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The *SP*-Displacement Measure for Assessing Line Simplification

B. Nakos

In line simplification the overlay of the original and the derived line produces a certain number of polygons. The area of these polygons is related to the areal distortion between the original and the derived line caused by simplification. A new cartometric measure of displacement associated with each polygon (sp-displacement) is introduced and tested, expressing the displacements caused by line simplification. Data consisting of ten coastlines are processed under successive simplification tasks over a wide range of tolerances by applying the Douglas and Peucker algorithm. All derived lines were overlaid with the original ones by applying a typical GIS function of union. The new measure is compared with other measures suggested in the literature.

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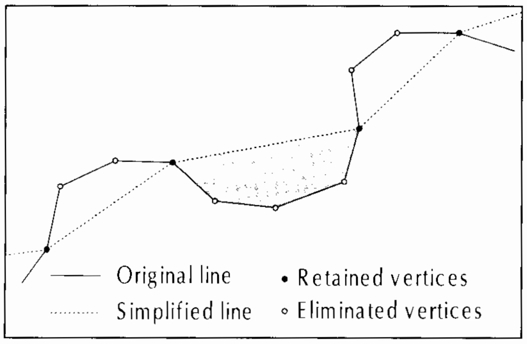
INTRODUCTION

Cartographers simplify lines by eliminating unnecessary details or by exaggerating several characteristics when they perform any generalisation task on the basis of their experience and knowledge. In general, the result of manual simplification preserves the shape or the visual character of any individual line in regard to the scale change and map purpose although its geometry is modified (Robinson *et al.*, 1995). Since line simplification is partially subjective (Keates, 1996; Robinson *et al.*, 1995), it is challenging to integrate the process of manual line simplification in a digital environment. In the past decades, many researchers with either cartographic or computer science backgrounds have tried to develop various line simplification algorithms, as well as algorithms for detecting critical points on digital cartographic lines -i.e. corner detection, polygonal approximation and a combination of them (Li, 1995). Among the line simplification algorithms, the most effective algorithms eliminate the unnecessary details – vertices along the line – by satisfying several local or global geometric criteria (McMaster, 1987; Li and Openshaw, 1992; Weibel, 1997). In most cases, line simplification algorithms are controlled by a tolerance (parameter), which is usually expressed in length units. The more the tolerance value is increased the more simplified the result becomes. Thus, users should tune the tolerance value in order to achieve an appropriate level of simplification for the given scale change or map purpose. As a result, the application of any line simplification algorithm retains a certain number of vertices of the original line. Thus, by overlaying the simplified versions of the same linear feature with the original version, several polygons are generated (Figure 1). The sizes of these polygons may express the displacement caused by the line simplification procedure. In a similar way, the same kind of polygon is created when digital maps are overlaid in a GIS environment – see for example: Burrough (1986), Cromley (1992), Jones (1997), or Heywood *et al.* (1998). Usually, these polygons are outlined by boundaries that are represented slightly differently in the source maps and are called sliver polygons (Chrisman, 1989; Goodchild, 1991). Goodchild (1978) studied the polygon overlay problem and concluded that the number of spurious (sliver) polygons generated by the superimposition of two versions of the same line (for example, a river and a boundary of a geological feature that should coincide with the river) depends on line complexity. Furthermore, the same author provided quantitative measures for estimating the number of these spurious polygons. In a recent study related to the same problem, a stochastic method is proposed and the results indicate that the number of polygons generated by the map overlay depends on the number of original polygons and their perimeters (Sadahiro, 2001). Since the differences of the two versions of the line (original and derived) are very small, most of the generated polygons are in general small in size and have a thin and elongated shape.

