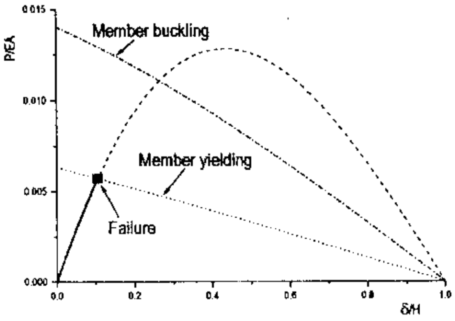


(b) member yielding

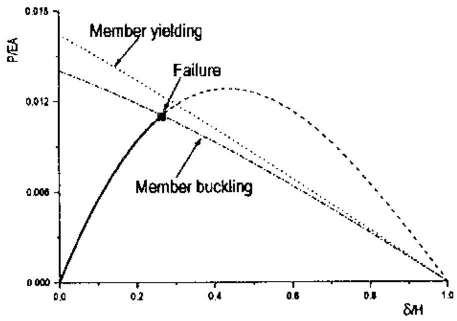


(c) member buckling

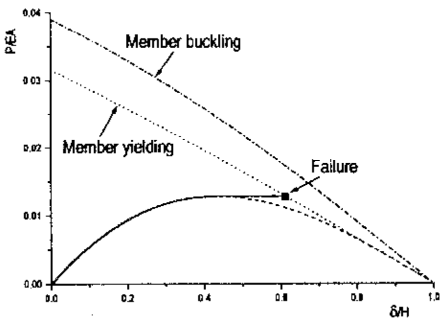
Fig 14 The three failure curves of von Mises truss for different span to height ratios



(a) due to member yielding on the prebuckling nonlinear equilibrium path



(b) due to member buckling on the prebuckling nonlinear equilibrium path



(c) due to snap through and subsequent member yielding

Fig 15 Three failure scenaria of von Mises truss introduce a material nonlinearity in the rotational spring. One way to do that is to assume that the spring is substituted by a very short joint made of elastic perfectly plastic material with yield stress f_y, and having a rectangular cross-section b by h (Kollár [11]). The elastic buckling response of this model is described by Eqn (1) in the perfect case and Eqn (8) in the presence of an initial imperfection θ_o. In order to investigate the interaction between geometric and material nonlinearity, we need the expressions P_{el} and P_{pl} of the load that correspond to the onset of yielding and full plastification of the cross-section, respectively. If the column remains vertical and is in pure compression, these expressions coincide and are equal to

$$P_{pl,o} = bhf_y \tag{13}$$

If there is a horizontal deflection of the tip, $w = \ell \sin\theta$, then the column is subjected to combined compression and bending. Then, by simple equilibrium considerations we obtain:

$$P_{el} = \frac{P_{pl,o}}{1+3\dfrac{w}{h/2}} \tag{14}$$

$$P_{pl} = P_{pl,o}\left[\sqrt{\left(\frac{w}{h/2}\right)^2+1}-\frac{w}{h/2}\right] \tag{15}$$

In the case of a perfect column, elastic buckling is critical if the critical load, given by Eqn (2) is smaller than the plastification load $P_{pl,o}$, otherwise yielding governs. The behaviour becomes more complex in the realistic case of an imperfect structure, for which the response is described by Fig. 16. If the structure has an initial imperfection θ_o, and is loaded gradually, then the response follows the elastic equilibrium path, given by Eqn (8). When this curve intersects the nondimensionalized P_{el}-curve, plastification begins. Eqn (8), which was derived on the basis of elastic behaviour, is no longer valid, and the stiffness of the structure is reduced. The equilibrium path deviates from the dotted curve of Eqn (8) and follows the dashed curve that approaches asymptotically the nondimensionalized full plastification P_{pl}-curve. Thus, the equilibrium path of a structure failing, in the perfect case, by stable symmetric bifurcation resembles, in the presence of combined imperfections and material nonlinearity, to that of a structure exhibiting limit point buckling.

A similar reasoning has been used to derive elasto-plastic buckling curves of real columns. Consider the simply supported column of Fig. 1b, with flexural stiffness EI and length ℓ. Its buckling response is obtained by writing the moment equilibrium equation in the deformed shape. Linear analysis yields the well-known expression of the Euler critical buckling load:

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} \tag{16}$$

which can be rewritten in terms of the critical buckling stress σ_{cr} :

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \tag{17}$$

where λ is the slenderness, defined as the buckling length divided by the radius of inertia of the cross-section. This relation is plotted in Fig. 17, where the stress is normalized with respect to the yield strength f_y and the slenderness with respect to

$$\lambda^* = \pi\sqrt{\frac{E}{f_y}} \tag{18}$$

that is the theoretical slenderness value at which

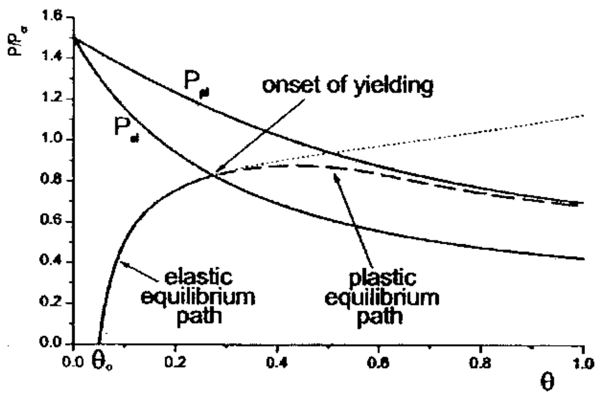


Fig 16 Interaction between strength and buckling failure in a simply-supported column

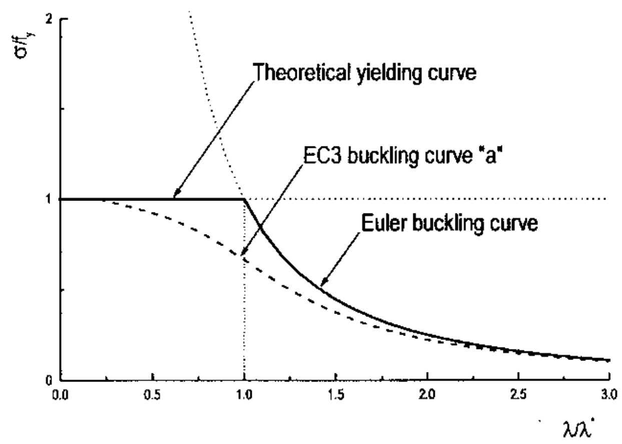


Fig 17 Interaction between strength and buckling failure in a simply-supported column

buckling and yielding occur simultaneously. For $\lambda<\lambda^*$ yielding is critical, while for $\lambda>\lambda^*$ buckling governs. In reality, however, the interaction between yielding and buckling is influenced by other factors, such as the real constitutive law of the material, residual stresses due to the manufacturing process, and initial imperfections. This is addressed by modern codes with the so-called buckling curves that have been obtained by combination of experimental, analytical and numerical considerations, and are material and cross-section dependent. One of the buckling curves proposed by Eurocode 3 [8] for steel columns is also shown in fig 17. It can be observed that the buckling curves differ significantly from the theoretical predictions only in slenderness values in the vicinity of λ^*, in other words in the region where theoretically failure due to pure buckling or pure yielding appear to coincide. This is in agreement to the observations of Fig. 16, where geometric nonlinearity is critical at the beginning of the curve, material nonlinearity at the end, while both phenomena influence the response in the intermediate part.

