

# Manipulation of Spatio-temporal Patterns in Networks of Relaxation Electrochemical Oscillators

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- ⇒ **Part I:** Electrochemical and Physiological Systems
- ⇒ **Part II:** Networks of Excitatory Electrochemical Oscillators
  - ✿ **Ring Networks:** Formulation
  - ✿ **Phase and Period Compensation**
  - ✿ **Fractured Synchrony and Perturbations**
- ⇒ **Part III:** From Excitation to Inhibition
  - ✿ **Coupled Electrode Pairs:** Formulation
  - ✿ Coupling in the **Oscillatory** Regime
- ⇒ **Part IV:** Conclusions



# PART I

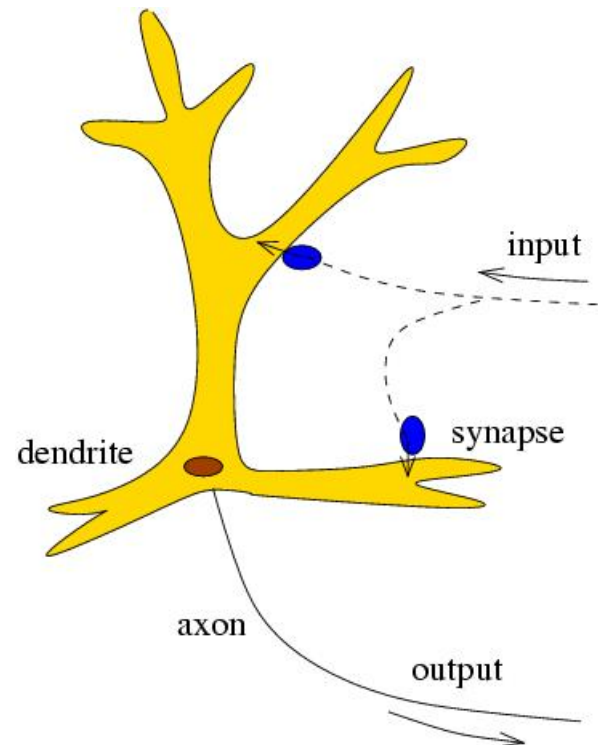
## Electrochemical and Physiological Systems



# An Amateur's Approach to Neurophysiology...

Hoppensteadt & Izhikevich, *Weakly Connected Neural Networks* (1997)

- Neurons are *cells* consisting of a *body* (soma), *dendrites* and an *axon* (not all neurons)
- Neurons generate *action potentials* (spikes) which propagate along the axon (not all neurons)
- Neurons *communicate* via action potentials
- Neurons are *functionally polarized* (not always)
- The junctions between the axon of one neuron and the dendrite of another is called *synapse* (chemical or electrical)
- A neuron or synapse can be *excitatory* (facilitates action potentials) or *inhibitory* (impedes action potentials)



# Neural Oscillators: Wilson-Cowan Model

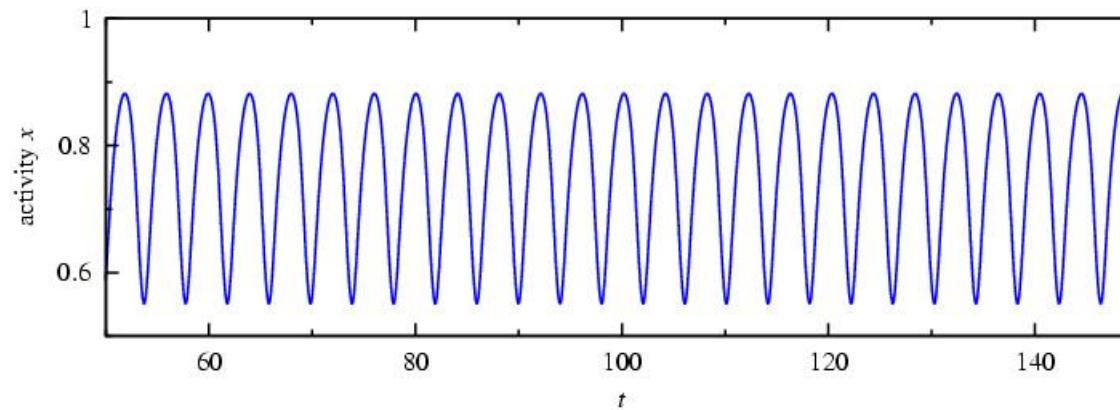
*Kybernetik* **13** (1973) 55

Consider an interconnected pair of an excitatory and an inhibitory neuron...

$$\dot{x} = -x + S(\rho_x + ax - by) \quad (1)$$

$$\dot{y} = -y + S(\rho_y + cx - dy) \quad (2)$$

where,  $S = \frac{1}{1+e^{-x}}$  and  $x, y$  the “activities” of each neuron.

