Local length ratio as a measure of critical points detection for line simplification

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Abstract

Critical points detection is a research topic related to cartographic generalization as well as computer science in disciplines like computer vision, pattern recognition, or signal processing. In the present paper a short review of existed critical points detection algorithms is given. Most of these algorithms are related to the estimation of curvature of the points defining the line. Instead of calculating the curvature, *local length ratio* (LLR) is introduced as a measure of critical points detection based on geometric principles. Four lines (three geomorphological lines and one arbitrary) selected from the existed cartographic literature were used in order to assess the new method of detecting critical points. Finally, the proposed measure of critical points detection is used to evaluate the results of two line simplification algorithms applied for six line simplification tasks on the coastline of a small island. Although the present research approached the problem at a preliminary stage, promising results have been produced. It is apparent that further research is required based on a more in depth analysis for concluding about the method power.

Key-words: critical/dominant points, local length ratio, line generalization, line simplification algorithms

Introduction

In a study of clarifying several concepts of visual perception in the context of information theory, Attneave (1954) concluded that the points of an object are divided into two categories according to the information they convey. The first category is partitioned in a portion of points that are dominant, and the remaining predictable from the former, thus highly redundant. It could be assumed, that the dominant points are able to describe and characterize the shape of the object, since they transmit more information than the others. These points are termed “critical points” in the cartographic literature and “dominant points” in the computer science literature (Li 1995). Marino (1979) underlines the impact of critical points in line generalization because these points are recognizable both by cartographers and non-cartographers at the same degree. Critical points according to Attneave (1954) are concentrated at those locations on a line where its direction changes more rapidly. Specifically he states that, “It is clear that subjects show a great deal of agreement in their abstractions of points best representing the shape, and most of these points are taken from regions where the contour is most different from a straight line” (Attneave 1954, p.185). Freeman (1978) includes in his definition of critical points the points along a digital line that are:

1. curvature maxima,
2. curvature minima,
3. end points, and
4. points of intersection.

Critical points have attracted interest in research of the cartographic generalization domain (Marino 1979, White 1985, Thapa 1987,1988a, 1988b, 1989, McMaster 1987, Jenks 1989, Visvalingam and Whyatt 1990, Li 1995). In the computer science domain, especially topics such as computer vision, pattern recognition, or signal processing, a large number of critical points detection algorithms have been developed mainly addressing the problems of line approximation, curve segmentation, or feature detection (Pavlidis and Horovitch 1974, Iñesta et al 1998, Cronin 1999, Neumann and Teisseron 2002). The majority of the existing algorithms of critical points detection are mainly focused on the estimation of the curvature of the contour of a graphical object at any location. In addition, the reader can find algorithms for detecting critical points based on wavelets (Antoine et al 1997), or adaptive algorithms that require no input parameters (Cornic 1997). In Li (1995) the reader can find a comprehensive review of critical points detection algorithms that could be applied on digital cartographic lines describing their advantages and disadvantages. Considering a cartographic line, that its digital representation through most of the software platforms is usually expressed in vector structure, consequently being a discrete representation, there is no way of applying directly the mathematical definition of curvature. In order to avoid the drawbacks of the discrete representation some studies smooth the original curve by a Gaussian filter before computing extreme curvature points (Cornic 1997). According to the same author, this approach includes two main problems: a rather small Gaussian filter width may lead to insignificant detections, whereas a large width may exclude certain critical points from detection (Cornic 1997).

In the present paper *local length ratio* (LLR) is introduced as a measure of curvature of a digital line based mainly on geometric principles. The LLR can be calculated for each vertex of the line and then be associated with every vertex. By scanning the values of LLR from one end point to the other, several fluctuations are observed with local maximum values at those locations that the line is most different from a straight line. By selecting a tolerance value, all vertices associated with higher LLR value than the tolerance are considered as critical points. In the special case of open lines the two end points are also considered as critical points. Three geomorphological lines from the study of Marino (1979), and the test line from the study of Thapa (1987) were used to select appropriate tolerance values for the application of the LLR measure. Finally, the proposed measure of critical points detection was used to assess the results of two line simplification algorithms applied for six line simplification tasks on the coastline of Peristera Island, a small island located at the center of the Aegean Sea, characterized as complex at a rather high degree.

