The Problem of Cartographic Generalization in the Context of Fractal Geometry

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ABSTRACT

In this paper the cartographic generalization procedure is reviewed from the viewpoint of the current achievements of information technology. Consequently, the basic concepts of fractal geometry theory are introduced and ways to be applied to cartographic problems are described. As a result empirical rules, utilized for a long period of time by cartographers, can be theoretically proved in the context of fractal geometry. Furthermore, techniques of line simplification can be applied by using fractal properties. Some examples of such applications are shown and discussed. As a conclusion the concept of including the fractal dimension of graphical objects as a parameter of object complexity when designing multi-scale spatial databases is introduced.

Keywords: Map generalization, fractal geometry, fractal dimension, line simplification and line complexity.

INTRODUCTION

There are different ways to represent real world, like: satellite images, aerial photographs or maps. Although these representation media are sharing common characteristics, for example the length of any real world entity is reduced according to a specific ratio (the scale), maps are distinguished by two fundamental factors. Maps represent real world with graphical objects – the symbols - according to a well established graphical code (Bertin, 1981), which is based on perceptual and cognitive aspects. Furthermore, maps represent a simplified part of real world according to the map generalization procedure.

Every symbol occupies on the map surface larger space than the physical dimensions of the represented entity, although it is reduced by the scale ratio. This main cartographic problem is caused by the need that each symbol should be clearly perceived visually by the map user. But the space of the map is limited and unfortunately the cartographer can not include into the map all natural objects of the real world. The procedure of deciding which piece of information should be included into the map, and how it can be simplified in order to be symbolized and visualized by the map user, is called generalization. According to the International Cartographic Association a formal definition of generalization is the following:

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> "Selection and simplified representation of detail appropriate to the scale and/or the purpose of the map", (I.C.A., 1973).

In the context of digital cartographic systems and Geographic Information Systems (GIS) generalization gained an even wider meaning. This meaning is very well described in the following definition of digital generalization, proposed by McMaster and Shea (1992):

> "Digital generalization can be defined as the process of deriving, from a data source, a symbolically or digitally-encoded cartographic data set through the application of spatial and attribute transformations. Objectives of this derivation are: to reduce in scope the amount, type, and cartographic portrayal of the mapped or encoded data consistent with the chosen map purpose and intended audience; and to maintain clarity of presentation at the target scale".

Cartographers in the digital era conceive generalization as a two fold cartographic operation transforming digital objects, which represent real world entities. The first one, the "bright side" of generalization, focuses on visualization issues and is called cartographic/graphic generalization. The second one, the "dark side" of generalization, refers to the design and development of multi-scale data models and is called model/non-graphic generalization. In addition, researchers in the field of cartography have focused on creating sets of rules incorporating the cartographers' knowledge of the generalization domain.

The main aim of this paper is to promote the idea that all these open cartographic problems of generalization may find an interesting environment in the context of fractal geometry theory.

BASIC CONCEPTS OF FRACTAL GEOMETRY

Mandelbrot (1982) studied sets of mathematical functions of both geometric and stochastic character and called them fractal sets. He showed, using computer graphic representations, that random fractal sets may be used in order to simulate natural objects like topographic surfaces, coastlines etc. (Mandelbrot, 1975). The main characteristic of both non-random and random fractal functions is that, although continuous, they are not differentiable at any point, at least for a certain range of scale changes (Mandelbrot, 1982).

The fractal sets are characterized by their fractal dimension D (Mandelbrot, 1982), whose value lies between the topological D_T and Euclidean D_E dimension ($D_T<D<D_E$). The fractal dimension of graphical objects may describe their complexity or, equivalently, the degree at which their projection fills the Euclidean space R^E.

An important property of fractal objects is their invariance to similarity transformation. These fractal objects are called self-similar. A bounded set of points $\boldsymbol{S}$ is self-similar with respect to a scaling ratio r if $\boldsymbol{S}$ is the union of N non-overlapping subsets $\boldsymbol{S}_1, \boldsymbol{S}_2,\ldots,\boldsymbol{S}_N$, each of which is identical after possible translations and/or rotations to the set $r(\boldsymbol{S})$ obtained from $\boldsymbol{S}$ by the similarity transform defined by $0<r<1$ (Feder, 1988). The fractal dimension of self-similar graphical objects is given by:

$$D = \frac{\ln N}{\ln \frac{1}{r}}.$$

Still, for several fractal objects self-similarity does not apply. However, these graphical objects are invariant to a more general form of transformation, the affine transformation, and they are called self-affine fractals. A bounded set of points $\boldsymbol{S}$ is self-affine with respect to a ratio vector $\boldsymbol{r} = (r_1,r_2,\ldots,r_E)$ if $\boldsymbol{S}$ is the union of N non-overlapping subsets $\boldsymbol{S}_1, \boldsymbol{S}_2,\ldots,\boldsymbol{S}_N$, each of which is identical after possible translations and/or rotations to the set $r(\boldsymbol{S})$ obtained from $\boldsymbol{S}$ by the affine transform defined by $\boldsymbol{r}$ (Feder, 1988). The fractal dimension of self-affine fractal graphical objects is not uniquely defined (Mandelbrot, 1985; Feder, 1988). Its global value $D=D_T$, which means that self-affine fractal objects are not globally fractal. But a local fractal dimension can be computed according to a standard procedure. Clearly, self-affinity can be conceptualized as a generalization of the self-similarity property.

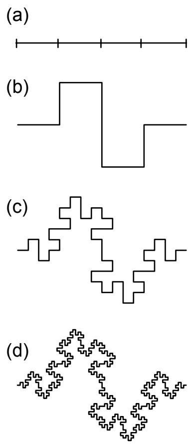

Figure 1: The Von Koch fractal curve.

Purely self-similar fractal curves can be created from geometric generators as shown in figure 1. The unit length in (a) is divided by the similarity ratio $r(N)$=1/4. The fractal curve in (b) comprises N=8 parts, while in (c) the procedure is repeated with $r(N)$=1/16 and N=64 parts and so on (see figure 1d). The fractal dimension of such a curve can be easily computed and is D=1.5. The exactly self-similar Von Koch curve, illustrated in figure1, is only a “crude” model of a naturally occurring graphical object, for example a coastline, and it differs in one significant aspect. Upon magnification, segments of a natural coastline look like – but are never exactly like – segments at different scales. The concept of fractal dimension, however, can also be applied to such statistically self-similar objects as the coastlines. Thus, a set $\boldsymbol{S}$ is statistically self-similar when $\boldsymbol{S}$ is the union of N distinct subsets each of which is scaled down by r from the original and is identical in all statistical respects to $r(\boldsymbol{S})$ (Feder, 1988).

All computational methods of fractal dimension of graphical objects involve the estimation of parameters of various statistical functions and data sampled over lines or analytical surfaces. In every method, the final step is the least-squares estimation of the slope of a linear function fitted to the data plotted on a double logarithmic diagram (Mandelbrot, 1982). The verification of the fractal character of a line (or a surface) and the reliability and the statistical significance of these parameters should be statistically tested, using a high confidence level (Nakos, 1990).

Depending on whether one deals with self-similar fractal graphical objects, like coastlines presented on maps or self-affine ones, like digital representations of terrain’s surface (e.g. DTM - Digital Terrain Model), their fractal dimension may be estimated by four methods (Nakos, 1990). These estimation methods are either pure geometrical (self-similar) or statistical (self-affine).

The first method of estimating the fractal dimension of self-similar graphical objects is based on the experimental research curried out by Richardson (1961). He studied the length of various curves (coastlines, frontiers etc.) derived from maps with different scales, by measuring their length using equal sized steps. Figure 2 presents the results of his study plotted on a double logarithmic diagram. As it can be seen in figure 2, all cases fall on a straight line with negative slope. Note the exception of the circle, which actually is not a fractal object and thus its slope is equal to zero (pure Euclidean shape). For Richardson the slope of each straight line had no theoretical interpretation. But according to Mandelbrot

(1982) the slope is an estimate of 1-*D*, where *D* is the fractal dimension. Thus:

$$L(\varepsilon) \approx \varepsilon^{1-D},$$

where: *L* is the length of the graphical object and: *ε* the equal sized step used to measure the length.

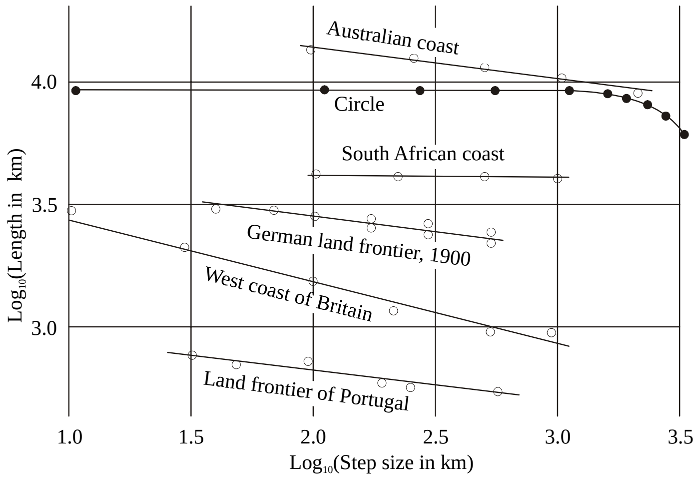

Figure 2: Length vs. step size (Richardson, 1961)

It is interesting to point out that the west coast of Britain has the largest slope among the lines illustrated in figure 2, being simultaneously the line with the highest degree of complexity.

