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THE USE OF EPSILON–CONVEX AREA FOR ATTRIBUTING BENDS ALONG A CARTOGRAPHIC LINE

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ABSTRACT

In this paper a method for detecting bends along cartographic lines is introduced. The method is based on the concept of epsilon-convex areas. By rolling iteratively circles of gradually decreasing sizes of diameter epsilon along the line, several bends are formed. The generated bends are represented in hierarchically structured trees and their attributes (size and shape) are calculated. The proposed method is implemented on Peristera Island coastline as a case study, digitised from a paper map of scale 1:50K. The results are compared and discussed with manually generalised coastlines of Peristera Island, digitised from maps of several smaller scales, i.e. 1:100K, 1:250K, 1:500K and 1:1M. Although the present research approaches the problem at a preliminary stage, promising results have been produced.

INTRODUCTION

Line generalisation may be considered as one of the most complex processes in the cartographic procedure due to its dependence on factors like the level of simplification, the purpose of the map and the character of the cartographic line. Cartographers ought to take into account all these factors in order to accomplish effectively any simplification task. Trying to suit the above factors, cartographers follow a holistic procedure when generalising manually a line. They examine the line globally as well as locally. The aim is the estimation of how the retention or the removal of each characteristic of the line can reflect to the neighbour location as well as to the whole line globally. This procedure takes place continuously and iteratively as line simplification proceeds.

Another characteristic of manual line simplification is the subjectivity of the procedure. Cartographers use personal logical and aesthetic criteria to form the generalised line. Nevertheless, studies have shown that there is a general principle that defines the way someone selects the positions that shape a generalised line. Marino (1979) curried out an empirical study in order to investigate how map users select positions which shape generalised lines. She presented six lines to a group of individuals (several of them were cartographers), and she asked them to select a set of points (positions), which they considered to be necessary and sufficient to retain the character of the line. The analysis of the results showed that the great majority of the points selected was located in lines' high slope change areas. With minor deviations, cartographers and non-cartographers chose the same positions. The results of Marino's study testify Attneave's concepts about visual perception. Attneave (1954) pointed out that each location of a drawn object contains (and conveys to map-readers) an amount of information. The greater amount of information is concentrated along the contours of a drawn object, and especially at these locations where contours' direction changes most rapidly. He concluded that these locations are able to characterise the shape of an object and he distinguished them from the rest that considered redundant. Based on this concept, cartographers consider that locations along a line at which its direction changes rapidly, are significant for the retention of the form and shape of the line, and so they must be retained after a simplification procedure.

In computer cartography, several research activities have the aim to formulate automated approaches on the problem of line simplification. A significant difference between analogue and digital cartography is the mode of representation of spatial features. In digital environment, cartographic lines are usually represented in vector structure - i.e. a discrete number of vertices connected by vectors. This way of line representation does not express the continuous character of real world objects. Cartographic lines (rivers, coastlines, etc) are continuous phenomena, each one having characteristic/individual physical and geometrical attributes, such as consecution, curvature, etc. In computer environment these attributes do not exist. Thus, cartographers ought to search for alternative methods of line analysis. They have to modulate their research to discrete representations. Most of line simplification algorithms that have been developed focus on the retention or the elimination of vertices constituting digital lines. The greatest part of them uses geometric criteria in order to select the points that shape the generalised line. Their structure is based on the retention of points located on high slope change parts of the line; that is algorithms' function is based on Attneave's theory. Among these algorithms, the most well known is the one proposed by Douglas and Peucker (1973).

The validity of these algorithms is a discussion topic for cartographers, mainly for three reasons. Firstly, the structure of each algorithm is based on specific geometric criteria and limitations set by users. Thus, they are not always efficient to operate well to all lines or to the different shaped parts of a specific line. Each line is encountered as an integral entity; its geometry is analysed globally according to the principles of each algorithm. In addition, cartographic lines (or parts of them) do not change in the same way during a simplification process. Depending on their geomorphological nature and character, lines behave differently to scale changes. Buttenfield (1989) states that linear features can be divided in two categories: those that their structure changes with scale (scale-dependent) and those that do not change (scale-invariant). Finally, having as functional principle the detection of high slope change positions of a line, several algorithms do not select points located in its smooth parts (silent points), which may be necessary for the line's shape retention (Dutton 1999).

Nowadays, it is a challenge for cartographers to create a 'total system' that will simulate the manual line simplification process. This system will examine and analyse the shape and the geometry of a line in global, as well as in local level. It will segment the line on the basis of common attributes (sinuosity, homogeneity, etc) and will apply in each part an appropriate line simplification algorithm using constant or different tolerances or different algorithms suitable for each part of the line. In addition, the choice of the above generalisation operators will depend on the level of simplification and the rate of scale reduction. Based on this concept, Dutton (1999, p. 36) points out that "by segmenting line features to be more homogenous, then applying appropriate algorithms and parameters to each regime individually, simplification results can always be improved".