The measure of local length ratio (LLR)

In a digital environment the measure is applied on lines with a vector structure -i.e. a discrete number of points connected by vectors. The criterion of detecting critical points along a line is chosen in such a way to locate those vertices with maximum change of the line slope. The central idea is to segment the line around each vertex and determine its curvature independently, using the length as geometric criterion. If two arbitrary points P1 and P2 are selected on the line, it is possible to estimate: L the length of the line along the two points, and S the chord between them (Figure 1). The measure of *local length ratio* (LLR) is defined as follows:

$$\mathrm{LLR} = \frac{\mathrm{L}}{\mathrm{S}}$$

Consider points P1 and P2 that are defined as the two consecutive intersections of the line with the circle. The circle center is located at each vertex of the line and its radius is fixed before the application. Thus, the line is equally segmented, on the basis of a common measure, around each vertex.

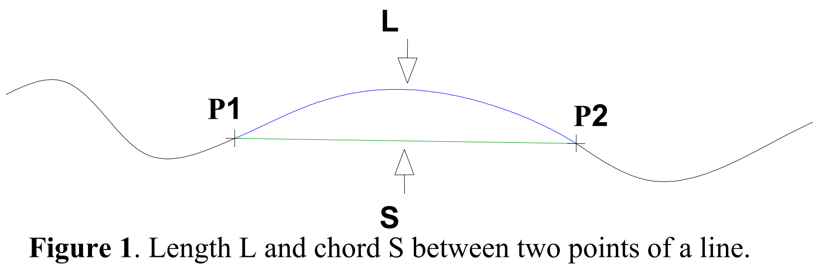

Figure 1. Length L and chord S between two points of a line.

The length L and the distance S are estimated for each segment associated with every vertex. Assuming that the digitization step of the line remains approximately constant and smaller than S, the length L and chord S depend on the lines shape between points P1 and P2. Obviously, the length L increases while the chord S decreases as the curvature of the line increases (Figure 2). Respectively, the LLR varies with the curvature and is always greater than 1. Therefore, LLR could be used as a measure of the curvature variation. The points of the line considered as critical are determined calculating LLR for each vertex and detecting the maxima (Figure 3). It must be stated that considering the calculation of LLR close to the end points of the line, a special modification of the proposed geometry must be applied, probably because there is only one point of intersection between the circle and the line. In such cases L is defined as the length along the line between the center of the circle and the point of intersection and S as being equal to the circle radius.

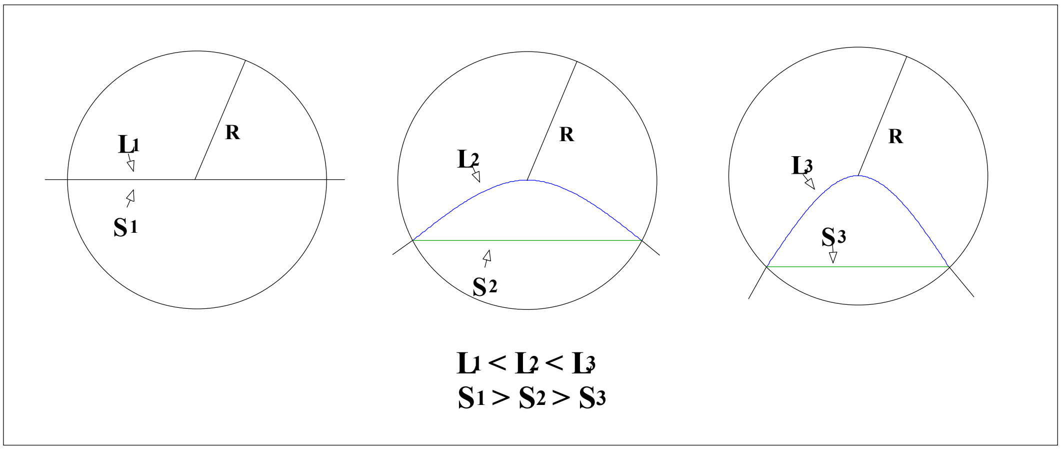

Figure 2. Variation of length L and chord S in regard to curvature.