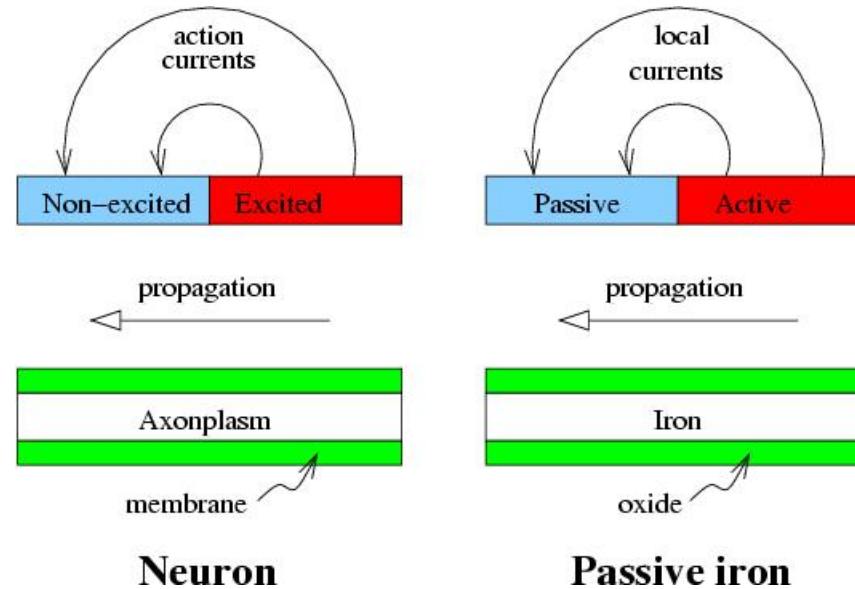


$\rho_x = 0.5, \rho_y = -8, a = 10, b = 10, c = 10, d = -2.$



# Ostwald model: From Neurons to Electrochemistry

*Z. Phys. Chem.* **35** (1900) 333, *Prog. Biophys. Chem.* **6** (1956) 171.



- The electrochemical interface is a membrane permeable to some of the components but not to others (V.G. Levich, *Physicochemical Hydrodynamics*, 1962).
- One of the oscillatory variables is electrical in nature (*Angew. Chem.* **17** (1978) 1).
- Non-excited (silent/passive) electrodes can activate when coupled with active electrodes (*J. Gen. Physiol.* **3** (1920) 107, *J. Gen. Physiol.* **7** (1925) 473).



# Electrochemical Rhythmic Activity

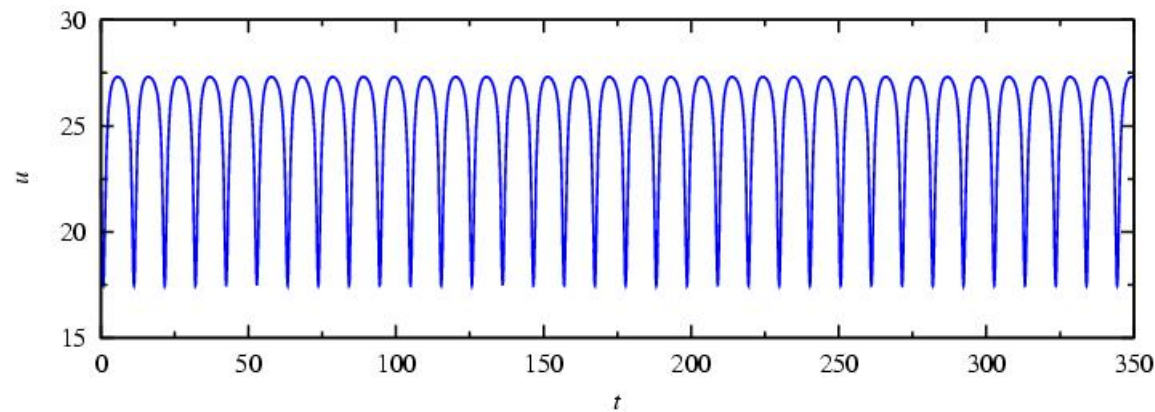
*Chem. Phys. Lett.* **347** (2001) 133.

Consider an electrochemical interface where ions are produced due an electrochemical *reaction* and transported due *diffusion* and *migration*, (or watch the [movie](#))

$$\varepsilon \dot{u} = \sigma(v - u) - f(u, c), \quad (3)$$

$$\dot{c} = 1 - c - \sigma(v - u) + \alpha f(u, c), \quad (4)$$

where ,  $f(u, c) = c(a_1 u + a_2 u^2 + a_3 u^3)$ .



$\sigma = 0.1$ ,  $\varepsilon = 0.055$ ,  $\alpha = 0.1$ ,  $V = 29.27$  and  $a_1 = 1.125$ ,  $a_2 = -0.075$ ,  $a_3 = 0.00125$ .