Having as background the above research directions and the concept of simulation of manual line simplification, two related research topics arise. Ways of line segmentation must be found and line analysis must take place locally as well as globally. In this context, it would be sounder for cartographers to examine sections and not isolate points on a line. The main objection of regarding points as the minor element of a line and using them for line's analysis is that it is difficult to take into account their local significance. Every point is strongly associated with its neighbours. Thus, the retention or not of a point may affect the significance of its predecessor or following points. And that may affect the final shape and the aesthetic and geographic validity of the generalised line. On the contrary, rule based defined parts on a line (e.g. bends) approach better its physical formation, regarding the geometrical analysis. By examining line parts, the assessment of the local importance is sounder, since their geometrical attributes can be calculated (even an approximation of them) and their topological relevance can be rated. Plazanet et al (1995) presented some rules for the characterisation of linear features. They defined objective criteria like sinuosity, homogeneity, density, and complexity, in different levels of perception (global, intermediate and local) that they used to describe the shape of a line. Based on these criteria, they proposed a method of segmentation of linear features. Finally, the geometrical attributes of the line pieces were calculated. In this way, Plazanet et al (1995) segmented and characterised a line in order to be properly analysed for a 'total system' to administrate it. In this context, Wang and Müller (1998) proposed a simplification algorithm, which is based on the detection of the bends of a line. Geometric principles were used for bends definition. Specifically, Wang and Müller defined that a bend is "that part of a line which contains a number of subsequent vertices, with the inflection angles of all vertices included in the bend being either positive or negative and the inflection of the bend's two end vertices being in opposite signs." (Wang and Müller 1998, p. 5) The attributes (size and shape) of each bend were calculated and the context with its neighbour bends was defined. The retained bends that shaped the resultant line, as well as, their final form were composed after the application of elimination, combination, and exaggeration operators.

Considering that the creation and the analysis of bends along a cartographic line is an important procedure for generalisation, a new method of detecting bends is introduced in the present paper. The method is based on the concept of epsilon–convex areas introduced by Perkal (1966a), for bends definition. The aim is the formation of bends along a line on the basis of a common measure and the computation of their attributes. The method may be useful in the process of designing a 'total method' of line generalisation, since one of its preconditions is the knowledge of quantitative means that characterise cartographic lines.

THE METHOD

Definition of Epsilon–Non-Convex Bends and Their Tree Representation

Perkal (1966a) introduced the concept of epsilon–convex areas in his effort to create a method of measurement of linear features' length. The formation and the implementation of his research concern analogue lines, that thier manner of structure is not known. Perkal defined that an epsilon–convex area of a line is the collection of all points on the plane not more than epsilon distant from the line. Theoretically, an epsilon–convex area is created when a circle of diameter epsilon rolls on both sides of a line. Its width depends on the size of epsilon. Based on this concept, he divided the lines (or parts of the lines) to epsilon–convex and epsilon–non-convex areas. A line is epsilon–convex "if a circle of diameter epsilon could fit on both sides of the arc" (Perkal, 1966a, p. 9). On the contrary, if an interruption exists between circle and line, this part of the line is epsilon–non-convex. An outgrowth of the epsilon–convexity concept is a region generalisation technique (called epsilon–generalisation), proposed by Perkal (1966b). Perkal considered two regions created by a close line, the internal (D) and the external (D΄). A circle of diameter epsilon roles separately on the inner and outer side of the line. "The set of all points having the property that they are contained within circles of diameter epsilon, which can be completely included in the region D, is called an epsilon–generalisation of the region D" (Perkal, 1966b, p. 4). The same procedure takes place to the region D΄. In other words, Perkal's method detects and retains the epsilon–convexes of the line. The epsilon–non-convexes are removed and replaced by the part of the circle that connects the intersection points.

Based on Perkal's concept Christensen (1999) proposed a line simplification technique. Applying Perkal's rolling circle, he broke the line to epsilon–convex and epsilon–non-convex sections. The edges of the epsilon–non-convex were connected with medial-axes arcs, and finally joined with the retained epsilon–convex sections, so as the simplified line to be formed.

In the present paper, the concept of epsilon–convexity is used for the detection of bends along cartographic lines. We will call bends the epsilon–non-convex sections of the line. Theoretically, the proposed epsilon–non-convex bends are created by a similar to Perkal procedure. A circle of diameter epsilon roles on both sides of a line, creating epsilon–convexes and epsilon–non-convexes parts of the line. The method is implemented in each line by applying iteratively graduated decreasing levels of diameter epsilon, from a maximum to a minimum value. In this way, each epsilon–non-convex bend created by a higher size of epsilon contains epsilon–non-convex bends created by smaller sizes of epsilon (sub-bends), or conversely, each epsilon–non-convex bend created by a small size of epsilon belongs to an epsilon–non-convex bend created by a bigger size of epsilon. Thus, there is a direct topological relation between the bends and their sub-bends, which can be transformed to a graph of hierarchically structured trees.