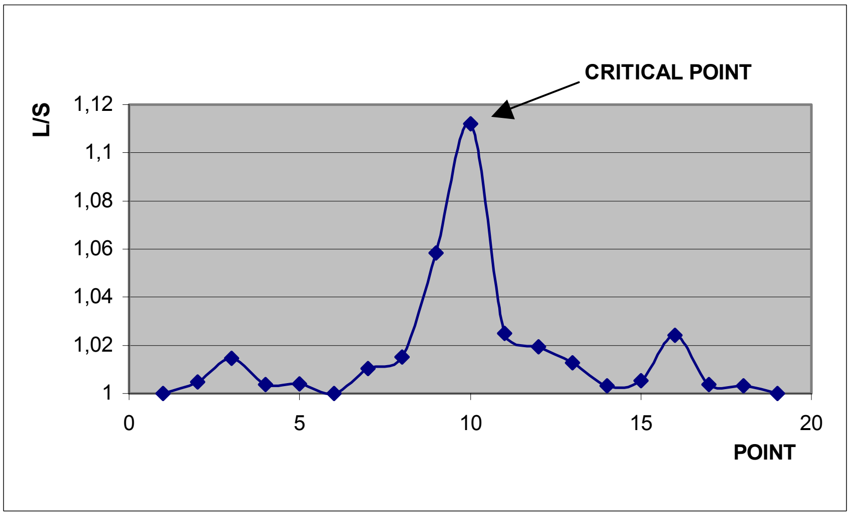

Figure 3. A critical point in a LLR diagram.

The size of the implemented radius of the circle around each vertex is a factor that directly affects the quality of the results. By increasing the radius it could be observed that both length L and chord S vary positively as long as the segment under consideration around the vertex of interest expands. To illustrate the above, an example of the variation of length L and chord S over a wide range of radii (R) is presented in Figures 4, 5 and 6; starting from a size equal to the average step of line digitization and advancing proportionally, for three characteristic points (position of sharp slope variation-point 172, peak of a curve-point 473 and edge of the following straight section-point 350) respectively.

Interpreting Figures 4, 5 and 6, it is observed that the rate of increase of length L relatively to chord S varies more in positions with high angularity. Specifically, the increase of length L, with regard to that of chord S, appears higher at high curvature positions (Figure 4). The difference between length L and chord S decreases at average curvature points (Figure 5) and converges to zero at straight sections (Figure 6). The growth rate of LLR over the range of different radii related to the average step size of the line varies respectively, as it is shown in Figure 7. The way that the LLR values change as it is presented in Figure 7, leads to the conclusion that circles with radius equal to two or better three times the average step of the line provide distinguishable values of LLR for critical points detection. Actually, by applying larger values of radii than three times the average step size, LLR tends to express more global than local characteristics of the shape of the digitized lines. In order to propose tolerance limits of applying LLR as a measure of critical points detection an empirical investigation is carried out in relation to local parameters of the shape of a digitized line. The values of LLR are correlated with the respective points on the digital line in order to be grouped according to the line shape characteristics. The following result came out from the empirical exploration:

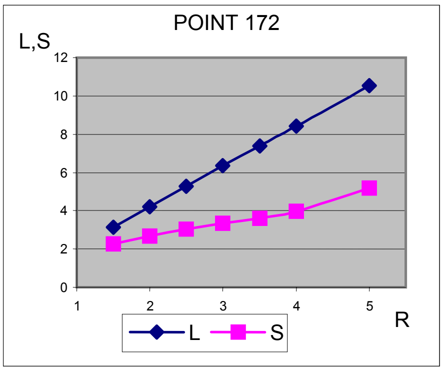

Figure 4: L and S variation over a range of different radii at a location of sharp slope variation.

