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D.E.M. DERIVED FROM EXISTING MAPS FOR USE IN GEOSCIENCES

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ABSTRACT:

Existing topographic maps in a variety of scales are a useful source for automated digitization to produce D.E.M. The reliability of the D.E.M. can be estimated with use of terrain classification methods to predict the appropriate sampling interval, as well as a model of its accuracy can be defined. These accuracy requirements are reviewed in concern of applications of D.E.M. in geosciences.

The applications of the concepts above are performed using topographic maps of 1:5000 scale in the area of Greece and results are commented.

TERRAIN MODELES DERIVES PAR CARTES TOPOGRAPHIQUES POUR UTILISATION EN SCIENCES DE TERRE

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RESUME:

Des cartes topographiques en une varieté des échelles, constituent une source utilisable à la dérivation des terrain modéles (D.E.M.). La reliabilité du ce modèle est éstimable par des méthodes de classification du terrain, aussi que la dérivation d' un modéle d'exactitude soit possible.

Les démandes d'éxactitude sont éxaminés en vue des applications du terrain modèle en sciences de terre.

En appliquant ces considérations/aux cartes topographiques en échelle 1:5000 de la région grecque, nous commentons les résultats.

D.E.M. Derived from existing maps for use in geosciences

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Introduction

Geosciences are concerned with the description of the Earth's form and shape as well as its genesis and the estimation of its internal structure. Anomaly potential field observations, like of the magnetic or gravity field are used as data. for various scopes, as geological interpretation, geophysical prospecting, geodetic positioning e.t.c.

There exists interaction between the observation surface on/or outside the earth's surface and the 3-D anomaly potential field (magnetic or gravity field). This interaction results to a distorted form of the potential anomalies due to the effect of the topographic relief (terrain) upon the potential field data, and it can be significant in rugged or in moderate relief areas. The problem is approached in two ways:

- The observational data are corrected for the terrain
- The observed potential field is continued to some constant elevation (upward/downward continuation of potential field)

For both approaches, the terrain information is needed in appropriate model form. The most traditional and frequent form of terrain representation in isoline maps, is not suitable for automated numerical analysis. However, traditional maps can be used for the derivation of a Digital Elevation Model (D.E.M.) which is a suitable form of the terrain representation in a variety of purposes and appropriate to compute the terrain parameters, like relief, slope, wavelength e.t.c. In most applications of geosciences, the potential field observations are terrain corrected with pre-existing terrain information, which can be less or more accurate than necessary.

Here we approach this problem in the inverse sense: The accuracy of a D.E.M. derived from existing maps can be checked whether it is appropriate for a specific use in geosciences, performing the test:

$$6^h_{D.E.M.} \leq 6^h_{g.science} \tag{1}$$

where $6^h_{D.E.M.}$ is the standard deviation of the D.E.M. estimated from chartographic rules and $6^h_{g.science}$ is the required accuracy in a specific application.

In this work, a software configuration to derive a D.E.M. from existing maps is presented. The output is an elevation or a slope matrix. The accuracy of the D.E.M. can be checked for its efficiency in the terrain correction computation (T.C.) in the gravity field problems. The various expressions of the T.C. can be put in form expressed with the terrain slope, which is a parameter of the terrain classification, and an estimation model for $6^h_{g.science}$ can be derived for the case of T.C. computation. Finally, an application of the software is presented and data about its production ability are given.

Terrain representation in form of D.E.M.

The terrain is considered as a continuously varying surface with traditional mapping representation in form of isolines. However, isolines are not suitable for numerical analysis and modeling. Therefore, the need for methods to use information about the continuous variation of altitude over space brough up various methods of this representation in form of D.T.M. These methods are classified in mathematical methods and image methods.

A D.T.M. consists of two components a) set of representative points of the terrain stored in the computer memory and b) algorithms to interpolate any new point of a given planimetric location and/or to estimate "other" data (Linkwitz, 1970). "Other data" means an other magnitude different from the elevation like the slope of the terrain surface. The slope of the terrain has been proposed as a criterion for the terrain classification (Ayeni, 1978).

The majority of D.T.M. applications concerns of Digital Elevation Model, (D.E.M.), i.e. a digital representation of the continuous relief variation over the space. The usefulness of a D.E.M. can be classified to the following applications:

1. Storage of the D.E.M. in national data bases (for geodetic and chartografic purposes)
2. Cut and fill problems in road design and in civil engineering
3. 3-D display of land forms (landscape architecture)
4. Statistical analysis and comparisons of different kinds of terrain
5. Computation of slope maps and slope profiles
6. Thematic information

Applications No. 4 and 5 concerns mainly of the geosciences.