The introduced tree structure uses one level for each circle diameter. If the original bend is a simple curved part of the line and the rolling circle diameter relatively short, then the whole bend is represented by a single tree node. This is the case in Figure 1(A & B). Each time the diameter is decreased, one single bend is discovered, since no curved parts of the line exist. In this trivial case, the tree constructed by levels associated to decreasing diameters, is a linear linked list. If the original bend has details that cannot be seen using a large rolling circle diameter, then as the diameter decreases, those detailed sub-bends are revealed. In this case, if N is the number of bends that are generated at a specific level, the node that represents the original bend becomes the parent of N nodes, and these nodes represent the generated sub-bends. This is shown in Figure 1C. When the rolling circle diameter is small enough the two sub-bends of Figure 1C will be discovered. The corresponding tree structure is shown in Figure 1D. This procedure of "decreasing the diameter - creating a new tree level with the visible bends" is repeated until no further bend details can be detected, which means that all nodes of the last tree level are expanded as linear linked lists. Of course, the selection of the circle diameter determines the detail that will be detected at each specific diameter level. Figure 1E shows a more complex line along with the corresponding tree structure in Figure 1F. It is worth noting in Figure 1 (E & F) that if we number node 1 as tree level 0 then tree level 1 corresponds to the detection of two sub-bends 1a and 1b. For the next diameter decrease, nothing is detected below node 1a, while two sub-bends are exposed at 1b. Nothing is exposed by the next decrease of diameter at both sub-bends, while the last one, exposes sub-bends 1a.1a and 1a.1b. It is obvious that the shape and the characteristics of the tree depend on the diameters selected.