Interaction between buckling modes

Having discussed the interaction between buckling and material nonlinearities, let us now look at possible interactions between different buckling modes. This phenomenon was first observed for the so-called Augusti's column (Bazant [6]), shown in Fig. 18. This model is a 3-D version of the structure of Fig. 2, consisting of a rigid bar of length L, hinged at the bottom and supported by two linear rotational springs in two vertical planes that are perpendicular to each other, with stiffness constants c_1 and c_2, respectively, subjected to a vertical concentrated load P at the top.

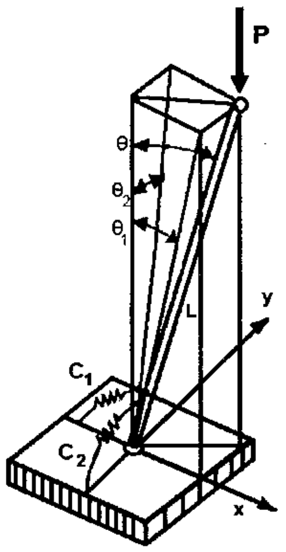


Fig 18 Augusti's column

By extending the findings for the model of Fig. 2 it can be concluded that, in the perfect case, this structure has two buckling modes, one with critical load $P_{cr,1}=c_1/L$, corresponding to rotation θ_1 in the plane of the spring c_1, and one with critical load $P_{cr,2}=c_2/L$, corresponding to rotation θ_2 in the plane of the spring c_2. The secondary equilibrium paths associated with both modes are stable.

Attempting to perform an optimum design that takes into account only the critical load, one would choose $c_1 = c_2$, so that $P_{cr,1}=P_{cr,2}=P_{cr}$. Then due to symmetry, the column's first buckling mode will be in the plane of symmetry $\theta_1=\theta_2$. By writing the moment equilibrium equations in the case of symmetric initial imperfections $\theta_{1,o}=\theta_{2,o}=\theta_2$, we obtain the following equation between the load P and the angles $\theta_1=\theta_2=\theta$, describing the equilibrium path:

$$P=\frac{c}{L}(\theta-\theta_o)\sqrt{1-2\sin^2\theta} \qquad (19)$$

This equilibrium path is plotted in Fig. 19 from which we can observe that buckling takes place via a limit point, and that the post-buckling response is unstable. In other words, the interaction between two stable buckling modes in the presence of initial imperfections resulted in a new, unstable buckling mode. Furthermore, the buckling load corresponding to the imperfect, limit point system is only approximately 35% of the critical bifurcation load of the perfect system.

3. PRACTICAL ANALYSIS AND DESIGN ISSUES

In this section it will be attempted to identify a number of issues that have to be accounted for during the analysis and design process of structures that are prone to instability, and propose pertinent guidelines. To that effect, the following steps are deemed necessary:

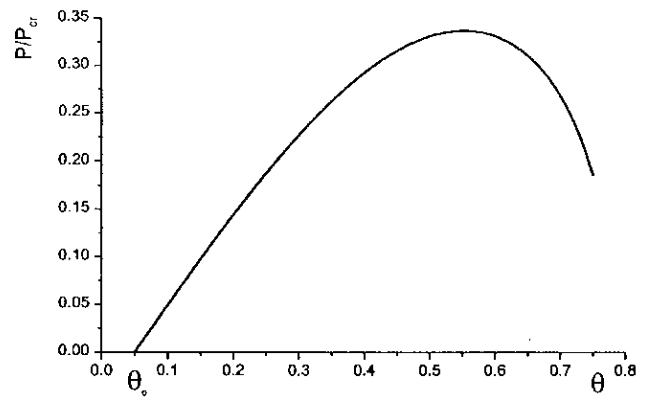


Fig 19 Equilibrium path of imperfect Augusti's column

- Realization of the design implications of the possible instability types
- Qualitative prediction of the type of possible instabilities for the specific structure
- Approximate assessment of buckling loads by simple analytical considerations
- Selection of appropriate modelling and analysis methods that account for all significant response factors
- Dimensioning with the aid of pertinent codes and/or other tools

These steps will be further analyzed next.

3.1 Main characteristics of the possible types of instability from a design perspective

As seen from the simple examples of section 2, one can distinguish between the following possible types of instability:

- stable symmetric bifurcation
- unstable symmetric bifurcation
- asymmetric bifurcation
- snap-through

This classification is of major significance, as it is the governing factor for deciding:

- whether the buckling loads can be evaluated satisfactorily with a linear analysis or a nonlinear formulation of the problem is required,
- whether buckling failure will be abrupt and violent or with a warning and smooth,
- whether imperfections will have a significant effect on the response, and, on the basis of the latter two points,
- what values of safety margins are appropriate for the problem.

Even more importantly, this qualitative knowledge at an early stage of preliminary design can be very helpful in deciding between alternative structural systems, thus avoiding solutions that are bound to have poor structural performance.

Generalizing the conclusions drawn from the previous examples, the following statements may be formulated with regard to the several types of instability:

Stable symmetric bifurcation

The main characteristics that are common to all structural systems that buckle through stable symmetric bifurcation are the following:

- The critical buckling load P_{cr} of perfect systems can be obtained accurately either with linear or nonlinear analysis.
- The stable nature of the secondary equilibrium path can only be predicted with nonlinear analysis.
- From a design point of view the stable secondary path means that with the onset of buckling the stiffness of the structure will decrease very rapidly, thus large deformations will develop, however, the structure possesses post-buckling strength, in other words it can carry some additional load beyond the critical load. Buckling failure is then usually inelastic, relatively smooth and provides a warning, especially in cases of significant post-buckling strength.
- The presence of imperfections in structures that fail via stable symmetric bifurcation can only be treated with nonlinear analysis, and reduces their stiffness, which may lead to serviceability problems, but does not alter significantly their ultimate capacity.

Unstable symmetric bifurcation

The main conclusions that can be drawn from the previous analysis, common to all structural systems that buckle through unstable symmetric bifurcation, are outlined next:

- The critical buckling load P_{cr} of perfect systems can be obtained accurately either with linear or nonlinear analysis, but evaluation of P_{cr} for imperfect systems requires nonlinear analysis.
- The unstable nature of the secondary equilibrium path can only be predicted with nonlinear analysis.
- From a design point of view this unstable secondary path means that with the onset of buckling not only will the stiffness of the structure decrease very rapidly, thus large deformations will develop, but also, the structure possesses no post-buckling strength, in other words it can not carry any additional load beyond the critical load and can not even sustain that level of loading. Buckling failure is then sudden and without warning.
- The presence of imperfections in structures that fail via unstable symmetric bifurcation not only reduces their stiffness and may lead to premature serviceability problems, but also changes their buckling mode from bifurcation to snap-through and reduces significantly their load bearing capacity. A geometrically

nonlinear analysis is necessary in order to quantify this. This reduction is more intense as the unstable post-buckling equilibrium path becomes steeper and has been reported to be up to the order of 50%, for example in problems of shell buckling.

