On the Assessment of Manual Line Simplification Based on Sliver Polygon Shape Analysis

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ABSTRACT

In the present paper, an attempt is made to analyse the shape of sliver polygons, generated by manual line simplification. Based on the shape analysis, an evaluation of line simplification is performed and the displacement caused by line simplification is estimated. More specifically, the shape of the sliver polygons is expressed quantitatively by the ratio between their perimeter and the square root of their area. Based on the estimated shape number, sliver polygons are modelled –normalised– as having rectangular shape with sides of ratio 1:n. The normalised sliver polygons are used to estimate the magnitude of displacement of line simplification. The developed method is applied to coastlines digitised from paper maps over a wide range of scales (1:50K, 1:100K, 1:250K, 1:500K and 1:1M). By assuming the coastlines of the larger scale (1:50K) as reference lines all the other versions of the same coastlines are overlaid successively by applying a typical GIS function of union. The generated sliver polygons are analysed on the basis of their shape over each scale change. Consequently, the magnitudes of line simplification displacement are estimated over each scale change, based on the shape analysis carried out. Finally, the outcome of the study is compared with the results of other relevant studies cited in literature. The results of the comparison overcome the underestimation degree of "true" displacement present in other global measures of displacement.

Introduction

Line simplification is a fundamental cartographic procedure of generalisation, which successively eliminates details or exaggerates specific characteristics along lines as the map scale decreases. Any simplification task has as a result to modify lines and simultaneously preserve their shape or their visual character. By overlaying different versions of the same linear feature over a range of scale, a large number of polygons is generated. These polygons may express the displacement caused by line simplification procedure. In a similar way the same kind of polygons are created when digital maps are overlaid in a GIS environment (see for example: Burrough [1986], Cromley [1992], Jones [1997], or Heywood et al. [1998]). Usually, these polygons are outlined by boundaries that are represented slightly differently in the source maps and are called sliver polygons [Chrisman 1989]. Goodchild [1978] studies the polygon overlay problem and concludes that the number of spurious –sliver– polygons generated by the superimposition of two versions of the same line depends on the line complexity. Furthermore, the same author provides quantitative measures for estimating the number of the spurious polygons. Since the differences of the two versions of the line are very small most of the generated sliver polygons are small in size and have a thin and elongated shape.

In the present study, an attempt is carried out towards analysing the shape of sliver polygons, generated by line simplification procedure. Based on the shape analysis, an evaluation of manual line simplification is performed and the magnitudes of displacement caused by line simplification are estimated. The present study can be seen as a contribution for evaluating manually generalised versions of existing maps, a need that it has already been expressed by Li [1993]. More specifically, coastlines (covering a central part of Greece) are digitised from paper maps over a wide range of scales (from 1:50K to 1:1M). By setting the coastlines of the larger scale (1:50K) as reference lines all the other versions of the same coastlines are overlaid successively by applying a typical GIS union function. The generated sliver polygons are analysed on the basis of their shape over scale changes. Finally, based on the carried out shape analysis, the magnitudes of line simplification displacement are estimated over scale changes.

On quantitative estimation of the shape

Assuming that any areal entity is outlined by a closed curve on a plane, we can realise that its shape property is invariant of any geometric transformation like translation, rotation, or scale. In general, sliver polygons, being areal entities, have a narrow and elongated shape and are characterised by small sizes. Three kinds of sliver polygons shapes can be identified: rounded polygon, elongated strip or crooked strip [Franklin and Wu 1987]. In a study related to area estimation by dot grids, Bonnor [1975] associates the error of the areas estimates with their shape and the grid density. The author classifies the areas shape into four categories: (a) areas with regular shape and boundaries, (b) areas with regular shape and somewhat irregular boundaries, or vice versa, (c) areas with regular shape or irregular and irregular boundaries, and (d) a combination or sums of individual areas. Although this classification schema is an interesting contribution to the problem of characterising the shape of areas, it is based on qualitative criteria and thus it is not applicable to quantify their shape.