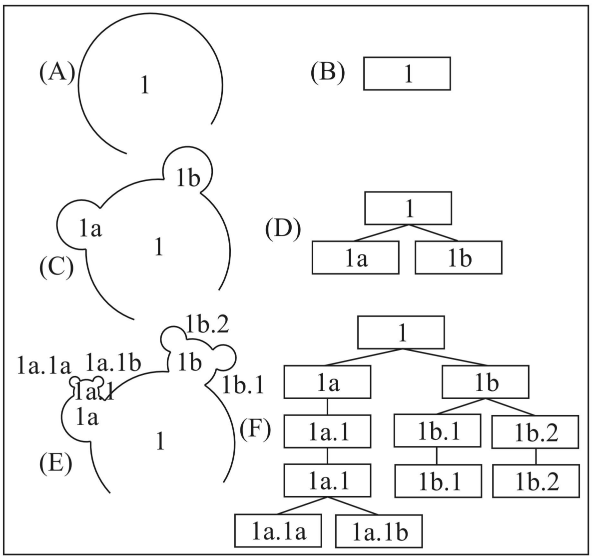

Figure 1. An example of bends' tree structure

Implementation of Perkal's Technique in Computer

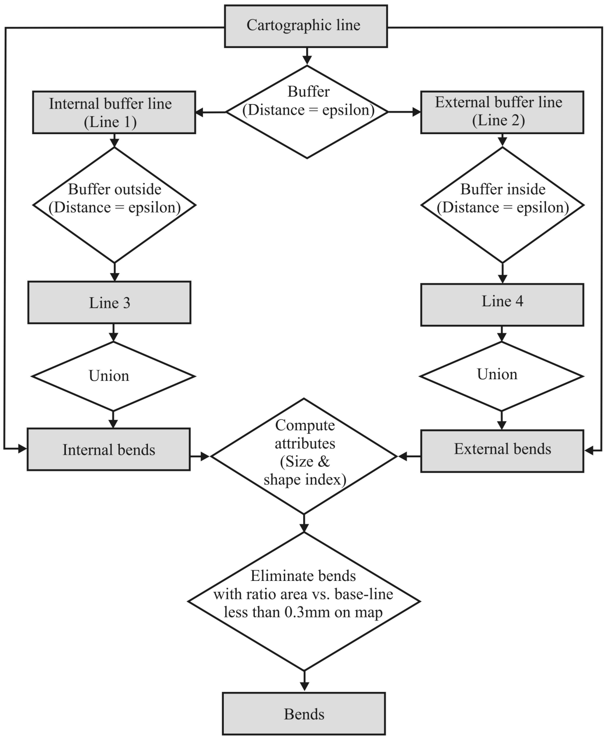

Figure 2. Implementation of Perkal's technique using a GIS software package

In computer environment, the implementation of Perkal's analytical procedure is accomplished using the ArcGIS v.9.0 software package (© ESRI). The rolling circle process is obtained by the buffering operation supported by the specific software. A buffer zone is created around each side of a line. The bandwidth of the buffer is equal to the half of the Perkal's circle diameter (epsilon). Then, a new buffer zone of width half of epsilon is created around the boundaries of the initial buffer zone. The inner boundaries of the new buffer zone intersect line in some positions. The bends shaped between two subsequent intersection points are the epsilon–non-convex parts of the line. It is worth mention that the described method is a satisfactory approximation of Perkal's technique. The second applied buffer simulates the rolling circle and the intersection between buffer and line corresponds to the tangent points of circle and line, as mentioned in Perkal's (1966a) study. The same procedure is iteratively repeated over different levels of decreasing epsilon in order to detect sub-bends inside each bend and to construct their topological structure. The process is obtained by the identity operation supported by the software package. Finally, by connecting the two end points of a bend a closed polygon is formed. The resultant polygons are used for the computation of the epsilon–non-convex bends quantitative attributes. Since several generated polygons are not visually observed, as being very small in size and narrow in shape, they are eliminated. All calculations are made using ArcGIS platform's environment. Figure 2 illustrates graphically the way the whole procedure is implemented inside the GIS software package.

Quantitative Attributes of the epsilon–Non-Convex Bends

The quantitative attributes of the epsilon–non-convex bends are the diameter (epsilon), size, and shape index. The size of diameter is common for all bends during each application. The shape index and the size of the created bends depend on it. For the computation of the size and shape index values that characterise each epsilon–non-convex bend, the closed polygon created from the base-line and the bend is used. The size of an epsilon–non-convex bend is defined as the area (A) of the polygon. The bends' shape can be described by a numerical expression. In this paper the shape index is determined by the ratio between the perimeter (L) of the polygon and the square root of its area (A): $k = L/\sqrt{A}$. The shape index k is dimensionless and independent of size of any areal entity. Nakos (2004) pointed out that the smallest value of k corresponds to a circle, which is considered as a 'perfect' shape. For rounded shapes the value of k increases slightly and takes high values on narrow and elongated shapes. Generally, the shape index k increases as the shape becomes more narrow and elongated.

CASE STUDY

The proposed method was implemented on the coastline of Peristera Island, a coastline characterised by high degree of complexity and shape irregularities. The coastline was digitised from a paper map of scale 1:50K. The average step of digitisation was approximately 0.1mm on the map (5m on the ground). Further on, the method is applied on Peristera Island coastline, presented on four smaller scale maps (1:100K, 1:250K, 1:500K, and 1:1M). The aim is the comparison of the results yielded from the method's application on manually generalised maps. The average step of digitisation was also approximately 0.1mm on the map for all generalised coastlines (i.e. 10m, 25m, 50m, and 100m on the ground, respectively). The raw data were cleaned up from duplicate vertices, spikes, or switchbacks after a 'weeding' process and they were smoothed in order to produce a working data set, as it is suggested by Jenks (1981).

The method was implemented on each coastline by applying iteratively eight decreasing sizes of diameter epsilon (1,400m, 1,200m, 1,000m, 800m, 600m, 400m, 200m, and 100m on the ground). The coastline's extent on scale 1:50K is taken into account for the choice of the epsilon sizes. Using the maximum diameter (1400m), the theoretical rolling circle adjoins the large bays of the coastline. Moreover, the epsilon–non-convex bends created by rolling circles of diameter lower than 100m are not visible or referable. In the paper only the results of applying the method on the outer side of the coastlines are presented and discussed.

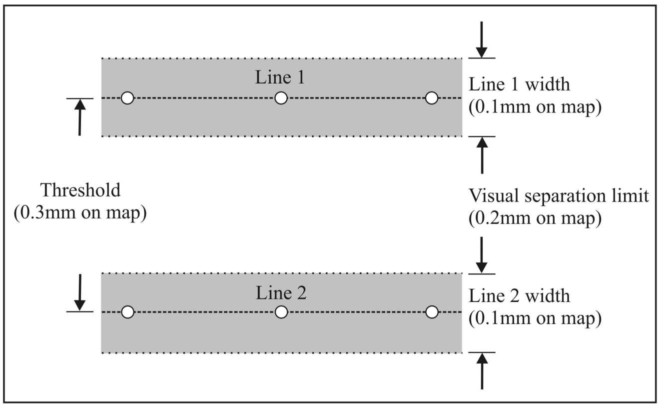

Figure 3. Selection of threshold for bends filtering

The epsilon–non-convex bends created by the implementation of the method on the five versions of the coastline are filtered in order to eliminate those bends that cannot be visually observed. The filtering is accomplished by using the bends' average depth as criterion. The average depth (D) of a bend is determined as the ratio between the area (A) of the polygon related to the bend and the length of the base-line (BL) that connects the two end points of the bend: $D = A/BL$. The eliminated bends are characterised by average depth less than a threshold value. In order to estimate a threshold, the width of a digital line and the visual separation limit must be considered. By assuming that a coastline is presented on a map with a line of 0.1mm width, and that the visual separation limit is 0.2mm (Rouleau 1984), 0.3mm is set as an appropriate threshold value (see Figure 3). By applying the filtering process a great number of small but noisy bends are eliminated.