Asymmetric bifurcation

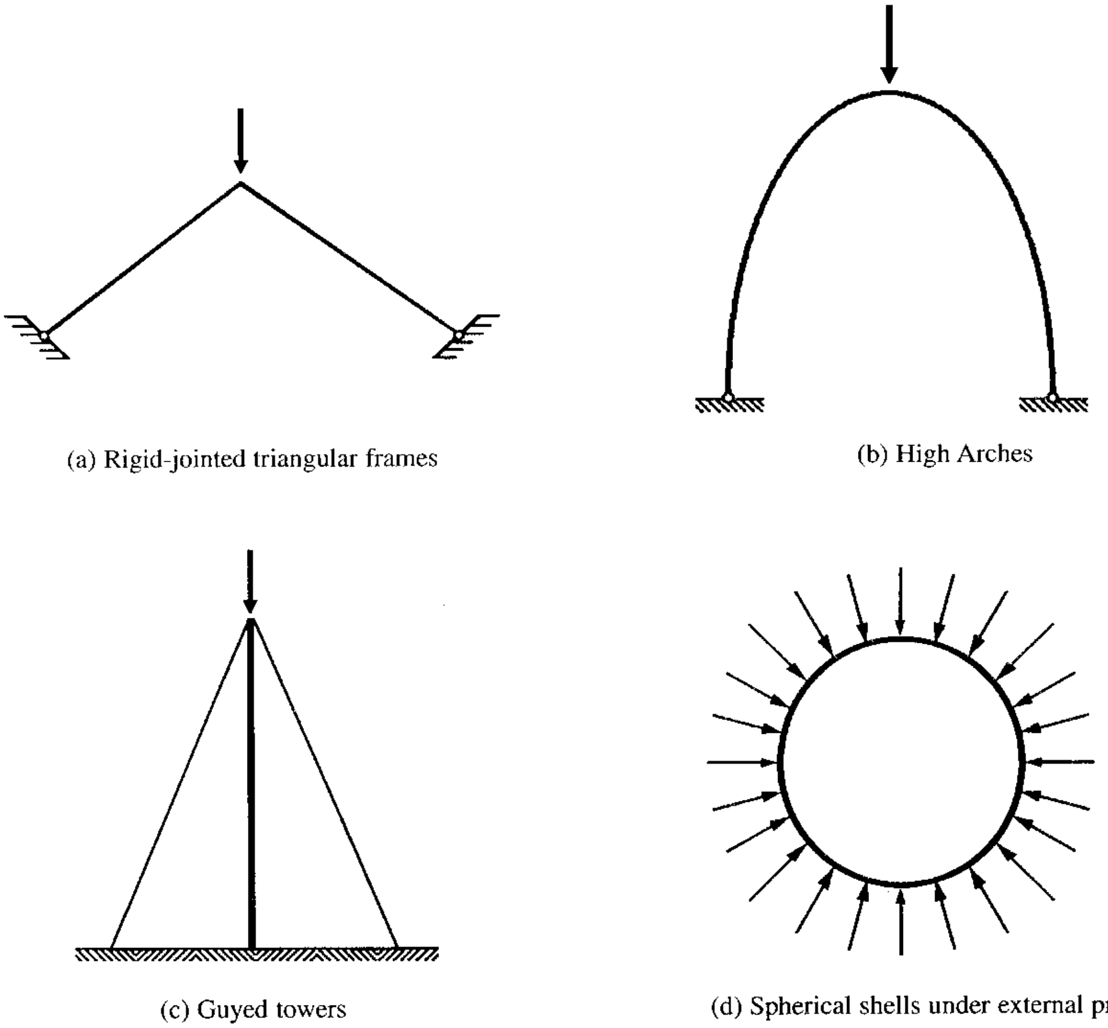
The main characteristics that are common to all structural systems that buckle via asymmetric bifurcation are:

- The critical buckling load P_{cr} of perfect systems can be obtained accurately either with linear or nonlinear analysis.
- The asymmetric nature of the secondary equilibrium path can only be predicted with nonlinear analysis.
- From a design point of view similar considerations hold for the perfect system as for systems exhibiting unstable symmetric bifurcations. However, the asymmetric nature of the response indicates that it could be appropriate to design the system with intentional imperfections that would lead it to buckle in a stable way, in other words to behave in a nonlinear manner with reduced stiffness, but would prevent its catastrophic snap-through buckling. Treatment of the imperfect system requires nonlinear analysis.

Snap-through buckling

The main characteristics that are common to all structural systems that exhibit snap-through buckling are:

- The critical buckling load P_{cr} can be obtained accurately only with nonlinear analysis, both for perfect and for imperfect systems.
- From a design point of view it is interesting to extend the observation of Fig. 14, comparing the failure envelopes of two trusses with different span to height ratios. The critical load corresponding to global buckling is reduced



(a) Rigid-jointed triangular frames

(b) High Arches

(c) Guyed towers

(d) Spherical shells under external pressure

Fig 20 Examples of structures that buckle via unstable symmetric bifurcation considerably s the truss becomes shallow. This conclusion can be extended to most cases of shallow or flat arches, shells, space trusses etc., for which snap-through buckling is often a critical failure mode. The flatness of the truss influences the yielding and local buckling failure modes as well, but less than the global buckling mode.

- The influence of initial imperfections on the response of structures that fail via snap-through buckling is, in most casts, not very significant from a design perspective.

3.2 Qualitative prediction of the type of possible instabilities

For the reasons mentioned in section 3.1 it is of great significance to have the ability to identify in a qualitative manner the types of possible instabilities at an early stage of the design process. One way to do this is to have knowledge of typical structural systems that are prone to each instability type, so that the engineers can relate these behavioural characteristics to their specific design problems.

The two main categories of real structures that buckle by stable symmetric bifurcation are slender columns and thin plates in compression (Fig. 1). The nonlinear post-buckling response of a simply supported column with flexural stiffness EI and length ℓ is given by the following relation between the load P and the maximum transverse displacement in the middle w_{max} (Bazant [6]):

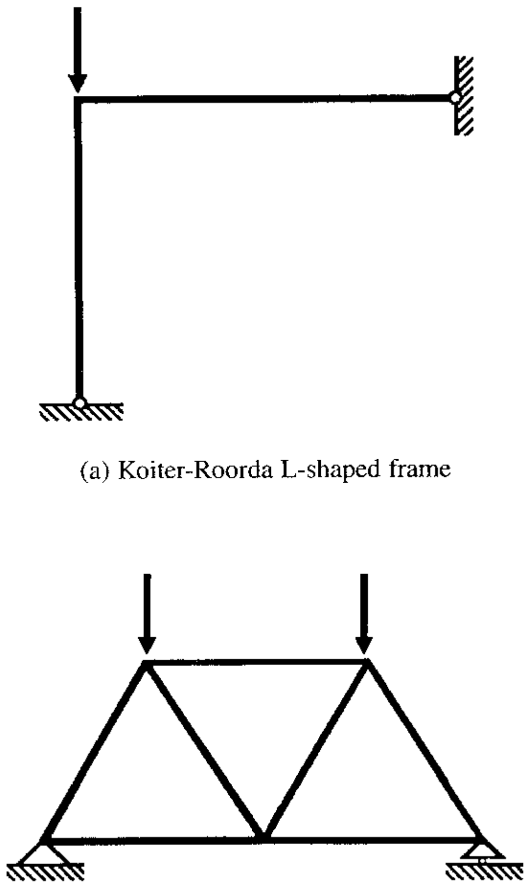
$$P = P_{cr}\left(1+\frac{\pi^2}{8\ell^2}w_{max}^2\right) \tag{20}$$

where Pcr is the critical Euler buckling load, given by Eqn (16). The positive w_{max}^2 term in the right-hand side indicates a stable symmetric post-buckling path.