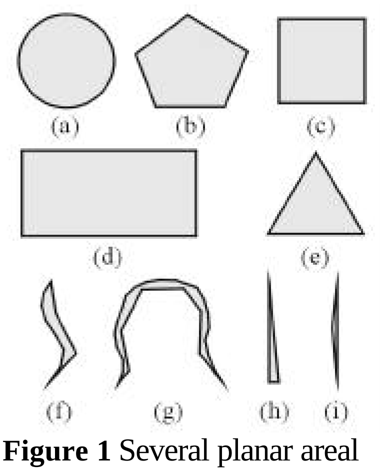

Figure 1 Several planar areal entities.

A numerical expression (k) appropriate to be used to describe the shape of a closed planar curve can be determined by the ratio between its perimeter (L) and the square root of its area (A), a number independent of its size [Mandelbrot 1983, Feder 1988, Maling 1989]:

$$k = \frac{L}{\sqrt{A}}. \qquad (1)$$

By applying the equation introduced above, the shape numbers (k) of all the planar areal entities illustrated in Figure 1, are: (a) k=3.5449 for the circle, (b) k= 3.8241 for the pentagon, (c) k= 4 for the square, (d) k=4.2426 for the rectangular having sides with ratio1:2, (e) k=4.5590 for the equilateral triangle, (f) k=7.6973 for the elongated strip, (g) k=12.6966 for the crooked strip, and (h) k=9.5649 or (i) k=12.5143 for the two type of spikes. One can observe that for the case of the circle – characterised as an areal entity of 'perfect' shape– the shape number has the smallest value. In addition, areal entities having a rounded shape (see Figure 1 b, c, d and e) can easily be distinguished from narrow and elongated shapes (see Figure 1 f, g, h and i) as having rather small or rather high values of k respectively. The mathematical problem behind this consideration is actually related to the classical isoperimetric problem [Rassias 1991]. The isoperimetric problem is related to the real questions [Rassias 1991, p. 1146]: "What shape must a close curve in the plane have if, with a given length it should enclose the greatest possible area? Or: When has a curve enclosing a given area the least possible length? The answer is that the curve has to be a circle". Thus, the isoperimetric problem provides a pure mathematical proof to the observations stated above.

In a study of measuring the length of closed geomorphic lines –like the shore lines of lakes– on various maps, Håkanson [1978] introduced an irregularity index describing the shape of these lines, named "shore development" (F). According to Håkanson [1978, p. 144] the "shore development" (F) is defined as: "the quotient of the length of the shore line to the length of the circumference of a circle with an area which is equal to that of the lake or the object enclosed by the given line". Thus, for any planar areal entity of perimeter (L) and area (A) its "shore development" (F) is given by:

$$F = \frac{L}{2\sqrt{\pi A}}.$$

However, Wang and Müller [1998] in developing line simplification algorithms with cartographic than geometric character based on the principle of preserving the overall structure with line bends, they were involved with the problem of quantifying the shape of the bends. In order to describe numerically the shape of the bends they used the "compactness index" (cmp), which is defined as: "the ratio of the area of the polygon over the circle whose circumference length is the same as the length of the circumference of the polygon" [Wang and Müller 1998, p. 7]. Assuming a bend polygon of perimeter (L) and area (A) its "compactness index" (cmp) is given by:

$$cmp = \frac{4\pi A}{L^2}.$$

By analysing these two shape indices, the "shore development" and the "compactness index", it can be seen that they are closely related each other. Furthermore, it can be assumed that both of them are alternative versions of the shape number (k) introduced above. Easily one can prove, that the "shore development" (F) is directly related to the shape number (k) with the equation:

$$F = \frac{k}{2\sqrt{\pi}},$$

and the "compactness index" (cmp) in turn is directly related to the shape number (k) with the following equation:

$$cmp = \frac{4\pi}{k^2}.$$

Summarising the considerations introduced above, we can accept that the shape number can be used successfully for quantifying the shape of planar areal entities in a similar way that analogous shape indices have already been used in relevant studies, especially for the case of sliver polygons, generated by the overlay of the original and the simplified line and being either narrow and elongated or rounded in shape.