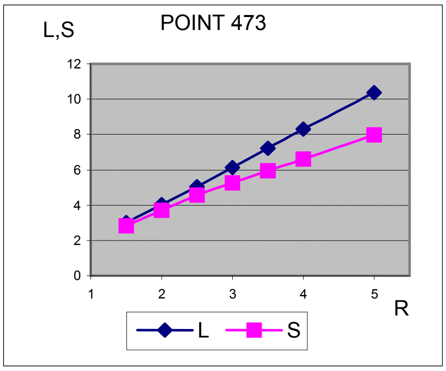

Figure 5: L and S variation over a range of different radii at a location of a curve peak.

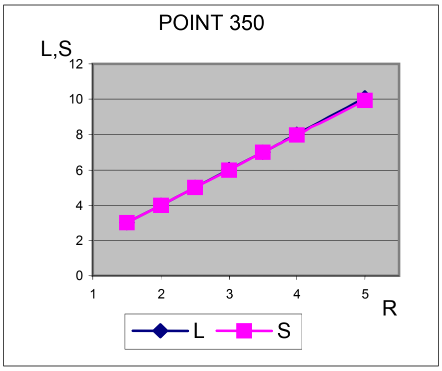

Figure 6: L and S variation over a range of different radii at a location of straight section.

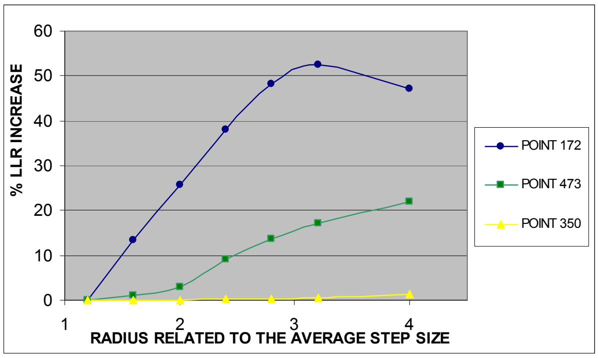

Figure 7. The rate of LLR change over different radii (R) at three different curvature positions (position of sharp slope variation-point 172, peak of a curve-point 473 and beginning of straight section-point 350).

- Group A:
  LLR values ranged between 1.04 and 1.15 correspond to locations of smoothed slopes (up to 120^o) that form fluctuations with basis vs. height ratio of 3.5:1 or smaller.
- Group B:
  LLR values ranged between 1.15 and 1.30 correspond to locations of sharp slope changes (90^o-120^o) that form "heavy" fluctuations with basis vs. height ratio between 3.5:1 to 2.5:1.
- Group C:
  LLR values greater than 1.30 correspond to locations of peaks with slopes less than 90^o that form peaks with basis vs. height ratio higher than 2.5:1.

Comparison with relevant research

The estimation of the credibility of the results of the proposed measure is implemented by applying the LLR measure on lines with critical points known in advance. The lines Mancos River, Shenandoah River, Cape Argo Coastline are selected from the study of Marino (1979) and Thapa's test line selected from his study (Thapa 1987). The first three lines represent natural phenomena and are different geographical and geomorphological samples of lines. The Shenandoah River line is assumed a sinusoidal shape, the Mancos River line is characterized by high complexity while the Cape Argo

Coastline includes both high curvature parts and straight sections. The critical points were derived from the empirical study of Marino (1979). According to her study, 161 individuals were asked to select a set of points which they consider to be important to retain the character of the line. The number of the points to be retained was predefined and the process was carried out for three different levels of generalization. In the present study the critical points derived from the first level of simplification are used in order to acquire a maximum possible sample. Thapa's (1987) test line is a geometrical model, designed to represent the maximum part of shapes (spikes, sharp slopes, straight sections, continuous long curves, etc.) that could be detected on a random line either independently or in combination. In the present study the critical points used have been produced from Thapa's mathematical model (Thapa, 1988b, p. 64, fig. 8).

The results of the implementation of the proposed LLR measure are compared with the known critical points from the studies of Marino (1979) and Thapa (1988b). The comparison of the former (LLR measure in respect to the proposed operational rules) with those referred in the above studies leads to satisfactory results. Table 1 shows that LLR measure succeeds to detect the same critical points of those determined by the compared studies at a higher of 90% percentage. However, there were appear cases that high values of the LLR measure were not considered critical in the Marino's (1979) or Thapa's (1988b) studies. After visual inspection some points were related to digitization errors, but a great deal of them to incorrect exclusion as critical. Probably they are encountered differently due to their inherent assumptions.