Similar is the response of axially compressed columns with different boundary conditions, however, the length ℓ in Eqns (16) and (19) is replaced by the buckling length $K\ell$, corresponding to the distance between two successive inflection points in the deformed shape of the buckled column. The K-factor depends on the boundary conditions, taking values between 0.5 for the case of a column with both supports clamped, to ∞ for a column-mechanism with one hinged and one free support.

Compressed plates (Fig. 1a) have a qualitatively similar response with the important practical difference that the stable secondary path is much steeper than for columns. Thus, plates possess a very significant post-buckling strength. This increase is attributed primarily to the membrane actions developing in the plates as they become curved due to buckling-related deflections. In addition, plates have the ability to redistribute load to parts than remain stiffer (for example due to proximity to supports) from other parts that become more flexible due to buckling. This is known as the "paradox of plate buckling" (Kollár [11]).

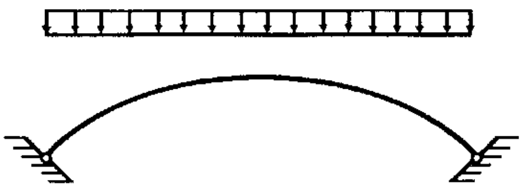
Examples of real structures that buckle by unstable symmetric bifurcation are shown in Fig. 20, including rigid-jointed triangular frames (Croll and Walker [3]), high arches (Bazant [6]), guyed towers (Chen and Lui [4]) and spherical shells under external pressure (Croll and Walker [3]). Typical examples of real structures that buckle by asymmetric bifurcation are the Koiter-Roorda L-shaped frame (Bazant [6]) and the truss (Croll and Walker [3]) shown in Fig. 21. Finally, snap-through buckling is often a critical failure mode for shallow or flat arches, shells, space trusses etc. (Fig. 22).



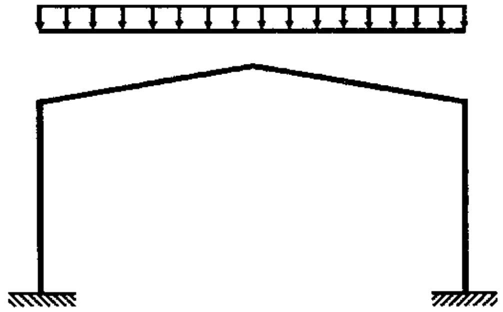
(a) Koiter-Roorda L-shaped frame

(b) Truss

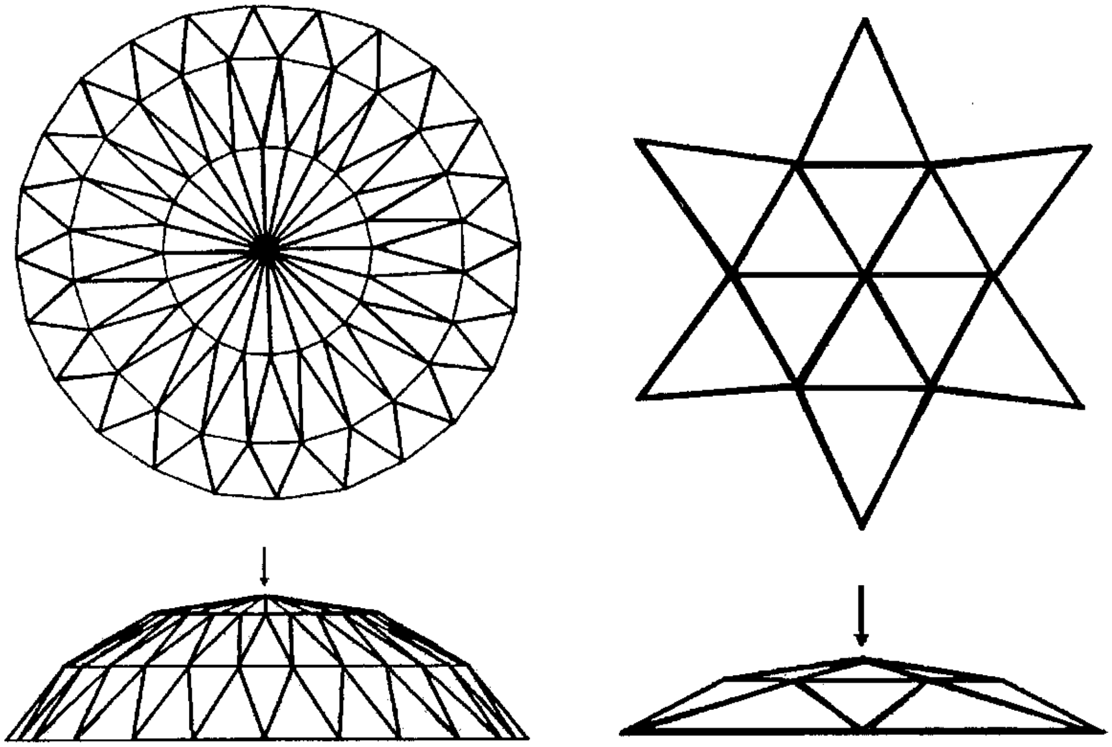
Fig 21 Examples of structures that buckle via asymmetric bifurcation



(a) Shallow arches



(b) Shallow pitched frames



(c) Shallow shells and domes

Fig 22 Examples of structures exhibiting snap-through buckling

Another way to identify in a qualitative manner the types of possible instabilities, particularly in simple structures, is by simple inspection and engineering considerations (Kollár [11]). For example, in the model of Fig. 2 we can observe that the overturning moment of the load P increases proportionally to sin θ, therefore slower than the restoring moment of the rotational spring, which is proportional to θ. Consequently, the post-buckling response must be of increasing nature, therefore stable.

Considering now the model of Fig. 4, the overturning moment of the load P increases again proportionally to sinθ, which is faster than the restoring moment of the translational sprig, which is proportional to sinθcosθ. Consequently, the post-buckling response must be of decreasing nature and, hence, unstable.

In the case of elastic bar structures the post-buckling response is increasing at a very slow rate and can be considered as almost constant for small deflections. The restoring moment is exerted by the internal bending moment of the bars due to their bending stiffness. If this stiffness is reduced by some effect, then the post-buckling response becomes decreasing. Material nonlinearity is such an effect that causes the internal bending moment to increase with increasing curvature slower than in the elastic range or not at all. This explains qualitatively the findings of section 2.3 about the interaction of yielding and buckling.