Figure 1. *Sliver polygon generation.*



In studies of assessing quantitative line simplification (McMaster, 1986; Müller, 1987; João, 1998; Veregin, 1999), a cartometric measure of areal displacement has been used to express the displacement caused by line simplification as a global quantity. This areal displacement measure is defined by the sum of the area of all polygons divided by the total length of the original line. Although in these studies the areal displacement measure is defined in the same way, several terms have been introduced by different researchers for naming this measure. For example, McMaster (1986) uses the terms: *total areal difference*. It should be mentioned that areal displacement gives an overall estimation of the displacement produced by line simplification for the line under examination. A more detailed view regarding the spatial distribution and size of displacements caused by line simplification would be an interesting and useful piece of information for assessing line simplification. This detailed view may be expressed by analysing the size of vector displacements associated with each vertex of the original line, but it is very difficult to identify the same point on the original line and the derived line, in order to define the vector displacement, especially for continuous lines without intersections, such as coastlines. Thus, it is a challenging idea to introduce a more comprehensive measure, associated with each individual polygon, as a cartometric measure for assessing line simplification. By analysing the shape of polygons created by line simplification, and classifying them into different classes according to their shape, a new measure of displacement can be calculated for each individual polygon which may express uniformly its areal displacement. In this paper the cartometric measure of *sliver polygon displacement* (*sp*-displacement) is introduced and tested through an empirical study. The new measure is applied to successive simplifications of ten coastlines, characterised as having low, moderate or high degree of complexity (Nakos, 1996), by applying the Douglas and Peucker (D-P) algorithm (Douglas and Peucker, 1973) over a range of seven tolerances. The D-P algorithm is selected since it appears as having wide use, incorporated in most of the existing software platforms. Also, it should be mentioned that this algorithm produces small displacements – see for example McMaster (1987), or Müller (1987) and, thus, it is interesting to test the new measure with it in order to estimate the sensitivity of the measure. The proposed *sp*-displacement is compared with several other relevant measures cited in the cartographic literature. The results of the comparison are regarded as promising, since the new measure is providing a more detailed view of the spatial distribution of areal displacement along the simplified line.

QUANTITATIVE INDICES OF SHAPE

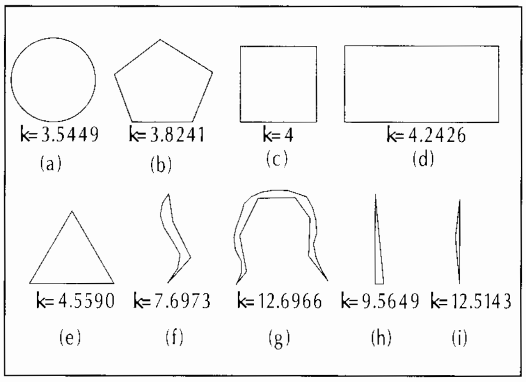
Assuming that areal entities are outlined by closed curves on a plane, it can be realised that their shape property is invariant to geometric transformations like translation, rotation or scale. In general, sliver polygons, being areal entities, have a narrow and elongated shape and are characterised by small sizes. More specifically, in a study on the polygon overlay problem, Franklin and Wu (1987) identified three kinds of sliver polygon shapes: rounded polygon, elongated strip and crooked strip. In a study related to area estimates by dot grids on paper maps, Bonnor (1975) associated the error of the area estimates with their shape and the grid density. He classified the area shapes into four categories: (a) areas with regular shape and boundaries, (b) areas with regular shape and somehow irregular boundaries, or vice versa, (c) areas with regular or irregular shape and irregular boundaries, and (d) a combination or sum of individual areas. Although this classification is an interesting contribution to the problem of characterising the shape of areas, it is based mainly on qualitative criteria and, thus, is not applicable to quantifying their shape.