The configuration of the D.E.M. production

The configuration of the D.E.M production consists of the use of already existing topographic maps to receive digitized coordinates (Fig. 1.). The registration of the digitized coordinates is done with an affine model of six parameters using the least squares method to obtain the parameters. The raster to vector conversion configures the scanner thinning and topology construction. The accuracy of scanner after propagation the model gives the following variance for X, Y ccordinates in a map system for scale 1:5000 (Nakos, 1989):

$$6^2_{x,y} = 0.27\ m^2 \qquad (2)$$

The digitized elevation data can be used in combination with some interpolation method to receive the D.E.M. The interpolation is performed assuming that the digitized data are randomly distributed points. (Fig. 2.).

The existing software can be combined with small commercial software packages for contour display and a printout or an orthographic or perspective view of D.T.M.

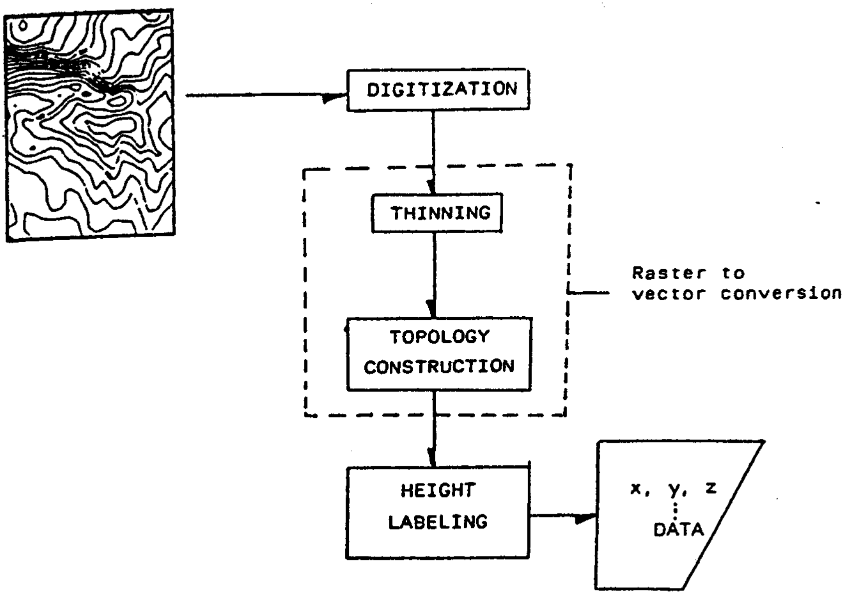

Figure 1. Digitization flow chart of existing maps

The hardware configuration of the used system consists of:

- An A.T. IBM compatible with 640k memory, 60 Mb H/D and EGA card.
- A HP scanjet/scanner with 300 dpi resolution
- A Digitizer Microgrid series II. Summagraphics with 1016 lines/inch resolution

In the application a non commercial software has been used, which can be described as follows:

- The digitization is held with automatic scanning of the map isolines.
- A raster to vector procedure transforms the digital image data of the contour lines in a vector format. (Nakos, 1988a).
- The interpolation is done (Nakos, 1988b), by the moving surface method (Schut, 1976).
- Magnitudes and slopes can be produced directly from the data.
- An interactive editing utility is tagging the height (elevation) values using also a digital tablet.

The output of the package is an elevation matrix and/or a slope matrix.

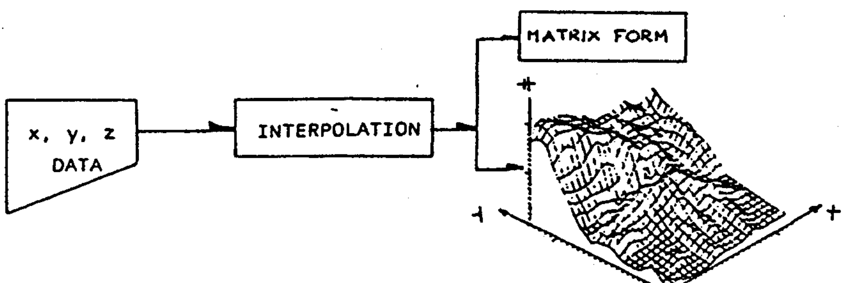

Fig. 2. D.T.M. production with interpolation of random distributed x, y, z data

The terrain information in geosciences

The terrain influence upon the intensity of the gravity of magnetic vector fields is taken into account either in form of terrain correction or in the form of the upward/downward continuation correction terms of the anomaly (gravity or magnetic) at the point level or at a constant surface level. This kind of corrections concern of varioos applications in geosciences. A case familiar to geodesists, is the geodetic use of the gravity field data, and especially the computation of T.C. in the numerical solutions of the Geodetic Boundary Value Problem (GBVP) (see Moritz, 1980)

The terrain influence upon the gravity vector is taken in form of T.C. applied to the Bouguer plate gravity reduction and in form of downward continuation correction terms of the Free Air (F.A.) gravity anomaly at the point or at the ellipsoid level.