A new measure of line simplification displacement

Cartographers' research was directed towards evaluating line simplification either on the perceptual level [Marino 1979, Wood, 1995] or by analysing statistically several cartometric measures [McMaster 1986, 1987, 1989, Müller 1987, João 1998, Veregin 1999] or both of them [White 1985, Jenks 1989]. Considering the methods of evaluating line simplification on the basis of mathematical measures a large number of cartometric measures have been developed. In a comprehensive statistical analysis of mathematical measures for line simplification, McMaster [1986] developed thirty measures discriminating them either as single attribute measurements of length, angularity, etc., or measures of displacement (vector displacement, areal displacement). One of the most commonly used cartometric measure is the "total areal difference" per unit length [McMaster 1986, 1987], referred also as "areal offset" by White [1985], or "total areal displacement" per unit length by João [1998], or even "uniform distance distortion" by Veregin [1999]. The measure of "total areal difference" per unit length is defined by the sum of the area of all sliver polygons divided by the total length of the original line. This measure can express as a global quantity the mean magnitude of displacement caused by line simplification. Additionally, the same cartometric measure, as defined above, is useful for evaluating line simplification because it can be associated with the needed accuracy standards for line generalisation. Since, "total areal difference" per unit length gives only an overall global estimation of the displacement magnitudes produced by line simplification, a more detailed view regarding the distribution of displacement magnitudes caused by line simplification would be useful.

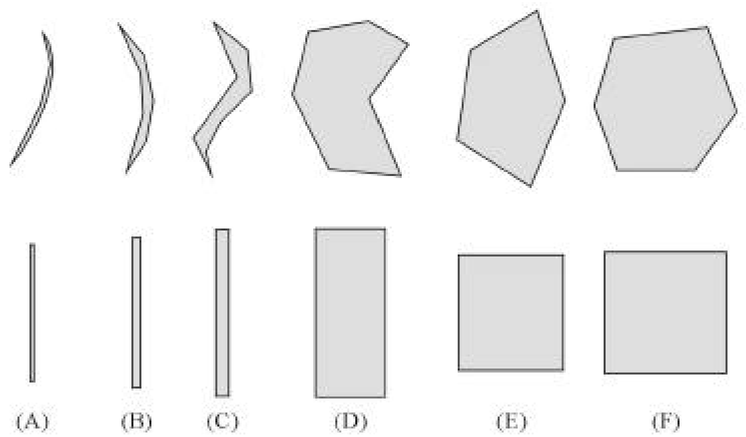

Figure 2 Different sliver polygons (above) and their equivalent normalised rectangular shapes with sides of ratio 1:*n* (below).

Based on the shape analysis introduced above, a new measure of displacement, named "sliver polygon (*sp*) displacement", is introduced associated to each individual sliver polygon generated by line simplification. By averaging the values of *sp* displacement over all sliver polygons, a global measure of displacement analogous to the "total areal difference" per unit length, can be estimated. In order to be able to estimate the *sp* displacement it is firstly suggested to normalise the form of the sliver polygons with rectangular shapes. All sliver polygons having a shape number of $k \geq 4$ are normalised as equivalent rectangular shapes having sides of ratio 1:*n* (see Figure 2 A-E). The *sp* displacement is defined as the basis of the rectangular shapes. All the remaining sliver polygons having a shape number $k<4$, and hence being rounded, are normalised as squares equal in area (see Figure 2 F). Their *sp* displacement is defined as the side of the square. It is obvious that by following this procedure all rounded sliver polygons are forced to have a normalised rectangular shape of sides with ratio 1:1.

Using equation (1) and substituting the perimeter (L) and the area (A) of the normalised rectangular shape the following equation is derived:

$$k = \frac{2\left(n+1\right)}{\sqrt{n}}.$$

Hence, resolving for the values of n, we get the following root as solution:

$$n = \frac{k^2 - 8 + k\sqrt{k^2 - 16}}{8}.$$

By examining the above equation it is obvious that we can model any individual sliver polygon with a rectangular shape of sides with ratio 1:*n*, if its shape number is $k \geq 4$. Considering the described approach the *sp* displacement can be calculated as follows:

$$sp = \sqrt{\frac{A}{n}} \text{ if } : k \geq 4 \text{ or}$$

$$sp = \sqrt{A} \text{ if } : k < 4,$$

where *A*: is the area of the sliver polygon.