## PART II

# Networks of Excitatory Electrochemical Oscillators



# Ring Networks: Formulation

Continuous case:

$$\frac{\partial c_m}{\partial t} = D_m \nabla^2 c_m + n_m F \gamma_m \nabla (c_m \nabla \Phi),$$

with boundary conditions,

$$j_{F,m} + j_{C,m} = -D_m \left. \frac{\partial c_m}{\partial z} \right|_{z=0} - n_m F \gamma_m c_m \left. \frac{\partial \Phi}{\partial z} \right|_{z=0},$$

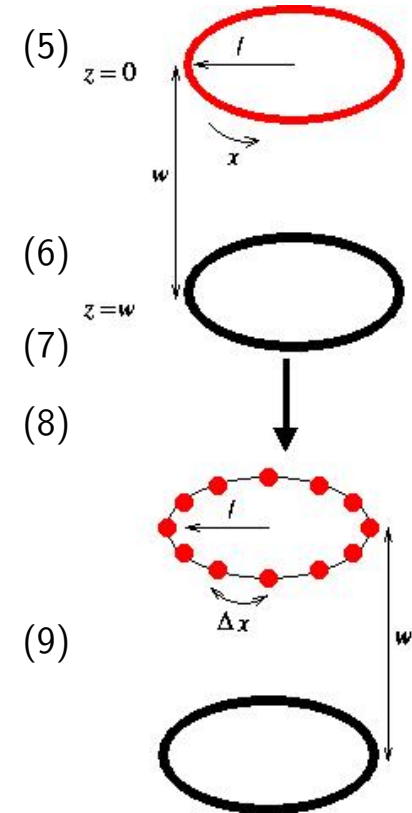
$$c_m(x, y, z = w, \tau) = c_{b,m},$$

$$\Phi(x, y, z = w, t) = 0,$$

Discrete case:

$$\varepsilon \frac{du^{(k)}}{dt} = \frac{\sigma}{\beta} (v - u^{(k)}) - i_F(u^{(k)}, c^{(k)}) + \frac{\sigma \beta}{3h} \sum_{\substack{m=[k-1] \\ m \neq k}}^{[k+1]} (u^{(m)} - u^{(k)}), \quad (9)$$

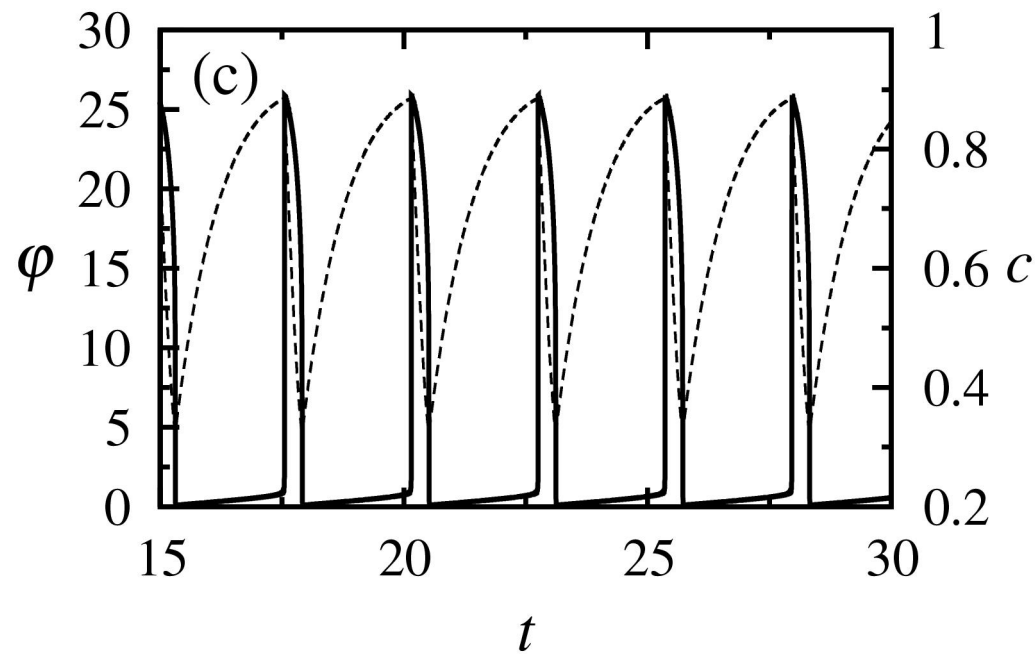
$$\frac{dc^{(k)}}{dt} = 1 - c^{(k)} - \frac{t_c \sigma}{\beta} (v - u^{(k)}) + j_F(u^{(k)}, c^{(k)}). \quad (10)$$