An appropriate numerical expression to describe the shape of closed planar curves can be given by the ratio between its perimeter (L) and the square root of its area (A) (Mandelbrot, 1983; Feder, 1988; Maling, 1989):

$$k = \frac{L}{\sqrt{A}}. \tag{1}$$

As defined above, the shape index (k) is a dimensionless metric, independent of the size of any areal entity. By applying the equation introduced above, the shape index (k) of the following planar closed curves (Figure 2), is calculated: circle, pentagon, square, rectangular having sides with ratio 1:2, equilateral triangle, elongated strip, crooked strip, and two types of spikes. One can observe that for the case of the circle – characterised as an areal entity of *perfect* shape – the shape index has the smallest value. In addition, areal entities, having a strong rounded shape in Figure 2 (b and c) can easily be distinguished from narrow and elongated shapes (Figure 2 f, g, h and i) as having rather small ($3.5449<k\leq4$) or rather high values ($k>4$) of shape index, respectively. The geometrical problem behind this consideration is actually related to the isoperimetric problem (Rassias, 1991). The isoperimetric problem is related to the questions (Rassias, 1991, p. 1146): *What shape must a closed curve in the plane have if, with a given length it should enclose the greatest possible area? Or: When has a curve enclosing a given area the least possible length? The answer is that the curve has to be a circle.* The isoperimetric problem provides a pure mathematical proof to the questions stated above.

Figure 2. *Several planar areal entities. (a) Circle, (b) Pentagon, (c) Square, (d) Rectangle having sides with ratio 1:2, (e) Equilateral triangle, (f) Elongated strip, (g) Crooked strip, (h) and (i) Two types of spikes.*



Furthermore, in a study of measuring the length of closed geomorphic lines – like the shorelines of lakes – on various maps, Håkanson

(1978) introduced an irregularity index, describing the shape of these lines, termed *shore development*. According to Håkanson (1978, p. 144), the *shore development* is defined as: *the quotient of the length of the shore line to the length of the circumference of a circle with an area which is equal to that of the lake or the object enclosed by the given line*. Thus, for any planar areal entity of perimeter (*L*) and area (*A*) its *shore development* (*F*) is given by:

$$F = \frac{L}{2\sqrt{\pi A}}. \quad (2)$$

However, in order to develop line simplification algorithms having a cartographic rather than a geometric character and, hence, being based on the principle of preserving the overall structure of the lines with line bends, Wang and Müller (1998) studied the problem of quantifying the shape of the bends. The same authors describe quantitatively the shape of the bends by introducing the *compactness index*, defined as: *the ratio of the area of the polygon over the circle whose circumference length is the same as the length of the circumference of the polygon* (Wang and Müller, 1998, p. 7). Assuming a bend polygon of perimeter (*L*) and area (*A*), its *compactness index* (*cmp*) is given by:

$$cmp = \frac{4\pi A}{L^2}. \quad (3)$$

By analysing the two shape indices introduced above (the *shore development* and the *compactness index*), it can be proven that they are closely related to each other. Furthermore, it can be assumed that both of them are alternative versions of the shape index (*k*) introduced earlier. One can easily prove that the *shore development* (*F*) is directly related to the shape index (*k*) with the equation:

$$F = \frac{k}{2\sqrt{\pi}}, \quad (4)$$

and the *compactness index* (*cmp*), in turn, is directly related to the shape index (*k*) with the following relation:

$$cmp = \frac{4\pi}{k^2}. \quad (5)$$

Summarising all the above considerations related to the problem of the numerical expression of the shape of areal entities, it can be assumed that the shape index (*k*) can be used in quantifying the shape of planar areal entities in a similar way that analogous shape indices have already been used in relevant studies.

THE *SP*-DISPLACEMENT MEASURE

Cartographic research in the past has been directed towards evaluating line simplification either on the perceptual level (Marino, 1979; Wood, 1995) or by statistically analysing several cartometric measures (McMaster, 1986; 1987; 1989; Müller, 1987; João, 1998; Veregin, 1999) or even both of them (White, 1985; Jenks, 1989). Considering the methods of evaluating line simplification on the basis of mathematical measures, several cartometric measures have been developed in the past. In a comprehensive statistical analysis of mathematical measures for line simplification, McMaster (1986) introduced thirty measures for evaluating line simplification, distinguishing them either as single attribute measurements of length, angularity, etc., or measures of displacement (vector displacement, areal displacement). One of the most commonly used cartometric measures for evaluating line simplification is the *total areal difference per unit length* (McMaster, 1986; 1987). This measure expresses, as a unique parameter, the mean value of displacements caused by line generalisation and, thus, gives only a global view of the displacements.