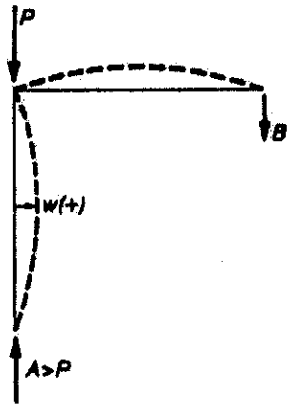
A further example is the asymmetric frame of Fig. 21a. If the frame buckles according to Fig. 23a, then the deformed shape of the beam indicates that at the right support a downwards reaction force B is needed (Kollár [11]). Thus, at the left support a reaction force A larger than P arises, which will also be the compressive force of the column. Taking into account that the post-buckling response of the column is approximately constant, after buckling P must decrease in order to keep the column compression constant. This is shown in the right, unstable part of Fig. 23c. The opposite is the case if the frame buckles according to Fig. 23b, resulting in the left, stable part of the curve in Fig. 23c.

A qualitative criterion in order to determine whether a structure will buckle via a bifurcation or a limit point is to study the nature of pre-buckling deformations. Bifurcation systems are loaded in such a way that before buckling their members exhibit only axial shortening, which is usually negligible. Only after buckling do these systems deform flexurally. Limit point systems on the other hand, deform in a flexural manner from the onset of loading, either at member level (Figs 22a,b) or at a macroscopic level, as is the case with the reticulated domes of Fig. 22c, which carry transverse loads via axial action of rods.

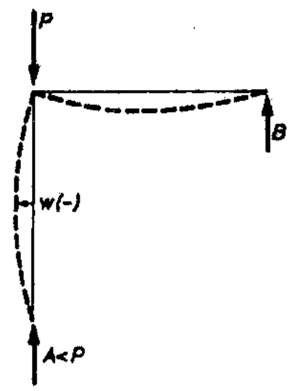
Finally, in most complex structures the interaction between local and global buckling, as well as geometric and material nonlinearity, renders, in the presence of unavoidable imperfections the post-buckling behaviour unstable. It is namely possible that the interaction of two stable buckling modes may generate an unstable coupled mode, rendering the member highly sensitive to imperfections (Davies [12], Dubina [13]). In such cases a significant reduction of the critical load may occur. As this influence is on the unsafe side, it must always be taken into consideration for design purposes.

This is particularly the case if the structural engineer attempts to perform a so-called "naive" optimization, in other words to design structures such that different failure modes occur at the same or similar values of the external loads, hoping that this will provide an optimum solution. Extreme caution is advised against this approach, due to possible strengthening of the coupling between buckling modes occurring at similar load values that may have adverse effects on the stability response. Such situations were identified to have been significant contributing factors in several catastrophic collapses (Bazant [6]).

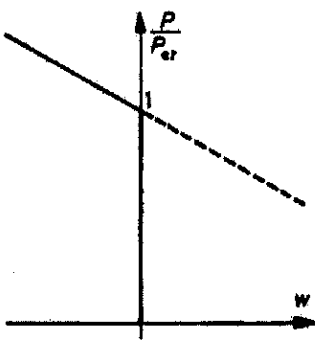
Thin-walled structures are particularly sensitive to such coupled instabilities due to the co-existence of local and global modes. Single-layer, reticulated shallow domes, like the ones of Fig. 22c, may exhibit coupling between local (nodal or member) instabilities



(a) Deformed shape for unstable buckling



(b) Deformed shape for stable buckling



(c) Equilibrium path

Fig 23 Asymmetric bifurcation buckling of the Koiter-Roorda L-shaped frame and global system snap-through buckling (Gioncu [14], Ivan and Gioncu [15]). Another example is the coupling between local and global buckling of built-up columns, illustrated in Fig. 24 (Galambos [5], Bazant [6]).

3.3 Approximate assessment of buckling loads by simple analytical considerations

The ability to estimate upper and lower bounds of the response is of utmost importance during preliminary structural design. It allows for a fast comparison between alternative solutions, for elimination of unsuitable structural systems and an initial selection of a shortlist of optimum ones. Furthermore, it provides valuable information for modelling purposes as well as values for comparison of numerical results.

In the case of instability problems such estimates are particularly difficult, because the nonlinear nature of the response makes scaling troublesome, and because the interactions between geometric and material nonlinearity as well as between different buckling modes cannot be easily account for. Nevertheless, it is still possible to obtain a lot of useful information by approximate analytical considerations.

The way to do is to find an analogy between the structure at hand and a "prototype" structure, for which the bearing capacity is known, either from the literature or from previous experience. To establish such an analogy it may be necessary to simplify our structure, for example by substituting individual members or parts by "equivalent" ones. In doing so,

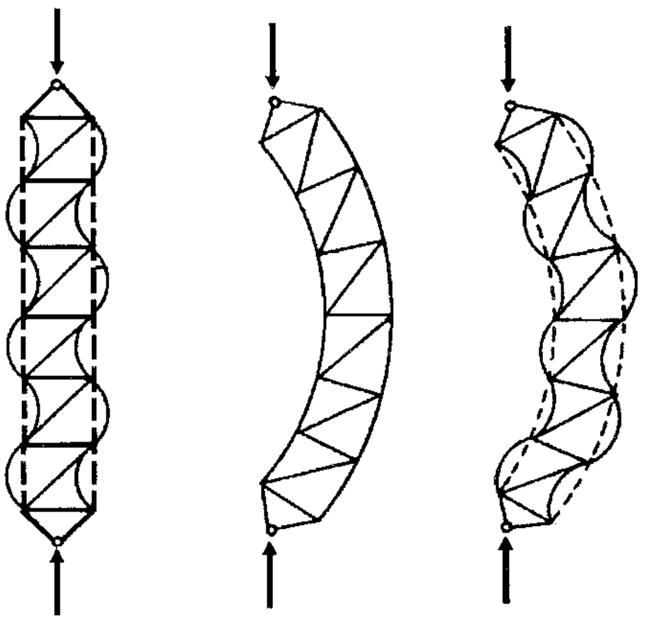


Fig 24 Local, global and compound buckling modes of built-up column

two things must be kept in mind:

- If our objective is to estimate buckling loads, and since buckling depends on structural stiffness, substitution must take place by criteria of "stiffness equivalence".
- The structural model resulting from the substitution process will, in general, consist of a smaller number of members than the real structure. The two structures will deform in approximately the same way, but only in a macroscopic sense, not locally. In stress analysis problems this results in inability to obtain information about local stress distributions and concentrations. Similarly, in buckling problems such simplified models can provide valuable information for global buckling modes, but are unable to capture local buckling effects. Nevertheless, this is still very useful during preliminary design, because in that early stage of the design process, when a structural system selection is the main objective, global buckling is decisive. Local instabilities can, in most cases, be treated at a later stage by appropriate member dimensioning.