Both corrections are equivalent under certain assumptions which concern of that no mass density change occurs within the region of study. This physical consideration is expressed analytically that F.A. anomalies and topographic elevations bear a linear relationship:

$$\Delta g_{F.A.} = a + b.h \qquad b = 2\pi k.\delta \qquad (3)$$

where $\Delta g_{F.A.}$ is the F.A. anomaly, h the topogr. elevation
a is the mean Bouguer anomaly within the region (assumed constant)
b is the Bouguer coefficient
k is the gravitational constant, δ the mass density of terrain

For the GBVP, the downward continuation of $\Delta g_{F.A.}$ is guaranteed by the theorem of Runge (Moritz, 1980, p.67) and practically it gives a ficticious gravity anomaly on a simple surface (level ellipsoid, some internal sphere e.t.c.) which generates a disturbing grav. potential T coinciding with the actual one on/outside the Earth.

The topographic information needed for the T.C. is known to be a function of which accuracy the "Bouguer" anomaly value is desired for any given gravity station. This information should not be necessarily in gridded format but its provision in gridded D.E.M. format is very atractive being efficient for any station location computations of T.C. (e.g. Forsberg 1984, Sideris 1985).

It should be noted that in the solutions of the GBVP based on approximation methods, the gravity field enters also as a function for modeling. Thus, the terrain information is needed to provide an overall smoothing to the field, and a D.E.M. which can be slightly different that the actual terrain is an appropriate form of the terrain information.

The slope of the terrain as a parameter of the T.C.

The spatial terrain variation is described with various parameters (e.g. Fredericksen et. al 1985). The most useful for gravity field problems are:
The relief, the slope, the wavelength, the power spectrum. Among these the most important is the slope, as it controls the gravitational forces. The value of slope is related to the convergence of the series which solve the GBVP (Moritz, 1973 and Moritz, 1980).

The slope is given in a 2-D function $z = z(x,y)$ as:

$$\text{slope} = \left[\left(\partial z/\partial x\right)^2 + \left(\partial z/\partial y\right)^2\right]^{1/2} \tag{4}$$

There exist various methods to compute the slope without need to approximate the terrain with a methematical function $z = z(x,y)$.

In the various forms of T.C. of gravity for the numerical solutions of the GBVP the slope enters as a parameter. The most frequent forms of T.C. are:

- The T.C. computed from integration of terrain mass (prism method):

$$T.C. = k\iiint_V \delta \frac{\cos z}{l^2}\, dv \qquad (5)$$

where k, δ as before and z, l as in Fig. 3.

Since:

$$\cos z = \frac{h-h_p}{l} \qquad (6)$$

$$T.C. = k\int_V \delta \frac{h-h_p}{l^3}\, dv \qquad (7).$$

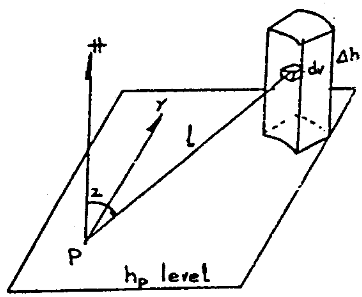

Fig. 3. Definition of T.C.

In this form T.C is used to compute the Faye anomalies (Δg_{FA}+T.C.) in the Stokes integral

- The G_1 term of Molodensky's solution
  In the planar approximation of the series solution (Molodensky's) of the GBVP it holds (Moritz, 1980):

$$G_1 = T.C. \qquad (8)$$

where G_1 is:

$$G_1 = \frac{1}{2\pi} \iint_{E} \frac{h(x,y)-h(x_p,y_p)}{\left[(x_p-x)^2+(y_p-y)^2\right]^{3/2}} \Delta_g(x,y)\,dxdy \tag{9}$$

note that: $A = \left[(x_p-x)^2+(y_p-y)^2\right]^{3/2}$ (10)

$$A = r(x,y) \tag{11}$$

The equality given by Eq. 8. holds if Eq. 3. is assumed. If, in addition, the elevation surface is assumed stochastic, G_1=T.C. can be put in form of convolution:

$$T.C. = G_1 = \frac{1}{2\pi}\left\{\left[h(x,y)*\Delta_g(x,y)\right] * r(x,y)-h(x,y)\left[\Delta_g(x,y)*r(x,y)\right]\right\}$$

(12)

This representation is the basis of FFT computations for T.C. (Forsberg, 1964, Sideris, 1985).