The empirical study

A data set consists of ten coastlines located at the central part of Greece (Figure 3) was created to test the introduced method of evaluating manual simplification. The ten cartographic lines (Figure 4) are presented on the topographic maps produced by the Hellenic Geographic Army Service (HAGS). The map series of HGAS cover the entire country over the scales of 1:50K, 1:100K, 1:250K, 1:500K and 1:1M. The selected coastlines are representative samples of typical manual line simplification from the larger scale to all the others. Table 1 illustrates the names of the coastlines with their associated ID's. These ten coastlines have been chosen as subjects of study, since they are considered as having a rather high level of line complexity (see Figure 4).

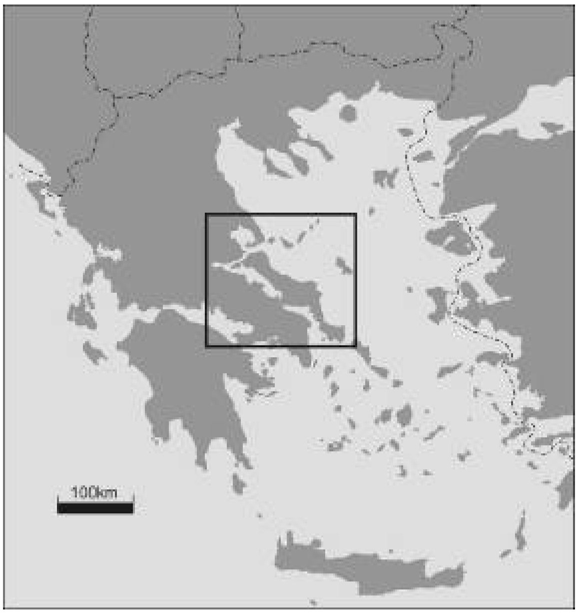

Figure 3 The location of the data set.

The data set was digitised with a resolution of 1016 lpi following the same standards for all the maps. All co-ordinates of the reference data set were transformed to the Greek Geodetic Reference System (Transverse Mercator projection, ellipsoid GRS-80) with less than 0.2mm RMS error per sheet on the map. The data set was edited and cleaned in order to link the parts of coastlines that share various map sheets. Any digitisation process of paper maps produces raw data with a number of redundant vertices like duplicate vertices, spikes, or switchbacks, etc., which should be removed by a "weeding" process [Jenks 1981]. The cleaning process should be carried out, by applying a data reduction algorithm with very small tolerance values. In similar studies, McMaster [1986] and João, [1998] suggest to apply Douglas and Peucker algorithm [1973] with tolerances of 0.002-0.05 mm on map, while Visvalingam and Whyatt [1990] suggest to apply the same algorithm with tolerance equal to half the width of the digitised line. In the present study, the raw data were cleaned by applying Douglas and Peucker algorithm with a tolerance of 0.01mm on map. The cleaned data set was eliminated from the unwanted vertices by an average of approximately 15%, while the length of the lines practically did not change (see Table 2).

Table 2 presents the results of the data cleaning process for the coastlines of mainland (C_100) and island of Evia (C_101) respectively. From Table 2, it can be observed how specific attributes –like number of vertices or length– of the manually simplified lines vary over the scales from 1:50K to 1:100K, 1:250K, 1:500K and 1:1M.

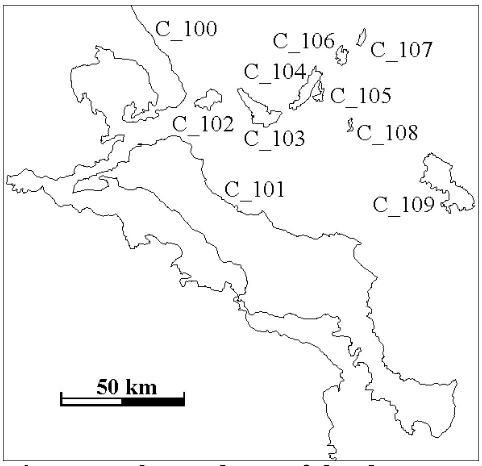

Figure 4 The ten lines of the data set.

Table 1 The studied coastlines.

	Coastline names	ID
1	Mainland	100
2	Isl. of Evia	101
3	Isl. of Skiathos	102
4	Isl. of Skopelos	103
5	Isl. of Allonissos	104
6	Isl. of Peristera	105
7	Isl. of Kyra-Panagia	106
8	Isl. of Gioura	107
9	Isl. of Skantzoura	108
10	Isl of Skyros	109

Table 2 The attributes of raw and reference data for the coastlines C_100 and C_101.