3.4 Choice of numerical model and analysis method

The examples of section 2 have been selected such that they are simple enough to have analytical solutions, so that emphasis could be placed on the different instability patterns. For most real structures, however, this is not the case. Even though simplified models have, as discussed in section 3.3, great value in capturing qualitative aspects of the response and estimating the bearing capacity, there is, in most cases, the need for a more 'exact' numerical analysis, which is usually carried out with the finite element method. Some basic considerations for the options that are available to structural engineers for such analyses are briefly outlined in this section.

It is a well-established fact that the results of a numerical analysis are as good and reliable as the model which is used for the analysis. In other words, it is extremely important to understand, prior to the analysis, the qualitative characteristics of the structural response, which should than be clearly reflected upon the chosen analysis model. This is true for all types of analysis, not just those aiming at predicting instability behaviour. The additional complication that is inherent in the latter case is the choice between several options in analysis type and corresponding nonlinear kinematic formulations, material models and incremental numerical solution algorithms. An excellent discussion of this issue is given by Bathe [16].

One point that cannot be over-emphasized is that one should start from as simple models and analysis types as possible and proceed to incorporating into the model more effects gradually. Thus, a number of preliminary analyses may be required before a final and appropriate finite element model is established. This approach has two major advantages:

- it is easier to isolate modelling errors, and
- it is possible to identify the influence of each parameter on the overall response.

Along these lines, a linear static analysis should always be the first step. This can then be followed by a so-called linearized buckling analysis, in which we solve the problem

$$\det(\mathbf{K}_{L} + \zeta\mathbf{K}_{NL}) = 0 \tag{21}$$

where $\mathbf{K}_L$ and $\mathbf{K}_{NL}$ are the linear and nonlinear stiffness matrices, respectively, usually corresponding to the initial, undeformed configuration of the structure. The physical meaning of this equation is that at buckling the structure experiences theoretically infinitely large displacements, therefore, the total stiffness matrix of the structure becomes singular. It is also assumed that the linear stiffness matrix does not change significantly prior to buckling and the nonlinear stiffness matrix is simply a multiple of its initial value by a factor ζ, denoting a load multiplier. The linearized buckling analysis can, therefore, be appropriately used to predict the load level, at which a structure becomes unstable, if the pre-buckling displacements and their effects are negligible, in other words if the pre-buckling response is linear or almost linear. This is the case for perfect bifurcation systems.

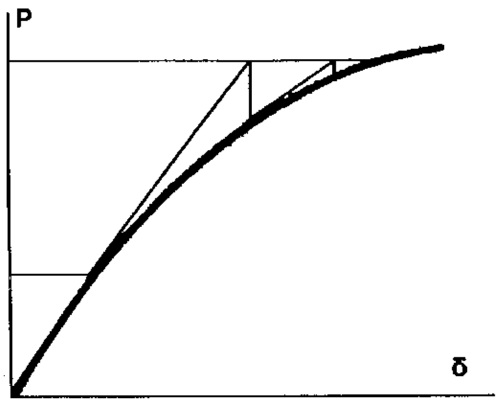
For limit point structures and imperfect bifurcation structures this is not the case, hence, a nonlinear analysis is necessary. Similarly to the way in which a physical experiment would be carried out in the laboratory, in a numerical nonlinear analysis the load is applied in steps. The final geometric configuration of each step is considered as initial configuration for the next one and is used for the evaluation of stiffness matrices and internal force vectors. In solid mechanics problems the motion of all particles of the body is followed from their original to their final configuration. This is called the Lagrangian formulation of the problem, in contrast to the Eulerian formulation, common in fluid mechanics problems, where attention is focused on the motion of the material through a stationary control volume. Furthermore, we differentiate between the total Lagrangian formulation, where all variables at all load steps are referred to the initial configuration, and the updated Lagrangian formulation, where all variables at each load step are referred to the configuration in the previous load step.

Within each step the response is assumed to be linear and iterations are carried out to achieve convergence, which is checked by comparing internal and external forces. The most common numerical iteration scheme is the so-called Newton-Raphson method (Bathe [16]), shown schematically in Fig. 25a for a one degree-of-freedom (1-DOF) system. The thick curve is the equilibrium path and the thinner lines denote the iterations within a load step. The stiffness matrix is updated at each iteration within a load step, which accelerates convergence in a reduced number of iterations but is computationally very intensive. Alternatively, the stiffness matrix may remain the same for all iterations within a step, thus reducing the cost per iteration but slowing down convergence and increasing the number of required iterations. This is called the Modified Newton-Raphson method (Bathe [16]) and is shown schematically for a 1-DOF system in Fig. 25b.

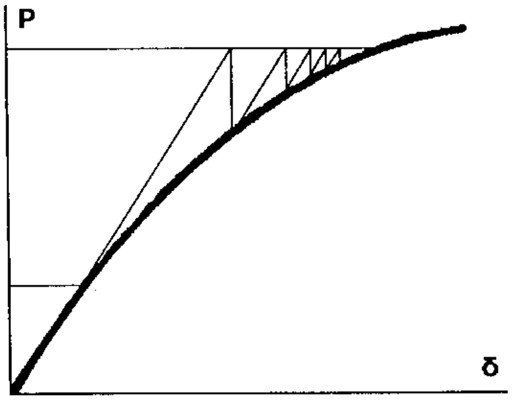
In order to trace the complete load-displacement path in limit point systems a displacement-controlled analysis must be carried out, in other words displacements must be imposed instead of loads, otherwise numerical instability in the vicinity of the limit point leads to convergence difficulties. Alternatively, load control is possible in conjunction with iterative algorithms known as arc-length methods (Crisfield [17], Bathe and Dvorkin [18]), shown schematically in Fig. 25c that adjust the size of the step to the degree of nonlinearity.

There are several improvements and variations of these basic methods but their description is not within the scope of this article. The selection of the optimum analysis method is problem dependent and requires a trial and error approach, even for experienced analysts.

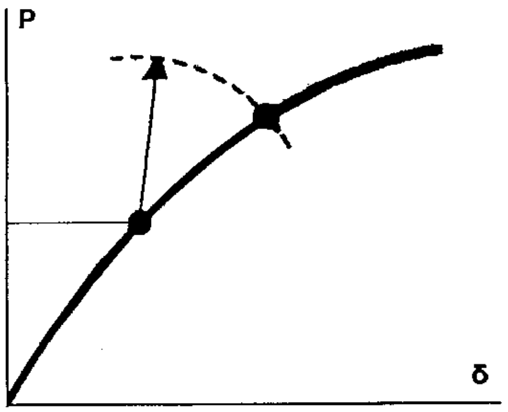
One further point that is worth making is the importance of linearized buckling analysis, even in cases of highly nonlinear pre-buckling behaviour, as a means for obtaining patterns of initial imperfections to be used for subsequent nonlinear analyses. It is common practice to consider imperfections with the shape of eigenmodes or linear combinations of them.



(a) Newton-Raphson method



(b) Modified Newton-Raphson method



(c) Arc-length method

Fig 25 Schematic representation of numerical algorithms for non-linear 1-DOF systems

This will be illustrated further with the example of section 4.