- The gradient correction

  The gradient correction (e.g. Groten, 1979. p.314) aims to replace G_1 but using also the difference:

$$\Delta_g(x,y) - \Delta_g(x_p,y_p) \tag{13}$$

The T.C. is:

$$T.C. = \frac{\partial\Delta g}{\partial h}\left[h(x,y) - h(x_p,y_p)\right] \tag{14}$$

where $\partial\Delta_g/\partial h$ is the F.A. gradient

$h(x_p,y_p)$ is the elevation of the point P where T.C. is to be computed

$h(x,y)$ is the elevation of the "running" point of the terrain.

The various forms of T.C. depend on the accuracy of the goal, the accuracy of the gravity survey and that of the topographic information.

An estimation model for 6_g^h science

From Eq. 7 the T.C. (in mgals) is of the form:

$$TC \sim \frac{\Delta g}{r^3} \Delta h . d6 \qquad (15)$$

where $\Delta h = h - h_p$ (elevation difference)

r is the distance

Δ_g mean or point anomaly

$d6$ the area of the base of prism element

For a required value of $6^2_{T.C.}$ (defined from the required accuracy for the specific application), 6^h will be evaluated. Applying the covariance propagation law in Eq. 15:

$$6^2_{T.C} \sim (\qquad)^2_h \, 6^2_h + (\qquad)^2 \, 6^2_{\Delta g} + (\qquad)^2 6^2_r \qquad (16)$$

which gives in detail:

$$6^2_{T.C.} \leq 6^2_{\Delta g} \sim \frac{\Delta g^2}{r^6} 6^2_h + \frac{\Delta h^2}{r^3} 6^2_{\Delta g} + \frac{3r^2 \Delta h^2 \Delta_g{}^2}{r^6} 6^2_r \qquad (17)$$

From the inequality:

$$6^2_{T.C.} \leq 6^2_{\Delta g}$$

The equality condition is chosen and after elementary algebra:

$$6_h^2 = \frac{(1-\Delta_h^{\ 2})}{\Delta g^2}\, 6_{\Delta g}^2 - \frac{3\Delta_h^{\ 2}}{r^2}\, 6_r^2 \qquad (18)$$

But $1-\Delta h^2 \approx \Delta h^2$ and $\Delta h/_r = k$ = slope and (18) is:

$$6_h^2 = \left(\frac{\Delta h}{\Delta g}\right)^2 6_{\Delta g}^2 - 3k^2 6_r^2 \qquad (19)$$

In Eq. 19 the variable factor is the slope and Eq. 19 can be used to evaluate 6_h^2 for various slopes of the terrain.

Application

For application of the D.E.M. configuration, a map of HAGS in scale 1:5000 was used with contour interval 4 m. The test area belongs to the western part of Greece and is of size 4 km x 2,5 km. The production ability was about 20 hours with use of the described non commercial software, divided as:

- about 15 hours for the digitization procedure (raster-to-vector conversions)
- about 5 hours for editing the D.E.M.

The file of digitized elevations was 8,54MByte while the final D.E.M. with 100 m sampling interval, consists of 26 rows and 41 columns.
Fig. 4, 5 and 6 represent respectively the D.E.M. the slopes and the isoline map after the D.E.M. drawn with isoline interval 20 m.
The elaboration of the digitization procedure was done after division of the area in 12 subregions. (Fig. 7. and 8.) where the mean, minimum and maximum value of elevation and slope are written respectively.

334,04 m 230,54 m 403,58 m	251,51 m 153,86 m 307,84 m	216,78 158,39 288,15	193,35 140,89 283,36
132,97 m 44,97 m 302,68 m	168,24 m 90,03 m 279,97 m	140,61 68,44 238,94	178,92 89,39 239,64
47,03 m 7,99 m 133,49 m	82,64 31,32 171,98	82,21 47,99 194,95	136,49 53,27 228,35

Fig. 7. Mean, minimum and maximum elevation of subregions

33,06 4,58E-04 116,80	20.57 0,16 49,55	32.00 0 74,59	27,63 0 70.35
39,39 1,88E-03 134,34	30,10 1.17E-03 99,33	25,41 0 76,85	24,62 2,93 72,97
16,18 1,79E-07 40,77	19,17 1,31E-05 54.09	17,07 1,90E-05 63,21	27,15 6.1E-03 130,37