Based on the shape index introduced in the previous section, the *sp*-displacement proposed here is associated with each individual polygon generated by the overlay of the original and the simplified line. In order to be able to calculate a uniform value of displacement for each polygon (Figure 3 - top), it is first suggested that each individual polygon be normalised to a rectangular shape. All polygons having a shape index of $k>4$, and thus being either weakly or strongly elongated, can be normalised as equivalent rectangular shapes having sides of ratio 1:*n* (Figure 3 - middle, A-E). The *sp*-displacement is defined as the basis of these rectangular shapes (Figure 3 - below). All remaining polygons having a shape index of $k\leq4$, and thus being strongly rounded, can be normalised as squares equal in area (*Figure 3* F) and their *sp*-displacement is defined as the side of the square.

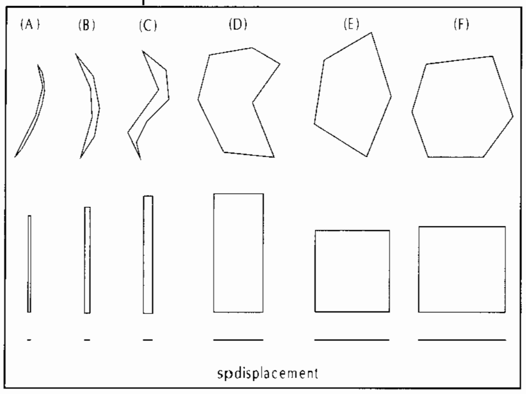


Figure 3. *Different polygons (above), their equivalent normalised rectangular shapes with sides of ratio 1:n (middle), and the respective sp-displacements (below).*

Using equation (1) and substituting the perimeter (*L*) and the area (*A*) of the normalised rectangular shape, the following equation is derived:

$$k = \frac{2(n+1)}{\sqrt{n}}. \tag{6}$$

Hence, solving for the values of *n*, the following root as a solution is achieved:

By examining the above equation it is obvious that any individual polygon with a rectangular shape of sides with ratio 1:*n* can be modelled, if its shape number is *k*≥4. Considering the approach described previously, the *sp*-displacement can be calculated as follows:

$$n = \frac{k^2 - 8 + k\sqrt{k^2 - 16}}{8}. \tag{7}$$

$$sp = \sqrt{\frac{A}{n}} \text{ if} : k > 4 \text{ or}$$
$$sp = \sqrt{A} \text{ if} : k \leq 4, \tag{8}$$

where *A*: is the area of the polygon.

Finally, by averaging the values of *sp*-displacement over all polygons along the line, a global measure of displacement for the line, analogous to the *total areal difference per unit length*, can be easily calculated.

AN EMPIRICAL STUDY OF TESTING THE NEW MEASURE

A data set consisting of ten coastlines located in the central part of Greece (Figure 4) is created to test the introduced method of evaluating line simplification. The ten cartographic lines (Figure 5) are presented on thirty-three topographic maps of scale 1:50 000 produced by the Hellenic Geographic Army Service (HAGS). In Table 1 the names of the coastlines with their associated identifiers (ID) are illustrated. These ten coastlines have been chosen as subjects of study, since they are considered as having low, moderate or rather high levels of line complexity (Nakos, 1996). The data set was digitised from paper map-sheets with a resolution of 1016 lpi following the same standards. The data set was edited and cleaned in order to link the parts of coastlines that share various map-sheets.