	Coastline of mainland (C_100)				Coastline of Isl. Evia (C_101)			
	Raw data		Reference data		Raw data		Reference data	
Scale	Vertices	Length m	Vertices	Length m	Vertices	Length m	Vertices	Length m
1:50K	31250	771424	26511	771408	29071	723665	25262	723653
1:100K	12654	742809	11510	742801	12339	702371	11274	702364
1:250K	3748	697803	3573	697801	3808	649457	3629	649454
1:500K	2974	644834	2633	644821	2654	596967	2315	596954
1:1M	997	610867	959	610865	1074	586306	1009	586302

Finally, the four versions of the ten coastlines of scale 1:100K, 1:250K, 1:500K, and 1:1M were overlaid successively with the ten coastlines of scale 1:50K with a typical GIS function of union and the sliver polygons were generated.

Analysis of the results

Considering the shape analysis of sliver polygons a statistical analysis was carried out. The frequencies of the shape numbers for each coastline over the four scale changes were calculated and classified into five groups. The classes' limits were defined in a way of discriminating as rounded in shape or narrow and elongated. Thus the limits of the five classes defined as: class *S1* with $k<4$ for sliver polygons with clear rounded shape, class *S2* with $4\leq k<4.5$ for sliver polygons with rounded shape, class *S3* with $4.5\leq k<6$ for sliver polygons with rounded and slightly elongated shape, class *S4* with $6\leq k<10$ for sliver polygons with narrow and elongated shape, and finally class *S5* with $k\geq 10$ for sliver polygons with narrow and highly elongated shape. The frequency distribution of sliver polygon shape numbers into the five groups expressed in percentage is presented in Table3 (see columns *S1* through *S5*).

Table 3 Frequency (%) classification of sliver polygon shape and displacement.

	Lines	*S1*	*S2*	*S3*	*S4*	*S5*	*D1*	*D2*	*D3*	*D4*	*D5*
	C_100	0	1	9	47	43	43	26	23	8	1
	C_101	0	0	9	55	36	49	30	19	2	0
	C_102	0	1	17	52	29	34	28	36	2	0
1:50K To 1:100K	C_103	0	1	13	63	23	36	31	28	5	0
	C_104	0	2	20	61	18	35	28	24	11	2
	C_105	0	0	13	61	27	43	27	29	1	0
	C_106	0	1	18	54	27	34	19	28	18	1
	C_107	1	1	15	61	22	43	26	21	8	1
	C_108	0	0	3	51	46	50	33	17	0	0
	C_109	0	0	10	59	31	49	34	17	0	0
	AVERAGE	0	1	13	56	30	42	28	24	6	1
	C_100	0	4	20	54	21	35	24	35	6	0
	C_101	0	3	26	52	19	35	23	36	5	0
	C_102	0	2	27	56	14	41	24	31	4	0
1:50K To 1:250K	C_103	0	3	25	59	13	40	28	29	2	0
	C_104	1	5	31	53	11	35	23	36	5	0
	C_105	0	2	17	68	13	42	24	32	2	0
	C_106	0	4	28	57	11	32	25	41	3	0
	C_107	0	6	44	43	6	28	27	42	2	0
	C_108	0	3	25	55	18	41	21	35	3	0
	C_109	2	7	28	53	10	28	18	32	19	2
	AVERAGE	0	4	27	55	14	36	24	35	5	0
	C_100	1	7	29	48	15	30	22	41	6	0
	C_101	1	6	31	48	14	33	21	37	9	0
	C_102	0	5	29	54	11	43	26	29	2	0
1:50K To 1:500K	C_103	0	3	34	54	8	46	27	27	1	0
	C_104	1	13	44	37	5	32	27	39	2	0
	C_105	0	5	38	51	7	35	32	33	0	0
	C_106	1	13	31	45	9	21	33	42	4	0
	C_107	3	3	36	50	8	31	17	39	14	0
	C_108	0	7	47	35	11	42	26	32	0	0
	C_109	0	11	38	47	4	25	24	40	10	0
	AVERAGE	1	7	36	47	9	34	26	36	5	0
	C_100	1	9	36	44	9	33	24	32	10	1
	C_101	1	7	39	43	10	38	21	28	12	1
	C_102	0	21	34	36	9	23	34	38	2	2
1:50K To 1:1M	C_103	0	6	31	49	14	63	19	18	0	0
	C_104	2	11	41	41	5	43	31	25	1	0
	C_105	0	9	53	30	7	33	33	33	2	0
	C_106	4	11	48	33	4	35	26	33	7	0
	C_107	0	15	40	40	5	20	20	55	5	0
	C_108	0	11	46	43	0	32	43	25	0	0
	C_109	0	8	31	53	8	36	20	32	12	0
	AVERAGE	1	11	40	41	7	36	27	32	5	0