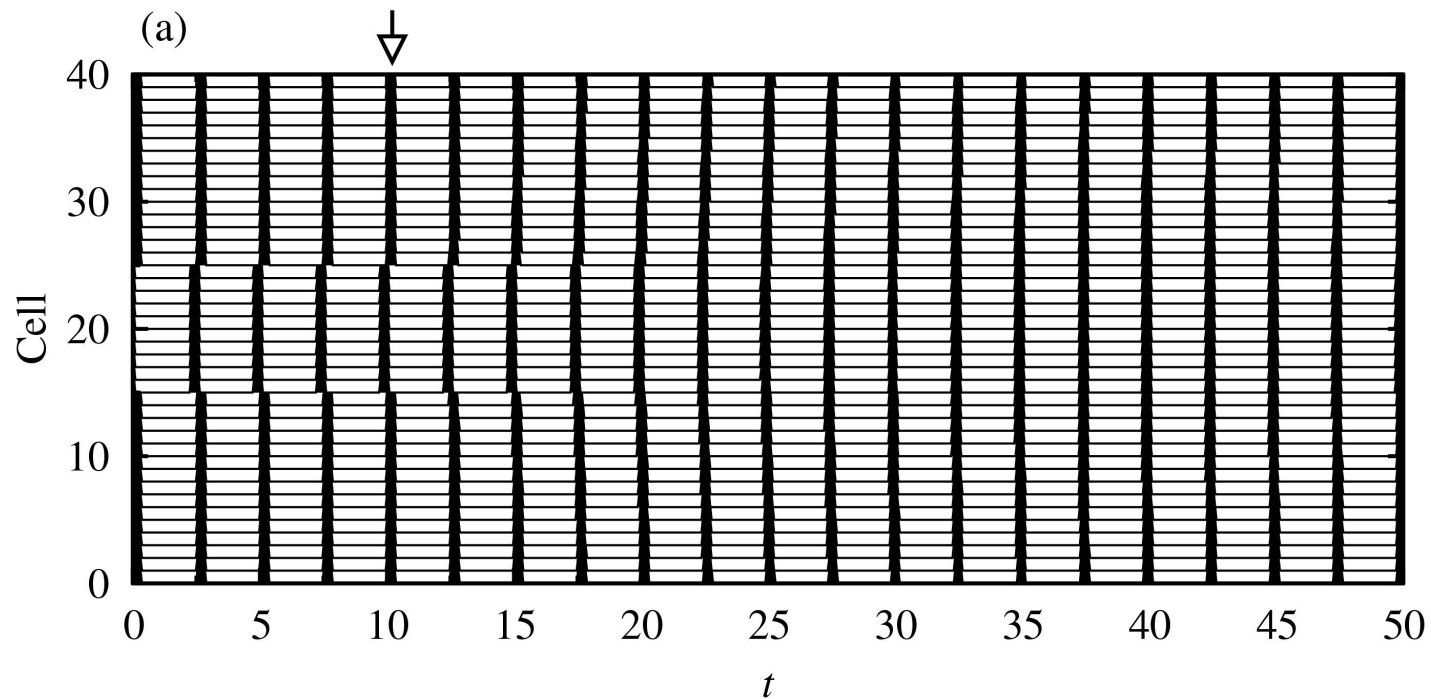
# Oscillations of a Single Cell

Relaxation oscillations of the potential  $\varphi$



# Phase Compensation

Group {16-25} oscillates with  $0.2\pi$  phase difference from the rest of the cells. At  $t = 10$  coupling is effective