Table 1. Comparison of detected critical points (C-P) with other studies cited in text.

Line	**Average Step Size** (drawing units)	**Tol.**	**Radius** (drawing units)	**Known C-P**	**Detected C-P**	**Common C-P** (%)
Thapa's Line	2.50	1.04	5	45	41	91
Mancos River	1.18	1.04	2,5	40	38	95
Shenandoah River	1.21	1.04	2,5	53	53	100
Cape Argo Coastline	1.22	1.04	2,5	53	49	92

Application of LLR measure for assessing line simplification

The proposed measure has been tested on six line simplification tasks with two algorithms on the coastline of Peristera Island, a coastline characterized by a high degree of complexity. The coastline was digitized from a paper map of scale 1:50K with an average step of 14.77 drawing units. The raw data were cleaned up from those vertices that are redundant -like duplicate vertices, spikes, or switchbacks- after a "weeding" process as it has been suggested by Jenks (1981) in order to produce the original line. The LLR measure was applied to the original line according to the above stated rules, with a radius of 30 drawing units and tolerance value of 1.04, producing 153 critical points (6% of the original). Figure 8 in the left illustrates the original coastline and on the right the 153 critical points connected with line segments.

By comparing the original with the resulting coastline (Figure 8), it is observed that the selected critical points satisfy the basic principle of retaining the shape and the character of the line. A thorough visual evaluation of the 153 points in local level leads to the following:

- Locations that are crucial in order to retain the character of the line are detected with high success. Thus, the basic shape of the line is represented precisely.
- Large curves of the line either are not detected or are detected with the minimum number of points (two ends and one peak).
- Locations of high degree of complexity are represented with high level of detail.

The coastline was generalized with two line simplification algorithms, the Douglas and Peucker (1973) and bendsimpify (Wang and Müller 1998). The aim was to compare them by using especially selected critical points. A qualitative and quantitative comparison of the retained vertices is discussed in the following. The generalization tasks include the simplification of the coastline at six different levels by using each algorithm and applying successively increasing tolerances. The tolerances were defined in such a way that the number of the retained vertices to be common in each level. Table 2 illustrates the parameters of the six levels of line simplification performed.

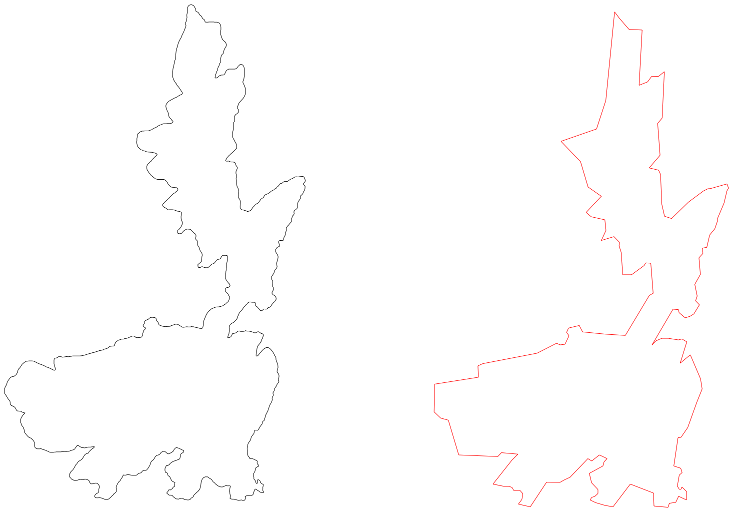

Figure 8. Peristera Island coastline (left) and its 153 critical points connected with line segments (right).

Table 2. The parameters of the six level of Peristera Island coastline simplification.