Fig. 8. Mean, minimum and maximum slope of subregions

	Mean	Minimum	Maximum
Elevation	147,25	7,99	403,58
Slope	25,13	0	130,37

Fig. 9. Mean, minimum and maximum elevation and slope of the test area

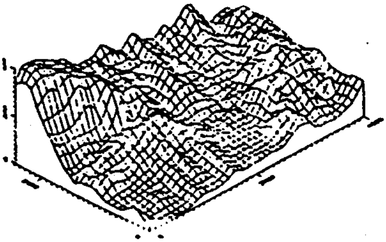

Fig. 4. D.E.M. of the application

Fig. 5. Slope matrix of the D.E.M.

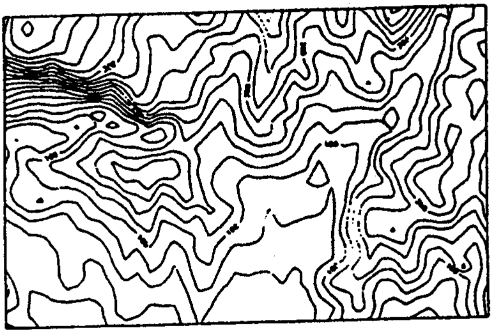

Fig. 6. Contour map created from the D.E.M. Contour interval 20 m

Comments and remarks

The production ability of the system for this large scale of map (1.5000) is rather cost effective (20 hours). However the system performed very well and it is of importance to point out that the produced D.E.M. has only a 10% of the original digitized data volume, with a very good result of accuracy.

For production purposes one could configurate smaller scales of maps and perhaps, the use of commercial interpolation software.

The excellent performance at this rather large scale shows that the system can be used for investigations of the T.C. computation as well as for numerical checks about the validity of downward continuation of the gravity field.

The output of a slope matrix is a very useful form to check, through Eq. 19 the efficiency of height information in the T.C. ~~computation, and~~ to visualise in which parts of the map more dense height information is needed.

The slope matrix can also be useful for investigations of geological structures (non visible faults e.t.c.) in relation with other potential geofunctions (gravity, magnetic e.t.c.).

July 1989

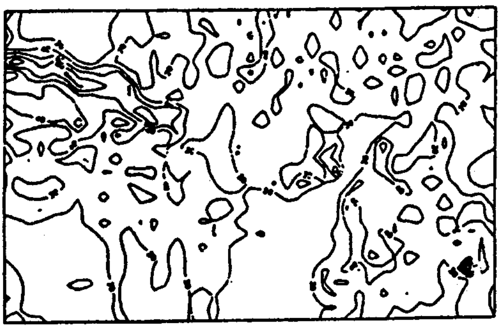

Fig. 10.Slope map created after the slope matrix
Contour interval 20% (not refered in text)

References

Ayeni,O.,O. (1978) Automated digital terrain models, Proceedings of the digital terrain models (D.T.M.) Symposium, May 9-11, Saint Lois Missouri, pp. 275-306

Forsberg,R. (1984) Gravity field terrain effect computations by FFT, A.G.U. Fall Meeting, San Francisco

Fredericksen,P., Jacobi,O., Kubik,K. (1985) A review of current trends in terrain modelling, I.T.C. Journal 1985, 2, pp. 101-106

Groten,E. (1979) "Geodesy and the Earth's gravity field" 2 vols, Dümmler, Bonn

Linkwitz,K. (1970) "Digital Geländemodelle", Bild Messung and Luft Bildwesen Reihe 38, No. 1.

Moritz,H. (1973) On the convergence of Molodensky's series Boll.Geod.Sci. Affini 32, pp. 125-144

Moritz,H (1980) "Advanced Physical Geodesy", Wichman, Verlag, Karlsruhe

Nakos,B. (1988a) Programms of map digitization (Digitization, Raster-to-vector conversion and Labeling) Laboratory of Chartography, National Technical University of Athens

Nakos,B. (1988b) Programms of 2-D interpolation, Laboratory of Chartography N.T.U. of Athens

Nakos,B. (1989) Ph. D Thesis, to be submitted to N.T.U. Athens

Schut,G.H. (1976) Review of interpolation methods for digital terrain models, The Canadian Surveyor, Vol. 30, no. 5, pp. 389-412

Sideris,M. (1985) A fast fourier transform method for computing terrain corrections, Manuscripta Geodaetica, 10 pp. 66-73.