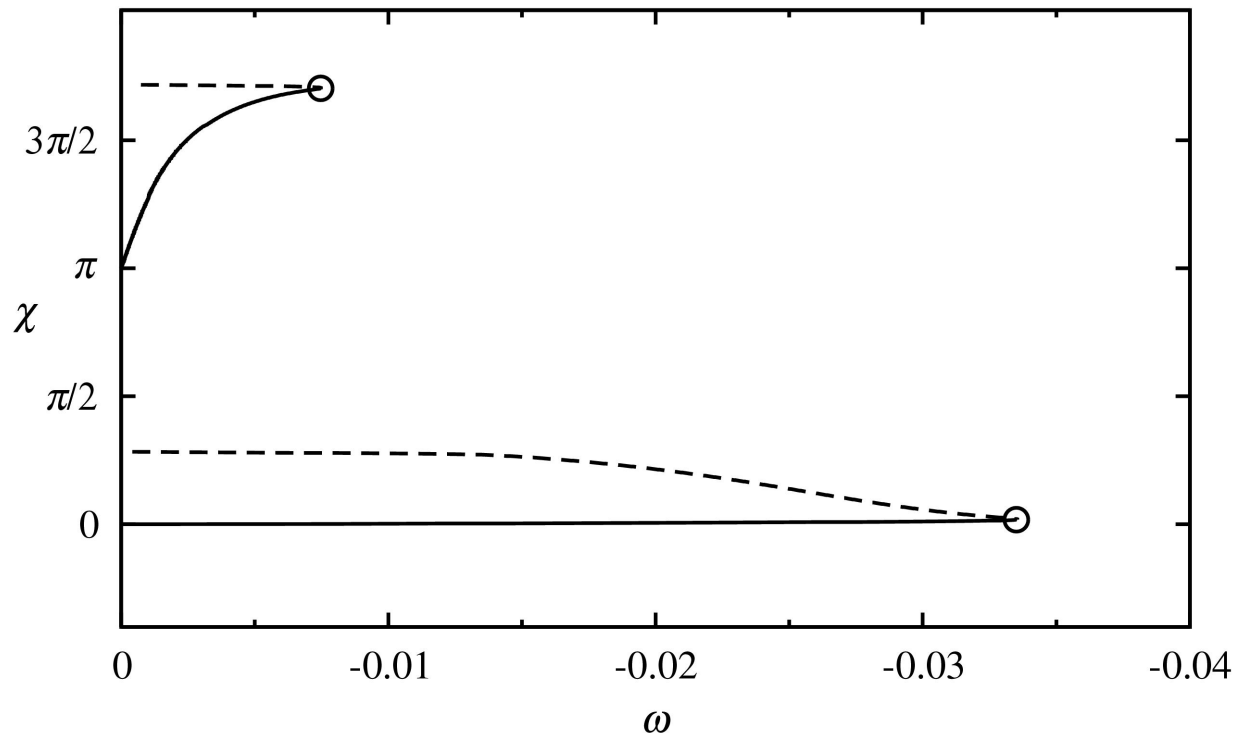


Noise reduction information processor for the in-phase state.



# Period Compensation

Relaxation oscillators also eliminate period differences. For small  $\omega$ , non-identical oscillators synchronize rapidly in-phase

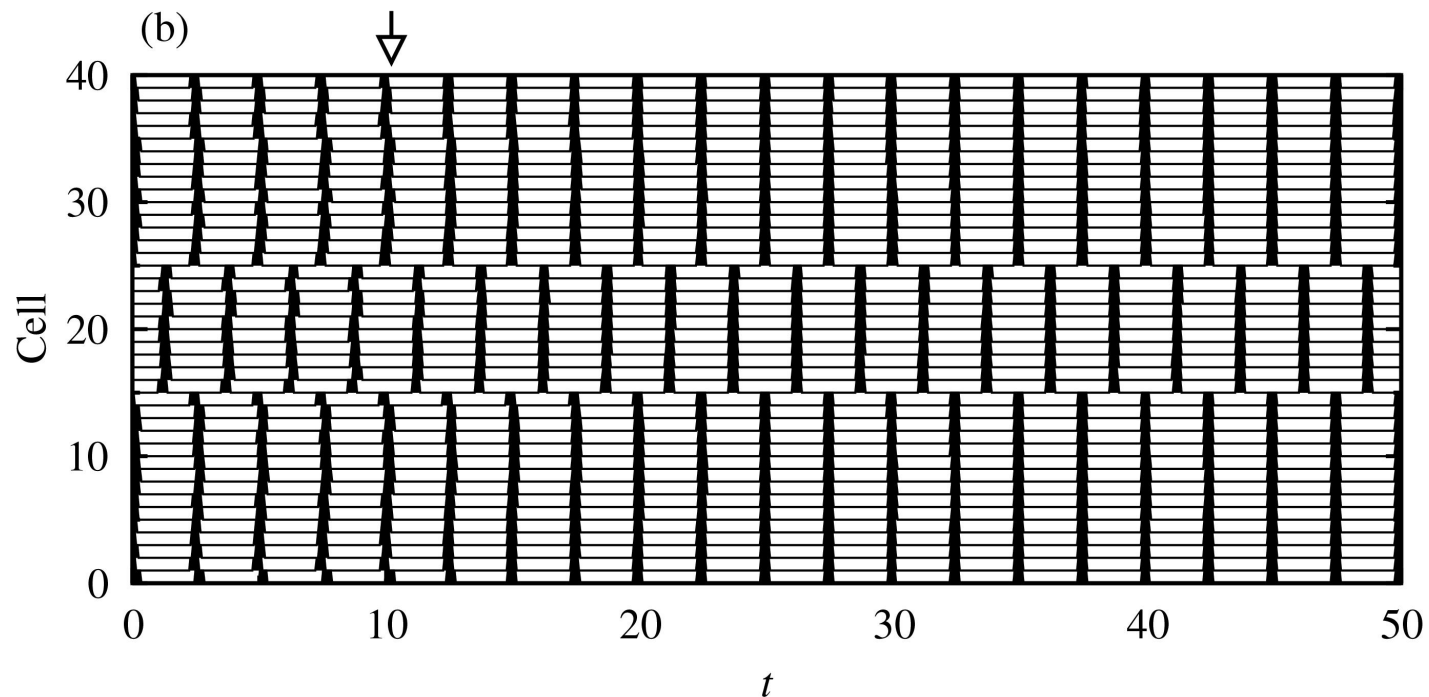


The in-phase state ( $\chi = 0$ ) remains over a wide range of period differences,  $\omega$ .