In conclusion, the following steps should be considered appropriate in most numerical analyses of instability problems:

- perform linear static analysis
- compare the results to approximate analytical solutions to verify the basic features of the model
- refine the mesh until satisfactory convergence for the static displacements is achieved
- perform linearized buckling analysis
- compare the results to approximate analytical solutions to verify the buckling aspects of the model
- refine the mesh until satisfactory convergence for the buckling loads is achieved
- if a limit point behaviour is expected,
  - perform a load-controlled, geometrically nonlinear, static analysis up to load levels slightly lower than the buckling loads of the linearized buckling analysis
  - identify the order of magnitude of the real buckling load and corresponding displacements from the occurrence of convergence difficulties
  - perform an arc length-controlled nonlinear static analysis up to load levels slightly higher than the non-convergence loads of the load-controlled nonlinear static analysis – alternatively, perform a displacement-controlled nonlinear static analysis up to displacement levels significantly higher than the non-convergence displacements of the load-controlled nonlinear static analysis – thus, trace also the part of the post-buckling equilibrium path that may be relevant from a design point of view
  - refine the mesh until satisfactory convergence for the limit point loads is achieved.
  - introduce initial imperfections having the shape of a linear combination of the first few buckling modes determined by the linearized buckling analysis, multiplied by appropriate weight factors, chosen so that the magnitude of the maximum imposed imperfection is reasonable for the material, skills of the construction team and level of quality control expected to be employed.
  - repeat the nonlinear analysis for the imperfect structure – study the effect of imperfections – change the imperfection pattern and magnitude to study their influence

- check the maximum stress levels occurring at all steps of the nonlinear analysis and compare to the linearly elastic range of the material – if necessary, carry out nonlinear analysis including material nonlinearity
- extract maximum allowable loads and maximum values of stresses and displacements, to be used for dimensioning purposes
- if a bifurcation point behaviour is expected,
  - repeat the steps proposed for the limit point systems, omitting the analyses of the perfect structure, as numerical algorithms, will, in most cases, fail to detect the bifurcation point
  - pay more attention to the influence of imperfections, which is expected to be more critical

3.5 Additional design issues

From the previous discussion it is evident that adequate consideration of potential instabilities is required for the design of spatial structures. The following points are of utmost importance and should be addressed by a future pertinent design code:

- The designer should have a good theoretical background of the possible types of instabilities and be able to identify which are pertinent for the structure to be designed. Accordingly, a set of required analyses should be performed, progressing gradually from simpler to more complex ones.
- In current design practice, in most cases a linear elastic analysis is carried out, and the results are used to check against member buckling using buckling curves and against local buckling using classification of cross-sections. Geometrically nonlinear effects are taken into account indirectly, by requiring that the displacements do not exceed certain limits or by multiplying internal forces by appropriate magnification factors. Imperfections are usually accounted for by applying additional actions.
- This approach seems appropriate for most common frame-type structures but is clearly insufficient for structures with unstable post-buckling paths. For such structures nonlinear analyses with consideration of imperfections associated with linearized buckling modes must be carried out.
- The interaction between different buckling modes as well as between geometric and material nonlinearities may result in unstable post-buckling paths and significant reduction of critical loads in many structures, particularly in the presence of imperfections and when different failure modes occur at similar levels of loading. Codes should warn against this type of optimization and require consideration of failure mode coupling.
- In recent years there is an ongoing discussion about abandoning the K-factor buckling design approach, and adopting advanced, second-order, inelastic analysis procedures even for simple frames (for example White and Hajjar [19–20]). Thus, residual stresses, geometric imperfections, nonlinearities and moment redistribution due to spreading of plastification can be accounted for, resulting in a more realistic assessment of the structure's capacity to resist loads. This approach can also be used for spatial structures to account for the interaction of actual behaviour of individual members to that of the structure (Richard Liew et al. [22]), for the effect of real joint behaviour (Kato et al. [23]), the effect of plasticity in the presence of imperfections (Chan and Zhou [24]) and all pertinent critical effects (Bridge et al. [25]). These developments point towards the future direction for the treatment of stability in a future design code for spatial structures.

4. ANALYSIS EXAMPLE

Consider the shallow, two-dimensional, three-hinge steel truss of Fig. 26 with a span of 80m and a height of 12m. The inclined flanges have a cross-section of 300cm^2, and the vertical and horizontal bars 200cm^2 and 100cm^2, respectively. This is a structure exhibiting all the qualitative characteristics of more complex three-dimensional spatial structures. From the preceding discussion four possible failure types can be qualitatively identified:

- snap-through buckling of the whole truss,
- Euler-type buckling of one of the two inclined built-up members of the truss,

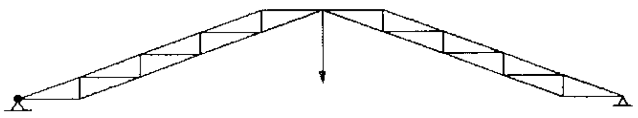


Fig 26 Undeformed configuration of shallow three-hinge truss

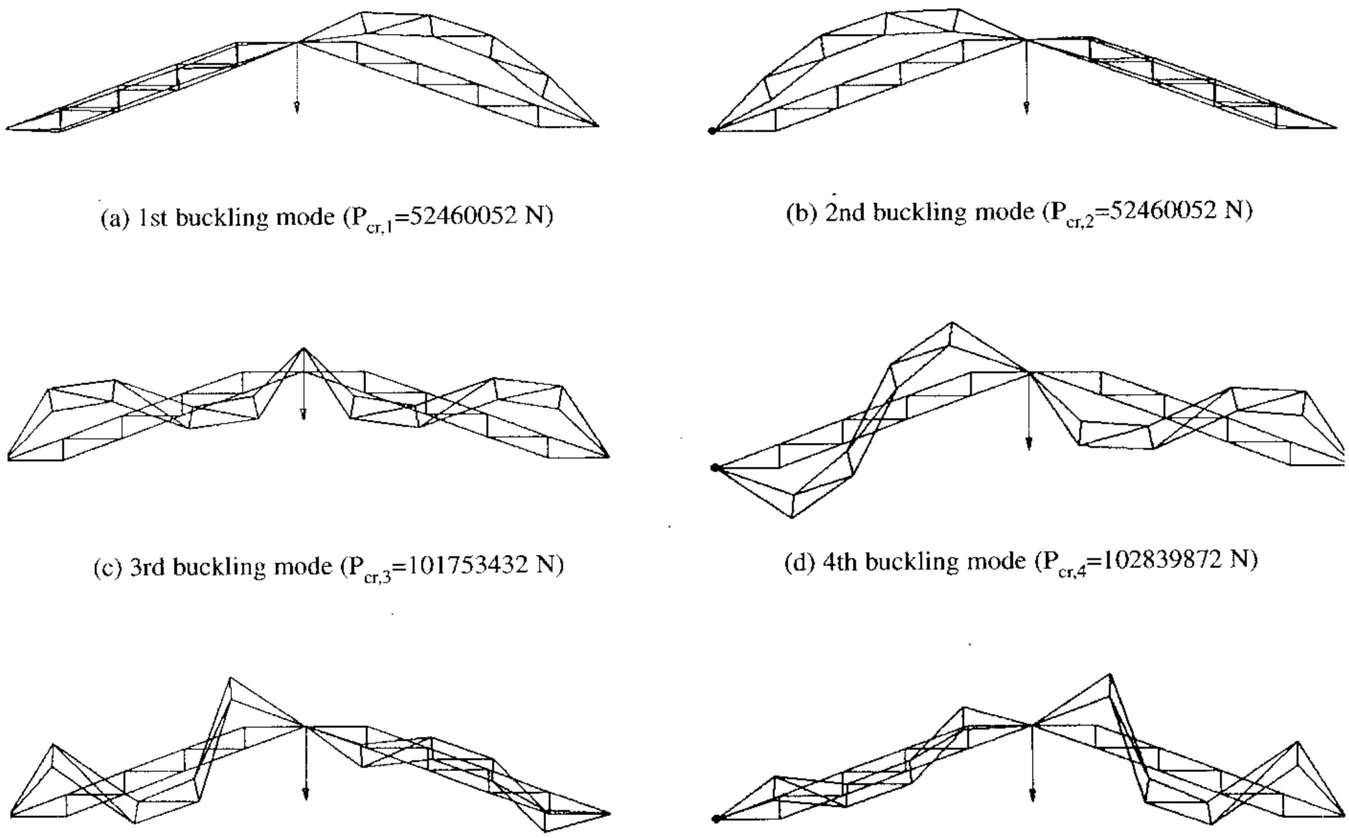
- Euler-type buckling of an individual bar, and
- Yielding of an individual bar.