DISCUSSION

Analysis of Bends' Attributes

The assessment of the results is based on ranges: the change of the number of bends and the attributes of the bends in the different levels of epsilon in a specific scale and their change according to scale for a specific level of diameter epsilon. The values of size have been divided into eighteen classes according to the frequency of appearance. Respectively, the values of shape k have been divided into four classes on the basis of their frequency of appearance: (a) $k<5$, (b) $5\leq k<6$, (c) $6\leq k<7$ and (d) $k\geq 8$. The bends that correspond to the lower classes are more rounded than those bends corresponding to higher classes that are more elongated and narrow.

By observing the number of epsilon–non-convex bends created in the five versions of the coastline, it is evident that as the scale increases, more bends are generated. In larger scales the coastline is represented in more detail and their shape is more complex. Thus, the rolling circles intersect the line in more positions, creating higher number of bends. On the contrary, the generalised versions of the coastline presented in small scales are smoother. As a result, the number of curved parts along the lines is smaller, and so the epsilon–non-convex bends become fewer. In addition, the number of epsilon–non-convex bends created by each diameter is assessed. In scale 1:50K, it is observed that applying rolling circles of diameters between 800m-200m on the ground the higher number of bends is created. This happens because these specific rolling circles have the appropriate size to enter inside the large curved parts of the line and detect smaller curved sections that they contain them. The large epsilon sized rolling circles mainly detect the large curved parts of the line and these of low diameters do not create epsilon–non-convexes. In the coastline presented at scale 1:100K, the number of bends created by diameters between 1400m-400m on the ground is almost the same. This flattering of the bends number, in regard to the previous scale, is the result of the smoother shape of the generalised line. A part of middle sized bends presented in scale 1:50K does not exist. Thus, the rolling circles of moderate diameters detect fewer bends. In addition, the number of bends corresponding to low levels of epsilon is fairly small. That is because the rolling circles do not create epsilon–non-convexes. The same situation appears in the rest versions of the coastline. The only difference is that as scale decreases, rolling circles of higher diameter create the greatest number of bends. For example, in the coastline presented in scale 1:100K, the greatest number of bends is created by rolling circles of diameters bigger than 600m on the ground, where this limit appears at the level of epsilon=1000m in the coastline of scale 1:1M. This situation also has to do with the smoother shape of the generalised versions of the line, as mentioned above.

For each scale, the way by which the epsilon–non-convex bends change in size for the eight levels of epsilon is analysed. In general, the bends created by rolling circles of diameter epsilon=1400m on the ground are the widest. As the size of diameter is reducing, the size of bends decreases. This way of change in size is prospective, since as the level of epsilon decreases the created bends are smaller. In the versions of the coastline presented in scales 1:50K, 1:100K, and 1:250K, the number of bends that corresponds to high levels of epsilon, is almost uniformly distributed in the middle and in high classes of size. By decreasing the size of diameter, the size of most bends belongs to the middle and lower classes. In the versions of the coastline presented at scales 1:500K and 1:1M, the size of bends belongs to the middle and mainly to the high classes, for the high levels of epsilon, having a trend to be accumulated in the middle classes as the level of epsilon decreases. In both cases, it is observed that the size of bends increases as scale decreases. The variation of bends size between large and small scales appears as long as the generalised versions of the coastline are smoother and less complex. In general, the size of the epsilon–non-convex bends decreases as the level of epsilon decreases and the scale increases.

The shape index *k* variations of the epsilon–non-convex bends generated by each level of epsilon, in the five versions of the coastline are examined. In the coastline presented at scale 1:50K, the *k* values of the bends created by rolling circles of high levels of epsilon (1400m-600m on the ground) dispread to all shape classes. In the versions of the coastline at scales 1:100K and 1:250K, these bends' *k* values tend to be accumulated in the two first classes, while at scale 1:500K and 1:1M, the majority of them belongs to the first shape class. This way of *k* change makes evident that the epsilon–non-convex bends created by rolling circles of high diameters become more rounded as scale decreases. That can be explained by the fact that the form of the simplified versions of the coastline presented in small scales is smoother. The *k* values of the bends created by rolling circles of low levels of epsilon change in a similar way according to scale. That is, as scale decreases the shape index *k* tends to be accumulated in the first shape classes. However, it is observed that the majority of the *k* values is concentrated in the first classes, even in the versions of the coastline presented at large scales. For example, in the initial coastline (scale 1:50K), most *k* values belong to the first two classes. Generally, by decreasing the size of diameter, the shape index *k* tends to be accumulated in the first classes. This change of *k* shows that the bends created by high sizes of diameter epsilon are more elongated or spiky than the bends created by small sizes of diameter, which are more rounded. In general, the shape of the epsilon–non-convex bends tends to be more rounded as scale and level of epsilon decreases.