The second method of estimating the fractal dimension of self-similar graphical objects is based on the correlation between the perimeter and the area of closed curves. According to Mandelbrot (1982) and Voss (1988), the fractal dimension *D* can be estimated by:

$$A \approx P^{\frac{2}{D}},$$

where: *A* is the area of closed curves and: *P* their perimeter.

Following the above method, Nakos (1996) measured the fractal dimension of the Greek islands' coastlines in an attempt to quantify their degree of complexity. He found as fractal dimension significantly high values (1.19-1.23), which verify their high degree of complexity, compared with the value of 1.25 which was measured by Mandelbrot (1967) for the case of west coast of Britain, a rather complex coastline.

The third method of the fractal dimension estimation deals with self-affine fractal objects and is based on the normalized variance function V_z. According to Mandelbrot (1982) and Voss (1988) the fractal dimension *D* is given by:

$$\frac{V_z(d)}{\sigma_z^{\ 2}} \approx d^{4-2D} \text{ where: } V_z(d) = E\left\{(z_i - z_j)^2\right\},$$

with: σ_z^2 the variance of each sample, where the sample size depends on the choice of the correlation distance *d* between points: *i* and *j*.

The last method cited here deals with self-affine fractal objects and is based on spectral analysis. The fractal dimension is estimated through the relation of the power spectrum $G(\lambda)$ versus the wavelength λ, (Mandelbrot, 1982; Voss, 1988) :

$$G(\lambda) \approx \lambda^{5-2D}.$$

There are certain aspects that should be considered when the fractal dimension is used as a geometric parameter in order to quantify the “roughness” or “complexity” of natural occurring graphical objects. Although in Mandelbrot’s work the notion of a constant *D*, or self-similarity, in the natural landscape occurs repeatedly, there are researchers that reject the concept of self-similarity over all possible scales (Goodchild, 1980; Mark and Aronson, 1984). But self-similarity is only one aspect of the fractal approach and most of the empirical studies show that self-similarity occurs for considerable wide scale ranges. This outcome allows technical applications in the field of cartography to be developed in the context of fractal geometry, i.e. line simplification (cartographic generalization). In addition, Goodchild (1987) is emphasizing that the fractal dimension provides a means of characterizing the effects of cartographic generalization and the recursive subdivision technique provides efficient ways of representing and organizing spatial data in digital form (model generalization).

LINE SIMPLIFICATION

Traditionally, the simplification procedure of cartographic generalization is supported by an empirical law, called “Principles of Selection”, and introduced by Töpfer and Pillewizer (1996). For the case of linear cartographic objects (for example coastlines), which are represented by the same width line symbols at all scales, the “Principles of Selection” can be expressed as follows (Jones and Abraham, 1987):

$$n_d = n_s \frac{m_d}{m_s},$$

where: n_d and n_s are the numbers of line segments at the derived and source scales, and: m_d and m_s the derived and source scales, respectively.

Actually, by applying “Principles of Selection” the cartographer can estimate the number of the retained vertices of the simplified cartographic line. The empirical law of Töpfer and Pillewizer answers to the query: *how many* objects should be retained on the derived map, and does not provide any information about the query: *which* objects should be retained (McMaster, 1989). In addition, “Principles of Selection” are based only on the scales ratio (of the source and derived map) and do not take into account the line complexity, which means that two linear objects with different degree of line complexity are treated exactly in the same way (Nakos, 1990). The first disadvantage of “Principles of Selection” may be overcome by applying existing line simplification algorithms, based on geometric criteria, which preserve the shapes of linear objects, like the Douglas and Peucker (1973) or Visvalingam and Wyatt (1993) algorithms. A functional description of the above mentioned algorithms can be found in Weibel (1997). The second disadvantage may be overcome by introducing fractal geometry and more specifically the property of self-similarity into line simplification procedure. Nakos (1990) proved that for self-similar linear cartographic objects, “Principles of Selection" can be expressed as follows:

$$n_d = n_s \left(\frac{m_d}{m_s} \right)^D ,$$

where: *D* is the fractal dimension of the linear cartographic object.