In the analysis that follows we will consider only the first two types, assuming that the latter two, which can actually be replaced by a single limit state of elastic or inelastic buckling, are checked manually at each analysis step. In Fig. 27 the first six buckling modes and the corresponding buckling loads are shown, as obtained with linearized buckling analysis. It can be seen that all modes are of the second type and that snap-through buckling cannot be captured by such analysis. The lowest buckling load appears to be 52460kN, corresponding to the non-symmetrical first and second modes. Then, a nonlinear analysis of the truss is performed using Newton-Raphson iterations and displacement control, by imposing a vertical displacement on the middle hinge. Fig. 28 shows an intermediate deformed configuration and Fig. 29 the complete load-displacement path, indicating a snap-through type of response and a critical load of approximately 47000kN, which is approximately 10% lower than the linearized buckling load.

In order to capture the possible reduction of the bearing capacity due to the presence of initial imperfections, additional nonlinear analyses are necessary. Two such analyses have been performed, using the first and the fourth linearized buckling modes as patterns for small initial geometric imperfections. Fig. 30 shows the resulting equilibrium paths and Figs 31 and 32 illustrate two corresponding intermediate deformed shapes, indicating that the truss deforms in a non-symmetrical pattern. Both analyses exhibit limit point loads in the order of 30000kN, which are approximately 40% lower than the linearized buckling load, and estimate more realistically the load bearing capacity of the truss. In a real design situation further nonlinear analyses would be required using the other modes and their combinations as initial imperfections.

5. SUMMARY AND CONCLUSIONS

It has been attempted to present the fundamental instability characteristics of spatial structures in a design perspective, suitable as a starting point for



(a) 1st buckling mode ($P_{cr,1}$=52460052 N)

(b) 2nd buckling mode ($P_{cr,2}$=52460052 N)

(c) 3rd buckling mode ($P_{cr,3}$=101753432 N)

(d) 4th buckling mode ($P_{cr,4}$=102839872 N)

(e) 5th buckling mode ($P_{cr,5}$=25083520 N)

(f) 6th buckling mode ($P_{cr,6}$=25083520 N)

Fig 27 The six lower buckling modes obtained from linearized buckling analysis of perfect shallow three-hinge truss

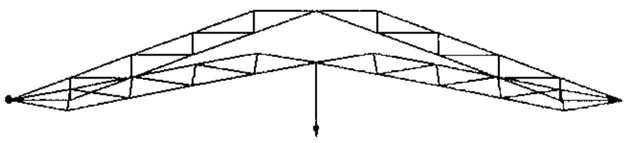


Fig 28 Intermediate deformed configuration resulting from nonlinear analysis of perfect shallow three-hinge truss

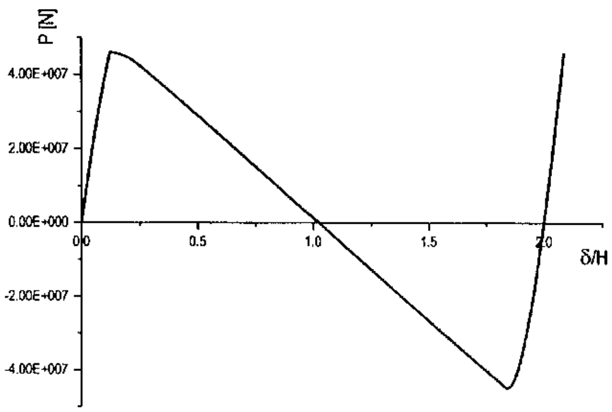


Fig 29 Equilibrium path resulting from nonlinear analysis of perfect shallow three-hinge truss

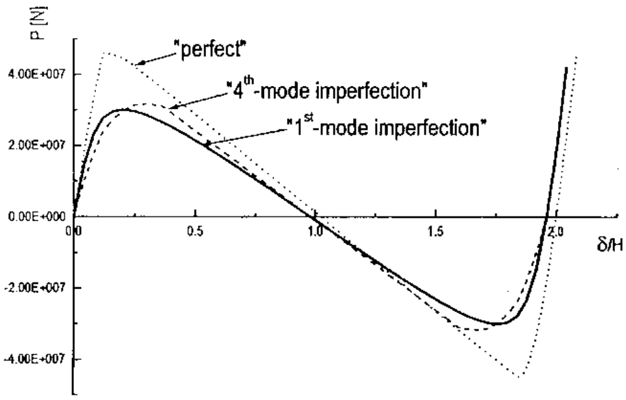


Fig 30 Equilibrium paths resulting from nonlinear analysis of perfect and imperfect shallow three-hinge truss

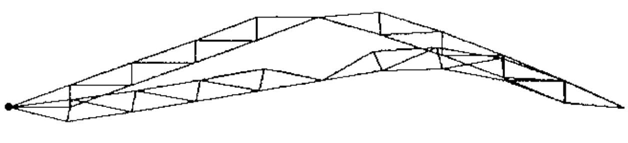


Fig 31 Intermediate deformed configuration resulting from nonlinear analysis of 1st-mode imperfect shallow three-hinge truss

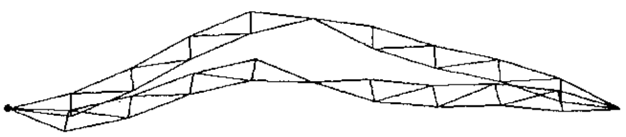


Fig 32 Intermediate deformed configuration resulting from nonlinear analysis of 4th-mode imperfect shallow three-hinge truss

discussion on a future design code. The nature of stable, unstable and asymmetric bifurcation as well as limit point buckling has been described by means of simple examples for which analytical solutions are available. Typical structures that buckle in each of these modes have been identified. The importance of linear and nonlinear analysis procedures and the effect of imperfections have been discussed. The interaction of buckling modes with material nonlinearities or other buckling modes has been addressed. Finally, structural analysis and design issues have been pointed out.

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