# Fractured Synchrony

Group {16-25} oscillates with almost  $\pi$  phase difference from the rest of the cells. At  $t = 10$  coupling is effective

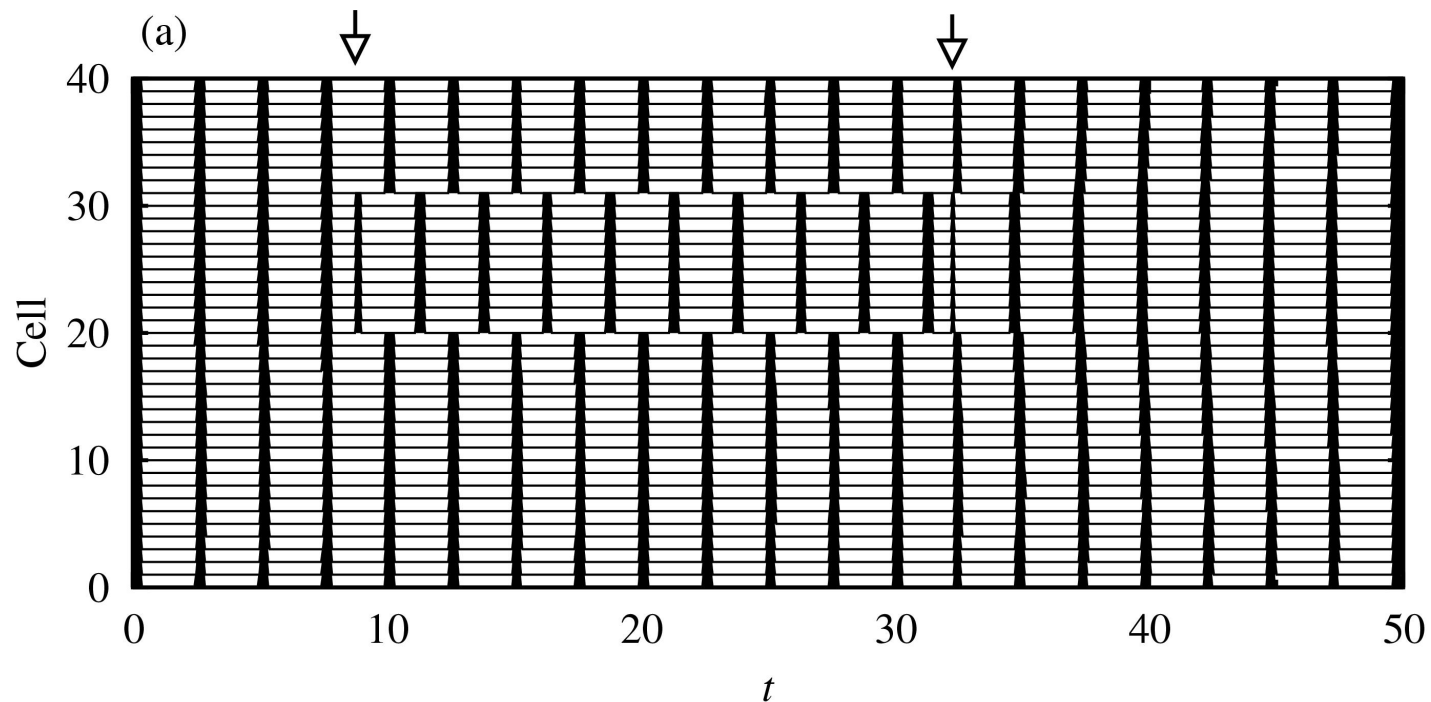


Noise reduction information processor for the anti-phase state. How about the [non-relaxation](#) case?



# Effect of Perturbations: Coding and Correcting

A perturbation on the in-phase state induces anti-phase synchrony. Group {20-30} is perturbed at  $t = 20$  and  $t = 32.25$ .



Coded information can be corrected also by perturbing a single cell.