By interpreting the results of the statistical analysis, in general the majority of the sliver polygons over all scale changes are narrow and elongated or highly elongated in shape. Furthermore, it is rare to observe sliver polygons rounded in shape or even very rare of a clear rounded shape. By examining the rate of scale change, we may observe that more sliver polygons of rounded shape are generated as the rate increases, while the sliver polygons of narrow and highly elongated in shape become fewer. Based on the above results, it could be stated that manual simplification preserves the shape of the lines as it has been accepted widely in the cartographic community (see for example Marino [1978], McMaster [1986], Jenks [1989], Visvalingam and Whyatt [1990]).

Considering the estimation of line simplification displacement the *sp* displacements measure referred to all sliver polygons for each coastline over the four scale changes were calculated. A statistical analysis of *sp* displacements was carried out based on frequencies classified into five groups. The classes' limits were defined according to the established cartographic standards of visual perception [Rouleau 1984, Keates 1996]. Thus the limits of the five classes defined as: class *D1* with $sp<0.05$mm on map for non-visually observable displacements, class *D2* with $0.05\text{mm}\leq sp<0.10$mm on map for limited observable displacements, class *D3* with $0.10\text{mm}\leq sp<0.25$mm on map for displacements within visual perception magnitude, class *D4* with $0.25\text{mm}\leq sp<0.50$mm on map for significant distinguished displacements, and finally class *D5* with $sp\geq0.50$mm on map for high magnitude displacements. The frequency distribution of *sp* displacements into the five groups expressed in percentage is presented in Table3 (see columns *D1* through *D5*). By interpreting the results of Table 3, it could be stated that there were no displacements of high magnitudes, as it was expected. The percentage of significant distinguished displacements was estimated approximately up to the level of 5%, which considering the coastlines' complexity seems reasonable. Furthermore, approximately one forth of the sliver polygons produces displacements within the visual perception magnitudes over small rates of scale change. When the rate of scale change is increased more then the sliver polygons of displacements within the visual perception magnitude become one third of the total. Finally, the significant majority of the generated sliver polygons (approximately 65%) do not produce visually observable displacements.

The introduced measure of *sp* displacement was compared with other global measures utilised for line simplification evaluation in the literature. For the need of the comparison the average values of *sp* displacement (MSP) for the ten coastlines over the four versions of scale change were calculated. In addition, the left and right *sp* displacement values (L_MSP and R_MSP respectively) across the lines were estimated. In addition, three global measures of line simplification cited in literature: the "total areal differences" (TAD) per unit length [McMaster 1986], the number of polygons (NP) per unit length [João 1998], and the percentage of the change in line length (LCH) [McMaster 1986] for the same lines and scale change were calculated. The results of all the cartometric measures are presented in Table 4. Interpreting data of columns L_MSP and R_MSP (Table 4) a balance between left and right displacements for all cases was observed, meaning that although the coastlines were digitised from different map series they were correctly superimposed. Although there are no significant differences between the "total areal differences" per unit length and the averages of *sp* displacement (columns TAD and MSP in Table 4 respectively), the later ones are assigned systematically higher values.

In general, we could assume that global measures like the last ones may express the generalisation error, caused by line simplification. By analysing more in depth the differences of the last two measures (TAD and MSP) we can estimate that systematically the "total areal differences" per unit length versus the averages of *sp* displacements are underestimated the generalisation error of line simplification up to a level of 20%. There are specific cases that the observed differences between these two measures are of 35%. Considering that the introduced measure of *sp* displacement is based on the shape number of each individual sliver polygon, it describes in more detail the "true" magnitude of displacement. So, it could be assumed that it is more closely to the generalisation error caused by line simplification.