It is known that any digitisation task of paper maps produces raw data with a certain number of redundant vertices like duplicate vertices, spikes or switchbacks, etc., which should be removed by a *weeding* process (Jenks, 1981). The cleaning process should be carried out by applying a data reduction algorithm with very

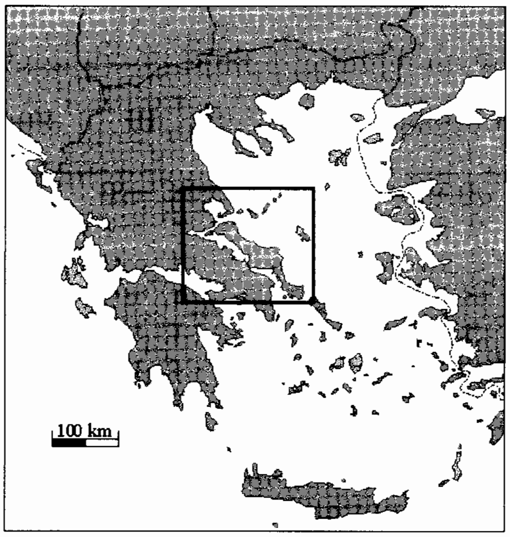


Figure 4. *The location of the data set.*

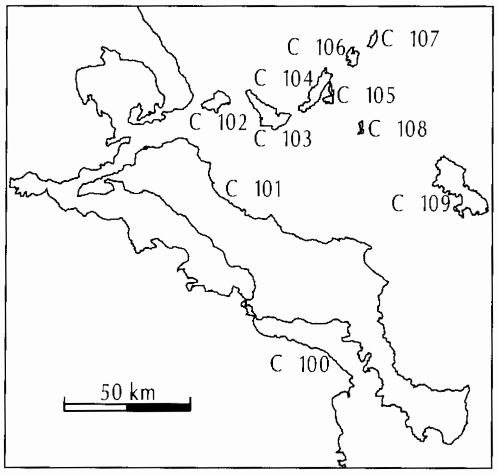


Figure 5. *The ten lines of the data set.*

small tolerance values. In relevant studies of the evaluation of line simplification dealing with digitised data sets, McMaster (1986) and João (1998) suggest to apply the D-P algorithm with very small tolerances (0.002-0.05 mm on the map), while Visvalingam and Whyatt (1990) suggest the same algorithm with tolerance values equal to half the width of the source line. In the present study, the raw data were cleaned by applying the D-P algorithm with a tolerance of 0.5m on the ground (0.01mm on the map), taking the above studies into account. The raw data were cleaned from the unwanted vertices amounting to an average of approximately 15 percent, while the length of the lines practically did not change (Table 1). Finally, the reference data, free from redundant vertices, consisted of the ten original lines of scale 1:50 000 for performing the simplification evaluation test.

The ten original coastlines were processed over successive simplification tasks by applying the D-P algorithm with seven tolerance values, varying from 2.5m to 250m on the ground (0.05mm–5mm on the map). The simplification tasks produced seven versions of derived lines associated with the seven tolerance values. The derived lines were subsequently overlaid with the original lines by applying a typical GIS union function. As a result, seven sets of polygons per each coastline were generated over the range of the selected tolerance values, in order to be analysed and discussed.

ANALYSIS AND DISCUSSION

Considering the shape analysis of polygons generated by line simplification, a statistical analysis was performed. The frequencies of the shape index for each coastline over the seven selected tolerances (2.5m, 5m, 10m, 25m, 50m, 100m and 250m on the ground) were calculated and classified into five groups. The class limits were defined as a way of discriminating various characteristic polygon shapes from rounded polygons to narrow and elongated ones. In Table

Table 1

Coastline name	ID	Raw data Vertices	Raw data Length (m)	Reference data Vertices	Reference data Length (m)
1 Mainland	C_100	31 314	774 758	26 625	774 742
2 Isl. of Evia	C_101	29 071	723 665	25 262	723 653
3 Isl. of Skiathos	C_102	2 815	49 641	2 333	49 639
4 Isl. of Skopelos	C_103	4 306	75 554	3 604	75 550
5 Isl. of Allonissos	C_104	4 067	79 035	3 522	79 033
6 Isl. of Peristera	C_105	1 721	35 874	1 491	35 873
7 Isl. of Kyra-Panagia	C_106	1 964	41 206	1 689	41 205
8 Isl. of Gioura	C_107	1 609	28 607	1 383	28 606
9 Isl. of Skantzoura	C_108	1 031	21 371	869	21 370
10 Isl of Skyros	C_109	5 683	133 737	5 092	133 735

Table 1. *The attributes of raw and reference data for the ten coastlines.*

2 the limits of the five classes (*S1*, *S2*, *S3*, *S4* and *S5*) are presented, respectively, for polygons of: strongly rounded, rounded, rounded and slightly elongated, narrow and elongated, and narrow and highly elongated shape.