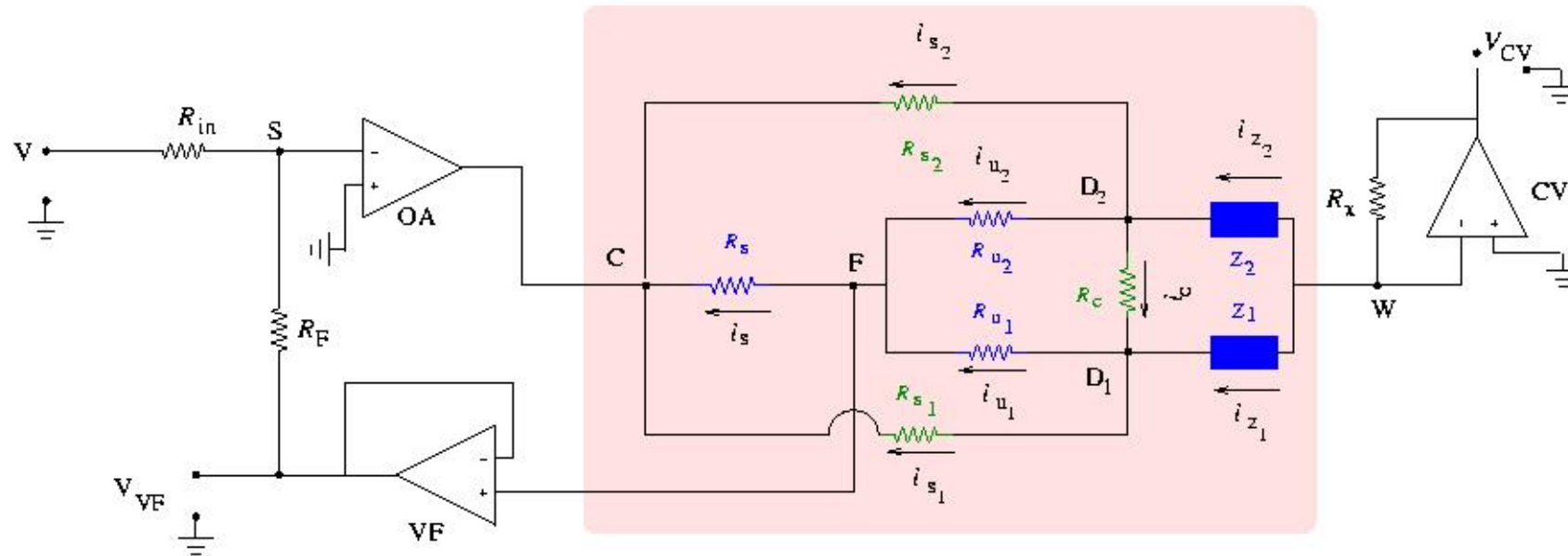


# PART III

## From Excitation to Inhibition



# Coupled Electrode Pairs: Formulation



If the reference electrode is a *point*,  $F$ , then we consider the “escape” currents,  $i_{s_1}$  and  $i_{s_2}$  flowing towards the counter  $C$ , without passing from  $F$ .



## Coupled Electrode Pairs: The Model

$$C_k \frac{dU_k}{dt} = \frac{V - U_k}{R_{u_k}} - i_{e_k}(U_k, c_k) + \frac{V - U_k}{R_{s_k}} + \frac{R_s}{R_{s_k}} \sum_{m=1}^2 \frac{V - U_m}{R_{u_m}} - \sum_{m=1}^2 (-1)^{m+k} \frac{U_m}{R_c}, \quad (11)$$

$$\frac{dc_k}{dt} = \frac{2D_k}{\delta_k^2} (c_{k,b} - c_k) - \frac{2}{\delta_k F A_k} \frac{V - U_k}{R_{u_k}} + \frac{2}{\delta_k F A_k} i_{q_k}(U_k, c_k), \quad (12)$$

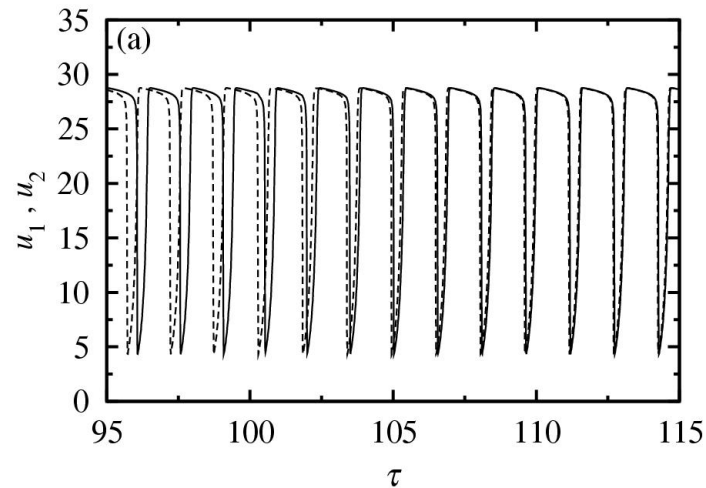
The connection can be tuned by varying  $R_s$ .