Analysis of Hierarchically Tree-Structured Bends

The hierarchical structured trees at all scale levels of Peristera Island coastline are generated. Each tree represents an epsilon–non -convex bend created by the rolling circle of maximum diameter and the sub-bends created by the smaller sizes of epsilon that are contained in it. Figure 4, illustrates the trees produced by the application of the method in the five versions of the coastline under study. Each node of a tree corresponds to an epsilon–non-convex bend. The upper nodes depict the bends created by the rolling circle of size epsilon=1400m and each one below, hierarchically, its sub-bends. In each node, the level of epsilon and the quantitative attributes of the bends are recorded. Figure 5 illustrates the coastline of Peristera Island at scale 1:50K, as well as, the bends generated by applying the rolling circle of epsilon=1400m.

Interpreting Figure 4, it is evident that the trees present significant differences among each other. There are one-column trees, trees that have two or more branches, trees that end to different levels of epsilon. In addition, the trees' branches start and end in various levels of epsilon and some of them have symmetrical structure while others not. By comparing the hierarchical structured trees of scale 1:50K with the coastline, as it is presented in Figure 5, it is observed, that there is a direct relation between the shape and the size of the bends and the formation of the corresponding trees. In general, the structure of each tree depicts effectively the form of the epsilon–non-convex bend.

Furthermore, it is observed that bends having a wide base-line in relation to their depth and small size (i.e. bends with IDs: 2, 19, 5 & 6) correspond to one-column trees that do not end to low levels of epsilon. One-column trees that extend to all levels of epsilon correspond to moderate in size and smoothed bends with wide base-line (i.e. bends with IDs: 22 or 31). The existence of branches implies irregularity, while their symmetry depicts the form of the irregularity. For example, the bend with ID: 14 is divided in two small sub-bends (see Figures 4 & 5). These sub-bends are detected at the level of epsilon=800m on the ground. Thus, the tree is divided in two branches at this level of epsilon. Moreover, since the bends are smooth and have almost the same size, the branches are symmetrical and end at the same level of epsilon.

By comparing the non-symmetrically structured trees (Figure 4) with the related bends (Figure 5), it is evident that they correspond to large in size and irregular bends (i.e. bends with IDs: 11, 16 & 25). As it is shown in Figure 4, the tree of the bend with ID: 25, at the level of epsilon=1000m on the ground is divided into three branches. The first branch depicts a small in size and smooth sub-bend and ends at the level of epsilon=400m on the ground. The second one corresponds to a smooth, big sized and rather deep sub-bend. Thus, it ends at level of epsilon=200m on the ground. The third branch depicts a big sized and complex sub-bend that contains smaller sub-bends. These sub-bends are depicted by two new branches on the tree, appearing at the level of epsilon=600m on the ground. In the same way, at the level of epsilon=400m, the one of the two branches is divided into two new branches that correspond to smaller irregularities of the coastline. It becomes obvious that, the level of epsilon at which a tree may branch, as well as, the last level of epsilon of each branch, depends on the form of the irregularities, the size, and the shape of the initial bend. Therefore, by observing the structure of a tree, as well as, the recorded bend's attributes, the character of the line may be predicted.