Table 2

Class	Limits	Shape
S1	$k < 4$	Strongly rounded
S2	$4 \leq k < 4.5$	Rounded
S3	$4.5 \leq k < 6$	Rounded and slightly elongated
S4	$6 \leq k < 10$	Narrow and elongated
S5	$k \geq 10$	Narrow and highly elongated

Table 2. *The five classes of polygons in regard to their shape index (k).*

Figure 6 presents the distribution of polygon frequencies over the five classes of shape (*S1* through *S5*) for the case of the coastline of mainland (C_100). By interpreting the results from Figure 6, it is found that a significant majority of the polygons over all tolerance values are narrow and elongated (class *S4*), or highly elongated in shape (class *S5*). Furthermore, polygons of rounded shape (class *S2*) are rarely observed, and even more rare are those of a strongly rounded shape (class *S1*). Finally, by examining the variation of tolerance value, it may be observed that more polygons of rounded shape are generated as the tolerance increases, while the polygons of narrow and highly elongated shape become even fewer. Similar results are observed in regard to the rest of the lines by analysing the shapes of the generated polygons.

In addition, the *sp*-displacement measure, associated with each polygon for the ten coastlines over the seven tolerance values, was calculated. A statistical analysis of *sp*-displacements was carried out based on frequencies classified into five groups for a given generalisation scenario of scale change from 1:50 000 to 1:100 000.

The group limits were defined according to the established cartographic standards of visual perception and map resolution under normal circumstances of observation (naked eye and

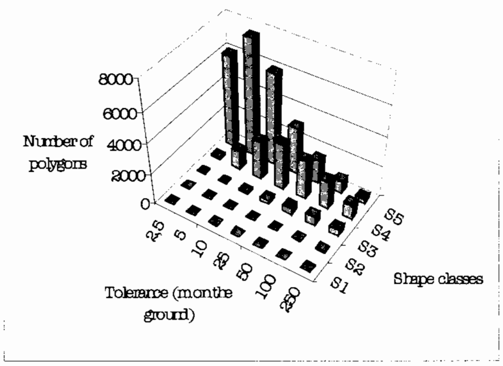


Figure 6. *Frequencies of polygons vs. shape classes for the line C_100.*

reading distance). Considering those standards for isolated elements on white paper, they should be theoretically greater than 0.1mm in diameter of a point symbol, or thicker than 0.06mm for a linear element on the map (Rouleau, 1984). In practice, the same author suggests the minimum diameter of a point symbol to be 0.2mm, the minimum thickness of a line symbol 0.1mm, the minimum length of the side of a solid square 0.4mm, and the minimum length of the side of an open square 0.6mm on the map. The above thresholds are in complete accordance with the greatest visual acuity of the human eye, suggested by Keates (1996) and MacEachren (1995). Moreover, Goodchild (1991) argues that 0.5mm can be assumed to be the typical resolution of an input map document, relating this threshold to the accuracy with which an average digitiser operator can position the crosshairs of a cursor over a point or a line feature. Thus, taking into account all the above stated thresholds, the limits of the five groups (*D1*, *D2*, *D3*, *D4* and *D5*) were defined as presented in Table 3 for: non-visually observable, limited observable, within visual perception limit, significant distinguished and high values of *sp*-displacements, respectively.