Table 4 Global indices of assessing the coastlines simplification.

	Lines	Polys	TAD	NP	LCH	MSP	L_MSP	R_MSP
			mm^2/mm	1/mm	%	mm	mm	mm
	C_100	3057	0.18	0.40	3.71	0.19	0.19	0.19
	C_101	4324	0.12	0.60	2.94	0.13	0.12	0.13
	C_102	242	0.17	0.49	8.86	0.19	0.16	0.23
1:50K	**C_103**	424	0.15	0.85	5.82	0.17	0.17	0.16
To	**C_104**	485	0.20	0.61	2.58	0.22	0.24	0.20
1:100K	**C_105**	254	0.11	0.71	4.58	0.12	0.13	0.09
	C_106	156	0.20	0.38	2.30	0.22	0.20	0.23
	C_107	174	0.18	0.61	3.66	0.19	0.19	0.19
	C_108	144	0.08	0.67	3.74	0.08	0.09	0.08
	C_109	1036	0.09	0.77	3.42	0.09	0.08	0.10
	C_100	1812	0.15	0.59	9.54	0.18	0.17	0.18
	C_101	1677	0.16	0.58	10.25	0.18	0.19	0.18
	C_102	135	0.14	0.68	16.06	0.16	0.15	0.16
1:50K	**C_103**	275	0.11	0.91	11.77	0.13	0.14	0.12
To	**C_104**	243	0.13	0.77	16.18	0.15	0.15	0.14
1:250K	**C_105**	116	0.10	0.81	10.51	0.12	0.12	0.11
	C_106	118	0.13	0.72	14.29	0.15	0.15	0.15
	C_107	99	0.11	0.87	19.91	0.14	0.12	0.15
	C_108	80	0.11	0.94	15.23	0.14	0.15	0.13
	C_109	290	0.21	0.54	12.04	0.25	0.26	0.24
	C_100	920	0.16	0.60	16.41	0.19	0.20	0.18
	C_101	865	0.17	0.60	17.51	0.20	0.17	0.22
	C_102	112	0.09	1.13	22.02	0.12	0.11	0.12
1:50K	**C_103**	169	0.10	1.12	17.07	0.12	0.11	0.14
To	**C_104**	168	0.11	1.06	24.40	0.14	0.11	0.15
1:500K	**C_105**	85	0.09	1.18	14.29	0.11	0.11	0.12
	C_106	67	0.13	0.81	24.22	0.17	0.18	0.16
	C_107	36	0.16	0.63	42.24	0.22	0.13	0.23
	C_108	57	0.09	1.33	24.71	0.11	0.10	0.13
	C_109	202	0.15	0.76	24.09	0.20	0.21	0.19
	C_100	510	0.19	0.66	20.81	0.23	0.25	0.21
	C_101	414	0.23	0.57	18.98	0.27	0.21	0.32
	C_102	47	0.12	0.95	28.20	0.16	0.15	0.17
1:50K	**C_103**	105	0.09	1.39	22.00	0.10	0.10	0.11
To	**C_104**	107	0.09	1.35	32.65	0.12	0.12	0.12
1:1M	**C_105**	43	0.09	1.20	27.50	0.12	0.13	0.10
	C_106	46	0.11	1.12	31.24	0.15	0.16	0.12
	C_107	20	0.12	0.70	39.91	0.16	0.13	0.19
	C_108	28	0.09	1.31	29.88	0.12	0.13	0.09
	C_109	97	0.17	0.73	18.41	0.21	0.21	0.20

Concluding remarks

A new method of assessing line simplification through a displacement measure based on sliver polygon analysis is described. The method is empirically tested with a reference data set consisting of several coastlines, which were manually simplified over a wide range of scale changes. The results show that the significant majority of the generated sliver polygons are narrow and elongated in shape, and one forth or one third of them –depending on the ratio of scale change– are characterised by magnitude of displacement within the visual perception limits.

Finally, the results of the carried out empirical study reveal, that the introduced measure overcomes the underestimation degree of "true" displacement, which is present in other global measures citied in literature.

However, the research must be extended including various complex cartographic lines in order to reach a wider approval. Additionally, this aim could be supplemented by studying various kinds of linear cartographic entities (i.e. roads, rivers, boundaries etc.) as well.

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