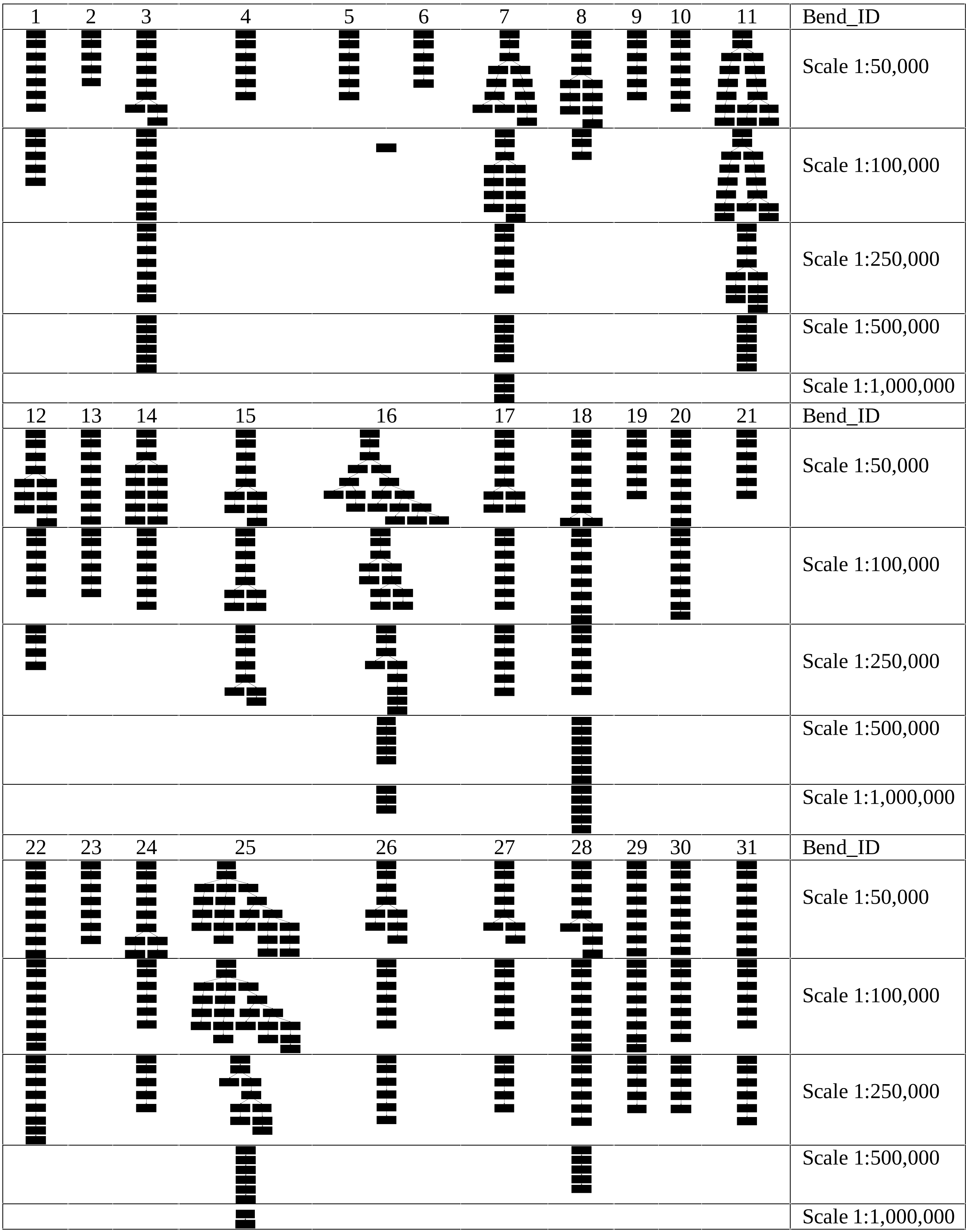

Figure 4. The hierarchically structured trees of the generated epsilon–non-convex bends