Level	Algorithm	Tolerance (drawing units)	Retained Vertices	Retained Vertices (%)
1	**D-P**	0.2	2131	88
	Bendsimplify	19.3	2131	
2	**D-P**	0.5	1733	71
	Bendsimplify	41.5	1728	
3	**D-P**	1	1333	55
	Bendsimplify	70	1333	
4	**D-P**	2	963	40
	Bendsimplify	124	960	
5	**D-P**	5	542	22
	Bendsimplify	240	546	
6	**D-P**	10	351	14
	Bendsimplify	325	353	

The remaining vertices after the successive simplification tasks were compared with the 153 detected critical points. Table 3 presents the number of the common points that share each level of simplification with the set of the critical points. For the first two levels of line simplification the number of retained vertices is high, therefore, the “success rate” of the two algorithms is almost absolute in selecting the entire set of critical points. By increasing the simplification level bendsimplify’s algorithm percentage decreases. At the last two levels less than half of the critical points are retained. On the contrary, in all cases, Douglas and Peucker algorithm retains, with minor deviations, the entire set of critical points. Similar results can be found in White (1985) research where several line simplification algorithms were assessed. This fact, in absolute numbers, it could be considered as a disadvantage for bendsimplify algorithm, since it does not retain a certain number of critical points.

Table 3. Critical points remaining after the six successive simplification tasks.

Level	Douglas-Peucker		Bendsimplify	
	Remaining C-P	**Remaining C-P (%)**	**Remaining C-P**	**Remaining C-P (%)**
1	153	100	153	100
2	153	100	148	97
3	153	100	131	86
4	152	99	112	73
5	151	99	76	50
6	147	96	56	37

For a further discussion, the retained vertices, being simultaneously critical points of each algorithm, are allocated in the three groups (A, B and C) of LLR values defined earlier. The results are presented in Table 4 for both algorithms and the six simplification tasks. From Table 4 it is obvious that the main volume of the points selected by bendsimplify algorithm corresponds to the first two groups (Group A: 1.04-1.15 and Group B: 1.15-1.30) of LLR values. In fact in the first group the majority of the values fluctuates at fairly low levels. Not adequate number of vertices with high level of LLR value is selected. This fact becomes more obvious at the last two levels of simplification. Practically, this implies the retention of the vertices that represent locations with smooth slope changes (these locations usually define the end points and the peaks of curves with large fluctuations) and characteristic slopes (with angularity approximately 90^o) that are necessary to keep the form of the line. The simplified lines are presented, in a large extent, smoothed. By reducing the level of simplification the retained vertices attribute to a greater detail of the final line.

Table 4. Number of the retained critical points into three groups of LLR values for the six levels of simplification.

	Douglas and Peucker algorithm						Bendsimplify algorithm					
Level	**1**	**2**	**3**	**4**	**5**	**6**	**1**	**2**	**3**	**4**	**5**	**6**
Group A	103	103	103	102	102	98	103	98	86	74	52	39
Group B	31	31	31	31	31	31	31	31	29	26	19	13
Group C	19	19	19	19	18	18	19	19	16	12	5	4

The Douglas and Peucker algorithm retains the majority of critical points for all six levels of simplification. Thus, the derived critical points distribution into the three groups is similar with the distribution into the same groups of the original critical points. Table 4 shows that the largest part of the vertices refers to the Group A. This leads to the retention of the characteristic slopes of the line of interest. Additionally, long curved parts of the line are detected with the minimum number of points. The line character is perfectly retained. The contemporaneous selection of vertices with high LLR values implies the representation of a large part of the line details. This fact is repeated for all six levels of simplification.

Concluding remarks

Considering the proposed measure of critical points detection it is found that, although it is simple in its conception and very easily implemented, it succeeded in detecting critical points very similar to those of other algorithms, like the one proposed by Thapa (1988b), or that based on perceptually detected critical points by cartographers and non-cartographers (Marino 1979). The LLR is also found useful to assess the two line simplification algorithms. Specifically, it came out that the Douglas and Peucker algorithm retains parts of the line that are of high complexity by preserving almost all the critical points at all levels of simplification. In contrast, the bendsimplify algorithm retains a minimum of the line complexity but the maximum of its curvature. Finally, it must be mentioned that the LLR values, associated with each vertex of the line and saved in a table, could be transformed into useful information for problems related to multi-scale representations of the line. Obviously, this final remark needs further consideration and deserves extensive research.

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