Figure 7 illustrates the polygon frequencies into these five groups over the tolerance values for the case of the coastline of the mainland (C_100). By interpreting the results of Figure 7, it could be stated that no high displacements were observed for tolerance values smaller than 100m on the ground, as was expected. The percentage of significant distinguished displacements above the level of 5 percent was observed for tolerance values larger than 50m

Table 3

Group	Limits (mm on the map)	Description
D1	$sp < 0.05$	Non-visually observable
D2	$0.05 \leq sp < 0.10$	Limited observable
D3	$0.10 \leq sp < 0.25$	Within visual perception limit
D4	$0.25 \leq sp < 0.50$	Significant distinguished
D5	$sp \geq 0.50$	High values

Table 3. *The five groups of sp-displacements.*

on the ground. Furthermore, approximately 15 percent of the polygons are associated with displacements within the visual perception limit for the tolerance value of 25m on the ground. Finally, the significant majority of the generated polygons do not produce visually observable displacements when the tolerance value is less than 10m on the ground. Similar results are observed in regard to the other lines by analysing their values of *sp*-displacements.

Finally, the introduced measure of *sp*-displacement was compared to other global measures utilised for line simplification evaluation in the literature. For comparison, the weighted average values of *sp*-displacement (MSP expressed in metres on the ground) for the ten coastlines over the seven selected tolerance values (2.5m, 5m, 10m, 25m, 50m, 100m and 250m on the ground) were calculated. Furthermore, the left and right *sp*-displacement values (L_MSP and R_MSP, respectively – expressed in metres on the ground) were calculated across the lines. In addition, three global measures of line simplification, cited in the literature, were calculated for the same lines and selected tolerances: the *total areal differences* (TAD) – in square metres per metre length

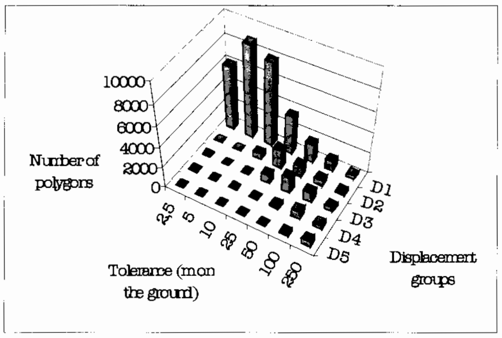


Figure 7. *Frequencies of polygons vs. displacement groups for the line C_100.*

– (McMaster, 1986), the *number of polygons* (NP) – per km length – (João, 1998), and the *percentage of the change in line length* (LCH) – expressed in percent – (McMaster, 1986).

Interpreting the results, in regard to the measures of L_MSP and R_MSP, a balance between left and right displacements was observed almost in all cases, except those associated with rather large tolerance values (more than 100m on the ground). This outcome indicates the D-P algorithm is not biased by the dominant shape of the line.

Regarding the measure of the number of polygons per km (NP), its graphical representation vs. the various tolerances for the coastline of mainland (C_100) is presented in Figure 8. As can be seen in Figure 8 the number of the generated polygons increases at a high rate, quickly reaches a critical maximum and finally decreases as the graph is scanned from left to right. Of course, the sizes of the generated polygons are very small and almost all of them have a strong narrow and elongated shape as the tolerances are kept very small in value. The sizes of the generated polygons increase gradually, as well as the proportion of polygons with rounded shape as the tolerance values become larger. This is evidence that the specific algorithm functions as a data reduction algorithm when it is applied with very small tolerances because it eliminates redundant co-linear vertices, and as a line simplification algorithm, when the tolerances become larger. The comment in regard to the D-P algorithm behaviour is verified for all the lines of the study.

Figure 9 presents the percentage change of the length vs. the various tolerance values for the coastline of mainland (C_100). It is observed that by increasing the tolerance value, the change in line length increases gradually. The same result is observed for the remaining lines of the study.