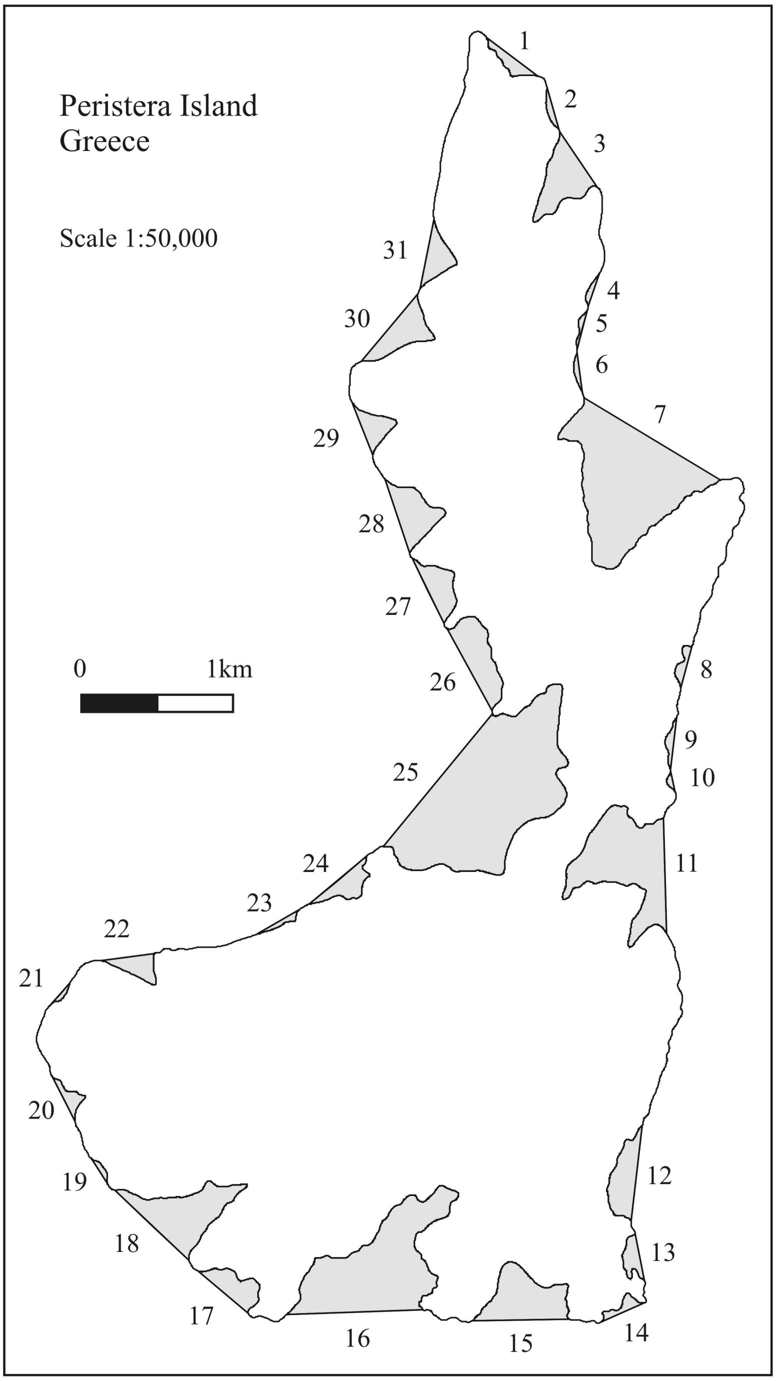

Figure 5. Peristera Island coastline and the generated epsilon–non-convex bends with the rolling circle of the highest diameter (the numbering of the bends corresponds to 'Bend_IDs' of Figure 3)

In Figure 4, it is clearly shown how many and which epsilon–non -convex bends are retained as scale decreases. In scale 1:50K thirty-one bends are created, while their number flows to twenty-three in scale 1:100K, seventeen in scale 1:250K, seven in scale 1:500K and just four in scale 1:1M. As mentioned in the previous section (Analysis of Bends' Attributes), that happens as long as lines become smoother and less complex as the degree of generalisation increases. By analyzing which non–epsilon non-convex bends retained as scale decreases, it is observed that one-column trees that do not have nodes in all levels of epsilon (like bends with IDs: 2, 9, 10, 19 or 23) are created only in scale 1:50K. These trees depict small in size and short in depth bends. On the contrary, one-column or branched trees that have boxes in all (or even in most) levels of epsilon in scale 1:50K are retained in some levels of simplification. In general, the retention of the trees depends on the attributes or on a combination of the attributes of the corresponding bends. For example, the two-branched tree that depicts the bend with ID: 14, appears only in scale 1:100K, mainly because its size is small. On the contrary, the less complex, but more large and deeper bend with ID: 18, is retained in all levels of simplification.

It is worth to be mentioned that the interpretation of the trees related to different simplification levels of the same line provides useful information about the character of the line. Thus, someone can analyse a specific bend at a specific scale, as well as, understand how its shape changes according to the level of simplification. A representative example is the case of bend with ID: 25. The bend in scale 1:50K is large in size and complex in shape, creating a tree of several branches as the level of epsilon decreases. It is observed that the tree in scale 1:100K has almost the same structure. That means that the complexity remains in the simplified line. The difference is that the size of the bends is smaller. The tree corresponding to the coastline of 1:250K is significantly different in structure. A great part of the branches presented in the initial bend are resolved. In addition, the remained branches are not detected in the lower levels of epsilon due to the smoother representation of the line. It is interesting that by analysing the tree, it is observed that the retained bends are those with the biggest size and the deeper depth. In reality, these branches depict deep bays of the coastline. The tree corresponding to the coastline presented in scale 1:500K has only one column. This one-column part of the tree depicts the deepest bay of the line. All the other irregularities of the initial coastline do not exist. In scale 1:1M the same bay is depicted only in the first two levels of epsilon, a fact that can be explained by the smooth shape of the line.

The way trees are transformed in various scales may offer interesting information about the significance of each part of the line under study. The proposed method during the case study is applied on an original and on four versions of the same line after simplification. By producing the hierarchically structured trees of the generated bends, we can easily observe the various irregularities of different parts of the line at each scale level. The bends corresponding to trees that retained in various scales or have nodes in various levels of epsilon, are candidate to contain line's critical parts. By analysing both the bends' attributes and the structure of the trees, we may result to a quantitative expression of the character of the line. Obviously, this final remark needs further consideration and extended research.

CONCLUDING REMARKS

Considering that the analysis of cartographic lines in separated sections of uniform characteristics is a useful research topic in line generalisation, a method of producing and attributing bends is proposed. The concept of epsilon–non-convex bends is theoretically defined and implemented in computer environment for any cartographic line. Each bend is characterised by quantitative attributes (i.e. size, and shape) and the implemented diameter. By applying the method iteratively over gradually decreasing diameters, interior bends are created into the bends of each previous level. The results of the case study show that the size of the bends decreases and their shape becomes more rounded as the level of epsilon decreases. As the scale reduces, the number and the size of the bends decrease, while their shape becomes more rounded. The fact that the bends of each level of epsilon are topologically enclosed to the bends of each previous level implies the capability to visualise the generalisation procedure through hierarchically structured trees. The hierarchical structured trees may be considered as an effective 'tool' for the analysis of cartographic lines, since they depict each bend, its attributes and its topological relation with the rest bends. Thus, these graphs of trees may assist to a direct observation of the bends' behaviour in regard to the level of epsilon or to the scale of representation. Based on above, important results about the significance of each part of the line for the whole line may yield. Further work is in progress to define quantitative measures for the hierarchical structured trees of bends and further exploit their potential.

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