Nakos (1990) proposed a methodology for applying fractal geometry in line simplification procedure with three statistical tests, which are based on the introduced methods of estimating the fractal dimension. All methods result in a linear model when data are plotted on a double logarithmic diagram. The first one verifies the fractal character of the specific cartographic linear object by checking the value of the correlation coefficient ρ with the null hypothesis: $H_0(\rho \neq 1)$ at 99% confidence level. With the second test the slope of the correlation line *b* is checked against the equivalent value for a Euclidean shape b_E by rejecting the null hypothesis: $H_0(b=b_E)$ at 95% confidence level. The last statistical test deals with the significance value of the estimated fractal dimension *D* of the cartographic object by rejecting the null hypothesis: $H_0(D=1)$ at 99% confidence level. Consequently, one can simplify the linear cartographic object by applying a line simplification algorithm by preserving the number of vertices, which have been calculated with the help of the estimated fractal dimension *D*.

Table1: Number of vertices of Ithaki Island coastline.

Scale	Fractal	Manual	"Principles of Selection"
(1:100.000)		(2935)	
1:250.000	1081	1227	1174
1:500.000	508	609	587
1:1.000.000	238	340	294

Following the above procedure an example is given here for simplifying the coastline of the island of Ithaki. The example uses a 1:100.000 scale source map and the simplification is examined over derived scales of 1:250.000, 1:500.000 and 1:1.000.000. By correlating the length of the coastline against the size of the step, the correlation coefficient is ρ=-0.9765 and the slope of the correlation line has a value of b=-0.090$\pm$0.005. The samples passed all statistical tests, which means that the coastline is self-similar at all scales with fractal dimension D=1.090. Table 1 gives the number of vertices of the coastline under study over all scales for the introduced method, manual simplification and "Principles of Selection". In figure 3 the three ways of simplifying the coastline from scale 1:100.000 to 1:250.000 are illustrated.

Comparing the three coastlines in figure 3, one can see that although the coastline (a), with fractal simplification, has less number of vertices (table 1) than the coastline (c), simplified with "Principles of Selection"; also, it preserves the shape of the manually simplified coastline (b). Furthermore, one can evaluate the result of simplification by determining various cartometric measures (Nakos, 1999). By comparing the coastline simplified with the fractal method (a) and "Principles of Selection" (c) using as reference line the manually simplified line (b), the outcome was that line (a) produces less mean area displacement

(Nakos, 1999) than line (c).

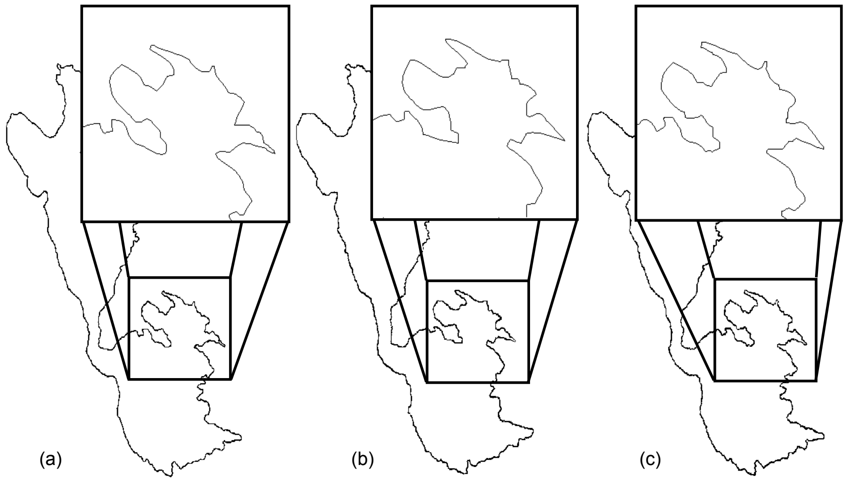

Figure 3: An example of line simplification using: (a) fractal geometry, (b) manual techniques and (c) “Principles of Selection”.

CONCLUSIONS

The coastlines have been chosen as subjects of study, since they are considered as having a rather high degree of complexity. However, the research must be extended to include various complex cartographic lines in order to reach a wider acceptance. Additionally, this aim could be supplemented by studying various kinds of linear cartographic features (i.e. roads, rivers, boundaries etc.) as well.

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