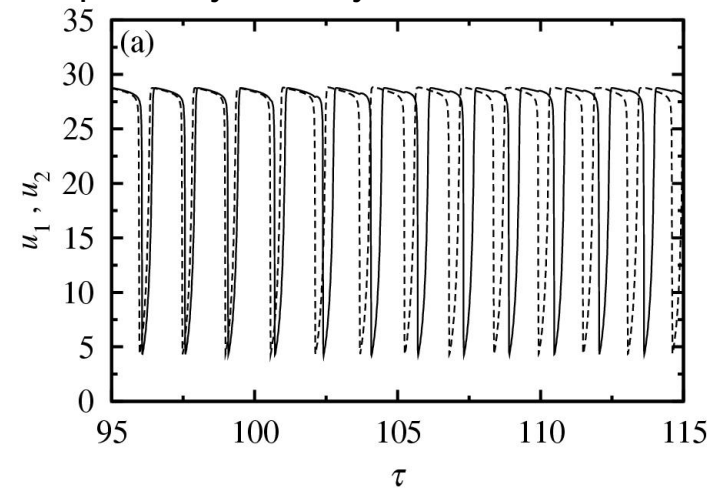
# Coupling in the Oscillatory Regime

In-phase synchrony



$$R_s = 40$$

Out-of-phase synchrony



$$R_s = 60$$

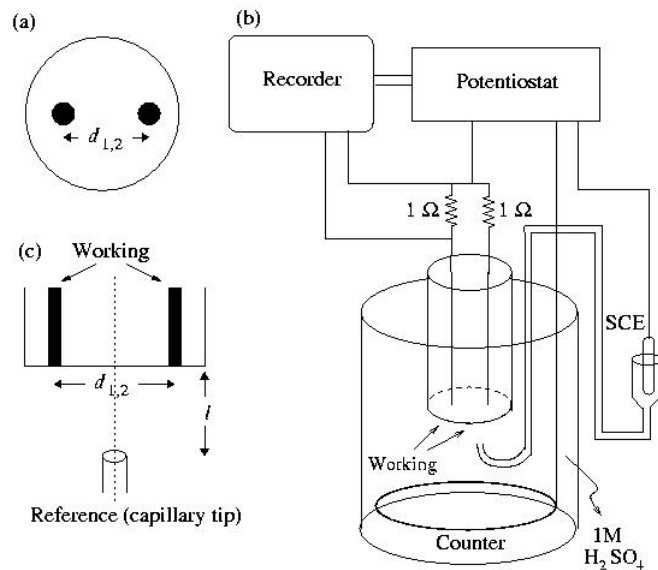
The transition can be seen in the [phase space](#). For  $t < 20$  inhibitory coupling ( $R_s = 60$ ) and for  $t > 20$  excitatory coupling ( $R_s = 40$ )



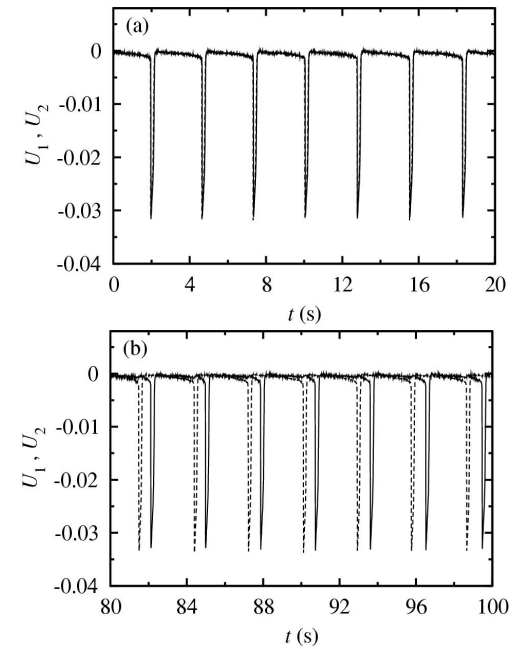
# Experimental Realization

The transition from excitation to inhibition was realized experimentally by varying the position of the *reference* electrode

Experimental setup



Transitions for  $l = 10$  and  $l = 2$  mm



# PART IV

## Conclusions



## Conclusions and Open Topics

- ⇒ Coupled relaxation electrochemical oscillators synchronize *rapidly* in-phase
- ⇒ Relaxation electrochemical networks compensate phase and period differences
- ⇒ Specific spatio-temporal patterns can be induced by appropriate perturbations
- ⇒ Perturbations can *erase* and *correct* the spatio-temporal patterns
- ⇒ The action of the connections (excitatory or inhibitory) can be tuned by varying the position of the reference
- ⇒ Electrochemical networks can perform some primitive information manipulation tasks

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