Figure 10 illustrates the results in regard to the measures TAD and MSP for the case of the mainland coastline (C_100). As shown in *Figure 10*, linear relationships between displacements and tolerance values are observed. The same

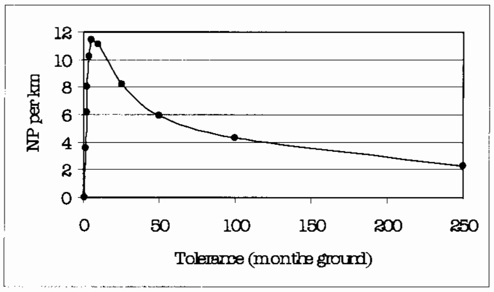


Figure 8. *The number of generated polygons (NP) per km vs. the tolerances values for the line C_100.*

linear behaviour is observed for all other lines of the study. Although there are no significant differences between the *total areal differences per unit length* and the weighted averages of *sp*-displacement (TAD and MSP, respectively), the latter present systematically higher values (Figure 10).

In general, it could be assumed that global measures – i.e. NP, LCH, TAD – may express as a uniform value the distortion of the line caused by simplification in any generalisation task. By analysing in more depth the differences between TAD and MSP, it can be observed that, contrary to the weighted average of *sp*-displacements, the *total areal differences per unit length* underestimates the distortion of the line caused by simplification up to a level of approximately 20 percent. There are specific cases where

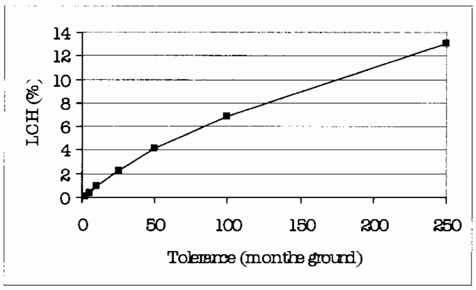


Figure 9. *The percentage of change in length (LCH) vs. the tolerances values for the line C_100.*

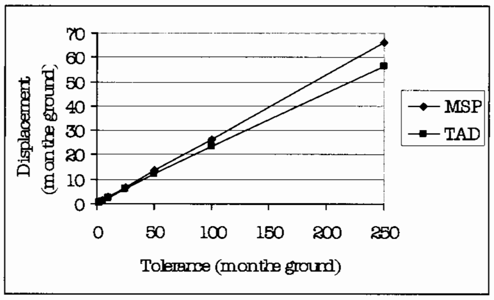


Figure 10. *The measures: total areal differences per unit length (TAD) and the weighted averages of sp-displacement (MSP) vs. the tolerances values for the line C_100.*

the observed differences between these two measures are more than 40 percent. Considering that the introduced measure of *sp*-displacement is associated with each individual polygon, it describes in more detail the size of displacements. So, it can be assumed that it is closer to the distortion caused by line simplification.

CONCLUDING REMARKS

A new method of assessing line simplification through a displacement measure associated with the polygons generated by the overlay of the original and the derived line is described. The method is empirically tested with a data set consisting of several coastlines over a wide range of tolerance values by applying the D-P algorithm. The results show that the significant majority of the generated polygons are narrow and elongated in shape, even for rather large tolerance values. Considering the displacements caused by the D-P algorithm, it is once more verified that it produces small displacements – see, for example, similar results in McMaster (1987), or even in Müller (1987). More specifically, the polygons generated by line simplification produce displacements within the visual perception limit or higher for tolerances larger than 25m on the ground in regard to the examined lines and assuming a generalisation scenario of scale change from 1:50 000 to 1:100 000.

The analysis performed provides a more detailed insight regarding the behaviour of the D-P algorithm as a data reduction or data simplification algorithm, specifically in terms of the tolerance variations.

Finally, the results of the empirical study reveal that the introduced measure diminishes the underestimation degree of the distortion of the simplified lines, present in other measures citied in the literature. In addition, the new measure provides the user with useful information about the spatial distribution of displacements along the line. For example, locations associated with displacements larger than the visual perception limit may be detected as possible locations of conflicts on the problem of conflict resolution.

However, the research must be extended to include other typical cartographic lines (i.e. roads, rivers, boundaries etc.) in order to reach wider acceptance.

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